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Strongly correlated fermions with nonlinear energy dispersion and spontaneous generation of anisotropic phases

Daniel G. Barci
Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, Rua São Francisco, Xavier 524, 20550-013, Rio de Janeiro, Brazil

Luis E. Oxman
Departamento de Física, Universidade Federal Fluminense, Av. Litorânea s/n, Boa Viagem, 24210-340, Niterói, Rio de Janeiro, Brazil

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Using the bosonization approach, we study fermionic systems with a nonlinear dispersion relation in dimension $d \geqslant 2$. We explicitly show how the band curvature gives rise to interaction terms in the bosonic version of the model. Although these terms are perturbatively irrelevant in relation to the Landau Fermi-liquid fixed point, they become relevant perturbations when instabilities take place. Using a coherent-state path-integral technique, we built up the effective action that governs the dynamics of the Fermi-surface fluctuations. We consider the combined effect of fermionic interactions and band curvature on possible anisotropic phases triggered by negative Landau parameters (Pomeranchuck instabilities). In particular, we study in some detail the phase diagram for the isotropic/nematic/hexatic quantum phase transition.

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I. INTRODUCTION

One commonly used approximation for studying strongly correlated fermions at low energies is the linearization of the fermion energy dispersion relation near the Fermi surface. For example, its implementation in the context of bosonization leads, together with the renormalization group (RG), to a powerful nonperturbative technique to deal with many-body problems. In one dimension this approximation gives rise to the Tomonaga-Luttinger model,[1] while in $d > 1$ the Landau theory of Fermi liquids comes up as a fixed point in the RG sense.[2–5] In both cases, the bosonized Hamiltonian is quadratic and the model can be solved exactly, while small perturbations can be studied using the RG. In particular, nonlinear terms in the dispersion relation are perturbatively irrelevant, that is, they do not modify the long-wavelength properties of the system.

However, in some cases, the linear approximation must be improved. Induced nonlinear terms in the quasiparticle energy dispersion become important when fermions are coupled to transverse fluctuating gauge fields. This is the case of some models of high-T_c superconductors[6] and gauge theories of the half-filled ($\nu = 1/2$) quantum Hall effect.[7]

More recently, the possibility of having anisotropic ground states driven by spontaneous rotational symmetry breaking (quantum liquid crystals)[8,9] was suggested. These new phases were proposed to describe transport properties of half-filled quantum Hall systems[9] and high-T_c superconductors.[8] They can be associated to Pomeranchuk instabilities[10] of the isotropic Fermi surface and for these novel ground states become stable, it is essential to consider nonlinear fermion dispersion relations.[11] An interesting related phenomenon was also pointed out in Ref. 12, where a Hubbard model is studied using RG techniques. There it was shown that for a certain region of the parameter space, strong forward-scattering interactions favor Pomeranchuk instabilities leading to a breakdown of the discrete rotational symmetry.

With these motivations in mind, we will present a systematic study of the bosonization of fermionic systems with a nonlinear energy dispersion relation in any number of dimensions.

Concerning the Luttinger model case ($d = 1$), the first work, included nonlinear dispersion terms that were taken into account, was carried out by Haldane.[1] He showed that the resulting bosonized Hamiltonian is modified by the addition of nonquadratic terms in the bosonic variables. Explicit corrections to the corresponding one-particle Green function were recently computed in Ref. 13. For dimensions greater than one, the influence of nonlinear dispersion terms on the one-particle Green function was studied by Kopietz in the framework of functional bosonization.[14]

In this work we are interested in studying the dynamics of the Fermi surface in dimension $d \geqslant 2$ and getting an explicit understanding of how the nonlinear dispersion relation could stabilize possible phases, other than the isotropic Fermi liquid one.

The idea of a Fermi surface as a dynamical quantum extended object was originally introduced by Luther[15] and improved by Haldane.[16,17] This concept was developed in great detail by Castro Neto and Fradkin[2,3] and by Houghton and Marston.[4] In Ref. 2, the bosonized theory is written in a coherent state basis $|\phi\rangle$ representing deformations of the Fermi surface. In this way, the quantum dynamics of the system (the partition function) is expressed as a Feynman path integral where the "sum over paths" corresponds to summing up the contributions coming from all possible deformations of the Fermi surface.

The first part of this paper is devoted to apply the above-mentioned formalism to explicitly show how nonlinear terms in the energy dispersion relation contribute with interacting nonquadratic terms in the bosonized action, in arbitrary dimensions. Equation (4.7) below is one of the main results of this paper, showing the effective low-energy Lagrangian of

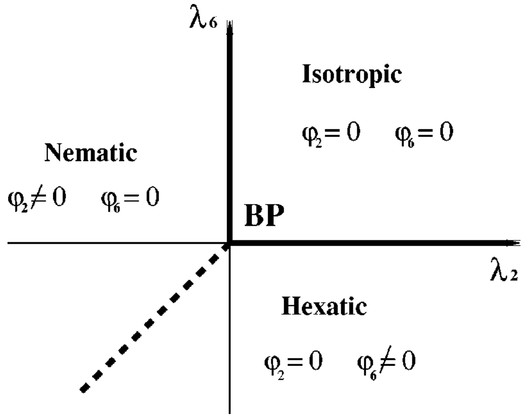

FIG. 1. Phase diagram of an electron gas with nematic and hexatic instabilities. In this case, $|F_n| \ll 1$ for $n \neq 2,6$. The bold and dotted lines represent second- and first-order transitions, respectively. The origin BP is a bicritical point

