Charged vortices in superfluid systems with pairing of spatially separated carriers

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It is shown that in a magnetic field the vortices in superfluid electron-hole systems carry a real electrical charge. The charge value depends on the relation between the magnetic length ℓ_B and the Bohr radius of electrons a_B^e and holes a_B^h. In double-layer systems at filling factors $\nu_e=\nu_h=\nu$ and for $a_B^e, a_B^h \gg \ell_B$ the vortex charge is equal to the universal value νe.

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I. INTRODUCTION

It is generally believed that the vortices in superconductors are connected with an applied magnetic field, while the magnetic field does not have any influence on the properties of the vortices in electrically neutral superfluid systems. The aim of this paper is to show that in superfluid systems subjected by a magnetic field the vortices have a real electrical charge (the compensating charge of the opposite sign appears on the surface of the system). In general case, the charge of the vortices is fractional. For the first time the fractional charge of the vortices was predicted by Laughlin[1] for the two-dimensional (2D) electron gas in a quantized magnetic field. Then it was established in Refs. 2 and 3 that in double-layer electron systems at half filling of the lowest Landau levels in each layer the vortex should carry the charge equal to $\pm e/2$ (here and below e is the absolute value of the electron charge).

It is found that in any superfluid system in a magnetic field the charge of the vortex is proportional to the polarizability of the particles and inversely proportional to their effective mass. For the superfluid phases of He isotopes and for the Bose gases of alkali metals, our estimates yield that even in strongest magnetic fields reachable now this charge has an unobservable small value, but it can be of the order of the electron charge in superfluid systems with pairing of spatially separated electrons and holes.

The authors of Ref. 4 call the possibility of the electron-hole pair superfluidity in question based on the fact that the interband transitions fix the phase of the order parameter and result in a transition into a dielectric state. But it was established in Refs. 5 and 6 that the exclusion of the electron-hole superfluidity does not take place in systems where spatially separated electrons and holes are coupled. In these systems, the interband transitions coincide with the interlayer ones and usually they are exponentially small. A superfluid state of the pairs with spatially separated components has both the superfluid and superconducting features. A superfluid flow of such electron-hole pairs is accompanied with real supercurrents flowing in opposite directions. Therefore, we will call these systems the condenser superconductors.

A pairing of a conduction-band electron from one layer with a valence-band hole from the other layer was considered in Refs. 5 and 6. Then in a number of theoretical papers,[2,3,7–10] a possibility of superfluidity of pairs composed from spatially separated electrons and holes belonging to the conducting band is shown. This possibility is realized in double-layer electron systems in a magnetic field normal to the layers for the case of the total filling factor $\nu_T=\nu_1+\nu_2=1$. During almost ten years there were many efforts to observe the condenser superconductivity experimentally.[11–15] Now it seems that these effort have been crowned with success.[16,17]

Let us consider a bilayer system, where in one conducting layer the carriers are electrons and in the other one the carriers are holes. The case of equal electron n_e and hole n_h densities ($n_e=n_h=n$) is specified and it is assume the electron effective mass m_e is much smaller than the hole effective mass m_h (in semiconductor heterostructures $m_e=0.067m_0$, $m_h\approx 0.4m_0$, where m_0 is the bare electron mass). We are interested in behavior of the system subjected by a strong, perpendicular to the layer, magnetic field B in case when the inequality $a_B^e \gg \ell_B$ is satisfied. Here, $a_B^e=\hbar^2\varepsilon/m_e e^2$ is the effective Bohr radius of the electron and $\ell_B=(c\hbar/eB)^{1/2}$ is the magnetic length. The similar inequality for the effective Bohr radius of the hole ($a_B^h=\hbar^2\varepsilon/m_h e^2$) is not required. The study is restricted to the case of low density n, when the filling factor $\nu=2\pi\ell_B^2 n \ll 1$. In this case, electrons and holes are paired in the coordinate space. Consequently, at low temperatures, the system behaves as a condensate of electron-hole pairs with the size of the pairs much smaller than the average distance between the pairs.

The properties of the condensate are principally dependent on the spectrum of the pairs. For the three-dimensional case the spectrum of the electron-hole pair in a strong magnetic field was found many years ago by Gor'kov and Dzyaloshinskii in their seminal paper.[18] For the 2D case, the same problem was considered for the first time in Ref. 19. While the consideration[18,19] does not contain any assumption about the ratio between the masses m_e and m_h, it is assumed in these papers that two inequalities $a_B^e \gg \ell_B$ and $a_B^h \gg \ell_B$ are satisfied. But the results of Refs. 18 and 19 become inapplicable for the case when at least one of these two inequalities is violated.[20]

We begin the consideration from the study of the eigenvalue problem for a single electron-hole pair and find the effective mass of the pair M_* as well as its electric polarizability α (Sec. II). Then, using the equation for the order parameter, the superfluid velocity of a gas of the pairs in the low-density limit and its dipole momentum as a function of electric and magnetic fields (Sec. III) is computed. Finally, taking into account the dependence of the dipole momentum on the superfluid velocity, the value of the electric charge of a vortex is found (Sec. IV).

