Photonic material for designing arbitrarily shaped waveguides in two dimensions

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(Received 13 March 2003; revised manuscript received 14 April 2003; published 26 June 2003)

We investigate numerically optical properties of novel two-dimensional photonic materials where parallel dielectric rods are randomly placed with the restriction that the distance between rods is larger than a certain value. A large complete photonic gap (PG) is found when rods have sufficient density and dielectric contrast. Our result shows that neither long-range nor short-range order is an essential prerequisite to the formation of PG's in the novel photonic material. A universal principle is proposed for designing arbitrarily shaped waveguides, where waveguides are fenced with side walls of periodic rods and surrounded by the novel photonic materials. We observe highly efficient transmission of light for various waveguides. Due to structural uniformity, the novel photonic materials are well suited for filling up the outer region of waveguides of arbitrary shape and dimension comparable with the wavelength.

DOI: 10.1103/PhysRevB.67.235109 PACS number(s): 71.55.Jv, 42.70.Qs, 42.25.Dd

Downsizing is an everlasting dream of researchers in the engineering field. Researchers in the field of optics are hoping to find a way to fabricate all-optic integrated circuits by using optical elements comparable with the wavelength. In fact, realization of miniature-sized optical waveguides should soon be possible due to the discovery of photonic crystals (PhC's).[1,2] Because of the periodicity of a dielectric constant, PhC's can be designed to have complete photonic gaps (PG's), a range of frequencies for which light in any direction cannot propagate within the PhC's. We can steer light through the waveguides made of PhC's having complete PG's. On the other hand, it is desired that the shapes of waveguides are structurally commensurate with the periodicity of the host PhC to avoid excess scattering due to incommensurability. Thus, PhC's are not suitable for the realization of arbitrarily shaped waveguides whose dimension is comparable with the wavelength.

Let us take two-dimensional waveguides of a PhC composed of periodic dielectric rods, for example. The waveguides are usually formed by removing rods along a line. Therefore, they are composed of a set of segments. Waveguides of 90° or 60° bends can be easily obtained from PhC's of square or triangular lattices. For a bend of an arbitrary angle, however, they become zigzag in shape and excess scattering occurs at the junctions of segments. Successive scattering reduces the transmittance of waveguides composed of many branches and bends. Arbitrarily shaped waveguides, therefore, require photonic materials of maximum structurally uniformity in addition to the complete PG's. In this article, we propose novel photonic materials in which parallel dielectric rods are randomly placed in a certain region provided that the distance between the centers of rods is larger than a certain value D_{min}:

$$|\mathbf{R}_i - \mathbf{R}_j| \geqslant D_{min}, \tag{1}$$

where $\mathbf{R}_i$ and $\mathbf{R}_j$ are the positions of ith and jth rod center. We label this new photonic material as uniformly distributed photonic scatterers (UDPS) in this article. It is noted that UDPS have neither long-range nor short-range order. Nevertheless, we show numerically that UDPS can have complete PG's if rods have sufficient density and dielectric contrast. We also propose a new design policy to fabricate arbitrarily shaped waveguides, i.e., we fence the waveguides with side walls of periodic rods and fill up the outer region with UDPS. We observe clear propagation of waveguide modes with large transmittance.

Two examples of UDPS are shown in Figs. 1(a) and 1(b) composed of $N=100$ and $N=200$ rods of radius a, respectively. Here, we set $D_{min}=4.0a$. As the figures show, the distribution of rods becomes more uniform with increase in rod density. Transmittance of UDPS is calculated by assuming the incidence of the plane electric field of wavelength λ from the upper side of Figs. 1(a) or 1(b). Incident light is scattered multiply by each rod. This scattering is treated analytically by solving the Maxwell equation.[3] The solution gives the distribution of electric field and energy flow (Poynting vector). From the average energy flow at line L in Fig. 1(a) or 1(b), we calculate the transmittance T normalized by that without rods. T is a function of normalized frequency $\Omega = 2\pi a/\lambda$ (known as size parameter) and becomes very small in the PG region. In some cases, T exceeds unity because of the diffraction due to the finite size of UDPS.