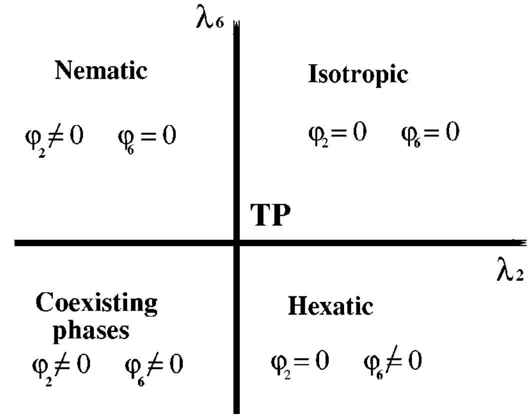

FIG. 2. Phase diagram of an electron gas with nematic and hexatic instabilities. In this case, the Landau parameters are not restricted to small values and $\mu_R > \gamma_R^2/16$. The bold lines represent second-order phase transitions and TB is a tetracritical point

the system. In Eq. (4.7), $\phi_q(\vec{k})$ represents deformations of the Fermi surface at the Fermi point $\vec{k}$ (particle-hole excitations with momentum $\vec{q}$). The second and third derivatives of the dispersion relation lead to the cubic and quartic bosonic terms, respectively. Notice that this general formulation stands for arbitrary smooth Fermi surfaces, being particularly suitable for studying phases where shape deformations are present.

These deformations can be classified according to the symmetries of an order parameter, similar to the classification of classical liquid crystals.[18] In fact, we can think about the quantum equivalent of smectic,[19,20] nematic,[21] or hexatic[11] phases and their corresponding quantum phase transitions.

In Ref. 11, Oganesian *et al.* showed for the first time that quantum isotropic/nematic and isotropic/hexatic phase transitions are possible in systems where the Landau parameters F_2 and F_6 of the usual Fermi-liquid theory[22] take large and negative values [for a definition of F_n's, see Eq. (5.5) below]. In Ref. 22, one important factor to stabilize the anisotropic states is the consideration of a nonlinear energy dispersion relation in the model Hamiltonian. The corresponding electronic properties are very promising, since the quantum nematic and hexatic states seem to present non-Fermi-liquid behavior.[11]

For these reasons, in the second part of this paper we use the nonperturbative bosonization approach to study quantum phase transitions to anisotropic electronic states in two-dimensional systems. In particular, we will concentrate in nematic and hexatic quantum liquid-crystal phases where the order parameter is invariant under π and $\pi/3$ rotations, respectively.

The main result of this paper is displayed by the phase diagrams in Figs. 1 and 2. By integrating out all the stable modes we obtain an effective free energy at zero temperature as a function of the Landau parameters F_2 and F_6. We find different behaviors depending on the relative values of the stable Landau parameters F_n ($n \neq 2,6$). When these parameters are small, the phase diagram has a tricritical point where two second-order phase transitions (isotropic/nematic, isotropic/hexatic) and a first-order one (nematic/hexatic) meet together. However, if the stable F_n's are not small, a coexisting nematic-hexatic phase with a tetracritical point is possible.

In the rest of the paper, we explicitly develop the mathematical details leading to these results. In Sec. II, we present our Hamiltonian model for spinless fermions. In Sec. III, we show how to apply the bosonization method to Hamiltonians with a nonlinear dispersion relation and explicitly compute the corresponding nonquadratic bosonized terms. Then, in Sec. IV, we analyze the Fermi-surface dynamics, building up a coherent-state path-integral formulation for the partition function of the system. Finally, in Sec. V, we analyze the possibility of isotropic-nematic-hexatic quantum phase transitions. Section VI is devoted to a discussion of the results and to the presentation of our conclusions.

II. THE HAMILTONIAN

We consider a fermionic system characterized by a smooth Fermi surface given by the set of Fermi points $\vec{k}_F$ satisfying $\epsilon(\vec{k}_F) = \mu$, where $\epsilon(\vec{k})$ is an arbitrary energy dispersion relation. The one-particle excitations are associated with a set of operators $c^\dagger_{\vec{k}}$ and $c_{\vec{k}}$, creating and destroying a fermion with momentum $\vec{k}$. These operators satisfy the usual fermionic anticommutation relations. For simplicity, we ignore the spin degree of freedom; however, the extension to spinfull fermions is straightforward.

In general, the Hamiltonian can be written in the form

$$H = H_0 + H_{\rm int}. \tag{2.1}$$

The free (quadratic) term is given by

$$H_0 = \sum_{\vec{k}} [\epsilon(\vec{k}) - \mu] c^\dagger_{\vec{k}} c_{\vec{k}}. \tag{2.2}$$

A general two-body interaction term can be written as

$$H_{\rm int} = \frac{1}{2V} \sum_{\vec{k}_F, \vec{k}'_F, \vec{q}} f_{\vec{k}_F, \vec{k}'_F}(\vec{q}) c^\dagger_{\vec{k}_F - (\vec{q}/2)} \times c_{\vec{k}_F + (\vec{q}/2)} c^\dagger_{\vec{k}'_F + (\vec{q}/2)} c_{\vec{k}'_F - (\vec{q}/2)}, \tag{2.3}$$

where $f_{\vec{k}_F,\vec{k}'_F}(\vec{q})$ is the scattering amplitude among two particle-hole pairs with momentum $\vec{q}$ at the Fermi points $\vec{k}_F$ and $\vec{k}'_F$.