II. ELECTRON-HOLE PAIR IN A MAGNETIC FIELD

The Schröedinger equation for the electron-hole pair has the form

$$\left[\frac{1}{2m_e}\left(-i\hbar\nabla_e+\frac{e}{c}\mathbf{A}_e\right)^2+\frac{1}{2m_h}\left(-i\hbar\nabla_h-\frac{e}{c}\mathbf{A}_h\right)^2 +e\mathbf{E}\cdot(\mathbf{r}_e-\mathbf{r}_h)-\frac{e^2}{\varepsilon\sqrt{|\mathbf{r}_e-\mathbf{r}_h|^2+d^2}}\right]\Psi(\mathbf{r}_e,\mathbf{r}_h) =\mathcal{E}\Psi(\mathbf{r}_e,\mathbf{r}_h), \tag{1}$$

where d is the interlayer distance. Equation (1) describes the case when a perpendicular to the layer magnetic field $\mathbf{B}=\mathrm{rot}\mathbf{A}$ and an electric field $\mathbf{E}$, directed parallel to the layers, are applied to the system.

It was shown by Gor'kov and Dzyaloshinskii[18] that the operator

$$\hat{\boldsymbol{\pi}}=-i\hbar\nabla_e-i\hbar\nabla_h+\frac{e}{c}(\mathbf{A}_e-\mathbf{A}_h)-\frac{e}{c}\mathbf{B}\times(\mathbf{r}_e-\mathbf{r}_h) \tag{2}$$

plays the role of the operator of the momentum of the electron-hole pair in a magnetic field. This operator commutes with Hamiltonian (1) and all its components commute with each other. It allows to parameterize the solutions of Eq. (1) by a c-number parameter $\boldsymbol{\pi}$. The parameter $\boldsymbol{\pi}$ is the eigenvalue of the operator $\hat{\boldsymbol{\pi}}$. Based on this observation the dependence of the energy $\mathcal{E}$ on the momentum $\boldsymbol{\pi}$ can be found.

It is convenient to formulate the problem using the representation for the wave function $\tilde{\Psi}=U\Psi$ with

$$U=\exp\left(\frac{ie\mathbf{A}(\mathbf{R})\cdot\mathbf{r}}{\hbar c}\right). \tag{3}$$

Here, $\mathbf{R}=(m_e\mathbf{r}_e+m_h\mathbf{r}_h)/(m_e+m_h)$ is the center-of-mass coordinate and $\mathbf{r}=\mathbf{r}_e-\mathbf{r}_h$ is the relative coordinate. The advantage of this representation becomes clear under the observation that the momentum operator in representation (3) has a simple form[21]

$$\hat{\tilde{\boldsymbol{\pi}}}=U\hat{\boldsymbol{\pi}}U^{-1}=-i\frac{\partial}{\partial\mathbf{R}}. \tag{4}$$

It follows from Eq. (4) that in this representation the wave function of the pair with the momentum $\boldsymbol{\pi}$ reads as

$$\tilde{\Psi}(\mathbf{R},\mathbf{r})=\exp\left(\frac{i\boldsymbol{\pi}\cdot\mathbf{R}}{\hbar}\right)\tilde{\Phi}_{\boldsymbol{\pi}}(\mathbf{r}). \tag{5}$$

Substituting Eq. (5) into the equation $\tilde{H}\tilde{\Psi}=\mathcal{E}\tilde{\Psi}$, one obtains the equation for the function $\tilde{\Phi}_{\boldsymbol{\pi}}(\mathbf{r})$. At $\boldsymbol{\pi}=0$ and $\mathbf{E}=0$, this equation has the form

$$[\hat{H}_0(\mathbf{r})+\hat{H}_1(\mathbf{r})]\tilde{\Phi}(\mathbf{r})=\mathcal{E}\tilde{\Phi}(\mathbf{r}). \tag{6}$$

Here, $\hat{H}_0(\mathbf{r})$ is the Hamiltonian for a free electron. In the symmetric gauge $\mathbf{A}=\frac{1}{2}\mathbf{B}\times\mathbf{r}$ it equals to

$$\hat{H}_0(\mathbf{r})=\frac{1}{2m_e}\left(-i\hbar\nabla+\frac{e}{2c}\mathbf{B}\times\mathbf{r}\right)^2. \tag{7}$$

