Communicating Josephson qubits

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We propose a scheme to implement a quantum information transfer protocol with a superconducting circuit and Josephson charge qubits. The information exchange is mediated by an *L*-*C* resonator used as a data bus. The main decoherence sources are analyzed in detail.

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I. INTRODUCTION

One of the main purposes of quantum information processing is the faithful transmission of quantum states between distant parties exploiting entanglement among the various subsystems. Examples include quantum teleportation[1] and dense coding,[2] both of them demonstrated using entangled photon pairs.[3] Until now, however, much has been done within the realm of quantum optics,[4] while comparatively little efforts have been devoted to study solid state implementations of entanglement and communication protocols. Interest in the problem also arises from recent proposals employing nano-electronic circuits for quantum hardware implementation.[5–7] In particular, superconducting devices seem to be promising since they combine the intrinsic stability of the superconducting phase with the possibility of controlling the circuit dynamics through manipulations of the applied voltages or magnetic fluxes.[8] Direct experimental evidence of their use as controllable coherent two level systems has already been provided.[9–11]

In this paper, we analyze an interconnection scheme among mesoscopic superconducting subcircuits, suitable for quantum communication. This setup is also relevant for the general task of finding a realistic way to probe dynamical aspects of entanglement in the solid state. In particular, we will illustrate a quantum state transfer protocol which can test the possibility of *controlled* interaction between superconducting devices and which gives an indirect way to check entanglement by just single-bit measurements performed on the target qubit.

Either the charge on the island or the phase difference at a junction can be used to store and manipulate quantum information.[12] Here, we concentrate on the charge regime and propose a setup allowing quantum information transfer between two such Josephson qubits.

In this scheme, two superconducting qubit are capacitively coupled to an electrical resonator playing the role of the data bus; see Fig. 1. The resonator can be either implemented by an *L*-*C* circuit or by a large Josephson junction. At present, a larger quality factor seems to be achievable through a large junction working in the harmonic regime, which is also easier to fabricate on chip. Therefore, we will concentrate on this case to give an estimation of the circuit parameters. This setup is flexible enough to allow for quantum state transfer from one qubit to another and for Bell state generation. We also give the necessary prescriptions to implement a universal set of quantum gates. The spirit of the proposal is very similar to the Cirac-Zoller scheme for trapped ion qubit.[13] However, the presence of a large number of low-energy environmental excitations, peculiar to the solid state implementation, requires a careful analysis of decoherence which we address in Sec. IV.

The coupling of a single charge qubit to a large Josephson junction has been proposed to perform an on chip quantum state measurement,[14] and recently implemented in the experiment of Vion *et al.*[10] The large junction is biased near the critical current and it operates essentially as a classical object. Since it is coupled to a reading port, its quality factor cannot be very large. Another recent work[15] studied the preparation of a mesoscopic Schrödinger cat state in a charge qubit plus large junction system. In this latter case the large junction is a quantum object, coupled to a qubit in the off-resonant (or dispersive) regime.

Our setup also exploits the interaction between a small and a large junction, but we use a superconducting quantum interference devices (SQUID) instead of a single junction in the qubit, to achieve a control of the coupling. This allows to use both the dispersive and the resonant regimes as required by the communication protocols described below. In particular, the use of the resonant regime allows to entangle the circuit elements on a shorter time scale, which is essential in view of the decoherence acting on the system.

II. MODEL

We first analyze the model of a single qubit coupled to the resonator. Letting φ_J be the effective phase for the SQUID, φ_r the phase difference across the resonator capacitance, and Q and P their conjugated charges, the system Hamiltonian reads ($\hbar=1$)

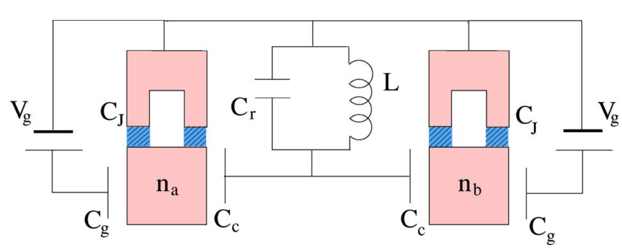

FIG. 1. Schematics of the superconducting circuit.