In Eq. (2.2), the energy dispersion relation $\epsilon(\vec{k})$ can be expanded in powers of $\vec{q}=\vec{k}-\vec{k}_F$,

$$\epsilon(\vec{k})=\mu+\vec{v}_F\cdot\vec{q}+\frac{1}{2}\left.\frac{\partial^2\epsilon}{\partial k_i\partial k_j}\right|_{\vec{k}=\vec{k}_F}q_iq_j+\frac{1}{3!}\left.\frac{\partial^3\epsilon}{\partial k_i\partial k_j\partial k_l}\right|_{\vec{k}=\vec{k}_F}q_iq_jq_l+\cdots, \qquad (2.4)$$

where $\vec{v}_F=\vec{v}_F(\vec{k}_F)=\vec{\nabla}\epsilon(\vec{k})|_{\vec{k}=\vec{k}_F}$ is the Fermi velocity.

III. THE BOSONIZED HAMILTONIAN

Bosonization is a powerful nonperturbative technique to deal with interacting fermions. In the case of two-dimensional parity breaking systems, it can be implemented in terms of a dual gauge theory (see, for instance, Ref. 23 and references therein). On the other hand, the bosonization of a parity preserving system at finite density can be accomplished by introducing a restricted Hilbert space of small energy particle-hole fluctuations around the Fermi surface. In this case, the general formalism was developed in Refs. 2–5.

In this section, we find a bosonic representation for the Hamiltonian system (2.1) when a general energy dispersion relation [Eq. (2.4)] is considered. In order to establish notation and to make this paper self-contained, we will first summarize the main concepts of bosonization by following Refs. 2 and 3.

We define a reference state $|FS\rangle$ by applying fermionic creation operators to the vacuum state $|0\rangle$ so as to occupy all the states up to the Fermi surface

$$|FS\rangle=\prod_{\vec{k}}^{\vec{k}_F}c^{\dagger}_{\vec{k}}|0\rangle. \qquad (3.1)$$

We use this state to normal order all the relevant operators of the theory according to

$$:\hat{O}:=\hat{O}-\langle FS|\hat{O}|FS\rangle. \qquad (3.2)$$

The low-energy behavior of the system is essentially described in terms of the particle-hole bosonic operator

$$n_{\vec{q}}(\vec{k},t)=c^{\dagger}_{\vec{k}-(\vec{q}/2)}(t)c_{\vec{k}+(\vec{q}/2)}(t), \qquad (3.3)$$

where $\vec{k}\approx\vec{k}_F$ and small $\vec{q}$ fluctuations are restricted to a thin shell around the Fermi surface. In fact, the approximation that defines the restricted Hilbert space of interest can be defined by the condition $q<D<\Lambda\ll k_F$, where D is the shell thickness and Λ is the width of the finite amount of patches used to cover the Fermi surface.[17] These restrictions mean that the physical Hilbert space considered corresponds to a subset of excitations above $|FS\rangle$ mainly generated by small-angle scattering processes.

In this space, the operators (3.3) satisfy the following commutation relation.[2]

$$[n_{\vec{q}}(\vec{k}),n_{-\vec{q}'}(\vec{k}')]=\delta_{\vec{k},\vec{k}'}\delta_{\vec{q},\vec{q}'}\vec{q}\cdot\vec{v}_F\delta(\mu-\epsilon_{\vec{k}}), \qquad (3.4)$$

where due to the last δ function, $\vec{k}$ is constrained to lie on the Fermi surface. For an arbitrary value of $\vec{q}$, the operators $n_{\vec{q}}(\vec{k},t)$ do not annihilate the reference state. However, we can define the operators

$$a_{\vec{q}}(\vec{k}_F)=\sum_{\vec{k}}\frac{\Phi(\vec{k},\vec{k}_F)}{\sqrt{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}}\{n_{\vec{q}}(\vec{k})\theta(\vec{q}\cdot\hat{n})+n_{-\vec{q}}(\vec{k})\theta(-q\vec{q}\cdot\hat{n})\} \qquad (3.5)$$

$$a^{\dagger}_{\vec{q}}(\vec{k}_F)=\sum_{\vec{k}}\frac{\Phi(\vec{k},\vec{k}_F)}{\sqrt{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}}\{n_{-\vec{q}}(\vec{k})\theta(\vec{q}\cdot\hat{n})+n_{+\vec{q}}(\vec{k})\theta(-\vec{q}\cdot\hat{n})\} \qquad (3.6)$$