The Hamiltonian $H_1(\mathbf{r})$ reads as

$$H_1=\frac{1}{2m_h}\left(-i\hbar\nabla-\frac{e}{2c}\mathbf{B}\times\mathbf{r}\right)^2-\frac{e^2}{\varepsilon\sqrt{r^2+d^2}}. \tag{8}$$

The eigenfunctions of the operator H_0 are numbered by the radial n_r and the azimuthal m quantum numbers. For the electron frozen in the lowest Landau level $n_r=0$ and $m\geqslant 0$ and the eigenfunctions have the form

$$\tilde{\Phi}_m=\left(\frac{1}{2^{m+1}\pi m!}\right)^{1/2}\frac{e^{-im\varphi}}{\ell_B}\left(\frac{r}{\ell_B}\right)^m\exp\left(-\frac{r^2}{4\ell_B^2}\right). \tag{9}$$

In the case considered, the inequalities $\hbar\omega_e\gg\hbar\omega_h$ and $\hbar\omega_e\gg e^2/\varepsilon\ell_B$ (equivalent to $m_h\gg m_e$ and $a_B^e\gg\ell_B$, correspondingly) are satisfied. It allows to take into account the Hamiltoniam $H_1(\mathbf{r})$ as a perturbation. At $d\ll\ell_B$, the spectrum of the pair obtained in the first order of the perturbation theory is

$$\mathcal{E}_m=\frac{1}{2}\hbar\omega_e+\hbar\omega_h\left(m+\frac{1}{2}\right)-\frac{e^2}{\sqrt{2}\varepsilon\ell_B}\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m+1)}. \tag{10}$$

Result (10) is valid for an arbitrary ratio between $\hbar\omega_h$ and $e^2/\varepsilon\ell_B$.

If the momentum of the pair $\boldsymbol{\pi}$ and the electric field $\mathbf{E}$ are nonzero, the Hamiltonian $\tilde{H}$ contains the additional term

$$\tilde{H}_{\boldsymbol{\pi},\mathbf{E}}=\frac{e}{Mc}(\boldsymbol{\pi}\times\mathbf{B})\cdot\mathbf{r}+e\mathbf{E}\cdot\mathbf{r}+\frac{|\boldsymbol{\pi}|^2}{2M}, \tag{11}$$

where $M=m_e+m_h\approx m_h$ is the mass of the pair. The terms in Eq. (11) linear in $\mathbf{r}$ do not contribute to the energy of the pair in the first order of the perturbation theory. In the second order of the perturbation theory, these terms yield the correction to the energy that depends on the momentum of the pair $\boldsymbol{\pi}$ and the electric field $\mathbf{E}$

$$\Delta\mathcal{E}=\frac{|\boldsymbol{\pi}|^2}{2m_h}-\frac{\left|\langle\tilde{\Phi}_1|e\left(\frac{\boldsymbol{\pi}\times\mathbf{B}}{m_hc}+\mathbf{E}\right)\cdot\mathbf{r}|\tilde{\Phi}_0\rangle\right|^2}{\mathcal{E}_1-\mathcal{E}_0} =\frac{|\boldsymbol{\pi}|^2}{2M_*}+\frac{M_B}{M_*}\boldsymbol{\pi}\cdot\mathbf{u}-\frac{1}{2}\frac{M_B}{M_*}m_hu^2. \tag{12}$$

Here, $\mathbf{u}=c(\mathbf{E}\times\mathbf{B})/B^2$ is the drift velocity of the pair in the crossed electric (parallel to the layers) and magnetic (perpendicular to the layers) fields. The mass

$$M_B=\frac{4}{\sqrt{2\pi}}\frac{\varepsilon\hbar^2}{e^2\ell_B}=\frac{4}{\sqrt{2\pi}}m_h\frac{a_B^h}{\ell_B}, \tag{13}$$

the effective mass of the pair is

$$M_*=M_B+m_h. \tag{14}$$

I emphasize that the value M_B depends on the magnetic field and does not depend on the masses m_e and m_h.

