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Periodic alternating 0- and π-junction structures as realization of φ-Josephson junctions

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We consider the properties of a periodic structure consisting of small alternating 0- and π-Josephson junctions. We show that depending on the relation between the lengths of the individual junctions, this system can be either in the homogeneous or in the phase-modulated state. The modulated phase appears via a second-order phase transition when the mismatch between the lengths of the individual junctions drops below the critical value. The screening length diverges at the transition point. In the modulated state, the equilibrium phase difference in the structure can take any value from $-\pi$ to π (φ-junction). The current-phase relation in this structure has very unusual shape with two maxima. As a consequence, the field dependence of the critical current in a small structure is very different from the standard Fraunhofer dependence. The Josephson vortex in a long structure carries partial magnetic flux, which is determined by the equilibrium phase.

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The minimum energy of Josephson junction (JJ) usually corresponds to the zero phase difference of the superconducting order parameter.[1] However, long time ago it has been predicted that the JJs with ferromagnetic interlayer (*S-F-S* junctions) may have ground state with phase difference equal to π ("π-junctions") (Ref. 2). Only recently this prediction has been experimentally verified[3–5] and now the controllable fabrication of *S-F-S* π-junctions becomes possible. Another realization of π-junctions became possible due to the *d*-wave symmetry of the order parameter in high-temperature superconductors.[6,7] Recent experiments on $YBa_2Cu_3O_7$-Nb zigzag JJs demonstrated that such junctions are composed of alternating facets of 0- and π- junctions.[8,9] In a pioneering work,[10] Bulaevskii, Kuzii, and Sobyanin demonstrated that the spontaneous Josephson vortex carrying flux $\Phi_0/2$ appears at the boundary between 0- and π- JJs. The structure of such semifluxon has been studied in more detail in Refs. 11, 12.

In the present work we study the properties of the periodic array of 0- and π- JJs. We show that depending on the ratio of lengths of 0- and π- JJs, such an array can be either in the homogeneous or the modulated state. The second-order phase transition between these states takes place when the length mismatch between 0- and π- JJs is small. At the transition point the screening length of the magnetic field diverges. In the modulated states the average phase difference φ_0 can take any value between $-\pi$ and π. Further, we call such systems φ-junctions. Such structures were first predicted by Mints[13] in the case of alternating 0- and π- JJs. We study the properties of φ-junctions: effective Josephson current, Josephson length, and Fraunhofer-like oscillations of the critical current. Magnetic properties of long φ-junctions are determined by two types of Josephson vortices, carrying partial fluxes $\Phi_0\varphi_0/\pi$ and $\Phi_0(\pi-\varphi_0)/\pi$ (see also Refs. 13, 14). We find, analytically, the shapes of these unusual Josephson vortices. We also demonstrate that at the boundary between φ- and usual JJs, a Josephson vortex appears carrying a partial flux $\Phi_0\varphi_0/2\pi$.

Consider a periodic structure composed of alternating 0- and π-JJs of lengths d_0 and d_π, respectively. The energy per period of such a structure is given by

$$F=\frac{\Phi_0 j_c}{2\pi c}\int_{-d_0}^{0}\left[\frac{\lambda_J^2}{2}\left(\frac{d\phi}{dx}\right)^2+1-\cos\phi\right]dx$$
$$+\frac{\Phi_0 j_c}{2\pi c}\int_{0}^{d_\pi}\left[\frac{\lambda_J^2}{2}\left(\frac{d\phi}{dx}\right)^2+1+\cos\phi\right]dx, \qquad (1)$$

where j_c is the Josephson current density, λ_J is the Josephson length of the individual junctions, $\lambda_J^2=c\Phi_0/(8\pi^2 t j_c)$, and $t=t_0+\lambda_1+\lambda_2$ is the effective junction thickness.[1] For simplicity, we focus on the case when critical current densities j_c are the same in 0- and π-JJs. Generalization for different critical currents is straightforward. Possible realization of such a system is the zigzag junction between high-T_c and conventional superconductors as well as *S-F-S* junctions with periodically modulated thickness of the ferromagnetic interlayer. In the case of the zigzag structure, x is the coordinate along the zigzag boundary. The ground-state phase distribution is determined by equation

$$\frac{d^2\phi}{dx^2}+j(x)\sin\phi=0,-d_0<x<d_\pi, \qquad (2)$$

where $j(x)=-\lambda_J^{-2}$ in 0-JJs and $j(x)=\lambda_J^{-2}$ in π-JJs. At the boundaries, the phase and its derivative have to be continuous. Periodicity of $\phi(x)$ implies that the solution is symmetric with respect to $x=-d_0/2$ and $d_\pi/2$, i.e., $d\phi/dx|_{x=-d_0/2}=d\phi/dx|_{x=d_\pi/2}=0$. Equation (2) always has homogeneous solutions $\phi=0$ and $\phi=\pi$. However, these solutions give the ground state only in some range of the ratio d_0/d_π. In general, Eq. (2) also allows for the inhomogeneous solution. Consider case $d_\pi<d_0$ and weak modulation around the $\phi=0$ state. In this case, Eq. (2) can be linearized and its solution is given by