Figure 1(c) shows values of T for UDPS of $N=100$, 150, and 200 when the electric field $\mathbf{E}$ is parallel to the rod axis (TM mode). In all the figures of this article, we fix the dielectric constant of rods at $\varepsilon = 12$, which corresponds to that of Si at 1.55 μm used worldwide in optical communications. We found no PG for UDPS of $N=100$ though there were two split dips at $\Omega = 0.35$. For UDPS of $N=150$, we observed a PG of $\Delta\Omega/\Omega_c = 30\%$, where Ω_c and $\Delta\Omega$ are the central frequency and width of the PG, respectively. Here, we define PG to be the frequency range continuously below $T=0.01$. This PG, however, is incomplete in that there appear spiky peaks in the gap region. At these spiky peaks we found an intrusion of energy flow through cracks (nonuniform re-

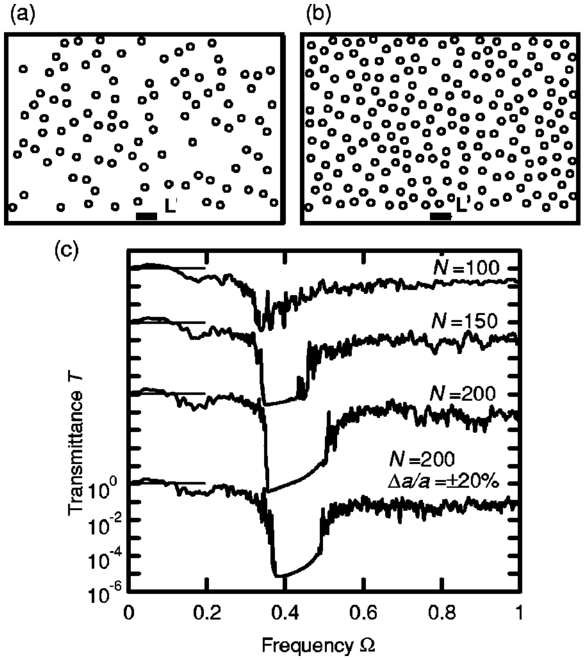

FIG. 1. Top view of distributions of rods under the condition $D_{min}=4.0a$ in (a), (b) and transmittance T in (c). Circles show rods of radius a and $\varepsilon=12$ in the rectangular region of width $W=84.6a$ and height $H=53.6a$ within the vacuum. Total number N and area fraction f of rods are $N=100$ and $f=0.069$ in (a) and $N=200$ and $f=0.138$ in (b). The electric field parallel to the rod axis (TM mode) of wavelength λ is incident from the upper side of (a) or (b). T is calculated as a function of $\Omega=2\pi a/\lambda$ by averaging the energy flow at line L in (a) or (b). Values of T for three cases of $N=100$, 150, and 200 are shown in (c). We also plot T for UDPS with $N=200$ and radius fluctuation of $\Delta a/a=\pm 20\%$. Each value of T is the average of 5 different configurations. Central frequency and width of PG for $N=150$ are $\Omega_c=0.398$ and $\Delta\Omega=0.119$, respectively. For $N=200$, $\Omega_c=0.431$, and $\Delta\Omega=0.159$. Introduction of $\Delta a/a=\pm 20\%$ only reduces $\Delta\Omega$ to $\Delta\Omega=0.128$.

gions) of UDPS. When N becomes 200, PG grows up to 37% with the suppression of spiky peaks. It was also verified that this PG is isotropic and therefore complete. We also calculated T of UDPS for various ε with common values of $D_{min}=4.0a$ and area fraction of rods $f=0.138$, and found PG for $\varepsilon \geqslant 5$. The presence of PG's was also confirmed by results of finite difference time domain (FDTD) calculation. Note that the rod radius becomes 0.11 μm if we correspond Ω_c of $N=200$ to $\lambda=1.55$ μm. This size can be prepared relatively easily using recently developed microfabrication technique.

The actual fabrication process inevitably involves certain fluctuation in rod position Δx and radius Δa. It is natural to expect from the construction rule of UDPS that the PG is unaffected by Δx. In contrast, the effect of Δa should be investigated. We have also plotted in Fig. 1(c) the transmittance of UDPS with $\Delta a/a=\pm 20\%$. It was confirmed that PG of $\Delta\Omega/\Omega_c=30\%$ can survive such large fluctuation. This means that the PG of UDPS is also considerably robust against radius fluctuation.