It is useful to compare the expression for the effective mass M_* (14) with Lerner and Lozovik expression[19] for the mass $M_*^{LL}=M_B$ obtained by the Gor'kov and Dzyaloshinskii method.[18] When $m_h\gg m_e$, the Gor'kov and Dzyaloshinskii method is applicable if $a_B^h\gg\ell_B$ and just the first-order correction in the perturbation theory using the parameter ℓ_B/a_B^h yields $M_*=M_*^{LL}$ (see Appendix A). The second-order correction yields $M_*=M_B+m_h$. But in the perturbation theory $m_h/M_B\sim\ell_B/a_B^h\ll 1$ and the second-order correction can be neglected. It is not the case in my approach, where the ratio ℓ_B/a_B^h is assumed to be arbitrary one and, as a consequence, the ratio m_h/M_B is arbitrary one as well. More arguments on the validity of Eq. (14) at arbitrary M_B and m_h are given below.

Introducing the pair polarizability

$$\alpha(B)=M_B\frac{c^2}{B^2}=\frac{4\varepsilon}{\sqrt{2\pi}}\ell_B^3, \tag{15}$$

one can rewrite the correction $\Delta\mathcal{E}$ in the form

$$\Delta\mathcal{E}=\frac{1}{2M_*}\left(\boldsymbol{\pi}+\alpha(B)\frac{\mathbf{E}\times\mathbf{B}}{c}\right)^2-\frac{\alpha(B)}{2}E^2. \tag{16}$$

Analogous expression was obtained in Ref. 22 for an electrically neutral atom in crossed fields for the case of small magnetic fields. In that case, Eq. (16) contains the zero magnetic-field polarizability of the atom $\alpha(0)$ instead of $\alpha(B)$ and the mass of the atom M instead of the mass of the pair M_*.

III. THE CONDENSATE OF THE ELECTRON-HOLE PAIRS IN CROSSED FIELDS

Replacing the momentum $\boldsymbol{\pi}$ with the operator $-i\hbar\boldsymbol{\nabla}$, one obtains from Eq. (16) the Hamiltonian for the electron-hole pairs. In the low-density limit, when the size of the pair is much smaller than the distance between the pairs and the exchange effects are inessential, the pairs can be considered as true bosons. At low temperatures, the rarefied Bose gas should form a superfluid state. The superfluid phase can be described by the order parameter Ψ. It satisfies the equation

$$i\hbar\frac{\partial\Psi}{\partial t}=\frac{1}{2M_*}\left(-i\hbar\boldsymbol{\nabla}+\alpha(B)\frac{\mathbf{E}\times\mathbf{B}}{c}\right)^2\Psi-\frac{\alpha(B)}{2}E^2\Psi+\gamma|\Psi|^2\Psi. \tag{17}$$

The last term on the right-hand side of Eq. (17) describes the interaction between the pairs. One can show that in the limit $d\ll\ell_B$, the interaction constant is equal to $\gamma=(\pi/2)^{3/2}e^2d^2/\varepsilon\ell_B$. The vanishing of the interaction constant at $d=0$ is a consequence of the exact compensation of the Coulomb forces between the pairs (compare with Ref. 23). Presenting the order parameter in the form $\Psi=|\Psi|e^{i\varphi(r)}$, one obtains from Eq. (17) the velocity of the superfluid component

$$\mathbf{v}_s=\frac{1}{M_*}\left(\hbar\boldsymbol{\nabla}\varphi+\alpha(B)\frac{\mathbf{E}\times\mathbf{B}}{c}\right). \tag{18}$$

To obtain the dipole momentum of the unit area $\mathbf{P}$ one takes into account that the r.h.s. of Eq. (17) is the variational derivative over $\Psi^*(\mathbf{r})$ of the Ginzburg-Landau-type functional of the energy of a superconductor. The derivative of that functional over the electric field $\mathbf{E}$ taken with the opposite sign is $\mathbf{P}$,

$$\mathbf{P}=\alpha(B)\left[\frac{m_h}{M_*}\mathbf{E}+\frac{\hbar}{M_*c}\boldsymbol{\nabla}\varphi\times\mathbf{B}\right]|\Psi|^2. \tag{19}$$

Expression (19) can be rewritten in the form

$$\mathbf{P}=\alpha(B)\left(\mathbf{E}+\frac{1}{c}\mathbf{v}_s\times\mathbf{B}\right)|\Psi|^2. \tag{20}$$

This result means that not only the electric field $\mathbf{E}$, but also the Lorentz force polarizes the pair, acting in opposite directions on the positive and negative charges of the pair. One should note that the expression for $\mathbf{P}$ (20) is valid within a linear accuracy in v_s/c.

