Relaxation time for magnetoresistance obtained from the band structure of a perfect cubic metal

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A well-known fact about the electrical resistance of a perfect crystal lattice is that this resistance is zero. The paper demonstrates that a different situation does apply for magnetoresistance: only a perfectly free-electron gas provides us with an infinite relaxation time and zero-magnetoresistance effect, but the presence of the crystal lattice makes the relaxation time equal to a finite quantity. The size of the product of the relaxation time for magnetoresistance and the electron gyration frequency is found to be a constant dependent on both the structure of electron states in a perfect lattice and the band filling. This property of constancy implies that the relaxation time is a quantity which becomes inversely proportional to the strength of the magnetic field applied to a crystal sample. Explicit calculations on the product of the relaxation time and the frequency of electron gyration are performed for the bands of the tightly bound *s* electrons in simple-cubic, body-centered-cubic, and face-centered-cubic lattices taken as examples.

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I. INTRODUCTION

A well-known property of a perfect crystal lattice is that its electrical resistance is zero.[1] But beyond the transport behavior of electrons in a purely electric field acting on a crystal, we have also the property of magnetoresistance, which is an equally well-known and extensively studied effect in metals.[2–7] Recently, the effects of magnetoresistance have attracted much study for the case of layered materials such as quasi-two-dimensional organic metals, magnetic multilayers, and manganese perovskites.[8–12] On the other hand, in considering magnetotransport in the metallic phase of the cuprate superconductors,[13–15] it has been pointed out that in fact the scattering times associated with the magnetoresistance and the zero-field resistance are different and have different temperature dependences.[16,17]

An explanation of magnetoresistance is usually based on the Lorentz equation. When a metal is submitted to the action of a constant magnetic field B_z alone, the electrons in a metal sample begin to circulate in the planes (x, y) perpendicular to B_z. For these planes the wave-vector parameter k_z is a constant. If—in the next step—an electric field E_x parallel to direction of x is applied and only a steady state of the electron velocities v_x and v_y is taken into account, so

$$\frac{dv_x}{dt}=\frac{dv_y}{dt}=0, \tag{1.1}$$

the conductivity σ of the electron system becomes an antisymmetric tensor:[7]

$$\sigma=\begin{pmatrix}\sigma_{xx} & -\sigma_{xy}\\ \sigma_{xy} & \sigma_{xx}\end{pmatrix}=\frac{\sigma_0}{1+(\Omega_0^{\mathrm{eff}})^2\tau^2}\begin{pmatrix}1 & \Omega_0^{\mathrm{eff}}\tau\\ -\Omega_0^{\mathrm{eff}}\tau & 1\end{pmatrix}. \tag{1.2}$$

The components of σ are strongly dependent on the relaxation time τ and the effective free-electron cyclotron frequency

$$\Omega_0^{\mathrm{eff}}=\frac{eB_z}{cm^{\mathrm{eff}}}; \tag{1.3}$$

here, m^{eff} is the effective electron mass. The conductivity σ_0 entering Eq. (1.2) is that obtained in the absence of a magnetic field:

$$\sigma_0=\frac{ne^2\tau}{m^{\mathrm{eff}}}; \tag{1.4}$$

here, n is the number of electron carriers.

An experimentally interesting tensor is that of resistivity, which is the inverted conductivity tensor σ. This tensor exhibits no effect of magnetoresistance in its diagonal components.[7] A contribution of B_z enters solely into the off-diagonal components of the resistivity tensor which lead to the Hall effect.

Evidently, τ plays a dominant role in any component of the conductivity tensor and the diagonal components of the resistivity tensor. Usually, τ is attributed to scattering events of the charge carriers on impurities, or defects, which lead to a finite τ. Our aim is now to obtain insight into new properties of τ.

II. RELAXATION TIME FOR MAGNETORESISTANCE DEDUCED FROM A FREE-ELECTRON BAND

Let us apply the Lorentz equation for the electron motion in the presence of a combined electric and magnetic field; for simplicity, we consider the electron mass m instead of the effective electron mass m^{eff}. In the relaxation-time approximation the carrier velocity $\vec{v}$ satisfies the equation[7]

$$m\left(\frac{d\vec{v}}{dt}+\frac{\vec{v}}{\tau}\right)=e\vec{E}+\frac{e}{c}\vec{v}\times\vec{B}. \tag{2.1}$$

For $\vec{B}=(0,0,B_z)$ we have $\vec{v}=(v_x,v_y)$. In this case Eq. (2.1) splits into two components

$$m\left(\frac{dv_x}{dt}+\frac{v_x}{\tau}\right)=eE_x+\frac{e}{c}v_yB_z, \tag{2.2}$$

$$m\left(\frac{dv_y}{dt}+\frac{v_y}{\tau}\right)=eE_y-\frac{e}{c}v_xB_z. \tag{2.2a}$$

In order to obtain insight into τ we assume a situation opposite to that presented in Eq. (1.1), viz.,

$$\frac{dv_x}{dt}\neq 0, \quad \frac{dv_y}{dt}\neq 0, \tag{2.3}$$

but simultaneously the electric field is assumed to be small, namely

$$|E_x|\ll\left|\frac{v_y}{c}B_z\right|, \tag{2.4}$$

$$|E_y|\ll\left|\frac{v_x}{c}B_z\right|. \tag{2.4a}$$

In a limiting case we assume that the terms eE_x and eE_y become negligible in comparison with the B_z-dependent terms entering the right-hand side of Eqs. (2.2) and (2.2a). In fact, in most experimental conditions—due to the high conductivity of metals—the second term entering the right-hand side of the Lorentz equation (2.1) is enormously greater than the first one.[3]

Because of the smallness of E_x and E_y, the electron velocities v_x and v_y approach those calculated in the presence of the field B_z alone. Then the electron wave vector $\vec{k}=(k_x,k_y)$ taken into account in a plane perpendicular to B_z satisfies the Lorentz equation[18]

$$v_x=\frac{\hbar c}{eB_z}\frac{dk_y}{dt}, \tag{2.5}$$

$$v_y=-\frac{\hbar c}{eB_z}\frac{dk_x}{dt}. \tag{2.5a}$$

Here,

$$k_x=a_0\cos(\Omega_0 t), \tag{2.6}$$

$$k_y=-a_0\sin(\Omega_0 t), \tag{2.6a}$$

because the wave vector $\vec{k}$ gyrates in a plane of reciprocal space perpendicular to k_z with the free-electron cyclotron frequency[6]

$$\Omega_0=\frac{eB_z}{cm}; \tag{2.6b}$$

here, a_0 is the amplitude of $\vec{k}$. We obtain

$$v_x=-\frac{\hbar c}{eB_z}a_0\Omega_0\cos(\Omega_0 t), \tag{2.7}$$

$$v_y=\frac{\hbar c}{eB_z}a_0\Omega_0\sin(\Omega_0 t), \tag{2.7a}$$

$$\frac{dv_x}{dt}=\frac{\hbar c}{eB_z}a_0\Omega_0^2\sin(\Omega_0 t), \tag{2.7b}$$

$$\frac{dv_y}{dt}=\frac{\hbar c}{eB_z}a_0\Omega_0^2\cos(\Omega_0 t). \tag{2.7c}$$