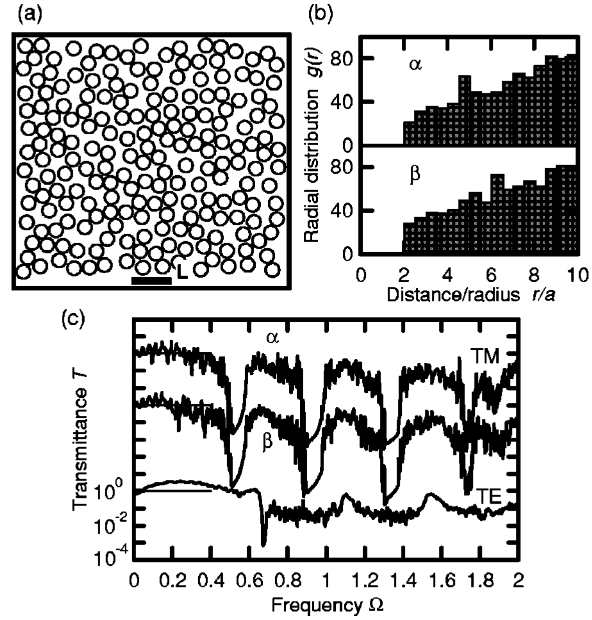

FIG. 2. (a) Top view of the distribution of rods, (b) two examples of radial distribution function $g(r)$, and (c) T for $D_{min}=2.1a$. Here, $N=200$, $W=37.5a$, and $H=33.3a$ (area fraction $f=0.503$). No radius fluctuation is introduced. Horizontal and vertical axes of (b) are the distance r between rod centers in units of a and its frequency $g(r)$, respectively. Two spectra indicated by α and β in (c) are transmittance of the TM mode corresponding to α and β in (b), and the lowest one is the average transmittance over five configurations for the TE mode (**E**$\perp$ rod axis). Gaps of TM and TE modes are respectively given by $0.50\leqslant\Omega\leqslant0.58$, $0.87\leqslant\Omega\leqslant0.98$, $1.28\leqslant\Omega\leqslant1.39$, and $0.67\leqslant\Omega\leqslant0.69$.

We have observed in Fig. 1 that an increase in rod density enlarges the PG width. We therefore examined a case of much higher density. Figure 2(a) shows one example of UDPS of $D_{min}=2.1a$, which includes no radius fluctuation. Two examples of radial distribution functions $g(r)$ are plotted in Fig. 2(b) for $D_{min}=2.1a$, where r is the distance between rod centers. For uniform distribution in two dimensions, $g(r)$ is proportional to r without showing any peak. Distributions in Fig. 2(b) are very similar to the uniform case. Nevertheless, we find three distinct PG's at $\Omega_c=0.54$, 0.93, and 1.34 for the TM mode in Fig. 2(c). It is remarkable that the UDPS have such wide PG's of higher frequencies. If one uses the third PG, a rod radius of 0.33 μm is required to utilize $\lambda=1.55$ μm. This facilitates the fabrication significantly. We also find a PG of TE mode (**E**$\perp$ rod axis) at $\Omega_c=0.68$.

Before discussing the origin of PG's in UDPS, let us use UDPS for various waveguides. For this purpose we plot in Fig. 3 the average transmittance T over five configurations of UDPS for three cases of sample thickness with common D_{min}. We can see that PG appears even for very thin UDPS containing three or four rods along the direction of light. This indicates that UDPS have wide applicability to build up waveguides of arbitrary shape and size comparable with the wavelength.