Let us give the proof of the mutual consistence of Eq. (14) for the effective mass of the pair M_* and Eq. (15) for the polarizability of the pair $\alpha(B)$. It follows from Eqs. (1) and (2) that

$$\hat{\boldsymbol{\pi}}=M\hat{\mathbf{v}}-\frac{e}{c}\mathbf{B}\times(\mathbf{r}_e-\mathbf{r}_h) \tag{21}$$

(for the first time expression (21) was given by Gor'kov Dzyaloshinskii[18]). Here, $\hat{\mathbf{v}}=(m_e\hat{\mathbf{v}}_e+m_h\hat{\mathbf{v}}_h)/(m_e+m_h)$—the operator of the average velocity of the pair. Using Eq. (21) one can find the density of the momentum of the superfluid medium of the electron-hole pairs

$$\boldsymbol{\Pi}=Mn\mathbf{v}_s+\mathbf{B}\times\mathbf{P}/c, \tag{22}$$

where n is the density of the medium and $\mathbf{P}=ne(\mathbf{r}_h-\mathbf{r}_e)$ is density the dipole momentum of the medium. The same result was obtained in Ref. 24 using the laws of transformation of the energy and the momentum of the medium under transition from one inertial system of reference into another one.

Let the electric field $\mathbf{E}=0$. Then, it follows from Eqs. (18) and (20) that $\mathbf{v}_s n=\boldsymbol{\Pi}/M_*$ and $\mathbf{P}=\alpha n(\mathbf{v}_s\times\mathbf{B})/c$, correspondingly. Substituting these expressions into Eq. (22) one can easily find that at $\boldsymbol{\Pi}\neq 0$ Eq. (22) is valid if

$$M_*=M+\alpha(B)\frac{B^2}{c^2}. \tag{23}$$

One should note that no assumptions about the value of the magnetic field was done under the derivation of result (23). At strong magnetic field ($a_B^e \gg \ell_B$) the polarizability $\alpha(B) = M_B c^2/B^2$ and, as it follows from Eq. (23), $M_* = M + M_B \simeq m_h + M_B$, which is consistent with Eq. (14).

IV. THE CHARGE OF THE VORTEX

The polarization charge ρ_{pol} can be found by taking the two-dimensional divergence of both sides of Eq. (19). To calculate the derivatives on the r.h.s. of Eq. (19), one takes into account that

$$\mathrm{curl}_z \nabla\varphi = 2\pi\sum_i \delta(\mathbf{r}-\mathbf{r}_i)n_i, \tag{24}$$

where $n_i = \pm 1$ and the upper (low) sign corresponds to the vortices rotating in the counter-clockwise (clockwise) direction, and the summation is over the vertex centers. Beside this one omits the terms containing the quantity $\nabla|\Psi|$. We are not interested here in the structure of the vortex core (It was considered as the mathematical point). At small fields E, the terms containing $\nabla|\Psi|$ are small out of the core. It allows to replace $|\Psi|^2$ with the pair density n. Finally, one obtains

$$\rho_{pol}(\mathbf{r}) = -\alpha(B)n\left[\frac{m_h}{M_*}\mathrm{div}_2\mathbf{E} + \sum_i 2\pi\frac{\hbar B}{M_* c}n_i\delta(\mathbf{r}-\mathbf{r}_i)\right]. \tag{25}$$

Thus, in a superfluid system subjected by a magnetic field the polarization charge emerges when the medium is polarized by the electric field with a nonzero divergence or the quantized vortices exist in the system. It also follows from Eq. (25) that the charge of the ith vortex is equal to the coefficient of $\delta(\mathbf{r}-\mathbf{r}_i)$, namely,

$$q=\pm 2\pi\frac{\hbar B}{M_* c}\alpha(B)n. \tag{26}$$

Equation (26) yields the vortex charge for the electron-hole bilayer systems in an arbitrary magnetic fields. The same result is valid for the electron-electron double-layer system with the substitution $M_* = M_B + 2m_e$.

In weak magnetic fields ($\ell_B \gg a_B^e$), the polarizability $\alpha = \gamma(a_B^e)^3$ with $\gamma \sim 1$ and the effective mass $M_* \simeq m_h + m_e \simeq m_h$. Then the vortex charge is equal to

$$q=\pm\frac{2\pi\gamma}{\varepsilon}\left(\frac{a_B^e}{\ell_B}\right)^2 a_B^e a_B^h n e. \tag{27}$$

In high magnetic fields ($a_B^e \gg \ell_B \gtrsim a_B^h$) using Eq. (15) for $\alpha(B)$, one obtains

$$q=\pm 2\pi\ell_B^2 n\frac{M_B}{M_*}e = \pm\frac{M_B}{M_*}\nu e. \tag{28}$$

Finally, in ultrahigh fields ($a_B^h \gg \ell_B$) the effective mass $M_* = M_B(1+\sqrt{2\pi}\ell_B/4a_B^h) \to M_B$ and the vortex charge obeys the universal relation $q = \pm\nu e$.

