

# Band structure for a one-dimensional photonic crystal containing left-handed materials

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(Received 19 November 2002; revised manuscript received 3 February 2003; published 13 June 2003)

The explicit dispersion equation for a one-dimensional periodic structure with alternative layers of left-handed material (LHM) and right-handed material (RHM) is given and analyzed. Some unusual phenomena such as spurious modes with complex frequencies, discrete modes and photon tunneling modes are observed in the band structure. The existence of spurious modes with complex frequencies is a common problem in the calculation of the band structure for such a photonic crystal. Discrete modes may exist regardless whether the optical length of the LHM layer cancels the optical length of the RHM layer or not. Physical explanation and significance are given for the discrete modes (with real values of wave number) and photon tunneling propagation modes (with imaginary wave numbers in a limited region).

DOI: 10.1103/PhysRevB.67.235103

PACS number(s): 78.20.Ci, 03.65.Ge, 42.70.Qs, 78.20.-e

## I. INTRODUCTION

Left-handed materials (LHM) with negative permittivity and negative permeability, which were first suggested in Ref. 1, have attracted much attention recently<sup>2-4</sup> thanks to the experimental realization of such materials<sup>5</sup> and the debate on the use of a LHM slab as a perfect lens to focus both propagating waves and evanescent waves.<sup>6,7</sup>

Multilayered structures containing LHM have been investigated through calculating the transmittance or reflectance of the structure.<sup>9,10</sup> Effects of photon tunneling and Bragg diffraction have been observed in these works. LHM has also been used to widen the stop band of a one-dimensional (1D) photonic crystal in the case of normal propagation.<sup>11</sup> In the present paper, we consider the unusual band structure of a photonic crystal formed by alternative layers of LHM and RHM. The band structure is investigated by analyzing the explicit dispersion equation. Some unusual phenomena such as spurious modes with complex frequencies, discrete modes (with real values of wave number) and photon tunnelling modes (with purely imaginary values of wave number) are found and explained in the present paper. The photon tunnelling modes exist only in the case of oblique propagation.

## II. ANALYTICAL DISPERSION RELATION AND SPURIOUS MODES

Consider a 1D periodic structure with alternating layers ( $\epsilon_1, \mu_1$ ) and ( $\epsilon_2, \mu_2$ ) as shown in Fig. 1.  $d_1$  and  $d_2$  are the widths of the two inclusion layers respectively, and  $a = d_1 + d_2$  is the period. Note that the conservation of energy requires that  $\epsilon_i \mu_i > 0$ ,  $i = 1, 2$ . The corresponding refractive index is given by  $n_i = \pm \sqrt{\epsilon_i \mu_i}$  (negative sign for LHM). We consider an oblique propagation of monochromatic electromagnetic field (with time dependence  $e^{-i\omega t}$ ) in a periodic structure with oblique wave vector  $\beta$  along the  $x$  axis. For the  $E$ -polarization case, one has

$$E_{1y} = e^{i\beta x} (e^{ik_1 z} + A e^{-ik_1 z}),$$

$$H_{1x} = \frac{-k_1}{\omega \mu_1} e^{i\beta x} (e^{ik_1 z} - A e^{-ik_1 z}), \quad (1)$$

$$H_{1z} = \frac{\beta}{\omega \mu_1} e^{i\beta x} (e^{ik_1 z} + A e^{-ik_1 z})$$

in region 1

and

$$E_{2y} = e^{i\beta x} (B e^{ik_2 z} + C e^{-ik_2 z}),$$

$$H_{2x} = \frac{-k_2}{\omega \mu_2} e^{i\beta x} (B e^{ik_2 z} - C e^{-ik_2 z}), \quad (2)$$

$$H_{2z} = \frac{\beta}{\omega \mu_2} e^{i\beta x} (B e^{ik_2 z} + C e^{-ik_2 z})$$

in region 2,

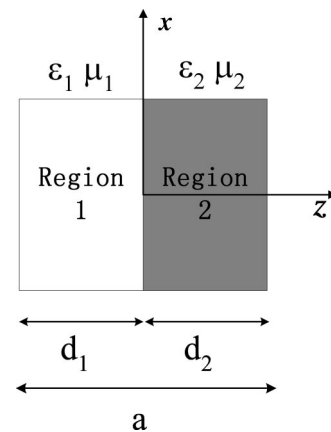


FIG. 1. A 1D periodic structure consisting of alternate layers of RHM and LHM inclusions.

where  $k_i$  is the component of the wave vector along the  $z$  axis in region  $i$  ( $i=1,2$ ), i.e.,  $k_i^2 = \omega^2 \epsilon_i \mu_i / c^2 - \beta^2$ . Here  $c$  is the speed of light in vacuum.