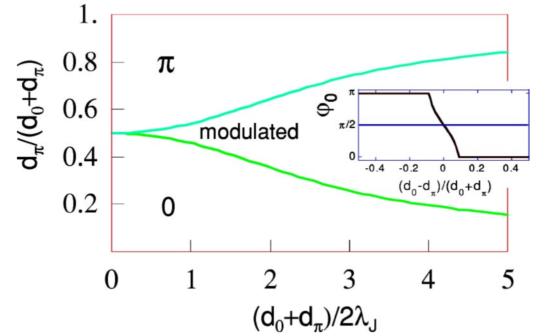

FIG. 1. (Color online) Phase diagram of the periodic alternating 0-π junction structure. Inset shows dependence of the equilibrium phase on the length mismatch, for $d_0 d_\pi = \lambda_J^2$.

$$\phi = \begin{cases} A\cosh\dfrac{x+d_0/2}{\lambda_J}, -d_0<x<0, \\ B\cos\dfrac{x-d_\pi/2}{\lambda_J}, 0<x<d_\pi. \end{cases} \tag{3}$$

Matching the logarithmic derivative $d\ln\phi/dx$ of these solutions at $x=0$, we obtain condition

$$\tanh(d_0/2\lambda_J) = \tan(d_\pi/2\lambda_J) \tag{4}$$

for the onset of the modulated solution. The energy analysis shows that the uniform $\phi=0$ solution is favorable at $\tanh(d_0/2\lambda_J) > \tan(d_\pi/2\lambda_J)$. In the opposite case $d_\pi > d_0$, the uniform $\phi=\pi$ solution is favorable at $\tanh(d_\pi/2\lambda_J) > \tan(d_0/2\lambda_J)$. In the case of a finite junction split into 0- and π-pieces, condition (4) was first derived in Ref. 10. These results are summarized in the phase diagram shown in Fig. 1. In limit $d_0 \gg \lambda_J$, the condition for the modulated solution is given by $d_\pi > (\pi/2)\lambda_J$. In the case $d_0, d_\pi \ll \lambda_J$, the region of the modulated solution, is given by

$$\frac{|d_0-d_\pi|}{d_0} < \frac{d_0^2}{6\lambda_J^2} \ll 1, \tag{5}$$

i.e., the modulated solution exists only for a very small length mismatch. Since λ_J is temperature dependent, the transition into the modulated state may occur with decreasing temperature.

In case $d_0 = d_\pi \equiv d$, the phase distribution is always modulated. Let us find this distribution. From symmetry $\phi(-x) = \pi - \phi(x)$, in particular, $\phi(0) = \pi/2$. The first integral of Eq. (2) for $x<0$ is given by

$$\frac{\lambda_J^2}{2}\left(\frac{d\phi}{dx}\right)^2 + \cos\phi = \cos\phi_0 \tag{6}$$

with $\phi_0 = \phi(-d/2)$. The solution is given by

$$x(\phi) = -\frac{\lambda_J}{\sqrt{2}}\int_\phi^{\pi/2}\frac{d\phi'}{\sqrt{\cos\phi_0 - \cos\phi'}}, \tag{7}$$

where ϕ_0 is determined by condition $x(\phi_0) = -d/2$. Dependence $\phi_0(d/\lambda_J)$ and shapes of phase variation $\phi(x)$ for different values of d/λ_J are presented in Fig. 2. For small

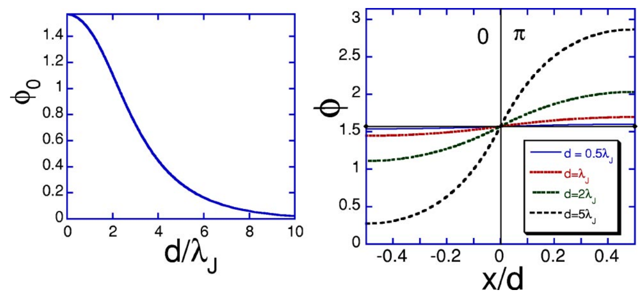

FIG. 2. (Color online) *Left panel*: dependence of the minimum phase on the length of junction, for $d_0 = d_\pi$. *Right panel*: shape of phase modulation at different junctions sizes.

length $d \ll \lambda_J$, it is easy to find that the phase distribution for $-d<x<0$ is $\phi(x) = \pi/2 + x(x+d)/(8\lambda_J^2)$, i.e., it varies weakly around $\pi/2$.