Let us present here some estimates. In the magnetic field $B = 10$ T, the magnetic length $\ell_B \approx 80$ Å. In GaAs herestuctures with the dielectric constant $\varepsilon = 13$, the magnetic mass is very small ($M_B \approx 10^{-28}$ g). Due to the smallness of this quantity, the vortex charge can be the of order of the electron charge only in superfluid systems with the boson mass of the order of the electron mass m_0. For the electron-hole bilayer system at $m_h = 0.4m_0$ and $m_e = 0.067m_0$, one obtains $M_B/M_* \approx 0.2$ and the vortex charge is $q \approx 0.2\nu e$. For the electron-electron double-layer system with the same m_e, one finds $M_B/M_* \approx 0.5$ and the vortex charge $q \approx 0.5\nu e$. The derivation of q from the universal value is connected with the ratio $\ell_B/a_B^e \approx 0.8$ and the strong inequality $\ell_B \ll a_B^E$ does not satisfy in this case.

The number of the vortices and their spatial distribution depends on the electric field **E**. Integrating both sides of Eq. (18) along a certain contour, one finds

$$M_* \oint \mathbf{v}_s d\mathbf{l} = 2\pi\hbar N_v - \frac{\alpha(B)}{c}\mathbf{B}\cdot\oint \mathbf{E}\times d\mathbf{l}. \tag{29}$$

Here, N_v is the number of the vortices inside the contour (the vortices of the same vorticity were considered). It follows from Eq. (29) that the velocity $\mathbf{v}_s$ decreases when the vortices emerge—it yields a gain in the kinetic energy of the system. When the vortex distribution is considered as a continuous one, the vortex density $n_v(\mathbf{r})$ can be introduced and

$$N_v = \int n_v(\mathbf{r})d\mathbf{r}. \tag{30}$$

Putting the r.h.s. of Eq. (29) to zero, one finds from Eqs. (29) and (30) the relation

$$n_v(\mathbf{r}) = \frac{\alpha(B)B}{2\pi\hbar c}\ \mathrm{div}_2\mathbf{E}. \tag{31}$$

A macroscopic number of the vortices with equal vorticites can also emerge in the absence of the electric field. It is realized when besides the uniform field $\mathbf{B}_z$ there is an extra field $\mathbf{B}_\tau$ with $\mathrm{div}_2\mathbf{B}_\tau \neq 0$ ($\mathbf{B}_\tau$ is parallel to the plane of the structure). Indeed, one can show (compare with Ref. 21) that in the field $\mathbf{B}_\tau$ the energy of the pair of spatially separated electron and hole is equal to

$$\Delta\mathcal{E} = \frac{1}{2M_*}\left(\boldsymbol{\pi} + \frac{ed}{c}\hat{z}\times\mathbf{B}_\tau\right)^2. \tag{32}$$

One can see that energy (32) differs from the expressions (16) only by that the induced dipole momentum $\alpha(B)\mathbf{E}$ is replaced with the spontaneous momentum $ed\hat{z}$. Therefore, the dipole momentum of the unit area can be obtained from Eq. (20) replacing the induced momentum with the spontaneous one. Then, taking the divergence of **P**, one finds

$$\rho_{pol}(\mathbf{r}) = -\frac{\alpha(B)Bn}{M_* c}\left[\frac{ed}{c}\mathrm{div}_2\mathbf{B}_\tau + \sum_i 2\pi\hbar n_i \delta(\mathbf{r}-\mathbf{r}_i)\right]. \tag{33}$$

It follows from this expression that in this case the vortex charge is equal to the value found above and in the continuous limit the vortex density is $n_v(\mathbf{r}) = (ed/2\pi\hbar c)\mathrm{div}_2\mathbf{B}_\tau$.

At nonzero temperatures, the charged vortices will emerge in condenser superconductors in a fluctuation way, in similarity with the same phenomena in a thin He-II film. The circumstance that the vortices are charged does not influence in the first approximation on the thermodynamic features of the system. It is connected with that the Coulomb correction to the interaction between the vortices falls down much faster (by the power law) than the bare logarithmic interaction between them. The last one, as is well known, results in a Kosterlitz-Thouless transition. Since the sign of the vortex charge is in one to one correspondence with the sign of the vorticity, at temperatures below the Kosterlitz-Thouless temperature the vortex-antivortex pairs should be electrically neutral. At temperature above the Kosterlitz-Thouless the vortices and antivortices decouple, and free electrical charges appear. It reveals itself in a principal change of the conducting properties of the system under the phase transition.