A substitution of these parameters into Eqs. (2.2) and (2.2a) (with $E_x=E_y=0$) gives

$$ma_0\frac{\hbar c}{eB_z}\left[\Omega_0^2\sin(\Omega_0 t)-\frac{\Omega_0}{\tau}\cos(\Omega_0 t)\right]=\frac{e}{c}\frac{\hbar c}{eB_z}a_0\Omega_0\sin(\Omega_0 t)B_z, \tag{2.8}$$

$$ma_0\frac{\hbar c}{eB_z}\left[\Omega_0^2\cos(\Omega_0 t)+\frac{\Omega_0}{\tau}\sin(\Omega_0 t)\right]=\frac{e}{c}\frac{\hbar c}{eB_z}a_0\Omega_0\cos(\Omega_0 t)B_z. \tag{2.8a}$$

Our idea is to calculate the square values of Eqs. (2.8) and (2.8a) and add them together. This gives

$$\left(ma_0\frac{\hbar c}{eB_z}\right)^2\left(\Omega_0^4+\frac{\Omega_0^2}{\tau^2}\right)=(\hbar a_0\Omega_0)^2 \tag{2.9}$$

or, because Eq. (2.6b),

$$(\hbar a_0)^2\left(\Omega_0^2+\frac{1}{\tau^2}\right)=(\hbar a_0)^2\Omega_0^2. \tag{2.9a}$$

Evidently, Eq. (2.9a) provides us with the result

$$\frac{1}{\tau^2}=0 \tag{2.10}$$

or

$$\tau=\infty. \tag{2.10a}$$

Therefore—in view of Eq. (1.4)—an infinite value is obtained for any component entering the conductivity tensor σ; see Eq. (1.2). Our aim is to demonstrate that the situation represented by Eqs. (2.10) and (2.10a) is fundamentally changed for the case of electrons moving in the field of a perfect crystal lattice.

III. BAND STRUCTURE OF A METAL SUBMITTED TO THE ACTION OF A MAGNETIC FIELD

A modification of Eq. (2.10) [or Eq. (2.10a)] due to the presence of a crystal lattice is dictated by the band structure of electrons in that lattice obtained for the case when a crystal sample is submitted to the action of a constant magnetic field. As our example, we choose bands of the tightly bound *s*-like electrons in three cubic—simple-cubic (sc), body-centered-cubic (bcc), and face-centered-cubic (fcc)—lattices. The band structure of such electrons in the absence of a magnetic field is represented by the well-known formulas[19]

$$E^{\mathrm{sc}}=\beta^{\mathrm{sc}}(\cos k_x+\cos k_y+\cos k_z), \tag{3.1}$$

$$E^{\mathrm{bcc}}=\beta^{\mathrm{bcc}}\cos k_x\cos k_y\cos k_z, \tag{3.1a}$$

$$E^{\mathrm{fcc}}=\beta^{\mathrm{fcc}}(\cos k_x \cos k_y+\cos k_z \cos k_x+\cos k_y \cos k_z). \tag{3.1b}$$

The positions of the nearest atomic neighbors in the crystal lattices were defined by the primitive translations (1,0,0), (0,1,0), and (0,0,1) in the sc lattice, the primitive translations (1,1,−1), (1,−1,1), and (−1,1,1) in the bcc lattice, and primitive translations (1,1,0), (1,0,1), and (0,1,1) in the fcc lattice. The terms β^{latt} refer to the nearest-neighbor atomic interaction integrals β of the tight-binding approach:[19]

$$\beta^{\mathrm{sc}}=2\beta, \quad \beta^{\mathrm{bcc}}=8\beta, \quad \beta^{\mathrm{fcc}}=4\beta. \tag{3.1c}$$

Some other constant terms entering the tight-binding expressions for the electron energy, especially $E^{(0)}$ and $\alpha^{(0)}$ (being, respectively, the eigenenergy of an atomic s state and the atomic interaction integral taken for the same atom[19]), have been neglected in Eqs. (3.1)–(3.1b).

But in the presence of a constant magnetic field, the tightly bound s electrons described by Eqs. (3.1)–(3.1b) behave in a different way. If the magnetic field is applied along axis z, being also parallel to one of the crystallographic axes, the crystal electrons begin to rotate along the planar orbits perpendicular to the field. For not too large electron energies the orbits in the reciprocal space are closed anisotropic curves of a constant energy put on the planes of $k_z=\mathrm{const}$. Let the length of the electron wave vector $\vec{k}$ in a plane perpendicular to the field B_z be equal to

$$a_0=(k_x^2+k_y^2)^{1/2}. \tag{3.2}$$

The electron energy along the curves of a constant energy can be expressed with the aid of a_0 (Ref. 20):

$$C^{\mathrm{latt}}=1-\cos a_0. \tag{3.3}$$

This dependence is valid for the tightly bound s electrons in all three cubic (sc, bcc, and fcc) lattices.

The C^{latt} given in Eq. (3.3) enter the formulas for the s-band electron energies E^{latt} in the following way:

$$E^{\mathrm{sc}}=\beta^{\mathrm{sc}}(2-C^{\mathrm{sc}}+\cos k_z), \tag{3.4}$$

$$E^{\mathrm{bcc}}=\beta^{\mathrm{bcc}}\cos k_z(1-C^{\mathrm{bcc}}), \tag{3.4a}$$

$$E^{\mathrm{fcc}}=\beta^{\mathrm{fcc}}[1+2\cos k_z-(1+\cos k_z)C^{\mathrm{fcc}}]. \tag{3.4b}$$

Here k_z is a constant parameter.

Therefore, the effect of a magnetic field in the reciprocal space is to choose only special k_x and k_y for the electron trajectory in a plane of a given $k_z=\mathrm{const}$ in order to make the electron energies of both kinds—that of Eqs. (3.1)–(3.1b) and that of Eqs. (3.4)–(3.4b), equal. An example of the equivalence of Eqs. (3.1)–(3.1b) and (3.4)–(3.4b) is attained for $k_x=k_y=k_z=0$: in this case $C^{\mathrm{latt}}=0$ for all lattices; see also Eqs. (3.2) and (3.3).