Consider a structure composed of short alternating 0- and π- JJs with a small length mismatch, $|d_0-d_\pi| \ll d_0$. In this case, function $j(x)$ in Eq. (2) oscillates rapidly and we can present $\phi(x)$ as $\phi(x) = \varphi(x) + \xi(x)$, where function $\varphi(x)$ varies slowly over the distance d_0 and $\xi(x)$ oscillates rapidly with $\overline{\xi(x)} = 0$. In limit $d_{0,\pi} < \lambda_J$, the oscillating term is small [as one can see from the right panel of Fig. 2, even at $d_0, d_\pi = \lambda_J$, (i) $|\xi(x)| \leqslant 0.13 \ll 1$ and (ii) the oscillating part is smaller than the average phase]. In this case one can use a coarse-grained approximation (see, e.g., Ref. 15), meaning that we can average out the rapidly oscillating function $\xi(x)$ and derive the equation for the slowly varying function $\varphi(x)$ only:

$$\frac{d^2\varphi}{dx^2} = -\frac{d_0 d_\pi}{24\lambda_J^4}\sin(2\varphi) + \frac{d_0-d_\pi}{(d_0+d_\pi)\lambda_J^2}\sin(\varphi). \tag{8}$$

Here, the first term in the right-hand side appears due to current modulations and the second term is proportional to the average Josephson current. The neglected terms in Eq. (8) are smaller by a factor of $(d_0/\lambda_J)^2$. Without the external magnetic field, when condition (5) is fulfilled, the equilibrium phase difference φ_0 is:

$$\cos\varphi_0 = f \equiv \frac{12\lambda_J^2(d_0-d_\pi)}{d_0 d_\pi(d_0+d_\pi)}, \tag{9}$$

and for $|f|<1$, the average phase difference lies in the region $0<|\varphi_0|<\pi$, i.e., we have the realization of a JJ with arbitrary ground-state phase difference (φ-junction). For $|f|>1$ homogeneous 0- or π-phase is realized. An example of the dependence of equilibrium phase φ_0 on length mismatch is shown in the inset of Fig. 1.

The energy in terms of the coarse-grained phase is given by

$$F = \frac{\Phi_0 j_c \lambda_J^2}{2\pi c}\int\left[\frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 + \frac{(\cos\varphi - f)^2}{2\lambda_\varphi^2}\right]dx, \tag{10}$$

with $\lambda_\varphi^2 = 12\lambda_J^4/d_0^2 \gg \lambda_J^2$. The coarse-grained Josephson current density flowing through the array is given by

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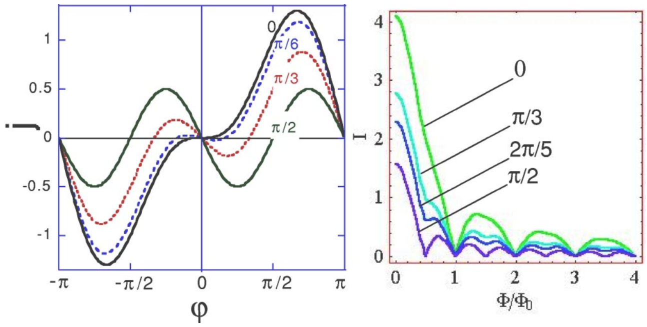

FIG. 3. (Color online) *Left panel*: current-phase relationships for different values of equilibrium phase φ_0 [see Eq. (11)]. Curves are marked by the value of φ_0. Unit of current density j in this plot is $d_0^2 j_c/(12\lambda_J^2)$. *Right panel*: dependencies of critical current of small 0- π-structure on the magnetic flux Φ through the structure, for different values of φ_0. The current unit at the vertical axis is $j_J S d_0^2/(12\lambda_J^2)$, where S is the cross-section area of the structure.

$$j(\varphi)=\frac{d_0^2 j_c}{12\lambda_J^2}\sin(\varphi)(f-\cos\varphi). \tag{11}$$

Equations similar to Eqs. (8), (10), and (11) were first dervied by Mints.[13] This current-phase relation is quite peculiar (see left panel in Fig. 3). The current has two maxima and two minima, which are achieved at $\cos\varphi_\pm=(f\pm\sqrt{f^2+8})/4$. The absolute maximum

$$j_{\max}=\frac{d_0^2 j_c}{48\lambda_J^2}\sqrt{\frac{1}{2}-\frac{f}{8}(f-\sqrt{f^2+8})(3f+\sqrt{f^2+8})}$$

is achieved at $\varphi=\varphi_-$.