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APPENDIX

Let us show how Eq. (1) can be solved by the Gor'kov and Dzyaloshinskii method.[18] In this method, the wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2)$ is sought in the form

$$\Psi(\mathbf{r}_1,\mathbf{r}_2) = \exp\left[\frac{i}{\hbar}\mathbf{R}\left(\boldsymbol{\pi} + \frac{e}{c}\mathbf{B}\times\mathbf{r}\right)\right]\exp\left(\frac{i\gamma\mathbf{r}\boldsymbol{\pi}}{2\hbar}\right)\Phi(\mathbf{r}-\mathbf{r}_0), \tag{A1}$$

where $\gamma = (m_h - m_e)/(m_h + m_e)$,

$$\mathbf{r}_0 = \left(\frac{\mathbf{B}}{B}\times\boldsymbol{\pi}\right)\ell_B^2\frac{1}{\hbar}. \tag{A2}$$

Substituting Eq. (A1) into the Schröedinger equation (1) taken in the symmetric gauge, one arrives to the following equation for the function of relative motion $\Phi(\mathbf{r})$:

$$\left[-\frac{\hbar^2}{2\mu}\triangle_{\mathbf{r}} + \frac{ie\hbar}{2\mu c}\gamma(\mathbf{B}\times\mathbf{r})\boldsymbol{\nabla}_{\mathbf{r}} + \frac{e^2}{8\mu c^2}B^2r^2 - \frac{e^2}{\varepsilon(|\mathbf{r}-\mathbf{r}_0|^2| + d^2)^{1/2}}\right]\Phi(\mathbf{r}) = \mathcal{E}\Phi(\mathbf{r}), \tag{A3}$$

where $\mu = m_e m_h/(m_e + m_h)$ is the reduced mass.

In zero-order approximation with respect to the interaction the wave function (A3) coincides with the function $\tilde{\Phi}_m$ (9) and the energy is equal to[19]

$$\mathcal{E}_{nm}^{(0)} = \hbar\frac{eB}{\mu c}\left[n + \frac{1}{2}(1 + |m| - \gamma m)\right]. \tag{A4}$$

In the case $m_h \gg m_e$ considered here the electron cyclotron frequency $\omega_e = eB/m_e c$ considerably exceeds the hole cyclotron frequency $\omega_h = eB/m_h c$. Therefore, we will further consider the $n=0$ and $m>0$ case when the zero-order energy is equal to

$$\mathcal{E}_m^{(0)} = \frac{\hbar\omega_e}{2} + \hbar\omega_h\left(m + \frac{1}{2}\right). \tag{A5}$$

The distance between the unperturbed energy levels is $\hbar\omega_h$, while the Coulomb energy (at small momenta π) is of the order of $e^2/\varepsilon\ell_B$. Therefore, if $\hbar\omega_h \gg e^2/\varepsilon\ell_B$ (which is equivalent to $a_B^h \gg \ell_B$) the Coulomb interaction term in Eq. (A3) can be treated as a perturbation. The first-order correction to the energy of the pair in the ground state $m=0$ is

$$\mathcal{E}_0^{(1)} = -\langle\tilde{\Phi}_0|\frac{e^2}{\varepsilon(|\mathbf{r}-\mathbf{r}_0|^2 + d^2)^{1/2}}|\tilde{\Phi}_0\rangle = -\frac{e^2}{2\pi\ell_B^2}\int d^2r\frac{e^{-r^2/2\ell_B^2}}{\sqrt{|\mathbf{r}-\mathbf{r}_0|^2 + d^2}}. \tag{A6}$$

Putting $r_0 = 0$ in Eq. (A6), one finds the bound energy of the pair as a function of ℓ_B and d (see Ref. 25)

$$\mathcal{E}_0^{(1)}(d) = -\left(\frac{\pi}{2}\right)^{1/2}\frac{e^2}{\varepsilon\ell_B}\exp\left(\frac{d^2}{2\ell_B^2}\right)\left[1 - \Phi\left(\frac{d^2}{2\ell_B^2}\right)\right], \tag{A7}$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt.$$

One should note that the bound energy does not depend on the electron and hole masses. Further on, we will be interested in the case of $d \ll \ell_B$. Putting in Eq. (A6) $d=0$, one finds[19]

$$\mathcal{E}_0^{(1)}(\pi) = -\left(\frac{\pi}{2}\right)^{1/2}\frac{e^2}{\varepsilon\ell_B}\mathbf{I}_0\left(\frac{|\pi|^2\ell_B^2}{4\hbar^2}\right)\exp\left(-\frac{|\pi|^2\ell_B^2}{4\hbar^2}\right). \tag{A8}$$