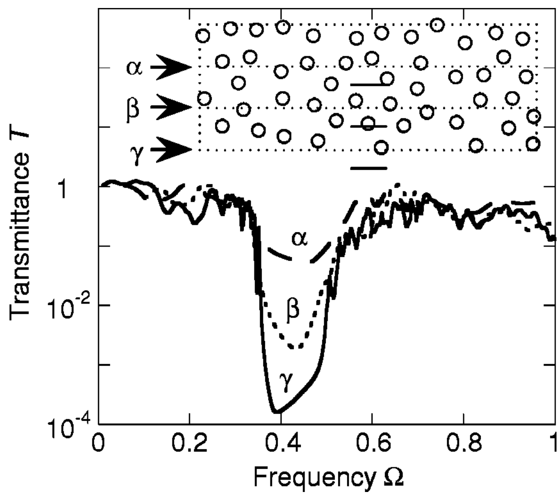

FIG. 3. Transmittance T of thin UDPS as a function of sample thickness ℓ. Rod density and incident light are the same as in Fig. 1(b). No radius fluctuation is introduced. We show in the inset one example of thin UDPS with $\ell = 20a$ defined by outer dotted lines. From this UDPS, two thinner UDPS are cut out at horizontal lines indicated by α or β, whose thickness is $\ell = 6.7a$ or $13.3a$. Rods on the cutting line are included when their centers are above the line. T of each UDPS is calculated by averaging the energy flow at line L of width $5.7a$ and $2.9a$ below each cutting line. Each value of T is the average over five different configurations. Central frequency and width of PG for $\ell = 13.3a$ and $20a$ are, respectively, given as $\Omega_c = 0.424$, $\Delta\Omega = 0.113$, and $\Omega_c = 0.429$, $\Delta\Omega = 0.146$.

To make best use of this property, we first decide the shape of a waveguide. It can be twisty, as will be shown in Fig. 5 below, to enable maximum flexibility in designing. Waveguides are separated by side walls from the surrounding medium. To avoid excess scattering from the side walls, they are chosen to be made of periodic rods in a line. Then, we fill up the outer region with UDPS. It is noted that UDPS are the best materials for the surrounding optical medium. PhC's are not suitable for this purpose because their periodicity conflicts with that of side walls. This mismatch causes nonuniformity of rod density, resulting in excess scattering. This is also the case not only for quasiperiodic PhC's[4] but also for photonic materials having short-range order.[5]

In the waveguides shown below, the density of UDPS is the same as that in Fig. 1(b) ($N = 200$), and the TM mode is assumed to be incident from the upper side. Let us first compare the transmittance and electric field distribution of the same shape waveguides of 90° bend made of PhC and UDPS with or without side walls. Figure 4(a) shows the distribution of electric field intensity in a waveguide of square lattice PhC with the same rod density as that of UDPS. Rods are shown by open circles. The intensity increases from blue to red. Energy flow is shown by white arrows. Corresponding frequency is indicated by the arrow in the transmittance in Fig. 4(b). We tried to simulate the same 90° bend waveguide by using only UDPS as shown in Fig. 4(c) and found that electromagnetic field does not flow smoothly. Note in this case that the width of waveguide fluctuates from place to place. It is well known in quantum mechanics that local bulges in the guide always gives rise to bound states in constant cross-sectional quantum waveguides provided that the wave function vanishes on the waveguide boundaries. This situation exactly applies to our UDPS waveguide because the electric field is almost completely confined within the waveguide.[6] The efficiency is greatly improved by introducing the side walls as shown in Fig. 4(e). Here, the side wall has the same periodicity with the square lattice PhC. It is intriguing that such a thin layer of periodic rods can suppress the excess scattering and exclude the bound states. We also note that UDPS with side walls show large transmittance exceeding two as shown in Fig. 4(f). This is not surprising, because we observe excess energy flow due to scattering from random configuration of rods at the entrance gate of the waveguide as shown by white arrows in Fig. 4(e). This also happens even in the case of PhC's in Fig. 4(b) in which the transmittance exceeds unity.

Let us next study the effect of radius fluctuation on the transmittance of waveguide made of UDPS. Figure 5(a) shows the distribution of electric field intensity in the same waveguide with that in Fig. 4(e) except for the inclusion of radius fluctuation $\Delta a/a = \pm 20\%$. UDPS in this case have the PG of $0.366 \leqslant \Omega \leqslant 0.494$ shown by the shaded region in the transmittance in Fig. 5(b). We can clearly observe the same propagation mode of large transmittance $T = 1.863$ with that in Fig. 4(e). The corresponding frequency is indicated by the arrow in Fig. 5(b). Comparison between Figs. 4(e) and 5(a) reveals that waveguides of UDPS have a wide tolerance for the fluctuation of rod radius which may occur in the fabrication process.

UDPS are not limited to waveguides composed of segments. They can also be used for waveguides with twists whose curvatures are comparable with λ. Figure 5(c) shows such an example composed of two quarter circles. The distribution of electric field intensity and energy flow are shown at the frequency noted by the arrow in the transmittance in Fig. 5(d). The fluctuation of rod radius is the same as that in Fig. 5(a). As can be seen in the figure, the electric field flows smoothly downward through the sample. While the corresponding value of T is not large (0.739), it can be increased by optimization.