$$H=\frac{Q^2}{2C_q}-\kappa_q V_g Q-E_J(\Phi_x)\cos(2e\varphi_J)$$
$$+\frac{PQ}{C_k}+\frac{P^2}{2C_p}+\frac{\varphi_r^2}{2L}-\kappa_p V_g P, \qquad (1)$$

where the capacitances are given by $C_q=C_g+C_J+(C_c^{-1}+C_r^{-1})^{-1}$, $C_p=C_r+[C_c^{-1}+(C_c+C_J)^{-1}]^{-1}$, $C_k=C_q(C_c+C_r)/C_c$, and where $\kappa_q=C_g/C_q$ and $\kappa_p=C_g/C_k$ are the attenuation parameters, whose values will also determine the strength of the coupling with the environment. The relevant energy scales are the charging energy, $E_{ch}=2e^2/C_q$, the resonator frequency $\omega_r=(LC_p)^{-1/2}$, and the effective Josephson coupling of the SQUID, $E_J(\Phi_x)=E_{J_0}\cos(2e\Phi_x)$, tunable via an external magnetic flux Φ_x.

Introducing creation and annihilation operator for the resonator via

$$\varphi_r=\frac{a+a^\dagger}{\sqrt{2\omega_r C_p}},\quad P=i\sqrt{\frac{\omega_r C_p}{2}}(a^\dagger-a), \qquad (2)$$

we can rewrite the Hamiltonian as

$$H=E_{ch}(n-n_g)^2-E_J\cos(2e\varphi_J)+\omega_r a^\dagger a$$
$$-ig(a-a^\dagger)(n-n_g), \qquad (3)$$

where $n=Q/2e$ counts the number of Cooper pairs on the island with respect to the off-set charge number $n_g=Q_g/2e$, while the coupling constant is given by $g=C_k^{-1}e\sqrt{2\omega_r C_p}$.

In the charge regime ($E_{ch}\gg E_J$), only the two lowest charge states ($Q=0,2e$) of the small island come into play, allowing to employ it as a qubit. The electrostatic splitting between these two states can be modified by the gate voltage V_g, which we fix by setting $Q_g\equiv C_g V_g=e$. The eigenstates of qubit Hamiltonian are then $|\pm\rangle=(|0\rangle\pm|2e\rangle)/\sqrt{2}$, which we use as logical states. In this basis, state preparation and read out can be performed as for the charge basis, by applying an ac voltage pulse to perform a $\pi/2$ rotation that maps the $|Q=0,2e\rangle$ states onto the $|\pm\rangle$ ones. The choice of electrostatic degeneracy, $Q_g=e$, is crucial for what follows; we will show that decoherence due to low-frequency noise is strongly quenched at this working point.

If the qubit and oscillator are tuned near resonance, $\delta=E_J-\omega_r\ll E_J+\omega_r$, and if the coupling is weak, $g\ll\omega_r,E_J$, one can perform the rotating wave approximation, neglecting the rapidly oscillating terms $|-\rangle\langle+|a$ and $|+\rangle\langle-|a^\dagger$. Equation (1) then reduces to the Jaynes-Cummings model[16]

$$H_{JC}=\frac{E_J}{2}\sigma_z+\omega_r a^\dagger a-ig(\sigma_+ a-a^\dagger\sigma_-), \qquad (4)$$

where the operators $\sigma_+=\sigma_-^\dagger=|+\rangle\langle-|$ and $\sigma_z=1-2\sigma_-\sigma_+$ have been introduced to describe the qubit.

H_{JC} generates Rabi oscillations between $|+,n_r\rangle$ and $|-,n_r+1\rangle$ at the frequency $2\mathcal{R}_{n_r}=\sqrt{\delta^2+4g^2(n_r+1)}$. We will need to take into account only oscillator states with at most $n_r=1$.

Exploiting the external flux dependence of E_J, it is possible to switch between nearly resonant ($\delta\ll g$) and dispersive regime ($g\ll\delta\ll\omega_r$). In the latter case, the time evolution of the system is generated by the effective Hamiltonian

$$H_{int}^{eff}=\frac{g^2}{\delta}[aa^\dagger|+\rangle\langle+|-a^\dagger a|-\rangle\langle-|], \qquad (5)$$

obtained by neglecting contributions of second and higher order in g/δ (see, e.g., Ref. 17 for details).

In the resonant regime it is possible to accomplish a quantum state transfer, whereas switching between the two regimes is required to perform a two-bit gate. If ω_r is very different from E_J, the coupling is effectively switched off, and the qubit and the resonator evolve independently.