The tangential electric and magnetic fields should be continuous at  $z=0$ , i.e.,

$$\begin{aligned} E_{1y}(z=0^-) &= E_{2y}(z=0^+), \\ H_{1x}(z=0^-) &= H_{2x}(z=0^+). \end{aligned} \quad (3)$$

To obtain the dispersion relation for this 1D photonic crystal, we need to use the following periodic conditions according to the Bloch theorem:

$$\begin{aligned} E_{2y}(z=d_2) &= E_{1y}(z=-d_1)e^{iqa}, \\ H_{2x}(z=d_2) &= H_{1x}(z=-d_1)e^{iqa}, \end{aligned} \quad (4)$$

where  $q$  is in the first Brillouin zone  $-\pi/a \leq q \leq \pi/a$ . Substituting systems (1) and (2) into systems (3) and (4), we obtain the following dispersion relation for the  $E$ -polarization case:

$$\begin{aligned} \cos(k_1 d_1) \cos(\tilde{n} k_1 d_2) - \frac{\tilde{n}^2 + \mu^2}{2\tilde{n}\mu} \sin(k_1 d_1) \sin(\tilde{n} k_1 d_2) \\ = \cos(qa), \end{aligned} \quad (5)$$

where  $\tilde{n} = k_2/k_1 = \pm \sqrt{n^2(k_1^2 + \beta^2) - \beta^2}/k_1$  (negative sign for  $n < 0$ ),  $\mu = \mu_2/\mu_1$ , and  $n = n_2/n_1$  is the index ratio of the two media.

Similarly, we can derive the following dispersion relation for the  $H$ -polarization case:

$$\begin{aligned} \cos(k_1 d_1) \cos(\tilde{n} k_1 d_2) - \frac{\tilde{n}^2 + \epsilon^2}{2\tilde{n}\epsilon} \sin(k_1 d_1) \sin(\tilde{n} k_1 d_2) \\ = \cos(qa), \end{aligned} \quad (6)$$

where  $\epsilon = \epsilon_2/\epsilon_1$ .

From analytical Eqs. (5) and (6), we can find out how  $k_1$  depends on  $q$ . The corresponding dispersion relation between  $\omega$  and  $q$  can then be obtained according to  $\omega^2 = c^2(k_1^2 + \beta^2)/(\epsilon_1 \mu_1)$  for each fixed  $\beta$  ( $\beta=0$  is associated to the normal propagation, i.e., the case when the propagation direction is along the normal of the material interfaces).

When the index ratio  $n < 0$ , the solution to the dispersion equation and the associated band structure have quite different behaviors as compared to those for the usual case when  $n > 0$ . For simplicity, we first illustrate this for the case when  $\beta=0$ . In this case,  $(\tilde{n}^2 + \mu^2)/2\tilde{n}\mu = (\tilde{n}^2 + \epsilon^2)/2\tilde{n}\epsilon$  and the dispersion equations for the  $E$  polarization and the  $H$  polarization have the same form (as expected).

*Spurious modes with complex  $\omega$ .* When one of the constitutive media is of LHM (i.e.,  $n < 0$ ), the analytical dispersion equation may have some complex solutions for  $k_1$ . These complex values of  $k_1$  lead to complex  $\omega$  since  $\omega^2 = c^2(k_1^2 + \beta^2)/(\epsilon_1 \mu_1)$ . Figure 2 gives the band structure when  $n = -2.5, \mu = -2, d_1 = d_2 = 0.5a$  for the normal propagation case (i.e.,  $\beta=0$ ). The real part of  $\omega$  varies little as  $q$  varies from