Let us discuss the origin of PG's in UDPS. In a study concerning effects of disorder on PG, it was found that there are two kinds of PG, one that is easily smeared out by disorder and the one that is very robust against disorder.[7] The former PG's are formed by the coherent interference of scattered waves from periodic rods like Bragg diffraction in x rays. The latter are formed by the bonding and antibonding states of Mie resonance states within each rod, similar to the electronic band gaps in semiconductors. Since the latter are formed by local interaction, they are not significantly affected by the fluctuations in position and radius. As a matter of fact, an isolated dielectric rod of $\varepsilon = 12$ has Mie resonance at $\Omega = 0.23$, 0.66, 1.06, 1.14, and 1.44 for the TM mode and $\Omega = 0.66$, 1.05, and 1.40 for the TE mode. It is likely that the PG of UDPS is a result of interaction of these modes. This is also evidenced by the appearance of PG in Fig. 3 for very thin UDPS, which is easily understood from the formation of bonding and antibonding states by local arrangement of rods. An important difference, however, exists between electrons

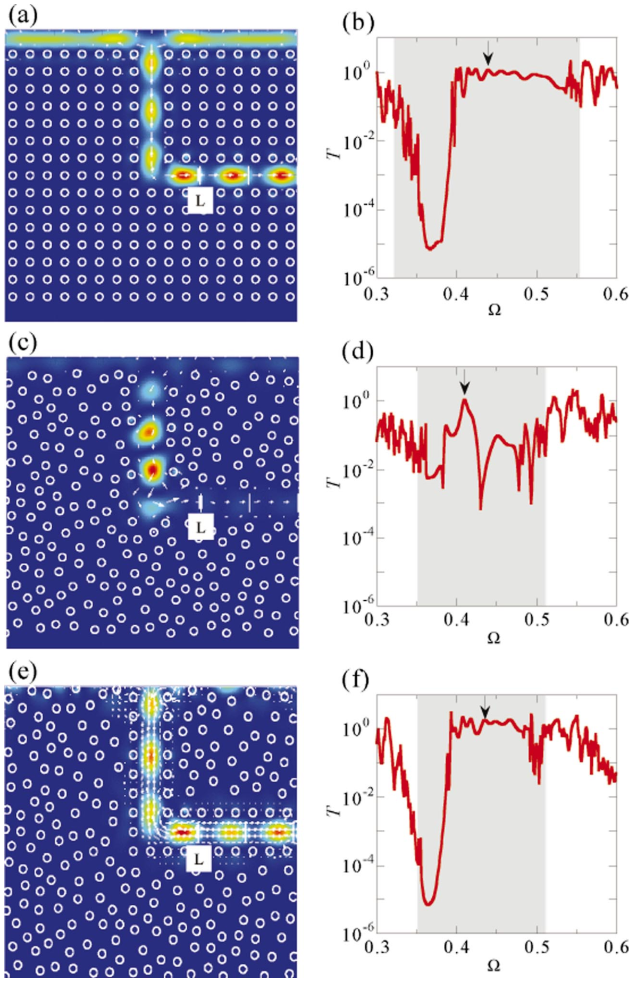

FIG. 4. (Color). Distributions of electric field intensity and energy flow in the waveguides of 90° bend made of square lattice PhC in (a), UDPS without side walls in (c), and that with side walls in (e). Field intensity increases from blue to red, and energy flow is indicated by the white arrows. Chosen frequency in each case is shown, respectively, by the black arrow in the transmittance T obtained at line L in (b), (d), and (f) where the shaded region is the PG. Maximum field intensity in units of incident light is 7.7 in (a), 55.7 in (c), and 12.8 in (e). Density of rods in (a), (c), and (e) is the same as that in Fig. 1(b) and no radius fluctuation is introduced. Periodicity of rods in (a) and side walls in (e) is $4.8a$. Dielectric constant of rods and incident light are the same as those in Fig. 1.