($\hat{n}$ is a unit vector normal to the Fermi surface at $\vec{k}_F$). The smearing function $\Phi(\vec{k},\vec{k}_F)$ is one, if $\vec{k}$ belongs to the patch labeled by $\vec{k}_F$ and zero otherwise. In the thermodynamic limit, we have

$$\lim_{D,\Lambda\to 0}\Phi(\vec{k},\vec{k}_F)=\delta_{\vec{k},\vec{k}_F}, \qquad (3.7)$$

the local density of states $N(\vec{k}_F)$ is given by

$$N(\vec{k}_F)=\sum_{\vec{k}}|\Phi(\vec{k},\vec{k}_F)|^2\delta(\mu-\epsilon(\vec{k})), \qquad (3.8)$$

and the operators $a_{\vec{q}}(\vec{k}),a^{\dagger}_{\vec{q}}(\vec{k})$ satisfy

$$a_{\vec{q}}(\vec{k}_F)|FS\rangle=0, \qquad (3.9)$$

$$[a_{\vec{q}}(\vec{k}_F),a^{\dagger}_{\vec{q}'}(\vec{k}'_F)]=\delta_{\vec{k}_F,\vec{k}'_F}(\delta_{\vec{q},\vec{q}'}+\delta_{\vec{q},-\vec{q}'}), \qquad (3.10)$$

generating the whole restricted Hilbert space of states. In this space, the fermion operator

$$\psi(\vec{r},\vec{k}_F)=\sum_{\vec{q}}e^{i\vec{q}\cdot\vec{r}}c_{\vec{q}}(\vec{k}_F), \quad q\ll\Lambda \qquad (3.11)$$

can be written in bosonic form as[3]

$$\psi(\vec{r},\vec{k}_F)=\sqrt{\frac{N(\vec{k}_F)}{\alpha}}U(\vec{k}_F)\times\exp\left[-\sum_{\vec{q}}\frac{e^{-i\vec{q}\cdot\vec{r}}}{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}n_{-\vec{q}}(\vec{k}_F)\right], \qquad (3.12)$$

or equivalently, in terms of $a_{\vec{q}}(\vec{k}_F)$,

$$\psi(\vec{r},\vec{k}_F)=\sqrt{\frac{N(\vec{k}_F)}{\alpha}}U(\vec{k}_F)\times\exp\left[-\sum_{\vec{q}\cdot\hat{n}>0}\frac{\{e^{-i\vec{q}\cdot\vec{r}}a^{\dagger}_{\vec{q}}(\vec{k}_F)-e^{i\vec{q}\cdot\vec{r}}a_{\vec{q}}(\vec{k}_F)\}}{\sqrt{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}}\right], \tag{3.13}$$

where α is an ultraviolet cutoff and $U(\vec{k}_F)$ are the "Klein factors" that guarantee anticommutation relations among operators with different $\vec{k}_F$ (for an explicit expression of the Klein factors, see Ref. 4).

Using the fermion-boson mapping (3.13) and the bosonic commutation relations (3.10) we can get the bosonized projection of any fermionic operator onto the restricted Hilbert space of states. For instance, the interacting part of the Hamiltonian [Eq. (2.3)] is simply bosonized, since it can be written in terms of $n_q(\vec{k}_F)$, provided we restrict the momentum space considering only small-angle scattering processes. By normal ordering of the projected Eq. (2.3), we find

$$H_{int}=\frac{1}{2}\sum_{\vec{k}_F,\vec{k}'_F,\vec{q}}F_{\vec{k}_F,\vec{k}'_F}(\vec{q})|\vec{q}\cdot\vec{v}_F|^{1/2}|\vec{q}\cdot\vec{v}'_F|^{1/2}\times\{a^{\dagger}_{\vec{q}}(\vec{k}_F)a_{\vec{q}}(\vec{k}'_F)\Theta(\vec{q}\cdot\vec{k}_F)\Theta(\vec{q}\cdot\vec{k}'_F)+a_{\vec{q}}(\vec{k}_F)a_{\vec{q}}(\vec{k}'_F)\Theta(-\vec{q}\cdot\vec{k}_F)\Theta(\vec{q}\cdot\vec{k}'_F)+\text{h.c.}\}, \tag{3.14}$$

where we have introduced the adimensional Landau function $F_{\vec{k}_F,\vec{k}'_F}(\vec{q})=N^{1/2}(\vec{k}_F)N^{1/2}(\vec{k}'_F)f_{\vec{k}_F,\vec{k}'_F}(\vec{q})$, and Θ is the usual Heaviside function.

The restriction of the Hilbert space is usually justified by RG arguments.[24] The long-angle scattering coupling constants flow to zero at long distances, leaving only a bosonized Hamiltonian that contains small-angle scattering excitations. That is, the long-angle scattering operators are perturbatively irrelevant in the renormalization-group sense. However, they renormalize the parameters in the Hamiltonian. For this reason, the couplings $F_{\vec{k}_F,\vec{k}'_F}(\vec{q})$ in Eq. (3.14) should be considered as phenomenological inputs with no trivial connection with the microscopic ones. This limitation is at the heart of the bosonization procedure. However, this technique gives very general and powerful results concerning phase diagrams and the universal structure of fermionic correlation functions. Of course, to make contact with microscopic models, nontrivial numerical computations are necessary.

Notice that for a fixed $\vec{q}$, the first term in Eq. (3.14) represents interactions among particle-hole pairs in the same hemisphere of the Fermi surface (with respect to the direction of $\vec{q}$), while in the second term the interaction mixes the two hemispheres. This second term does not contribute to the asymptotic fermionic correlation functions,[3] however, we will keep this term, since it could become relevant in the case of nested Fermi surfaces.