In fact, C^{latt} is a discontinuous function of parameter a_0 because different a_0 define the positions of different Landau levels in a crystal lattice. But for a not too strong magnetic field, the Landau levels are so numerous that a_0 can be considered as a quasicontinuous variable. If we label the Landau levels by the indices n_x, the density of the Landau levels versus energy C^{latt} is[21]

$$\frac{\partial n_x}{\partial C^{\mathrm{latt}}}=\frac{2\pi}{\omega}, \tag{3.5}$$

where

$$\omega=\frac{\Omega^{\mathrm{latt}}}{\Omega_0} \tag{3.6}$$

is the ratio of the electron gyration frequency Ω^{latt} in the crystal lattice to the free-electron gyration frequency (2.6b). Evidently, for free electrons we have

$$\omega=\omega_0=1. \tag{3.6a}$$

The solutions of the nonlinear equations calculated for k_x and k_y obtained in the case of the cubic crystal lattices[20] seem to be more conveniently represented with the aid of ω and Ω_0 than Ω^{latt} alone. We have

$$k_x=a_1\cos(\omega\Omega_0 t)+a_3\cos(3\omega\Omega_0 t)+a_5\cos(5\omega\Omega_0 t)+\cdots, \tag{3.7}$$

$$k_y=b_1\sin(\omega\Omega_0 t)+b_3\sin(3\omega\Omega_0 t)+b_5\sin(5\omega\Omega_0 t)+\cdots. \tag{3.7a}$$

There exists a relation between a_i and b_i:

$$b_i=(-1)^{(i+1)/2}a_i. \tag{3.8}$$

Let us note that i is an odd integer number. For the tightly bound s electrons in the sc and bcc lattices the coefficients a_i and b_i and the frequency factor of ω can be calculated independently of the value of k_z.[20] On the other hand, the a_i, b_i, and ω for the fcc lattice depend on k_z; see Sec. V. The constants a_i, b_i, and ω—obtained for perfect crystal lattices submitted to the action of a constant magnetic field—depend strongly on the structure of these lattices. For example, for the less tightly bound s electrons than those obtained due to the nearest-atomic-neighbor interactions in cubic lattices the coefficients a_i and b_i, as well as those entering ω, would be modified in comparison to those calculated for the case of the tightly bound s electrons in the nearest-neighbor approximation. In particular, the dependence on k_z could enter a_i, b_i, and ω for other cubic lattices than solely the fcc one, in distinction from the situation obtained in the nearest-neighbor case.

IV. RELAXATION TIME FOR MAGNETORESISTANCE DEDUCED FROM THE ELECTRON BAND STRUCTURE OF A PERFECT CRYSTAL

The Lorentz equations (2.5) and (2.5a) can be applied equally for the case of k_x and k_y calculated in Eqs. (3.7) and (3.7a). Let us add again the square values of Eqs. (2.2) and (2.2a) for $E_x=E_y=0$; we obtain

$$\left(\frac{dv_x}{dt}\right)^2+\left(\frac{dv_y}{dt}\right)^2+\frac{1}{\tau^2}(v_x^2+v_y^2)=\Omega_0^2(v_x^2+v_y^2) \tag{4.1}$$

TABLE I. Products $\Omega_0\tau$ and $\Omega_0^{\rm latt}\tau$ calculated as functions of a_0 [see Eq. (3.2)] for the band structure parameters of the tightly bound *s* electrons in the sc lattice; see Eqs. (3.6) and (4.6).

a_0	$\Omega_0\tau\approx\Omega^{\rm latt}\tau$	a_0	$\Omega_0\tau$	$\Omega^{\rm latt}\tau$
0.001	2×10^3	0.3	6.7	6.6
0.005	4×10^2	0.6	3.4	3.3
0.01	2×10^2	0.9	2.4	2.1
0.05	4×10^1	1.2	1.8	1.5
0.1	2×10^1	1.5	1.5	1.2

because of Eq. (2.6b). The cross terms entering Eq. (4.1), namely, those represented by

$$v_x\frac{dv_x}{dt},\quad v_y\frac{dv_y}{dt}, \tag{4.2}$$

remain out of phase because of Eqs. (2.5),(2.5a) and (3.7),(3.7a) and can be omitted. In the next step, the same property of the phase difference can be applied in the calculation of v_x^2 and v_y^2 or $(dv_x/dt)^2$ and $(dv_y/dt)^2$ themselves. Owing to Eqs. (3.7) and (3.7a), we have

$$\begin{aligned}v_x^2+v_y^2=\left(\frac{\hbar c}{eB_z}\right)^2\omega^2\Omega_0^2[&b_1^2\cos^2(\omega\Omega_0t)+b_3^2 3^2\cos^2(3\omega\Omega_0t)\\&+b_5^2 5^2\cos^2(5\omega\Omega_0t)+\cdots+a_1^2\sin^2(\omega\Omega_0t)\\&+a_3^2 3^2\sin^2(3\omega\Omega_0t)+a_5^2 5^2\sin^2(5\omega\Omega_0t)+\cdots]\end{aligned} \tag{4.3}$$

because the terms being out of phase have been neglected. In the same way,

$$\begin{aligned}\left(\frac{dv_x}{dt}\right)^2+\left(\frac{dv_y}{dt}\right)^2=\left(\frac{\hbar c}{eB_z}\right)^2\omega^4\Omega_0^4[&b_1^2\sin^2(\omega\Omega_0t)\\&+b_3^2 3^4\sin^2(3\omega\Omega_0t)\\&+b_5^2 5^4\sin^2(5\omega\Omega_0t)+\cdots\\&+a_1^2\cos^2(\omega\Omega_0t)+a_3^2 3^4\cos^2(3\omega\Omega_0t)\\&+a_5^2 5^4\cos^2(5\omega\Omega_0t)+\cdots].\end{aligned} \tag{4.4}$$

Expressions (4.3) and (4.4) averaged over the time period of electron gyration,

TABLE II. Products $\Omega_0\tau$ and $\Omega_0^{\rm latt}\tau$ calculated as functions of a_0 [see Eq. (3.2)] for the band structure parameters of the tightly bound *s* electrons in the bcc lattice; see Eqs. (3.6) and (4.6).

a_0	$\Omega_0\tau\approx\Omega^{\rm latt}\tau$	a_0	$\Omega_0\tau$	$\Omega^{\rm latt}\tau$
0.001	1.4×10^3	0.3	4.8	4.7
0.005	2.8×10^2	0.6	2.4	2.2
0.01	1.4×10^2	0.9	1.7	1.4
0.05	2.8×10^1	1.2	1.4	0.9
0.1	1.4×10^1	1.5	1.2	0.5

TABLE III. Products $\Omega_0\tau$ and $\Omega^{\rm latt}\tau$ calculated as function of a_0 [see Eq. (3.2)] for the band structure parameters of the tightly bound *s* electrons in the fcc lattice [see Eqs. (3.6) and (4.6)]; here, $k_z=0$.

a_0	$\Omega_0\tau\approx\Omega^{\rm latt}\tau$	a_0	$\Omega_0\tau$	$\Omega^{\rm latt}\tau$
0.001	1.6×10^3	0.3	1.16	0.58
0.005	3.3×10^2	0.6	1.08	0.39
0.01	1.6×10^2	0.9	1.03	0.24
0.05	3.3×10^1	1.2	1.01	0.12
0.1	1.6×10^1	1.5	1.00	0.02

$$T=\frac{2\pi}{\Omega^{\rm latt}}=\frac{2\pi}{\omega\Omega_0}, \tag{4.5}$$

and substituted into Eq. (4.1) give

$$\begin{aligned}&\omega^4\Omega_0^4(a_1^2+3^4a_3^2+5^4a_5^2+\cdots)\\&\quad+\frac{\omega^2}{\tau^2}\Omega_0^2(a_1^2+3^2a_3^2+5^2a_5^2+\cdots)\\&\quad=\omega^2\Omega_0^4(a_1^2+3^2a_3^2+5^2a_5^2+\cdots)\end{aligned} \tag{4.1$'$}$$

because of the time averages of $\sin^2(i\omega\Omega_0t)$ and $\cos^2(i\omega\Omega_0t)$ equal to $\frac{1}{2}$ and relation $b_i^2=a_i^2$ obtained in view of Eq. (3.8).