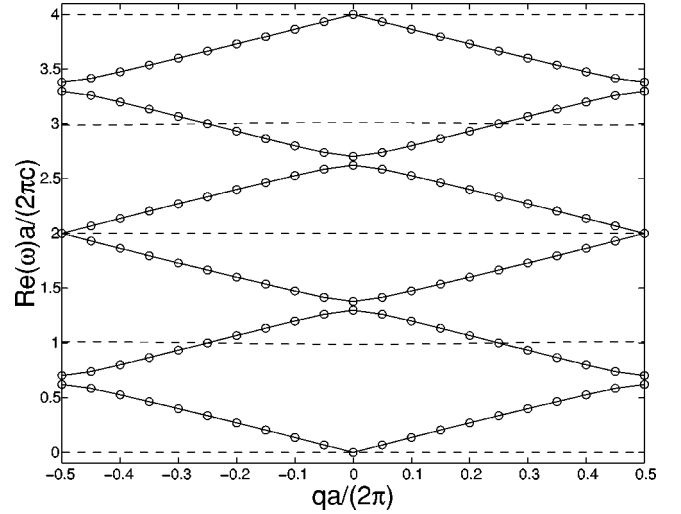


FIG. 2. The band structure for the normal propagation (i.e.,  $\beta = 0$ ) with  $\epsilon_1 = 1$ ,  $\mu_1 = 1$ ,  $n = -2.5$ ,  $\mu_2 = -2$ ,  $d_1 = d_2 = 0.5a$ . The circles show the real  $\omega$ , while the dashed lines indicate the real parts of the complex  $\omega$ .

$-\pi/a$  to  $\pi/a$ . The problem of complex  $\omega$  will also appear when  $\beta \neq 0$  (i.e., the case of oblique propagation). Obviously, these complex  $\omega$  have no physical significance since physically the frequency  $\omega$  must have a real value.

Traditionally, the pass bands of a conventional 1D periodic structure can be obtained when the absolute value of the left-hand side of the dispersion relation [Eq. (5) or (6)] is less than one (this means that one can obtain a real wave number  $q$  for the normal propagation). On the other hand, when the left-hand side of Eq. (5) or (6) is bigger than 1, the frequency belongs to a stop band. For a photonic crystal consisting of only usual media, whose permittivity and permeability are real and positive, the solution to the dispersion relation [Eq. (5) or (6)] for the transverse wave number  $k_1$  is always real and thus one can enforce the wave number to be real in the dispersion relation. However, this is no longer true when the photonic crystal contains both RHM and LHM. Some complex value of  $k_1$  can still make the left-hand side of Eq. (5) or (6) smaller than 1 and these complex solutions of  $k_1$  may have physical significance. For example, a purely imaginary solution for the transverse wave number  $k_1$  corresponds to a photon tunnelling mode for the case of oblique propagation (see the discussion on photon tunnelling modes below). Therefore, we cannot enforce the solution to be real in the present situation.

Furthermore, in a general case or some numerical methods for calculating the band structure of a 2D or 3D photonic crystal, we usually end up with solving a complex equation (due to the introduction of the  $e^{-i\omega t}$  convention) for a complex wave number and thus consequently we can not enforce the solution to be real in these cases.

The existence of the complex solutions is a common phenomenon and even a very serious problem for some numerical methods. For example, one would meet a similar problem of complex  $\omega$  in the calculation of the band structure using other methods such as the plane wave expansion method (one needs to remove these spurious complex  $\omega$  in the band

structure). In the plane wave expansion method, the  $\omega$  values are determined from the eigenvalue equation  $(1/\mu)\nabla \times [(1/\epsilon)\nabla \times \mathbf{H}] = (\omega^2/c^2)\mathbf{H}$ . The existence of complex  $\omega$  indicates that the operator  $(1/\mu)\nabla \times [(1/\epsilon)\nabla]$  is not a Hermit operator or has no equivalent Hermit operator. One can also expect that these complex  $\omega$  (associated with an exponential increasing factor of time) will make the finite-difference time domain (FDTD) method (with a periodic boundary condition) divergent in the calculation of the band structure of a photonic crystal consisting of alternate RHM and LHM inclusions (with constant permittivity and permeability).<sup>12-14</sup> We have verified numerically with the plane wave expansion method and the FDTD method the existence of the spurious modes with complex  $\omega$ .

The fields associated with the spurious complex  $\omega$  satisfy Maxwell's equations, the boundary condition and the enforced periodic condition. Therefore, an additional requirement that the frequency  $\omega$  must be real should be applied to make the solution significant in physics.