As suggested in Refs. 14 and 15, and experimentally realized by Vion *et al.*,[10] a large current biased Josephson junction can be used to implement the resonator. For bias current I well below the critical value I_c, the phase φ_r of the large junction is trapped in one of the minima of the tilted washboard potential $U(\varphi_r)=-(I_c/2e)\cos\varphi_r-(I/2e)\varphi_r$, so that the system approximately behaves as an harmonic oscillator with

$$\omega_r=\sqrt{\frac{2eI_c}{C_p}}\left(1-\frac{I^2}{I_c^2}\right)^{1/4}. \qquad (6)$$

One can take advantage of the dependence of ω_r on the current I to have a second (and independent) mechanism to go from the resonant to the dispersive regime. However, since the dependence on I is very weak, one cannot use this mechanism to switch off the coupling.

In the two-qubit setup of Fig. 1, for simplicity we take $C_c\ll C_q$, so that the direct electrostatic interaction between the two qubits ($\sim C_c^2/C_q^2$) can be neglected and they only interact through the resonator (in fact, C_p, the C_q's and g are slightly modified, but the changes are negligible for small C_c).

III. PROTOCOLS

To illustrate the use of the oscillator as a data bus, we show how the quantum state of qubit a can be transferred to b. Let us suppose that the three subsystems are initialized independently with qubit b in $|-\rangle$ and the resonator in its ground state:

$$|\psi,0\rangle=(c_+|+\rangle+c_-|-\rangle)\otimes|-\rangle\otimes|0\rangle. \qquad (7)$$

In the first step, the state of qubit a is transferred to the data bus by resonantly coupling them for a time $\tau=\pi/2g$. This leads to the state $|\psi,\tau\rangle=|-\rangle\otimes|-\rangle\otimes(c_+|1\rangle+c_-|0\rangle)$. Then, decoupling qubit a and performing the same operation on qubit b, the system is led to

$$|\psi,2\tau\rangle=|-\rangle\otimes(c_+|+\rangle+c_-|-\rangle)\otimes|0\rangle. \qquad (8)$$

Thus, the state of one qubit has been transferred to the other (target) one via the intermediary action of the resonator.

In a similar way, a maximally entangled singlet state can be obtained by employing a protocol already realized with atoms and cavity.[18] The underlying idea is very simple: first entangle *a* and *r*, then swap the entanglement by just "exchanging" the states of the oscillator and of qubit *b*. With the system prepared in $|+\rangle_a \otimes |-\rangle_b \otimes |0\rangle_r$, we first let island *a* and the resonator to interact resonantly for a time $\tau/2 = \pi/4g$ and then allow for the same coupling (but lasting a time τ) to be experienced by island *b*. This procedure gives rise to the EPR state $1/\sqrt{2}(|+-\rangle - |-+\rangle)\otimes|0\rangle$.

Note that, although the oscillator is left in the ground state after the operations, it actively mediates between the qubits. From the physical point of view, this is the main difference with respect to the scheme of Shnirman *et al.*,[6] where the oscillator is only virtually excited. As a consequence, when evaluating dephasing effects, the oscillator needs to be included explicitly (see below).

Besides quantum state transfer and entanglement generation, a universal set of logic gates can be implemented. Indeed, single-bit rotations are obtained by applying ac voltage pulses on the qubit gate electrodes. Furthermore, a two-bit gate (equivalent to the control phase up to a one-bit operation) can be accomplished through the following four steps: (i) couple qubit *a* to the oscillator in the dispersive regime for a time t_1 (with *b* decoupled and *r* initially prepared in $|0\rangle$). This leaves the state $|-\rangle_a$ unaffected, while appending the phase factor $e^{-i\theta}$ to $|+\rangle_a$, with $\theta = (g^2/\delta)t_1$; (ii) transfer the state of *a* to the oscillator as before (i.e., let the two systems interact for a time τ); (iii) *a* being de-coupled, let *r* and *b* interact dispersively, again for time t_1; (iv) transfer back the state of the oscillator to qubit *a* [same operation as in the step (ii)]. The resulting gate is represented in the basis $\{|\pm_a, \pm_b\rangle\}$ as

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\theta} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix}. \tag{9}$$

This operation is equivalent to the control phase gate up to a single-bit operation.