A transformation of Eq. (4.1$'$) gives

$$\begin{aligned}&\frac{\omega^2(a_1^2+3^4a_3^2+5^4a_5^2+\cdots)-(a_1^2+3^2a_3^2+5^2a_5^2+\cdots)}{a_1^2+3^2a_3^2+5^2a_5^2+\cdots}\\&\quad=-\frac{1}{\tau^2\Omega_0^2}\\&\quad=-\frac{1}{\tau^2B_z^2}\frac{c^2m^2}{e^2}.\end{aligned} \tag{4.6}$$

This relation defines the dimensionless quantity $\tau\Omega_0$ in terms of the band structure parameters a_1, a_3, a_5,... and ω. Evidently, for the free-electron case—in view of Eq. (3.6a) and the result

$$a_3=a_5=\cdots=0 \tag{4.7}$$

implied by Eqs. (2.6) and (2.6a)—we arrive at Eq. (2.10).

The products $\Omega_0\tau$ and $\Omega^{\rm latt}\tau$ calculated from Eqs. (4.6) and (3.6) for the sc, bcc, and fcc lattices are presented in Tables I–III. In the first step, for the fcc lattice solely the case of $k_z=0$ is considered. More extended calculations for the fcc lattice—those based on Sec. V and Tables IV and V—are given in Table VI.

V. RANGES OF PARAMETERS APPLIED IN THE CALCULATIONS

The acceptable intervals of $C^{\rm latt}$ given in Eq. (3.2) are dictated by the requirement that a planar electron trajectory along a surface of a constant energy should be a closed curve, or—more precisely—we assume that the curvature χ along the electron trajectory should be a positive number.

TABLE IV. Coefficients $a_1, a_3, a_5 \ldots$ entering the time-dependent wave-vector components k_x and k_y [see Eqs. (3.7) and (3.7a)] calculated for tightly bound s electrons gyrating in the fcc lattice. The coefficients are functions of the amplitude a_0 [see Eq. (3.2)] and the wave-vector component k_z. The b_i of k_y are referred to a_i of k_x by the formula (3.8). The lattice parameter $a^{\mathrm{latt}}=1$. For $k_z=0$ the results transform into those calculated before for the fcc lattice (Ref. 20).

$$a_1=a_0+\frac{-\cos k_z+2}{96(\cos k_z+1)}a_0^3+\frac{-88\cos k_z-7\cos 2k_z+129}{15360(4\cos k_z+\cos 2k_z+3)}a_0^5$$
$$+\frac{-39183\cos k_z-1200\cos 2k_z-53\cos 3k_z+46384}{10321920(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}a_0^7$$
$$+\frac{-2032637\cos k_z+88184\cos 2k_z+2209\cos 3k_z+758\cos 4k_z+2065090}{743178240(56\cos k_z+28\cos 2k_z+8\cos 3k_z+\cos 4k_z+35)}a_0^9+\cdots$$

$$a_3=\frac{\cos k_z-2}{96(\cos k_z+1)}a_0^3+\frac{10\cos k_z+\cos 2k_z-15}{1536(4\cos k_z+\cos 2k_z+3)}a_0^5+\frac{1701\cos k_z+75\cos 2k_z+11\cos 3k_z-2053}{368640(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}a_0^7$$
$$+\frac{2070436\cos k_z-77956\cos 2k_z+8716\cos 3k_z+515\cos 4k_z-2129927}{594542592(56\cos k_z+28\cos 2k_z+8\cos 3k_z+\cos 4k_z+35)}a_0^9+\cdots$$

$$a_5=\frac{-4\cos k_z-\cos 2k_z+7}{5120(4\cos k_z+\cos 2k_z+3)}a_0^5+\frac{-43\cos k_z-6\cos 2k_z-\cos 3k_z+58}{49152(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}a_0^7$$
$$+\frac{-9831\cos k_z-188\cos 2k_z-173\cos 3k_z-16\cos 4k_z+10900}{11796480(56\cos k_z+28\cos 2k_z+8\cos 3k_z+\cos 4k_z+35)}a_0^9+\cdots$$

$$a_7=\frac{13\cos k_z+8\cos 2k_z-\cos 3k_z-24}{229376(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}a_0^7$$
$$+\frac{142\cos k_z+52\cos 2k_z-6\cos 3k_z-\cos 4k_z-203}{1572864(56\cos k_z+28\cos 2k_z+8\cos 3k_z+\cos 4k_z+35)}a_0^9+\cdots$$

$$a_9=\frac{-40\cos k_z-44\cos 2k_z+8\cos 3k_z+\cos 4k_z+83}{9437184(56\cos k_z+28\cos 2k_z+8\cos 3k_z+\cos 4k_z+35)}a_0^9+\cdots$$

For the magnetic field B_z parallel to one of the crystallographic axes the limits of C^{latt} and a_0 dictated by the requirement of $\chi>0$ are[20]

$$0<C^{\mathrm{sc}}<2, \quad 0<a_0<\pi, \tag{5.1}$$

$$0<C^{\mathrm{bcc}}<1, \quad 0<a_0<\pi/2, \tag{5.1a}$$

$$0<C^{\mathrm{fcc}}<2, \quad 0<a_0<\pi. \tag{5.1b}$$

The last relation, given for the fcc lattice, holds solely for the plane of $k_z=0$.

For reasons of the convergence of solutions obtained for the sc lattice, further calculations for that lattice were limited to the interval

$$0<a_0<\pi/2. \tag{5.1c}$$

The same interval applied for the bcc lattice [see Eq. (5.1a)] corresponds to the filling of a half of the planar area of the first Brillonin zone at $k_z=0$ attained in the absence of a magnetic field.[19] Simultaneously, the upper limit of the interval (5.1b) for the fcc lattice valid for the plane of $k_z=0$ leads to the situation for which the frequency ratio $\omega\to 0$.[21] This means that the density of the Landau levels, or the planar density of the Bloch states versus energy calculated in the absence of a magnetic field, tends to infinity; cf. here Eq. (3.5). The plots of ω as functions of a_0 are given in Ref. 20, Fig. 5. In the calculations done in Tables I–III we applied the intervals (5.1c), (5.1a), and (5.1b) for the sc, bcc, and fcc lattices, respectively. The accuracy of calculations of ω for the sc lattice is examined in Ref. 21, for the bcc and fcc lattices in Sec. VI.