In the rest of the paper, we consider only situations when  $\omega$  is real. When  $\beta=0$ , a complex solution of  $k_1$  always leads to a spurious complex  $\omega$ . However, when  $\beta \neq 0$  some complex solutions of  $k_1$  may correspond to real  $\omega$  and thus have physical significance. Since  $\omega^2 = c^2(k_i^2 + \beta^2)/(\epsilon_i\mu_i)$ , a complex solution of  $k_i \equiv k_{iR} + jk_{iI}$  (with  $k_{iI} \neq 0$ ) should satisfies the following conditions in order to make  $\omega$  real:

$$k_{iR} = 0, \quad k_{iI}^2 < \beta^2. \quad (7)$$

In other words, an imaginary solution of  $k_1$  can have physical significance when  $\beta \neq 0$  (i.e., the oblique propagation case). Imaginary solutions of  $k_1$  correspond to evanescent waves. In a periodic structure consisting of alternate RHM and LHM layers, the special property that the evanescent waves (in both directions) decrease in the RHM layer and increase in the LHM layer<sup>8</sup> or vice versa permits the evanescent waves to propagate in the periodic structure (i.e., a propagation mode with imaginary  $k_1$  and real  $q$ , for which the energy density does not decay over a period; we will call it a photon tunnelling mode and will discuss it in more details below). In the present work all possible solutions (including complex solutions) to the analytical equation (5) are obtained by using the Matlab program "solve" for solving a nonlinear equation.

### III. DISCRETE MODES AND PHOTON TUNNELLING MODES

As discussed before, imaginary solutions of  $k_1$  may have physical significance when  $\beta \neq 0$ . This will be illustrated by the unusual photon tunnelling modes in the band structure for a photonic crystal consisting of alternate RHM and LHM layers. Another unusual (and interesting) phenomenon is that some discrete modes (with real values of wave number) are also found in the band structure.

To simplify the analysis, we consider a special case when  $n = -1$  for the  $E$  polarization. Then  $\tilde{n} = -1$  (independent of  $\beta$ ) and Eq. (5) is simplified to

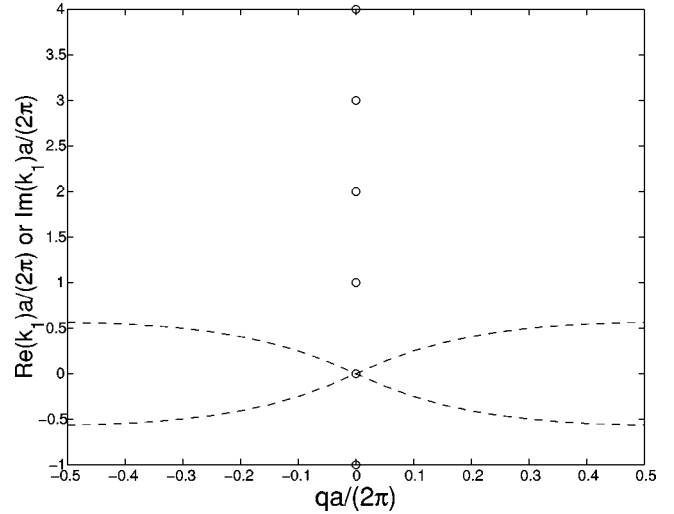


FIG. 3. The solutions (with physical significance) of  $k_1$  to Eq. (8) with  $\epsilon_1=1$ ,  $\mu_1=1$ ,  $n=-1$ ,  $\mu_2=-2$ ,  $d_1=d_2=0.5a$ . The circles indicate the real solutions, which exist discretely at  $q=0$ . The dashed lines show the imaginary solutions. The imaginary solutions are continuously distributed in a region from  $-0.561j \times 2\pi/a$  to  $0.561j \times 2\pi/a$ .

$$\cos(k_1 d_1) \cos(k_1 d_2) - \frac{1 + \mu^2}{2\mu} \sin(k_1 d_1) \sin(k_1 d_2) = \cos(qa). \quad (8)$$

One notices that the above equation for  $k_1$  is independent of  $\beta$ . This indicates that different  $\beta$  may have the same solutions of  $k_1$ . The solutions of the dispersion equations only give the corresponding wave number  $k_1$ . The frequencies and the band structure of the photonic crystal should be given for a fixed  $\beta$ . Furthermore, one should pay special attention that only  $\beta \neq 0$  can permit the existence of some photon tunnelling modes (see discussed below).

We discuss situations when  $\mu \neq -1$  (so that the RHM and LHM layers have different values of impedance). It is particularly interesting when the two inclusion layers have the same width (i.e.,  $d_1 = d_2 = 0.5a$ ) since in this case only discrete modes and photon tunnelling modes exist (conventional propagating band disappears in the band structure).