IV. EFFECT OF THE ENVIRONMENT

The treatment given so far has to be extended to account for unwanted decoherence effects, whose major sources are electromagnetic fluctuations of the circuit and noise originating from bistable charged impurities located close to the islands. To estimate the time scales for relaxation and decoherence *during* operations, we focus on the single-qubit plus resonator scheme depicted in Fig. 2.

We first consider noise due to the impedances $Z_\alpha, \alpha = 1,2$, which are modeled by LC lines corresponding to sets of quantum harmonic oscillators,[12,19,20] with Hamiltonian H_α. Following Ref. 20 it is possible to derive a Hamiltonian of the Caldeira-Leggett kind[21]:

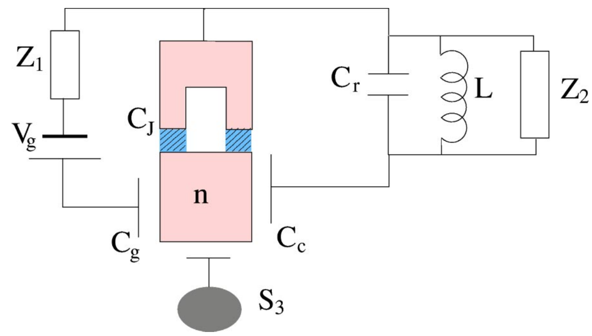

FIG. 2. One qubit coupled to the resonator in the presence of the decohering reservoirs, the third of which represents a bath of fluctuating charge impurities.

$$\delta H = \sum_{\alpha=1}^{2} H_\alpha^{env} - \sum_{\alpha=1}^{2} \hat{K}_\alpha \hat{E}_\alpha + B(Q,P,\varphi_r). \tag{10}$$

The coupling term contains system operators, $\hat{K}_1 = \kappa_q Q + \kappa_p P$, and $\hat{K}_2 = \varphi_r$, and environment operators, $\hat{E}_\alpha$. Also a counter-term, *B*, is generated, which we disregard from now on since it does not affect the decoherence rates. All the information on the reduced system dynamics is contained in the fluctuation spectra of the operators $\hat{E}_\alpha$ for the environment alone,[21] which are found to be:

$$S_1(\omega) = \omega \,\mathrm{Re} \frac{Z_1(\omega)}{1 + i\omega Z_1(\omega) C_{eff}(\omega)} \coth \frac{\beta\omega}{2},$$

$$S_2(\omega) = \omega \,\mathrm{Re}[Z_2(\omega)]^{-1} \coth \frac{\beta\omega}{2},$$

with $C_{eff}(\omega) \simeq C_g$.

This result can be understood by using classical circuit theory. The main effect of the impedances is to produce electromagnetic fluctuations, accounted for by stochastic voltage (E_1) and current (E_2) sources.[19] The corresponding power spectra derived in Eq. (11) describe these fluctuations as seen at the impedance Z_1 and Z_2, respectively. The interaction Hamiltonian of Eq. (10), then, couples voltage noise to the charges *Q* and *P* via the attenuation parameters $\kappa_{q,p}$, and current noise to the phase of the resonator, φ_r.

To evaluate the effect of δH on the eigenstates of H_{JC}, we suppose a weak coupling with the environment (κ_q, κ_p and the impedances can be chosen to fulfill this condition). The spectrum of H_{JC} is made up of a ground state, $|g\rangle = |-,0\rangle$, and a series of dressed doublets,

$$|a(n_r)\rangle = \cos\theta_{n_r}|+,n_r\rangle + i\sin\theta_{n_r}|-,n_r+1\rangle,$$

$$|b(n_r)\rangle = i\sin\theta_{n_r}|+,n_r\rangle + \cos\theta_{n_r}|-,n_r+1\rangle,$$

having eigenenergies $(n_r + 1/2)\omega_r \pm \mathcal{R}_{n_r}$, and where we defined $\tan 2\theta_{n_r} = 2g\sqrt{n_r+1}/\delta$. Only $|g\rangle$ and the first doublet, $\{|a\rangle, |b\rangle\}$, are involved in the coherent operations described so far. In the secular approximation,[22] relaxation and dephasing rates in this subspace can be expressed in terms of the quantities

TABLE I. Relevant matrix elements of the coupling operators with the electromagnetic ($\alpha=1,2$) and the $1/f(\alpha=3)$ environments. Diagonal elements are equal, e.g., $\langle a|\hat{K}_1|a\rangle=\langle b|\hat{K}_1|b\rangle=\langle g|\hat{K}_1|g\rangle=e\kappa_q$. Matrix elements $\langle a|\hat{K}_\alpha|b\rangle$ vanish. Here $c=\cos\theta_0$, $s=\sin\theta_0$, and $\chi=\sqrt{C_p\omega_r/(2e^2)}$.