For the fcc lattice the energy expression (3.1b) implies that the upper limit for parameter k_z within the first Brillouin zone is

$$k_z=\pi \tag{5.2}$$

($a_{\mathrm{latt}}=1$). Correspondingly, the position coordinates of the corners of the square being at the boundary of the Brillonin zone at $k_z=\pi$ are[22]

$$W=\left(\pm\frac{\pi}{2},0,\pi\right) \quad \text{or} \quad \left(0,\pm\frac{\pi}{2},\pi\right). \tag{5.3}$$

This makes $\pi/2$ equal to a half of the length of the square diagonal at the zone boundary and, consequently, the shortest distance from the square center to the square edge becomes equal to

$$a_0^s=\frac{\pi}{2\sqrt{2}}. \tag{5.4}$$

In Table IV we present the expansion coefficients a_i for the wave-vector component k_x of the fcc lattice dependent on parameter k_z; similar coefficients b_i can be calculated for the component k_y. The relation between a_i and b_i remains iden-

TABLE V. Frequency parameter ω for a tightly bound s electron gyrating in the fcc lattice [see Eq. (3.6)] calculated as a function of a_0 given in Eq. (3.2) and the wave-vector component k_z. For $k_z=0$ the result for ω transforms into that calculated before for the fcc lattice (Ref. 20).

$$\omega=1+\frac{-\cos k_z-2}{8(\cos k_z+1)}a_0^2+\frac{48\cos k_z+5\cos 2k_z+13}{768(4\cos k_z+\cos 2k_z+3)}a_0^4+\frac{629\cos k_z-316\cos 2k_z-17\cos 3k_z-1052}{92160(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}a_0^6$$
$$+\frac{604480\cos k_z-122908\cos 2k_z+9152\cos 3k_z-55\cos 4k_z-445285}{82575360(56\cos k_z+28\cos 2k_z+8\cos 3k_z+\cos 4k_z+35)}a_0^8+\cdots$$

tical to that given in Eq. (3.8). In the next step, Table V presents the dependence of ω on k_z.

Expansions given in Tables IV and V represent exact solutions of the equations of motion. This property can be demonstrated for example by a check of accuracy with which the solution of Table IV satisfies the boundary condition

$$k_x(t)|_{t=0}=a_0. \tag{5.5}$$

For, at $t=0$, because of the well-known property of the cos-like functions, we have [see Eq. (3.7)]

$$k_x(0)=a_1+a_3+a_5+a_7+\cdots. \tag{5.6}$$

On the basis of Table IV the expansion (5.6) becomes

$$k_x(0)=a_0+\frac{-\cos k_z+2+\cos k_z-2}{96(\cos k_z+1)}a_0^3+\frac{(-88+100-12)\cos k_z+(-7+10-3)\cos 3k_z+129-150+21}{15360(4\cos k_z+\cos 2k_z+3)}a_0^5$$
$$+\left[\frac{(-39183+1701\times 28-43\times 210+13\times 45)\cos k_z}{10321920(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}+\frac{(-1200+75\times 28-6\times 210+8\times 45)\cos 2k_z}{10321920(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}\right.$$
$$\left.+\frac{(-53+11\times 28-210-45)\cos 3k_z}{10321920(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}+\frac{46384-2053\times 28+58\times 210-24\times 45}{10321920(15\cos k_z+6\cos 2k_z+\cos 3k_z+10)}\right]a_0^7+\cdots=a_0. \tag{5.7}$$

Since all terms entering the second step in Eq. (5.7)—beyond the first one—give exactly zero, we obtain precisely the result required by condition (5.5). The property is checked to hold also for further powers of a_0.

It is evident from Table IV that the case of Eq. (5.2) leads to divergent $a_1, a_3, \ldots$ and—because of Eq. (3.8)—also to divergent $b_1, b_3, \ldots$. A similar divergence applies to ω for the same case; see Table V. This makes the calculations outlined in Sec. IV inapplicable to k_z near Eq. (5.2). Nevertheless, it is possible to examine the behavior of $\Omega_0\tau$ and $\Omega^{\text{latt}}\tau$ within the interval of k_z between $k_z=0$ and $k_z<\pi$. We choose the maximal a_0 considered for any $k_z=$const slightly smaller than a_0 for the line defined by the points

$$K=\left(\frac{3}{4}\pi,\frac{3}{4}\pi,0\right) \tag{5.8}$$

and

$$K'=\left(\frac{\pi}{4},\frac{\pi}{4},\pi\right) \tag{5.8a}$$

lying on the boundary of the first Brillouin zone of the fcc lattice. A reason for the slight decrease of the maximal a_0 examined in comparison to a_0 defined by the line KK' is due to the fact that for the K point in Eq. (5.8) (the plane of $k_z=0$) we obtain

$$a_0=\sqrt{\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^2}\,\pi=\sqrt{\frac{18}{16}}\,\pi, \tag{5.9}$$

which is a number slightly larger than the upper limit of $a_0=\pi$ derived in Eq. (5.1b). The data for $k_z=0$ are presented in Table III. The $\Omega_0\tau$ and $\Omega^{\text{latt}}\tau$ calculated for a_0 lying in the planes of $k_z=\pi/6$, $\pi/3$, $\pi/2$, $\frac{2}{3}\pi$, and $\frac{5}{6}\pi$ are given in Table VI. The a_0 examined for different k_z are limited to $a_0=2.95$ for $k_z=\pi/6$, $a_0=2.55$ for $k_z=\pi/3$, $a_0=2.20$ for $k_z=\pi/2$, $a_0=1.80$ for $k_z=\frac{2}{3}\pi$, and $a_0=1.45$ for $k_z=\frac{5}{6}\pi$. For small a_0 we obtain $\Omega_0\tau\approx\Omega^{\text{latt}}\tau$. The dashes near the bottom of the table replace imaginary $\Omega_0\tau$ and $\Omega^{\text{latt}}\tau$ obtained from the formulas (3.6) and (4.6). A characteristic point is that products $\Omega_0\tau$ considered as functions of a_0 behave—for not too large a_0—in the way much similar to $\Omega_0\tau$ presented in Tables I–III. But for $k_z\geqslant\pi/2$ and a_0 very close to the zone boundary the products of $\Omega_0\tau$ and $\Omega^{\text{latt}}\tau$ become equal to imaginary numbers.