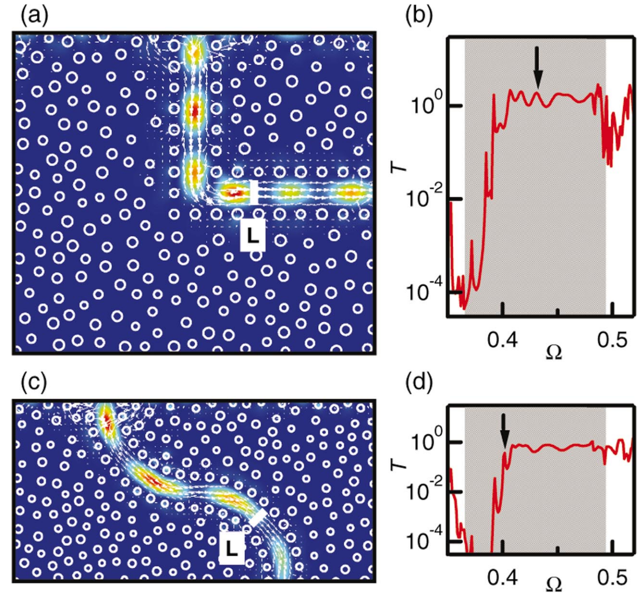

FIG. 5. (Color). Distributions of electric field intensity and energy flow in various waveguides made of UDPS. Notations and definitions are the same as those in Fig. 4. A waveguide (a) has $9.5a$ width and bends by 90° with $N=229$, $W=72.0a$, and $H=81.3a$. A waveguide (c) is composed of two quarter circles with $N=238$, $W=106.7a$, and $H=53.3a$. Outer and inner radii of the circles are arbitrarily chosen as $31.4a$ and $21.9a$, respectively. Maximum field intensity in units of incident light is 15.5 in (a) and 11.9 in (c). In all the cases, density of rods of UDPS is the same as that in Fig. 1(b) and radius fluctuation $\Delta a/a=\pm 20\%$ is introduced. Dielectric constant of rods and incident light are the same as those in Fig. 1.

and photons in that resonance wave functions of photons are not localized exponentially. Rather, they decay in inverse power and have a long-range nature. This long-range nature is responsible for the formation of PG's in UDPS that does not require even a short-range order. There are numerous studies concerning the effect of disorder such as randomness of radii, positions, or dielectric constants of rods on the PG's[8] and waveguides made of PhCs.[9] PGs are observed even when disorder is introduced, but they are obviously vestiges of PG's of the underlying lattices. In contrast, there is no underlying lattice for UDPS and no peak is observed in radial distribution functions $g(r)$ as shown in Fig. 2(b).

One might think that UDPS is deeply related with two-dimensional disordered systems which are used to investigate Anderson localization of light.[10] Let us discuss this point. In three dimensions, Anderson localization takes place only when the disorder is strong enough to satisfy the Ioffe-Regel criterion. In contrast, even a very small amount of disorder is sufficient in one and two dimensions to invoke Anderson localization. We have evaluated the localization length ℓ_{loc} in various UDPS samples of common $D_{min}=4.0a$ and different sample thickness ℓ. We assume the form $T=T_0\exp(-\ell/\ell_{loc})$, and found that within the PG region ℓ_{loc} is roughly $2a$ comparable with the surface distance ℓ_{surf} between nearest neighbor rods. Note generally that $\ell_{loc}\gg\ell_{MFP}\gg\ell_{surf}$ (usually $\ell_{loc}\gg\ell_{MFP}$), where ℓ_{MFP} is the mean free path. Therefore, it would not be appropriate to conclude that Anderson localization can explain the observed localization length within the PG's. In fact, PG's have not been detected in two-dimensional disordered systems which show localization of light.[11] On the other hand, ℓ_{loc} above PG's is estimated to be $30a$–$40a$ comparable with the sample thickness. The effect of localization is usually obvious when ℓ_{loc} is comparable with or less than the sample thickness ℓ. Therefore, the thickness dependence of T outside PG reflects the effect of Anderson localization.

We investigated numerically optical properties of novel two-dimensional photonic materials called UDPS and found a large complete PG when rods have sufficient density and dielectric contrast. Due to structural uniformity, UDPS are shown to be well suited for producing an arbitrarily shaped waveguide of wavelength dimension fenced with side walls of periodic rods. Future researches of air-hole type UDPS to make use of TE modes and better design policy to increase uniformity are needed for further development of UDPS.

This research was supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science, and Technology. One of the authors (H. M.) expresses his sincere thanks to R. Ohkawa for his continuous encouragement. We are grateful to K. Ohtaka for valuable discussions.

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