#### A. Discrete modes

Since  $\cos(k_1 d_1) \cos(k_1 d_2) + [(1 + \mu^2)/2|\mu|] \sin(k_1 d_1) \times \sin(k_1 d_2) \geq 1$  (when  $k_1$  is real), real solutions of  $k_1$  to Eq. (8) exist only when  $q=0$  (notice that  $-\pi/a \leq q \leq \pi/a$ ), which corresponds to discrete solutions  $k_1 = 2N\pi/a$ ,  $N=0, \pm 1, \pm 2, \dots$ . As an example, the discrete real solutions of  $k_1$  to Eq. (8) are shown by the circles in Fig. 3 (with  $\mu = -2$ ). This unusual phenomenon (existing for any  $\beta$ ) can be utilized to make a very narrow filter with no side lobe (the ripples outside the main lobe of the transmission spectrum), which is quite different from any conventional type of filters. From these discrete solutions, one can obtain the corresponding eigen field distributions by deriving the coefficients  $A$ ,  $B$ , and  $C$ . For example, when  $k_1 = 2\pi/a$  one has  $B = \frac{3}{2} - A/2$  and  $C = -\frac{1}{2} + 3A/2$  [here  $A$  is an arbitrary constant repre-

senting the amplitude of the left-going wave excited in region 1; note that the right-going wave excited in region 1 has been normalized according to Eq. (1)]. The arbitrariness of  $A$  indicates that any linear combination of the left- and right-going waves can exist in region 1 (since in such a case each layer is transparent to the neighboring medium as discussed below).

These discrete solutions satisfy the Fabry-Perot resonating condition of  $k_1 d_1 = N\pi$  and  $k_2 d_2 = -N\pi$  (note that  $k_2 = \tilde{n}k_1 = -k_1$  for the special case  $n = -1$  considered here). When this Fabry-Perot condition is fulfilled, each layer is transparent to the neighboring medium at these discrete wave numbers (corresponding to some discrete frequencies for a fixed  $\beta$ ), and thus an incident field at one of these frequencies will surely pass through the periodic structure and becomes a propagating mode. To excite such a discrete mode by an external field and to detect by an external probe, one has to truncate this infinitely extended periodic structure with a finite number of periods. When truncating this periodic structure with finite periods and terminating it with any one of the two inclusion media at both sides, the structure of finite periods will also be transparent at these frequencies (in this finite slab case  $A = 0$  at these frequencies since there is only a right-going incident wave in the left half-space, which contains the same medium as in region 1; if one wishes to excite modes with  $A \neq 0$ , an incident wave from the right half-space should also be used). On the other hand, fields with any other wave number (i.e., any other frequency for the fixed  $\beta$ ) satisfies the Bragg reflective condition  $k_1 d_1 + k_2 d_2 = 0$  (independent of the frequency since  $k_2 = \tilde{n}k_1 = -k_1$  for the special case considered here) and will be reflected from this finite periodic structure. Therefore, there will be a very steep transmission peak around the discrete frequency in the transmission spectrum.

For  $k_1 d_1 + k_2 d_2 \neq 0$  (with  $n < 0$ ), we can still have the unusual discrete mode under some conditions. When an incident field is launched into a 1D periodic structure with a finite number of layers, it will be reflected from two kinds of interface. The reflected waves at different interfaces sum up to the total reflected wave. If the periodic structure contains both LHM and RHM, there are some unusual narrow transmission bands in the transmission spectrum around some special points. At these special points both the Fabry-Perot condition (i.e.,  $k_1 d_1 = N\pi$  and  $k_2 d_2 = -L\pi$ ) and the Bragg condition (i.e.,  $k_1 d_1 + nk_1 d_2 = M\pi$ ) are satisfied. Since  $n < 0$  and  $\tilde{n} < 0$ , one can have  $|M| < |N|$  and thus the Bragg condition dominates (i.e., reflected waves at different periods sum up to an increased reflected field at the first interface) when the wave number deviates from these discrete points. In some later part of the present paper we will give the solutions for the case  $d_1 \neq d_2$  (as an example of  $k_1 d_1 + k_2 d_2 \neq 0$ ) and show the discrete modes in the band structure.

### B. Photon tunnelling modes

The present photon tunneling refers the tunneling through a part of a period (complex  $k_1$ , but real  $q$ ), but not the tunneling through the entire hetrostructure (complex  $q$ ). Optical field which decreases in a RHM layer (or a LHM layer)

is amplified in a LHM layer (or a RHM layer) and does not decrease in amplitude after propagating through a period [since  $q$  has a real value; see Eq. (4)].