The bosonization of the free fermionic Hamiltonian is less trivial. First, let us see what kind of terms appear in H_0 [Eq. (2.2)]. The tensor structure of the Hamiltonian is conveniently written in terms of a local reference frame defined by the unit vectors $\hat{n}$ and $\hat{t}$, normal and tangent to the Fermi surface, respectively. In terms of these directions, it is easy to see that when considering expansion (2.4), the first term in H_0 only contains the normal field derivative

$$\int d^n x\,\psi^{\dagger}(x,\vec{k}_F)\hat{n}\cdot\vec{\nabla}\psi(x,\vec{k}_F). \tag{3.15}$$

For the second-order derivatives of the dispersion relation, we can write

$$\left.\frac{\partial^2\epsilon}{\partial k_i\partial k_j}\right|_{\vec{k}=\vec{k}_F}=\xi_1 n_i n_j+\xi_2 t_i t_j+\xi_3(n_i t_j+n_j t_i), \tag{3.16}$$

where ξ_1, ξ_2, and ξ_3 are functions of $\vec{k}_F$. A similar expression can be written for the third-order rank tensor containing the third-order derivatives. Therefore, the x-space representation of the dispersive part of the Hamiltonian contains normal $(\hat{n}\cdot\vec{\nabla})$ as well as tangential field derivatives $(\hat{t}\cdot\vec{\nabla})$.

In order to obtain the bosonized form of H_0, we consider the point-splitted product of the fermion field and its adjoint along a general direction $\vec{a}$. Using Eq. (3.12) and the Baker-Hausdorff formula

$$e^{\hat{A}}e^{\hat{B}}=:e^{\hat{A}+\hat{B}}:e^{\langle\hat{A}\hat{B}+(1/2)(\hat{A}^2+\hat{B}^2)\rangle}, \tag{3.17}$$

we obtain

$$:\psi^{\dagger}\left(\vec{r}-\frac{1}{2}\vec{a},\vec{k}_F\right)\psi\left(\vec{r}+\frac{1}{2}\vec{a},\vec{k}_F\right):=e^{G(\vec{a},\vec{k}_F)}:e^{-2i\sum_{\vec{q}}\frac{e^{i\vec{q}\cdot\vec{r}}}{N(\vec{k}_F)V\vec{q}\cdot\vec{v}_F}\sin\left(\frac{\vec{q}\cdot\vec{a}}{2}n_{\vec{q}}(\vec{k}_F)\right)}, \tag{3.18}$$

where

$$G(\vec{a},\vec{k}_F)=\sum_{\vec{q}\cdot\hat{n}>0}\frac{(e^{i\vec{q}\cdot\vec{a}}-1)}{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}. \tag{3.19}$$

Let us first consider a direction $\vec{a}$ locally tangent to the Fermi surface $(\vec{a}=\epsilon\hat{t})$. In this case, when summing in Eq. (3.19) over the normal $(q_N=\vec{q}\cdot\hat{n})$ and tangent $(q_T=\vec{q}\cdot\hat{t})$ components, we get

$$G(\epsilon,\vec{k}_F)=\frac{1}{N(\vec{k}_F)Vv_F}\sum_{\vec{q}_T}(e^{i\epsilon\vec{q}_T}-1)\sum_{q_N>0}\frac{1}{|\vec{q}_N|}, \tag{3.20}$$

and considering $\epsilon\vec{q}_T\ll 1$, we obtain

$$G(\epsilon,\vec{k}_F)=-\frac{(\epsilon\Lambda)^2}{2v_F}\int_0^D dq_N\frac{1}{|\vec{q}_N|} \tag{3.21}$$

(for an anisotropic Fermi surface $v_F=|\vec{v}_F|$ is $\vec{k}_F$ dependent). Note that the remaining integral in q_N has a logarithmic infrared divergence, $G(\epsilon,\vec{k}_F)\propto-\ln(V)$, and since the normal order in Eq. (3.18) is a regular function we conclude that in the thermodynamic limit,

$$:\psi^\dagger\left(\vec{r}-\frac{1}{2}\epsilon\hat{t},\vec{k}_F\right)\psi\left(\vec{r}+\frac{1}{2}\epsilon\hat{t},\vec{k}_F\right):=0. \tag{3.22}$$

This result implies that the tangent derivatives do not contribute to the projected bosonized Hamiltonian. In this regard, we recall that the bosonic particle-hole excitations, tangent to the Fermi surface, do not contribute to the asymptotic form of the correlation function;[3] however, they do contribute to the density of states and they are crucial to obtain the correct specific heat and other thermodynamic properties.[2]

We will now concentrate on the bosonization of fermionic terms containing normal derivatives. In this case, we can generalize a calculation proposed by Haldane in one spatial dimension[1] to the case of arbitrary dimensions. Let us consider the integral

$$A=\int d\vec{r}:\psi^\dagger\left(\vec{r}-\frac{1}{2}\epsilon\hat{n},\vec{k}_F\right)\psi\left(\vec{r}+\frac{1}{2}\epsilon\hat{n},\vec{k}_F\right):. \tag{3.23}$$

Introducing Eq. (3.11) into Eq. (3.23) and expanding in powers of ϵ, we find on one hand,

$$A=\sum_n\frac{(-i)^n}{n!}\epsilon^n\sum_{\vec{q}}(\vec{q}\cdot\hat{n})^n:c^\dagger_{\vec{q}}(\vec{k}_F)c_{\vec{q}}(\vec{k}_F):. \tag{3.24}$$