TABLE VI. $\Omega_0\tau$ and $\Omega^{\text{latt}}\tau$—calculated from the band structure parameters of the tightly bound s electrons in the fcc lattice—compared for different k_z. The data for $k_z=0$ are presented in Table III. For small a_0 we have $\Omega_0\tau\approx\Omega^{\text{latt}}\tau$, for $a_0\geqslant 0.3$ column (i) refers to $\Omega_0\tau$ and column (ii) refers to $\Omega^{\text{latt}}\tau$.

a_0	$k_z=\pi/6$	$k_z=\pi/3$	$k_z=\pi/2$	$k_z=\frac{2}{3}\pi$	$k_z=\frac{5}{6}\pi$
0.001	1.6×10^3	1.5×10^3	1.4×10^3	1.2×10^3	6.9×10^2
0.005	3.2×10^2	3.1×10^2	2.8×10^2	2.3×10^2	1.4×10^2
0.01	1.6×10^2	1.5×10^2	1.4×10^2	1.2×10^2	6.9×10^1
0.05	3.2×10^1	3.1×10^1	2.8×10^1	2.3×10^1	1.4×10^1
0.1	1.6×10^1	1.5×10^1	1.4×10^1	1.2×10^1	6.9

	$k_z=\pi/6$		$k_z=\pi/3$		$k_z=\pi/2$		$k_z=\frac{2}{3}\pi$		$k_z=\frac{5}{6}\pi$	
a_0	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
0.3	5.4	5.3	5.2	5.1	4.8	4.7	3.9	3.8	2.5	2.2
0.6	2.8	2.6	2.7	2.5	2.4	2.2	2.1	1.8	1.2	0.4
0.9	1.9	1.6	1.8	1.5	1.7	1.4	1.5	1.0	—	—
1.2	1.5	1.1	1.5	1.0	1.4	0.9	1.1	0.3	—	—
1.5	1.3	0.8	1.2	0.7	1.2	0.5	—	—		
1.8	1.15	0.6	1.13	0.5	1.03	0.1	—	—		
2.1	1.07	0.4	1.05	0.2	—	—				
2.4	1.03	0.2	1.00	0.02						
2.7	1.00	0.07								

VI. ACCURACY OF CALCULATIONS OF $\Omega_0\tau$

The calculations presented in Tables I–III and Table VI are based on the formula (4.6). The factor of ω^2 entering Eq. (4.6) can be obtained from the expansion of ω given in Ref. 20. But these ω can be checked by comparing them with ω approached on the basis of the Bloch density of states.[21] This check could be done because, on the one hand, we have the relation (3.5) referring ω to the density of semiclassical quantum states of the electron gyration performed in a constant magnetic field. This density of states is obtained from the action function J^{latt} calculated for a given cubic lattice.[20,21] But, on the other hand, we have that[21]

$$\frac{\partial n_x}{\partial C^{\text{latt}}}=\eta^{\text{latt}}, \tag{6.1}$$

where η^{latt} is a planar density of the Bloch quantum states in a perfect cubic lattice.[21] The formula (6.1), combined with Eq. (3.5), provides us with a check of ω calculated from the direct solutions of Ref. 20 and gives insight into their convergence.

The equivalence presented in Eq. (6.1) has been demonstrated explicitly for the case of the sc lattice.[21] Because of the importance of ω in the present calculations, we show in Tables VII and VIII that Eq. (6.1) holds also for the bcc and fcc lattices. A poorer agreement of Eqs. (3.5) and (6.1) in the bcc case is dictated by a slower convergence of the expansion (3.5) obtained in this case.

The accuracy of $\Omega_0\tau$ is influenced also by the power expansions entering Eq. (4.6) and can be examined in a numerical way. This is done by calculating $\Omega_0\tau$ from the power expansions in Eq. (4.6) combined successively of two-, three-, and four-component terms. For small a_0 the plots of $\Omega_0\tau$ obtained in this way remain indistinguishable. For

TABLE VII. Density of electron states distributed on the Landau levels, $\partial n_x/\partial C^{\text{bcc}}$, compared with the planar density of the Bloch states, η^{bcc}, in the bcc lattice for selected values of electron energies C^{bcc}; cf. Eqs. (3.5) and (6.1).

C^{bcc}	$\frac{1}{4}\frac{\partial n_x}{\partial C^{\text{bcc}}}=\frac{\pi}{2}\omega^{-1}$	η^{bcc}
0.00	1.571	1.571
0.05	1.611	1.611
0.10	1.655	1.655
0.15	1.701	1.701
0.20	1.751	1.751
0.25	1.805	1.804
0.30	1.863	1.863
0.35	1.926	1.926
0.40	1.995	1.994
0.45	2.071	2.069
0.50	2.156	2.152
0.55	2.250	2.243
0.60	2.355	2.344
0.65	2.463	2.456
0.70	2.588	2.581
0.75	2.732	2.720
0.80	2.888	2.875
0.85	3.072	3.050
0.90	3.278	3.245
0.95	3.523	3.465
1.00	3.823	3.713

TABLE VIII. Density of states on the Landau levels, $\partial n_x/\partial C^{\mathrm{fcc}}$, compared with the planar density of the Bloch states, η^{fcc}, in the fcc lattice for selected values of electron energies C^{fcc}; here, $k_z=0$.

C^{fcc}	$\frac{1}{4}\frac{\partial n_x}{\partial C^{\mathrm{fcc}}}=\frac{\pi}{2}\,\omega^{-1}$	η^{fcc}
0.00	1.571	1.571
0.05	1.601	1.601
0.10	1.632	1.632
0.15	1.665	1.665
0.20	1.700	1.700
0.25	1.736	1.736
0.30	1.774	1.774
0.35	1.814	1.814
0.40	1.856	1.856
0.45	1.900	1.900
0.50	1.947	1.947
0.55	1.996	1.996
0.60	2.048	2.048
0.65	2.104	2.104
0.70	2.163	2.163
0.75	2.227	2.227
0.80	2.294	2.294
0.85	2.366	2.366
0.90	2.444	2.444
0.95	2.527	2.527
1.00	2.617	2.616
1.05	2.714	2.713
1.10	2.819	2.818
1.15	2.933	2.931
1.20	3.056	3.054
1.25	3.190	3.188
1.30	3.337	3.333
1.35	3.496	3.492
1.40	3.670	3.665
1.45	3.861	3.854
1.50	4.070	4.061
1.55	4.299	4.288
1.60	4.550	4.537
1.65	4.826	4.809
1.70	5.128	5.108
1.75	5.460	5.436
1.80	5.826	5.797
1.85	6.228	6.192
1.90	6.669	6.627
1.95	7.154	7.104
2.00	∞	∞

larger a_0, Figs. 1–3 show also a rapid convergence of $\Omega_0\tau$: the two-term expansions represented by the dashed lines give $\Omega_0\tau$ very close to that calculated from the four-term expansions represented by the solid lines. The dotted line of the three-term expansions (see Fig. 2) merges with $\Omega_0\tau$ of the four-term expansions in Figs. 1 and 3. The merging of $\Omega_0\tau$ calculated from any kind of expansion to the