	$\|i\rangle=\|g\rangle$	$\|i\rangle=\|a\rangle$	$\|i\rangle=\|b\rangle$
$\alpha=1$	$e\kappa_q$	$-e(\kappa_q c-\chi\kappa_p s)$	$-ie(\kappa_q s+\chi\kappa_p c)$
$\alpha=2$	0	$i(2\chi e)^{-1}s$	$(2\chi e)^{-1}c$
$\alpha=3$	eC_q^{-1}	$-e(C_q^{-1}c-\chi C_k^{-1}s)$	$ie(C_q^{-1}s+\chi C_k^{-1}c)$

$$\gamma_{if}^{\alpha}(\omega)=2|\langle f|\hat{K}_\alpha|i\rangle|^2 S_\alpha(\omega). \tag{11}$$

For instance, the transition rates for the populations are given by $\Gamma_{i\to f}=[1+\exp(-\beta\omega_{if})]^{-1}\Sigma_\alpha\gamma_{if}^{\alpha}(\omega_{if})$, the standard golden rule result.

Each relaxation rate is made up of two parts. If Z_2 is purely resistive, its contribution becomes $\propto 1/(Z_2C_p)$ (see Table I), describing the finite quality factor of the resonator. On the other hand, voltage fluctuations affect both the qubit and the oscillator charges, so that Z_1 couples to the overall system through two interfering channels.

Explicitly, the contribution of the qubit impedance at temperatures $T\ll E_J$, are found to be

$$\Gamma_{a\to g}^{(1)}\propto\omega_{ag}\left[g\kappa_q-\kappa_p\chi\left(\mathcal{R}_0-\frac{\delta}{2}\right)\right]^2, \tag{12}$$

$$\Gamma_{b\to g}^{(1)}\propto\omega_{bg}\left[\kappa_q\left(\mathcal{R}_0-\frac{\delta}{2}\right)+g\kappa_p\chi\right]^2, \tag{13}$$

where $\omega_{ag}=\omega_{bg}+2\mathcal{R}_0=(\omega_r+E_J)/2+\mathcal{R}_0$, while $\chi=\sqrt{C_p\omega_r/(2e^2)}$.

Due to destructive interference, these relaxation rates can be substantially reduced for certain parameter values. For example, the transfer rate out of state $|a\rangle$ is quenched if $\chi\kappa_p/\kappa_q\simeq 1$ for $\delta=0$. Even if this condition is not met, one of the two eigenstates can be made more stable by choosing an optimum δ.

Concerning the dephasing rates, we point out two important consequences coming from the structure of the matrices $\langle f|\hat{K}_\alpha|i\rangle$ reported in Table I. First, all matrix elements between the states of the doublet vanish; thus, fluctuations at the relatively small frequency ω_{ab} never come into play. As a consequence, coherence is well preserved in the usual temperature regime of operation, $g<T\ll E_J$. A second property is that each $\hat{K}_\alpha$ has equal diagonal matrix elements. This implies that the dephasing rates Γ_{ij} do not contain the so called *adiabatic terms*,[22] which describe coherence suppression without energy exchange. These contributions are proportional to the squared difference of the diagonal matrix elements of the coupling operators, $|\langle i|\hat{K}_\alpha|i\rangle-\langle j|\hat{K}_\alpha|j\rangle|^2$, which is zero in our case. As a result, the largest off-diagonal damping rate at the temperatures of interest is found to be

$$\Gamma_{ab}=\frac{1}{2}(\Gamma_{a\to g}+\Gamma_{b\to g})\approx\frac{1}{2}\sum_\alpha\left[\gamma_{ag}^{\alpha}(\omega_{ag})+\gamma_{bg}^{\alpha}(\omega_{bg})\right]. \tag{14}$$

The above important properties of the K's directly result from the choice $Q_g=e$. Therefore, the choice of this optimal working point allows to eliminate at the same time two kinds of unwanted terms, namely (i) the adiabatic terms, proportional to the zero frequency noise spectra, $S_\alpha(\omega=0)$, which are very dangerous in solid state implementations because of the presence of many low-energy excitations in the environment (e.g., $1/f$ noise), and (ii) relaxation and decoherence contributions due to fluctuations at frequencies comparable with the small separation, ω_{ab}, between the entangled doublet.