On the other hand, choosing $\vec{a}=\epsilon\hat{n}$ in Eq. (3.18), replacing into Eq. (3.23), and expanding in powers of ϵ we find the bosonic version of A. Thus, comparing these two expressions order by order in ϵ, we find the bosonized projected Hamiltonian. When expanding the dispersion relation up to the third order in the derivatives, we get

$$H_0=\sum_{\vec{k}_F}h_1(\vec{k}_F)+h_2(\vec{k}_F)+h_3(\vec{k}_F) \tag{3.25}$$

with

$$h_1=\frac{v_F}{2}\sum_{\vec{q}\cdot\hat{n}>0}|\vec{q}\cdot\hat{n}|a^\dagger_{\vec{q}}(\vec{k}_F)a_{\vec{q}}(\vec{k}_F), \tag{3.26}$$

$$\begin{aligned}h_2=&\frac{\beta}{2[N(\vec{k}_F)Vv_F]^{1/2}}\sum_{\vec{q}_i}|\vec{q}_1\cdot\hat{n}|^{1/2}|\vec{q}_2\cdot\hat{n}|^{1/2}|\vec{q}_3\cdot\hat{n}|^{1/2}\\&\times\delta(\vec{q}_1+\vec{q}_2+\vec{q}_3)\{a^\dagger_{\vec{q}_1}(\vec{k}_F)a^\dagger_{\vec{q}_2}(\vec{k}_F)a_{\vec{q}_3}(\vec{k}_F)\\&\times\Theta(-\vec{q}_1\cdot\hat{n})\Theta(-\vec{q}_2\cdot\hat{n})\Theta(\vec{q}_3\cdot\hat{n})+\text{h.c.}\},\end{aligned} \tag{3.27}$$

$$\begin{aligned}h_3=&\frac{\gamma}{4!N(\vec{k}_F)Vv_F}\sum_{\vec{q}_i}|\vec{q}_1\cdot\hat{n}|^{1/2}|\vec{q}_2\cdot\hat{n}|^{1/2}|\vec{q}_3\cdot\hat{n}|^{1/2}|\vec{q}_4\cdot\hat{n}|^{1/2}\\&\times\delta(\vec{q}_1+\vec{q}_2+\vec{q}_3+\vec{q}_4)\{4a^\dagger_{\vec{q}_1}(\vec{k}_F)a_{\vec{q}_2}(\vec{k}_F)a_{\vec{q}_3}(\vec{k}_F)a_{\vec{q}_4}(\vec{k}_F)\\&\times\Theta(-\vec{q}_1\cdot\hat{n})\Theta(\vec{q}_2\cdot\hat{n})\Theta(\vec{q}_3\cdot\hat{n})\Theta(\vec{q}_4\cdot\hat{n})\\&+3a^\dagger_{\vec{q}_1}(\vec{k}_F)a^\dagger_{\vec{q}_2}(\vec{k}_F)a_{\vec{q}_3}(\vec{k}_F)a_{\vec{q}_4}(\vec{k}_F)\Theta(-\vec{q}_1\cdot\hat{n})\\&\times\Theta(-\vec{q}_2\cdot\hat{n})\Theta(\vec{q}_3\cdot\hat{n})\Theta(\vec{q}_4\cdot\hat{n})+\text{h.c.}\},\end{aligned} \tag{3.28}$$

where

$$\beta=\frac{\partial^2\epsilon(\vec{k})}{\partial k_j\partial k_i}\hat{n}_i\hat{n}_j|_{\vec{k}=\vec{k}_F}, \tag{3.29}$$

$$\gamma=\frac{\partial^3\epsilon(\vec{k})}{\partial k_l\partial k_j\partial k_i}\hat{n}_i\hat{n}_j\hat{n}_l|_{\vec{k}=\vec{k}_F}. \tag{3.30}$$

The coefficient β is related to the particle-hole asymmetry. In fact, in a system invariant under charge conjugation, $\epsilon(\vec{k}_F+\vec{q})-\mu$ is odd under the transformation $\vec{q}\to-\vec{q}$. In this case, the even derivatives of ϵ at the Fermi level ($\vec{k}=\vec{k}_F$) vanish. For stability reasons, we will consider $\gamma>0$. If γ happens to be negative, then we should continue expanding the fermion dispersion relation until a well-defined result be achieved.

Equations (3.25)–(3.28) display the main results of this section. The h_1 term corresponds to the bosonized free-fermion Hamiltonian when a linear dispersion relation is considered and coincides with the one computed in Refs. 2 and 4. Then, the final bosonized Hamiltonian contains a quadratic part ($\Sigma_{k_F}h_1+H_{\text{int}}$) plus a nonquadratic ($h_2+h_3$) term which is related to dispersive effects on free fermions. In order to calculate any observable, the dispersive part should be treated by means of perturbation theory. In the Fermi-liquid regime, the nonquadratic terms are irrelevant, although interesting effects were studied in the context of the Landau theory.[25]

Here, we are interested in understanding the role played by the presence of dispersion effects when the interacting system is otherwise unstable. The next two sections are devoted to study this issue.

IV. DYNAMICS OF THE FERMI SURFACE

As shown in Ref. 2, the Fermi-surface deformations can be associated to collective particle-hole excitations described by coherent states in the bosonized theory. In the stable case, contact is made with the Landau theory for Fermi liquids. As we will see, this procedure applies equally well to the general case where dispersion effects must be taken into account.