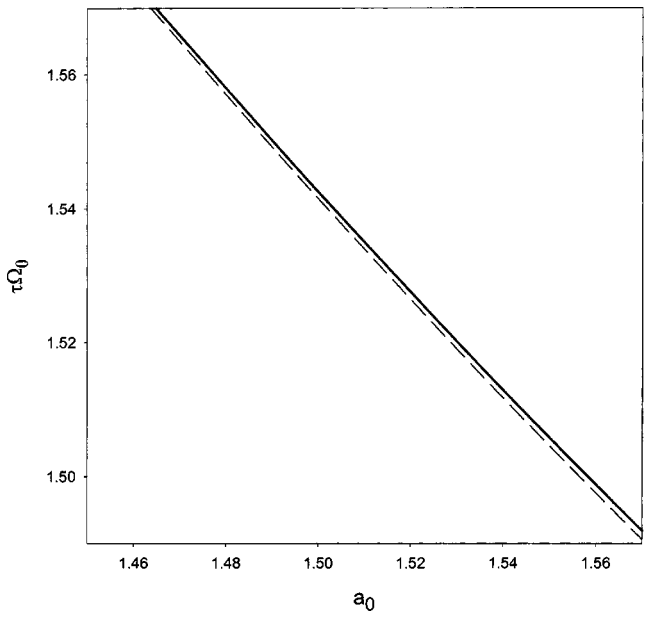

FIG. 1. Product $\Omega_0\tau$ calculated for large a_0 in the sc lattice. Dashed line: two-term power expansions applied in Eq. (4.6). Solid line: three-term and four-term power expansions applied in Eq. (4.6).

value of $\Omega_0\tau=1$ at $a_0\approx\pi$ for the fcc lattice (see Fig. 3) is due to the limiting behavior of $\omega\to 0$ at $a_0\to\pi$.

VII. DISCUSSION

Historically, the problem of magnetoresistance has attracted much interest of experimentalists. Kapitza[23,24] pointed out that a change of resistance in a magnetic field is

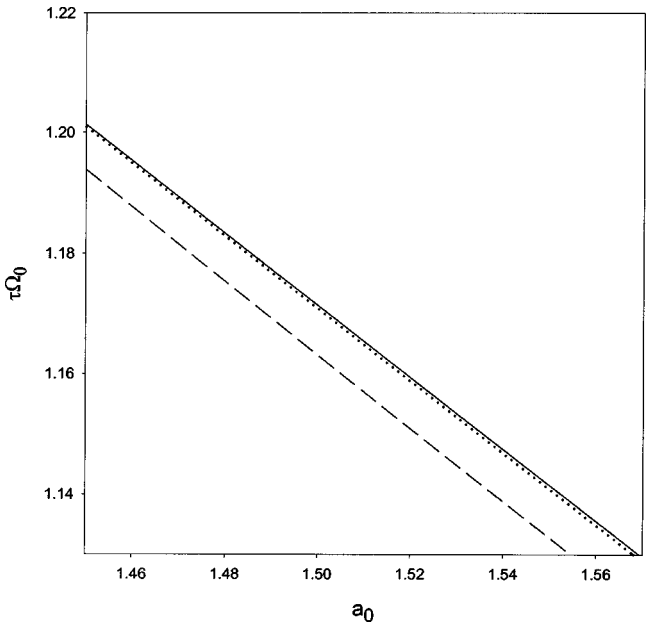

FIG. 2. Product $\Omega_0\tau$ calculated for large a_0 in the bcc lattice. Dashed line: two-term power expansions. Dotted line: three-term power expansions. Solid line: four-term power expansions applied in Eq. (4.6).

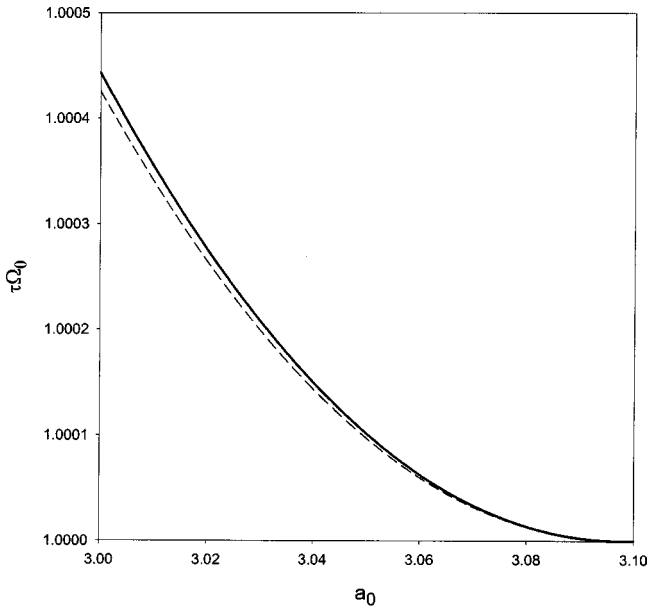

FIG. 3. Product $\Omega_0\tau$ calculated for large a_0 in the fcc lattice. Dashed line: two-term power expansions. Solid line: three-term and four-term power expansions applied in Eq. (4.6). At $a_0 \to \pi$ the $\Omega_0\tau$ obtained from all expansions merge to the value of $\Omega_0\tau = 1$ because of $\omega \to 0$; see Sec. VI.

relatively very small for numerous metals: proportionally to results of the performed measurements, but with the problem of magnetic breakdown neglected, a change of the magnetic field by 10^6 G is found to make a change of the resistance of a metal by about some 10%. Simultaneously, some exceptions in the behavior of metals have been discovered: for example, for a given change of the magnetic field the change of resistance of metallic Bi was found to be more than 1.5×10^4 times larger than a similar change of resistance of metallic Li.

Gradually, a saturation of magnetoresistance for a large field B_z became a well-established experimental fact observed in many metals, especially those which possess closed Fermi surfaces (In, Al, Na, Li).[6] Also, some alloys having more than two components—for example, Cu-Ni alloys with the addition of Sn or Zn (Ref. 25)—exhibit this saturation property. On the other hand, for a single crystal of Sn the saturation is observed with the current directed along the c axis and a special orientation of the $\vec{B}$ field in the basal plane.[5,26] As a consequence, a typical problem concerning magnetoresistance became its rapid change with direction of the magnetic field when this field is tilted to the crystallographic axes.[4,6] In high fields the observed magnetoresistance tends to saturate except in those directions in which open orbits exist.[27–30] This effect of open orbits is usually quite pronounced, providing us with an indication concerning the shape of the Fermi surfaces. A limit of this behavior is the strength of the magnetic field for which the breakdown effects for the circulating electrons may occur. Briefly, the experiments indicated an important factor of the shape of the Fermi surface for magnetoresistance or, more precisely, the importance of the shape of the orbits along which the electrons circulate. A statement that for metals there is no change in magnetic resistance $R(B_z)$ due to the change of B_z when the electrons occupy one band only[2] has been formulated.