The determination of an optimal working point has been exploited in Ref. 10 to achieve a spectacular noise suppression. The criterion they propose is to chose a working point such that the level splitting weakly depends on the external control parameters, which corresponds to our property (i). This is a sufficient condition for minimizing low-frequency noise in a single qubit device, provided that the splitting between energy levels can be made large, as it is in the device of Ref. 10. However, this is not enough in many-qubit devices, due to the occurrence of small energy differences. In these cases, the search for favorable operating conditions should lead to a working point where the contributions of *both* zero and small frequencies are absent from the decoherence rates. This criterion is satisfied in our case, because of the selection rule (ii). In general, the reduced sensitivity to the environment could be achieved, by "engineering" the coupling operators (K's in our notation), i.e. by searching for a computational basis in which their matrix elements have the "good" properties discussed above.

The quenched sensitivity to low frequency fluctuations is crucial in the case of dephasing due to charged impurities lying close to the island, responsible for $1/f$ noise, which is believed to be the most relevant problem for Josephson charge qubits. For such an environment, correlation times are usually too long for a master equation approach to be valid. Indeed,[23] slower fluctuators show a distinctive behavior directly related to their discrete character and strongly contribute to decoherence when adiabatic terms enter the dephasing rates. However, as shown above, dephasing due to low-frequency fluctuations is minimized at $Q_g=e$. In this case, the Gaussian approximation turns out to give a quite correct answer and therefore an estimate of the order of magnitude of the effect can be obtained if the coupling with the environment is treated to second order,[23] which is equivalent to mimic the fluctuating impurities with a suitable oscillator environment. Then, Eqs. (11) and (14) are still valid, with $\hat{K}_3=C_q^{-1}Q+C_p^{-1}P$ and

$$S_3(\omega)\equiv S_Q(\omega)=\pi Ae^2\omega^{-1} \tag{15}$$

where $S_Q(\omega)$ is the power spectrum of the charge fluctuations in the island, whose amplitude can be inferred from independent measurements.[24]

V. DISCUSSION AND CONCLUSION

We now give some estimates of the relevant parameters of the setup, and show that state transfer and entanglement generation can be obtained with routinely fabricated circuits. We take a large Josephson junction as a resonator (this choice is preferable at present, as it allows for larger quality factors), with $C_r = 1$ pF, $\omega_r \simeq 37$ μeV. A low-temperature subgap resistance $R_2 \gtrsim 600$ $K\Omega$ (here modeled by the parallel impedance) can be easily achieved with Nb-based junctions, which yields a quality factor $\omega_r R_2 C \gtrsim 4\times 10^4$. For the box, we take $E_J = 40$ μeV, $C_J = 0.5$ fF, $C_g = 20$ aF, and $R_1 = 50$ Ω. Furthermore, by taking $C_c = 50$ aF, we obtain $g \simeq 0.5$ GHz which allows operations on a time scale $\lesssim 2$ ns. These choices give $E_{ch} \simeq 0.6$ meV$\gg E_J$ and $g \ll \omega_r$, ensuring the validity of the rotating wave approximation. Moreover, g turns out to be much larger than the level broadening, which guarantees the correctness of the secular approximation leading to Eqs. (11) and (14). These parameters lead to estimate the dephasing times as $\tau_{\phi 1} \approx 1$ μs and $\tau_{\phi 2} \approx 1.20$ μs. For background charge noise, $A = 10^{-7}$ in Eq. (15) gives $\tau_{\phi 3} \approx 1$ μs. The resulting overall decoherence time is then $\tau_\phi = 1/\Gamma_{ab} \approx 376$ ns, allowing for the two communication protocols. To perform the two-bit gate, a dephasing time one order of magnitude larger is required, which could be obtained by improving the quality factor of the resonator.

In conclusion, we presented a quantum data bus scheme connecting two Josephson qubits and implementing protocols for quantum state transfer, Bell states generation and two-bit gates. Favorable working conditions can be found, where decoherence is substantially reduced despite the presence of small level splitting and strong low-frequency environment fluctuations, typical of the solid state. The quantum state transfer protocol (implementable within the present technology) provides an indirect probe of the dynamics of entanglement, less demanding as compared to the realization of a two-bit gate.

After submission of this paper, we become aware of a related proposal by Blais *et al.*[25]

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