Following Ref. 2, we define a many-body state that is a direct product of coherent states parametrized by a complex field $\phi_{\vec{q}}(\vec{k})$:

$$|\phi\rangle=U(\phi)|FS\rangle, \tag{4.1}$$

where

$$U(\phi)=e^{-\Gamma(\phi)} \tag{4.2}$$

and

$$\Gamma(\phi)=\sum_{\vec{k}_F,\vec{q}\cdot\vec{k}_F>0}\left(\frac{1}{N(\vec{k}_F)V\vec{q}\cdot\vec{v}_F}\right)^{1/2}\{\phi_{\vec{q}}(\vec{k}_F)a^{\dagger}_{\vec{q}}(\vec{k}_F)-\phi^{*}_{\vec{q}}(\vec{k}_F)a_{\vec{q}}(\vec{k}_F)\}. \tag{4.3}$$

The "deformation" fields satisfy $\phi^{*}_{\vec{q}}(\vec{k}_F)=\phi_{-\vec{q}}(\vec{k}_F)$, implying $\Gamma^{*}(\phi)=-\Gamma(\phi)$ and $U^{\dagger}(\phi)=U^{-1}(\phi)$.

The system's partition function can be written in terms of this overcomplete coherent-state basis by means of the path integral

$$Z=\int\mathcal{D}\phi\mathcal{D}\phi^{*}e^{iS(\phi,\phi^{*})}, \tag{4.4}$$

where $S(\phi,\phi^{*})=\int dt\mathcal{L}(\phi,\phi^{*})$ and

$$\mathcal{L}(\phi,\phi^{*})=\sum_{\vec{k}_F,\vec{q}}\frac{i}{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}\left(\phi^{*}_{\vec{q}}(\vec{k}_F,t)\frac{\partial\phi_{\vec{q}}(\vec{k}_F,t)}{\partial t}\right)-\langle\phi|H|\phi\rangle. \tag{4.5}$$

The evaluation of $\langle\phi|H|\phi\rangle$ is straightforward, since the bosonized Hamiltonian [Eq. (3.25)] is normal ordered and the coherent states are eigenvalues of the destruction operator $a_{\vec{q}}(\vec{k}_F)$:

$$a_{\vec{q}}(\vec{k}_F)|\phi\rangle=\frac{[\phi_{\vec{q}}(\vec{k}_F)\Theta(\vec{q}\cdot\vec{k}_F)+\phi^{*}_{\vec{q}}(\vec{k}_F)\Theta(-\vec{q}\cdot\vec{k}_F)]}{\sqrt{N(\vec{k}_F)V|\vec{q}\cdot\vec{v}_F|}}|\phi\rangle. \tag{4.6}$$

Using Eqs. (3.14), (3.25), and (4.6), we find the following Lagrangian [after the field redefinition $\phi\rightarrow(VN(\vec{k}_F)v_F)^{1/2}\phi$]:

$$\begin{aligned}\mathcal{L}=&\sum_{\vec{q}\vec{k}}\left(\frac{i}{|\vec{q}\cdot\hat{n}|}\right)\phi^{*}_{\vec{q}}(\vec{k},t)\frac{\partial\phi_{\vec{q}}(\vec{k},t)}{\partial t}\\&+\frac{1}{2}\sum_{\vec{q}\vec{k}\vec{k}'}\phi^{*}_{\vec{q}}(\vec{k})\{v_F\delta_{\vec{k},\vec{k}'}+F_{\vec{k},\vec{k}'}(\vec{q})\sqrt{v_Fv'_F}\}\phi_{\vec{q}}(\vec{k}')\\&+\mathrm{Re}\left(\frac{\beta}{3!}\sum_{kq_i}\phi_{q_1}(k)\phi_{q_2}(k)\phi_{q_3}(k)\delta(q_1+q_2+q_3)\right.\\&\left.+\frac{\gamma}{4!}\sum_{kq_i}\phi_{q_1}(k)\phi_{q_2}(k)\phi_{q_3}(k)\phi_{q_4}(k)\delta(q_1+\cdots+q_4)\right),\end{aligned} \tag{4.7}$$

where $\vec{k}$ and $\vec{k}'$ lie on the Fermi surface, and we have absorbed powers of $N(0)Vv_F$ (Ref. 26) into the definition of β and γ.

The Lagrangian in Eq. (4.7) controls the low-energy dynamics of the Fermi surface. Note that this dynamics is extremely nonlocal due to the factor $1/|q\cdot\hat{n}|$ in the kinetic term. In the region where the quadratic term is stable (positive definite), the first two lines represent the usual Landau theory of Fermi liquids. This is a fixed point in the RG sense, that is, the nonquadratic dispersive terms do not modify the asymptotic correlation functions. However, if interactions are such that the quadratic term is unstable, the nonquadratic terms become relevant, thus stabilizing the theory in another fixed point.

A word of caution is necessary to understand the validity of Eq. (4.7). According to the renormalization-group theory, not only the couplings $F_{\vec{k},\vec{k}'}(\vec{q})$ but also the parameters β and γ will be renormalized by irrelevant operators. Therefore, although we showed how the band curvature generates the nonquadratic terms in Eq. (4.7) [see Eqs. (3.29) and (3.30)], the actual calculation of these parameters for a given band structure is not at all trivial.