This fact—which presented a difficulty in a former theory[4]—is confirmed by our calculations. For, in our case, the well-known expression[4,6,31]

$$\frac{R(B_z)-R(0)}{R(0)} = (\Omega^{\mathrm{latt}}\tau)^2 = (\omega\Omega_0\tau)^2 \qquad (7.1)$$

remains a term independent of B_z. This property is due to $\Omega_0\tau$ calculated in Eq. (4.6), which is a number dependent solely on the band structure parameters. But the band should have a certain anisotropy in order to provide us with a finite $\Omega_0\tau$. For a small anisotropy (small a_0) the product $\Omega_0\tau$ remains still very large, decreasing gradually with an increase of a_0. A property of the independence of the field B_z applies to all parameters entering the left-hand side of Eq. (4.6) including ω.[20]

The result of the constancy of Eq. (4.6) or (7.1) projects on the behavior of the relaxation time τ for magnetoresistance. In numerous textbooks it is assumed that the change of B_z, or Ω_0, would imply a similar change of $\Omega_0\tau$—and consequently $\Omega^{\mathrm{latt}}\tau$—because τ is independent of B_z.[3,4,6,7,29,32,33] This seems to be not true: the product of $\Omega_0\tau$ should be a constant term as far as the band structure remains uninfluenced by the strength of the applied magnetic field. The property of the constancy of Eq. (4.6), or Eq. (7.1), makes any change of the magnetic field accompanied by a corresponding reciprocal change of the value of τ. The point, shown to be sound for the transversal magnetoresistance measured in a plane being parallel to a crystallographic plane, may be, however, different for a magnetic field tilted to the crystallographic axes; see remarks below Eq. (7.3).

The assumption of the closed orbits is fully taken into account in our considerations on the electron motion in a magnetic field and the derivation of Eq. (4.6). An immediate insight into the behavior of $\Omega_0\tau$ for a single-band model can be obtained from some special values of ω; see Eq. (4.6). If $\omega \to 0$—as it is obtained for the s-band states on a concave Fermi surface in the fcc lattice which is going to touch the boundary of the Brillouin zone [see Eqs. (3.5), (5.1b), and (6.1) and Table VIII]—the product $\Omega_0\tau$ should necessarily tend to unity.

The formula (7.1) can be applied to identify the measurements done on the magnetoresistance of metals[25,34,35]—for example, that of Na—with the results of our calculations. The experimental saturation of the magnetoresistance of Na begins roughly at $\Omega^{\mathrm{latt}}\tau \approx 1$. In our theory developed for the bcc lattice, for parameter a_0 taken rather close to the boundary of the Brillouin zone, the product $\Omega^{\mathrm{latt}}\tau$ attains a value not far from unity; see Table II. In reality, the s electrons in the bcc lattice of an alkali metal are far from being strongly tightly bound and an exact comparison between experiment and theory requires much more developed calculations of the band structure than those outlined above.[36–39] Moreover, the temperature effects neglected in the calculations of the present paper may become sound.

A more general approach to magnetoresistance than that represented by Eq. (7.1) is given by the formula[3]

$$\frac{R(B_z)-R(0)}{R(0)}=A_t\tau^2B_z^2, \tag{7.2}$$

where A_t is usually a complicated expression dependent on the band structure.[3] In the case when Eq. (7.1) is put equal to Eq. (7.2) we have

$$A_t=\frac{e^2}{c^2m^2}\omega^2. \tag{7.3}$$

In fact, the band structure—calculated for the tightly bound *s* electrons in the bcc lattice and a magnetic field normal to the crystallographic plane—makes A_t several times smaller than Eq. (7.3).[40] It should be noted, however, that in the quoted references 3 and 40 the coefficient A_t was not separated from τ^2, but formed a joint term equal to

$$B_t=A_t\tau^2. \tag{7.4}$$

Simultaneously, the property of the constancy of the product τB_z entering the right-hand side of Eq. (7.2) remained unnoticed. Nevertheless, the calculations of A_t (or ω) separated from those on τ may become important for a magnetic field tilted to the crystallographic axes. Experimentally, the problems of magnetotransport phenomena in a tilted magnetic field have been raised recently in organic metals.[41–43]

An immediate application of our theory is the cyclotron resonance effect where the frequency of the external microwave field becomes equal to that of the electron orbital frequency $\Omega^{\text{latt}}=\omega\Omega_0$ attained in a constant magnetic field.[4] The impedance Z in a metal skin layer in the presence of a magnetic field B_z is modified in comparison to Z for $B_z=0$ by the formula[6]

$$Z(B_z)\cong Z(0)\left[1-\exp\left(-\frac{2\pi}{\Omega^{\text{latt}}\tau}-i\frac{2\pi\Omega}{\Omega^{\text{latt}}}\right)\right]^{1/3}, \tag{7.5}$$

where Ω is the frequency of the applied external field. The constant $\Omega^{\text{latt}}\tau$ is that calculated in the present paper (Tables I–III and VI). On each electron revolution the current is multiplied by the phase factor

$$e^{-w}\equiv\exp\left(-\frac{2\pi}{\Omega^{\text{latt}}\tau}-i\frac{2\pi\Omega}{\Omega^{\text{latt}}}\right), \tag{7.6}$$

so the factor of the total current arising from all cycles becomes

$$1+e^{-w}+e^{-2w}+\cdots=\frac{1}{1-e^{-w}}. \tag{7.7}$$

This expression enters the formula of the effective conductivity,[6] which—in its turn—modifies the impedance of a metal skin layer in the way indicated in Eq. (7.5).

Let us note that the property of constancy obtained for product

$$\xi=\Omega^{\text{latt}}\tau, \tag{7.8}$$

which holds independently of the scattering processes in the bulk of a metal, can be suggested also by an elementary reasoning. The ratio between components σ_{xy} and σ_{xx} of the conductivity tensor in a constant magnetic field becomes[6]

$$\frac{\sigma_{xy}}{\sigma_{xx}}=\Omega_0^{\text{eff}}\tau. \tag{7.9}$$

On the other hand, in a high-field limit of two carriers model we have[4,6]

$$\sigma_{xy}=(n_1-n_2)\frac{ec}{B_z} \tag{7.10}$$

because of the relation $\sigma_{xy}=-\sigma_{yx}$. Let us assume that $n_2 \ll n_1$, so n_2 can be practically neglected. For the Hall field the value of the electric current j_x in the direction of x remains unchanged in comparison to that in the absence of a magnetic field; therefore,

$$\sigma_{xx}=\frac{ne^2\tau}{m^{\text{eff}}}. \tag{7.11}$$

By dividing Eq. (7.10) for $n_2\approx 0$ by σ_{xx} entering Eq. (7.11) and by putting $n_1\approx n$ we have

$$\frac{\sigma_{xy}}{\sigma_{xx}}=\frac{nec}{B_z}\frac{m^{\text{eff}}}{ne^2\tau}=\frac{cm^{\text{eff}}}{B_ze\tau}=\frac{1}{\Omega_0^{\text{eff}}\tau}. \tag{7.12}$$

A comparison of Eqs. (7.9) and (7.12) gives the result

$$(\Omega_0^{\text{eff}}\tau)^2=1. \tag{7.13}$$

Rather surprisingly, the products $\Omega_0\tau$ calculated in the present theory for a_0, which are not far from the boundary of the first Brillouin zone, become equal to a number not much different from 1; see Tables I–III and VI. This property and still smaller numbers obtained for $\Omega^{\text{latt}}\tau$ provide us with the problem of broadening of the Landau levels present at the mentioned a_0.[7]

ACKNOWLEDGMENT

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