It follows from Eq. (A8) that at small momenta $(|\pi| \ll \hbar/\ell_B)$

$$\mathcal{E}_0^{(1)}(\pi) = \mathcal{E}_0^{(1)}(0) + \frac{|\pi|^2}{2M_B} \equiv \epsilon_0 + \frac{|\pi|^2}{2M_B}, \tag{A9}$$

where $\epsilon_0 = -(\pi/2)^{1/2}e^2/\varepsilon\ell_B$ and M_B is determined by Eq. (13). Thus, in that approximation the effective mass of the pair does not depend on m_e and m_h. The situation is changed if one takes into account the higher-orders correction terms. In the second order of the perturbation theory the correction to the energy is equal to

$$\mathcal{E}_0^{(2)}(\pi)=\frac{|V_{10}|^2}{\mathcal{E}_0^{(0)}-\mathcal{E}_1^{(0)}}, \tag{A10}$$

where the matrix element V_{10} at $d=0$ reads as

$$V_{10}=-\frac{1}{2\sqrt{2}\pi}\frac{e^2}{\ell_B^3}\int d^2r\frac{e^{-i\varphi}}{\varepsilon|\mathbf{r}-\mathbf{r}_0|}r\ \ \exp\left(-\frac{r^2}{2\ell_B^2}\right). \tag{A11}$$

At small momenta the integral in Eq. (A11) can be easily evaluated and

$$V_{10}=-\frac{\sqrt{\pi}e^2}{4}\frac{|\pi|}{\hbar}.$$

Substituting this expression into Eq. (A10), one obtains proportional to $|\pi|^2$ correction to the energy:

$$\mathcal{E}_0^{(2)}(\pi)=-\frac{|\pi|^2}{\hbar\omega_h}\frac{\pi e^4}{16\hbar^2}. \tag{A12}$$

This correction results in a renormalization of the effective mass of the pair. Collecting the $|\pi|^2$ depending terms in $\mathcal{E}_0^{(1)}$ (A9) and $\mathcal{E}_0^{(2)}$ (A12), one obtains

$$\frac{|\pi|^2}{2M_B}-\frac{\pi e^2}{16\hbar^2\hbar\omega_h}|\pi|^2\equiv\frac{|\pi|^2}{2M_*}. \tag{A13}$$

It follows from Eq. (A13) that

$$M_*=M_B\left(1-\frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{\ell_B}{a_B^h}\right)^{-1}. \tag{A14}$$

Expanding Eq. (A14) in series with respect to the parameter $\ell_B/a_B^h\ll 1$ and taking into account the linear in ℓ_B/a_B^h terms, one finds the following expression for the effective mass of the pair:

$$M_*=M_B+m_h. \tag{A15}$$

This result coincides with Eq. (14). It seems that in the Gor'kov and Dzyaloshinskii method, Eq. (A15) is valid only with a linear in ℓ_B/a_B^h accuracy, because, as it follows from Eq. (A14), the higher order in ℓ_B/a_B^h terms may modify the result obtained. Nevertheless, one can show that an accurate accounting of all terms of the order of $(\ell_B/a_B^h)^2$ leaves relation (A15) unchanged.

The third-order correction to the energy is equal to:[26]

$$\mathcal{E}_0^{(3)}=\frac{V_{01}V_{11}V_{10}}{\hbar^2\omega_{10}^2}-V_{00}\frac{|V_{10}|^2}{\hbar^2\omega_{10}^2}, \tag{A16}$$

where $\hbar\omega_{10}=\mathcal{E}_1^{(0)}-\mathcal{E}_0^{(0)}=\hbar\omega_h$. One can easily find that $V_{11}=V_{00}/2=\epsilon_0/2$. Therefore, the third-order correction term is reduced to

$$\mathcal{E}_0^{(3)}=-\frac{\epsilon_0}{2}\frac{|V_{10}|^2}{\hbar^2\omega_{10}^2}. \tag{A17}$$

Using the expression for $|V_{10}|^2$ found above and adding energy (A17) to expression (A13), one can find the renormalized value of M_*. To do this one can expand Eq. (A14) up to the terms of the order of $(\ell_B/a_B^h)^2$ and takes into account the same order terms emerged from correction (A17). One can see that in this expansion the terms of the order of $(\ell_B/a_B^h)^2$ compensate each other exactly and relation (A15) survives. Based on this result and taking into account that relation (A15) can be obtained by another method without implying the smallness of $|Sj_B/a_B^h$ (see the body of the paper) one can conclude that in the Gor'kov and Dzyaloshinskii method expression (A15) should be valid in all orders of the perturbation theory.

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