From Eq. (8) one sees that only complex  $k_1$  can be the solutions if the real  $q$  is not equal to 0. Then according to condition (7) the solutions  $k_1$  with physical significance must be imaginary. The expression on the left-hand side of Eq. (8) always decreases when  $k_{1I}$  increases or decreases away from 0 (a symmetric function of  $k_{1I}$  with a maximum at  $k_{1I} = 0$ ). Therefore, only two conjugate imaginary solutions exist. The dashed lines in Fig. 3 show the imaginary solutions of  $k_1$  to Eq. (8) for an example. When  $q (\neq 0)$  varies in the first Brillouin zone, the two conjugate imaginary solutions vary continuously in a region from  $-0.561j \times 2\pi/a$  to  $0.561j \times 2\pi/a$ . Note that this figure is independent of  $\beta$  for this special case. For oblique propagation with  $\beta^2 > k_{1I}^2$ , the imaginary solutions correspond to special propagation modes — photon tunneling modes (as discussed before). For this example, evanescent waves with  $k_{1I}$  outside the region  $[-0.561j \times 2\pi/a, 0.561j \times 2\pi/a]$  can not propagate inside the periodic structure. From Fig. 3 one can also see that  $k_{1I}$  approaches 0 when  $q$  approaches 0. Thus, any  $\beta \neq 0$  can permit the existence of some photon tunnelling modes since  $k_{1I}$  can be arbitrarily small to satisfy  $k_{1I}^2 < \beta^2$ . From the above discussion, one sees that for any  $\beta \neq 0$  there always exist some evanescent waves which can propagate inside the periodic structure while some other evanescent waves can not (one notices that in the case of a perfect lens<sup>8</sup> all the evanescent waves can go through the lens). This limitation of the photon tunnelling modes is due to the enforced periodic condition. Thus, the problem of square nonintegrable field<sup>6</sup> will not occur in the present situation (this is also true for the next example when  $d_1 \neq d_2$ ).

It is known that the photon tunnelling effect exists in the following two cases: long-range surface plasma waves support by thin metallic films (with negative dielectric constant)<sup>15,16</sup> and optical modes of long wavelength in superlattices of, e.g., GaAs-AlAs.<sup>17-19</sup> At the interface of two materials with one material having a negative dielectric constant and the other a positive dielectric constant, a surface plasma wave that decreases on either side of the surface can be excited. The first case of photon tunnelling effect is due to the coupling between the surface plasma waves at the two interfaces through the evanescent wave in the metallic film. The second case of photon tunnelling effect is due to the plasmaphonon (transverse and longitudinal optical modes) oscillations excited by an external optical field. In both cases, the optical field pass through the structure indirectly, i.e., by the coupling of surface plasma modes or by the excitation of phonons. However, in the present case, the photon tunnelling effect is based on a very different mechanism—the amplification of the evanescent wave in the LHM (in which both the permittivity and the permeability are negative). As discussed earlier, evanescent waves that decrease in RHM (or LHM) are amplified in LHM (or RHM) and become propagation modes in the periodic structure. In the present situation, the fields (photons) pass through the periodic structure directly.