Consider screening of the weak magnetic field by the periodic structure. In the modulated phase linearizing Eq. (8) near φ_0, $\varphi=\varphi_0+\tilde{\varphi}$, we obtain a linear equation for phase variation $\tilde{\varphi}$

$$\frac{d^2\tilde{\varphi}}{dx^2}=\frac{\sin^2\varphi_0}{\lambda_\varphi^2}\tilde{\varphi}. \tag{12}$$

One can see that the effective screening length

$$\lambda_{\text{eff}}\equiv\lambda_\varphi/\sin\varphi_0=\lambda_\varphi/\sqrt{1-f^2}, \quad \text{for} \quad |f|<1 \tag{13}$$

diverges at the transition point. In homogeneous phase $|f|>1$, similar derivation gives

$$\lambda_{\text{eff}}=\lambda_\varphi/\sqrt{|f|-1}, \quad \text{for } |f|>1. \tag{14}$$

Because $\lambda_\varphi(T)$ diverges at $T\to T_c$ faster than $f(T)$, $\lambda_{\text{eff}}(T)$ is nonmonotonic above the transition point. The minimum value of λ_{eff}, $\lambda_{\text{eff}}=2d_0^2/\sqrt{3}(d_0-d_\pi)$ is reached at $f=2$.

Exactly at the transition point $f=1$, screening is nonlinear and the phase variation obeys equation

$$2\lambda_\varphi^2 d^2\tilde{\varphi}/dx^2=\tilde{\varphi}^3. \tag{15}$$

The general solution of this equation is given by $\tilde{\varphi}=\pm 2\lambda_\varphi/(x+C)$. Using the phase-field relation $H=(\Phi_0/2\pi t)d\tilde{\varphi}/dx$, we derive from this solution that the magnetic field decays inside the structure as

$$H(x)=H_0\left(1+x/\sqrt{\frac{\lambda_\varphi\Phi_0}{\pi t H_0}}\right)^{-2}, \tag{16}$$

where H_0 is the external magnetic field. It is interesting to note that the field tail at large x does not depend on the applied field

$$H(x)=\frac{\lambda_\varphi\Phi_0}{\pi t x^2}, \quad \text{for} \quad x\gg\sqrt{\frac{\lambda_\varphi\Phi_0}{\pi t H_0}}. \tag{17}$$

In addition to the usual 2π-degeneracy, in case $f<1$, the ground-state energy is also degenerate with respect to sign change of the equilibrium phase; $\varphi_0\to-\varphi_0$. This degeneracy leads to the appearance of two new kinds of solitons in which the phase sweeps either from $-\varphi_0$ to φ_0 or from φ_0 to $2\pi-\varphi_0$. Rewriting Eq. (8) in a more convenient form

$$\frac{d^2\varphi}{dx^2}=\frac{1}{\lambda_\varphi^2}\sin(\varphi)(\cos\varphi_0-\cos\varphi),$$

we derive the shapes of the two solitons as

$$\varphi_1(x)=2\arctan\left[\tanh\left(\frac{x\sin\varphi_0}{2\lambda_\varphi}\right)\tan\left(\frac{\varphi_0}{2}\right)\right], \tag{18a}$$

$$\varphi_2(x)=\pi+2\arctan\left[\frac{\tanh(x\sin\varphi_0/2\lambda_\varphi)}{\tan(\varphi_0/2)}\right]. \tag{18b}$$

The soliton energies per unit length are given by

$$\varepsilon_1=\varepsilon_{J0}(\lambda_J/4\lambda_\phi)(|\sin\varphi_0|-\varphi_0\cos\varphi_0), \tag{19a}$$

$$\varepsilon_2=\varepsilon_{J0}(\lambda_J/4\lambda_\phi)(|\sin\varphi_0|+(\pi-\varphi_0)\cos\varphi_0), \tag{19b}$$

where $\varepsilon_{J0}=4\Phi_0 j_c\lambda_J/(\pi c)$ is the energy of a single soliton in a uniform JJ. At the transition point $f=1$ the first soliton vanishes and the second one acquires the following shape: $\varphi_2(x)=\pi+2\arctan(x/\lambda_\varphi)$. Even though for $d_0>d_\pi$ the first energy is smaller than the second one, both solitons play a role in the magnetic properties of the φ-junction. This is because without the second solitons, the system cannot contain two or more first solitons of the same sign due to topological constrains. In the external magnetic field H, the energy of the system with a small number of solitons of both kinds, N_1 and N_2, can be written as