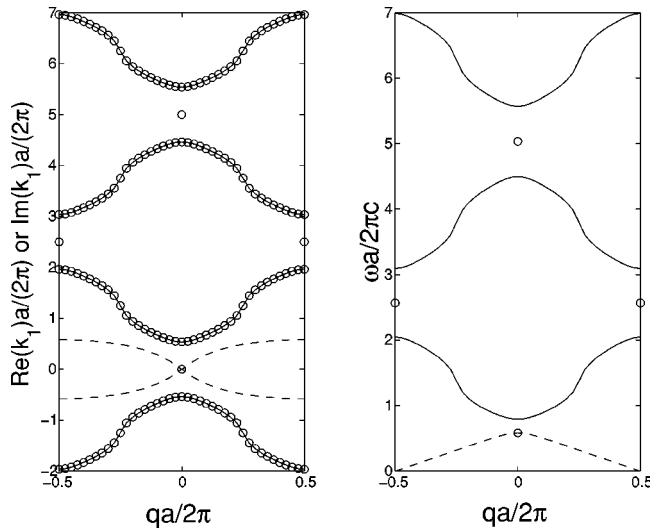


FIG. 4. An oblique propagation case with  $d_1 \neq d_2$ . The two inclusion layers have  $\epsilon_1=1$ ,  $\mu_1=1$ ,  $n=-1$ ,  $\mu_2=-2$ ,  $d_1=0.4a$ , and  $d_2=0.6a$ . (a) The solutions of  $k_1$  with physical significance. The circles show the real solutions. The real solutions have discrete values (at  $q=0$  and  $q=\pm\pi/a$ ) as well as continuous bands. The dashed lines indicate the imaginary parts of the imaginary solutions. (b) The band structure when  $\beta=0.583 \times 2\pi/a$ . The dashed lines show the frequencies of the photon tunnelling modes. The circles indicate the discrete frequencies of the discrete modes. The solid lines show the continuous bands.

Both discrete modes and photon tunnelling modes exist when  $\beta \neq 0$ . However, when  $\beta=0$  (as in the above example), which corresponds to normally incident waves (i.e., the normal propagation case), only discrete modes can exist (this means that only fields at discrete frequencies can exist in the structure).

Figure 4(a) shows the solutions of  $k_1$  for an example of  $d_1 \neq d_2$  (so that  $k_1 d_1 + k_2 d_2 \neq 0$ ). The discrete solutions are at  $k_1 = (2.5 + 5N) \times 2\pi/a$  for  $q = \pm\pi/a$  and at  $k_1 = 5N \times 2\pi/a$  for  $q=0$ ,  $N=0, \pm 1, \pm 2, \dots$ . They are located in the forbidden band gaps [see Fig. 4(b) for the corresponding band structure for a given  $\beta$ ]. From this figure one sees that nondiscrete real  $k_1$  solutions [corresponding to some continuous bands shown in Fig. 4(b)] exist in addition to some discrete modes and photon tunneling modes.

#### IV. DISCUSSION AND CONCLUSION

In the above we have discussed only the ideal situations when the periodic structure consists of ideal materials with lossless and nondispersive negative  $\epsilon$  and negative  $\mu$  ( $\epsilon$  and

$\mu$  are the relative permittivity and permeability of the two inclusion media). In a nonideal situation with material loss or dispersion the unusual phenomena such as spurious modes with complex  $\omega$ , discrete modes and photon tunnelling modes can also be expected to occur for the following reasons.

If the inclusion media are dispersive,  $\epsilon$  and  $\mu$  will be frequency dependent. Since the solutions can be obtained from Eq. (5) in a point-by-point manner, they are correct locally (i.e., in a small frequency range where the frequency dependence of  $\epsilon$  and  $\mu$  can be neglected). Thus, the unusual phenomena can still exist in certain small frequency region. Furthermore, the frequency in the band structure is scalable and thus we can scale it to a frequency at which the material parameters have desired values in order to observe these unusual phenomena.

If the LHM inclusion is lossy the relative  $\epsilon$  or  $\mu$  has a very small positive imaginary part (e.g.,  $10^{-5}j$ ; as claimed in some papers that LHM must have some loss<sup>6</sup>). The unusual phenomena can still occur since the dispersion equation [Eq. (5)] will not give any rapid change when a very small imaginary part is added to  $\epsilon$  or  $\mu$ . Note that for the photon tunnelling modes  $q$  should be complex (corresponding to a pseudoperiodic field with a damping factor along the propagation direction) when  $\epsilon$  or  $\mu$  is complex. Since the imaginary parts of these parameters are very small, Eq. (5) is still valid when all the parameters (including  $\epsilon$ ,  $\mu$  and  $q$ ) are replaced with their real parts.

In conclusion, we have found some unusual phenomena such as spurious modes with complex  $\omega$ , discrete modes and photon tunneling modes in the band structure by analyzing the explicit dispersion equation for a 1D periodic structure with alternating LHM and RHM layers. One will meet the problem of complex  $\omega$  in the calculation of the band structure using other methods such as the plane wave expansion method and the FDTD method. The physical significance of the discrete modes and photon tunneling modes has been explained. The discrete modes can be utilized to make a very narrow filter with no side lobe. For an oblique wave propagation there exist some evanescent waves which can become propagating modes (photon tunneling modes) while some other evanescent waves cannot propagate inside the periodic structure.

#### ACKNOWLEDGMENTS

The partial support of National Natural Science Foundation of China (under a key project grant, Grant No. 90101024) is gratefully acknowledged.

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