$$E=\varepsilon_1 N_1+\varepsilon_2 N_2-\left(N_1\frac{\varphi_0}{\pi}+N_2\frac{\pi-\varphi_0}{\pi}\right)\frac{H\Phi_0}{4\pi}, \tag{20}$$

and has to be supplemented by the topological constraint $|N_1-N_2|\leqslant 1$. The penetration scenario is determined by the ratio of soliton energy ε_α to its flux Φ_α. Simple analysis shows that for $d_0>d_\pi$ these ratios are arranged in order $\varepsilon_1/\Phi_1<(\varepsilon_1+\varepsilon_2)/\Phi_0<\varepsilon_2/\Phi_2$. These inequalities imply that a single first soliton appears at field $H_1=4\pi^2\varepsilon_1/\varphi_0\Phi_0$. The soliton lattice, composed of alternating solitons of two types, penetrates into the structure at higher field $H_{c1}=4\pi(\varepsilon_1+\varepsilon_2)/\Phi_0$. In case $d_0=d_\pi$, fields H_1 and H_{c1} coincide.

An interesting possibility to have an equilibrium φ_0 Josephson vortex is realized at the boundary between an alternating structure of 0- and π-JJs and the usual 0-JJ. We assume that the alternating structure occupies region $x<0$, while 0-JJ, with Josephson penetration length Λ, occupies a positive semiaxis $x>0$. In this case the phase difference must vary continuously from φ_0 for $x\rightarrow -\infty$ to zero for $x\rightarrow +\infty$. Due to condition $\lambda_\varphi \gg \Lambda \sim \lambda_J$, value $\varphi(0)\ll 1$, which means that the boundary vortex is almost completely localized in region $x<0$ and it is equivalent to half of the first soliton (18a). The metastable $\pi-2\varphi_0$ boundary vortex is also possible and in limit $\lambda_\varphi \gg \Lambda$, its shape corresponds to the second soliton (18b).

Consider a structure with total length L smaller than screening length $\lambda_{\rm eff}$, in an external magnetic field H smaller than $\Phi_0/(d_0+d_\pi)t$. In this case the coarse-grained approximation for the phase is justified and we can use the current-phase relation (11), where the coarse-grained phase depends on the coordinate as $\varphi(x)=hx+\beta$ with $h=2\pi tH/\Phi_0$ and an arbitrary phase shift β. The total current per unit thickness, flowing through the structure is given by

$$I(\beta)=\frac{j_c d_0^2}{12\lambda_J^2}\int_{-L/2}^{L/2}dx\left(f\sin(hx+\beta)-\frac{\sin[2(hx+\beta)]}{2}\right).$$

Calculating the integral, we reduce it to the form

$$I(\beta)=\frac{I_0}{\eta}\sin(\beta)\sin\eta[f-\cos(\beta)\cos\eta], \tag{21}$$

where $I_0=j_cLd_0^2/(12\lambda_J^2)$ and $\eta=hL/2=\pi\Phi/\Phi_0$, and $\Phi=tLH$ is the total flux through the structure. The current reaches a maximum at

$$\cos(\beta)=\frac{\cos\varphi_0-\sqrt{\cos^2\varphi_0+8\cos^2(\eta)}}{4\cos(\eta)}. \tag{22}$$

Shapes of the field dependencies of the critical current at different values of φ_0 are plotted in the right panel of Fig. 3. These dependencies differ significantly from the Fraunhofer dependence in usual JJs. They have a large component with half of the main period. At $\varphi_0=\pi/2$ the dependence has the Fraunhofer shape but the period is two times smaller than in usual case. Note that finite critical current observed in the alternating 0-π junction structure[8] at zero magnetic field is naturally obtained in the framework of our analysis. The theoretical analysis in Refs. 8 and 16, giving zero current at $H=0$, is incomplete because it does not take into account the current term $\propto \sin(2\varphi)$ coming from the rapidly oscillating phase.

In conclusion, the possibility to realize the transition into the φ-junction state, by decreasing the temperature from T_c, may be very helpful for experimental verification of the predicted effects. In particular, the observation of the striking nonmonotonous variation of the screening length with temperature would provide an unambiguous proof of such a transition. Also, the studies of the fine structure of the critical current dependence versus magnetic field and the observation of a periodicity two times smaller than that expected for standard JJ, could be of considerable interest. Finally, note that the scanning magnetic probe microscope may directly probe the specific shapes of the partial flux vortices.

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