Also, in the present paper we are interested in understanding how the band curvature can drive the system to a new rotational symmetry-breaking ground state. For this reason, in Eq. (4.7), we have only considered small-angle scattering processes with small momentum transfer, disregarding any irrelevant term coming from the interaction Hamiltonian. On the other hand, it is well known that some interaction channels, although irrelevant, could give rise to different instabilities. For instance, the Kohn-Luttinger instability[27] comes from the competition of forward scattering with a BCS channel, even for repulsive interactions. Recently, a similar dynamical effect in two-dimensional Fermi liquids was reported in Ref. 28. The interplay among the anisotropic phases studied here and other interaction channels is a very interesting issue, and the multidimensional bosonization technique described in this paper seems to be a promising tool to handle it.

With these comments in mind, we can study, for instance, the static deformations of the Fermi surface described by the system's free energy, which can be computed as the action per unit time, when setting to zero the kinetic term in Eq. (4.7). Introducing the Fourier-transformed field

$$\varphi(x,\vec{k})=\int\frac{d^dq}{(2\pi)^d}\phi_{\vec{q}}(\vec{k})e^{-i\vec{q}\cdot\vec{x}} \tag{4.8}$$

at each point of the Fermi surface, the expression for the free energy is simplified to

$$F=\frac{1}{2}\int dxdx'\sum_{\vec{k}\vec{k}'}\varphi(x,k)M(\vec{k}-\vec{k}',x-x')\varphi(x',k')+\sum_{\vec{k}}\int dx\left\{\frac{\beta}{3!}\varphi(x,k)^3+\frac{\gamma}{4!}\varphi(x,k)^4\right\}, \tag{4.9}$$

where

$$M(\vec{k}-\vec{k}',x-x') =\{v_F\delta_{\vec{k},\vec{k}'}\delta(x-x')+F_{\vec{k},\vec{k}'}(x-x')\sqrt{v_Fv'_F}\}. \quad (4.10)$$

In the following section, using the framework provided by bosonization, the problem of spontaneously broken rotational invariance is explored.

V. THE ISOTROPIC-NEMATIC-HEXATIC QUANTUM PHASE TRANSITIONS

The order parameter of the two-dimensional nematic phase is a second-rank traceless antisymmetric tensor. It is odd under $\pi/2$ spatial rotations, and the two independent components can be cast in the form of a ‘‘headless’’ two-dimensional vector called the ‘‘director.’’ The director is invariant under a π rotation, characterizing in this way the nematic state.[18] On the other hand, a hexatic phase is characterized by an order parameter that is invariant under $\pi/3$ rotations. These states are of great interest, since they are candidates to present non-Fermi-liquid behavior in two dimensions.[11]

By means of the bosonization approach, we will study them as particular instabilities of an isotropic Fermi surface. The important role played by the fermionic interactions as well as the dispersion effects for the establishment of these phases will become explicit.

Let us begin by considering an isotropic Fermi surface in two dimensions and a short-ranged interaction. In this case, the Landau function only depends on the angle between the two Fermi vectors $\vec{k}$ and $\vec{k}'$:

$$F_{\vec{k},\vec{k}'}(x-x')=F_{\theta-\theta'}\delta(x-x'). \quad (5.1)$$

With these considerations, the ground state of Eq. (4.9) becomes homogeneous, $\langle\varphi(x,k)\rangle=\varphi(\theta)$, and the free energy is simplified to

$$F=\frac{v_F}{2}\int d\theta d\theta'\varphi(\theta)\{\delta_{\theta-\theta'}+F(\theta-\theta')\}\varphi(\theta') +\int d\theta\left\{\frac{\beta}{3!}\varphi(\theta)^3+\frac{\gamma}{4!}\varphi(\theta)^4\right\}. \quad (5.2)$$

Now, introducing the representation

$$\varphi(\theta)=\sum_n \varphi_n e^{in\theta}, \quad (5.3)$$

$$F(\theta)=\sum_n F_n e^{in\theta}, \quad (5.4)$$

($\varphi_n^*=\varphi_{-n}$) and considering a particle-hole symmetric system (β=0), we obtain

$$F=\frac{v_F}{2}\sum_n (1+F_n)\varphi_n\varphi_n+\frac{\gamma}{4!}\sum_{nmlp}\varphi_n\varphi_m\varphi_l\varphi_p\delta_{n+m+l+p}. \quad (5.5)$$

In the case where $F_n\geqslant-1$, it is clear that the only minimum of Eq. (5.5) corresponds to $\varphi_n=0$ for all n. This is the usual criterion for the Fermi-liquid stability.[2,10,16,29] However, if for some channel n it happens that $F_n<-1$, then it is possible to develop quantum phase transitions to anisotropic states. Assume, for instance, that we have a ground state characterized by $\varphi_2\neq0$, and $\varphi_n=0$ for all $n\neq2$. In this case, the deformation field would have the form

$$\varphi(\theta)=4\varphi_2\{2\cos^2(\theta)-1\}, \quad (5.6)$$

characterizing a two-dimensional nematic order.[18] This would correspond to a transition where, because of the interactions, the otherwise circular shape of the Fermi surface is deformed into an ellipse.

In general, as we mentioned above, it is interesting to study instabilities in the nematic/hexatic sector. They are expected to occur when F_2 and F_6 are large and negative.[11] This case can be treated by integrating over all the stable modes φ_n, $n\neq2,6$. As a result of this integration, all the relevant terms that are compatible with the symmetry will be generated, obtaining the following effective free energy for the modes ϕ_2 and ϕ_6:

$$F=\lambda_2\varphi_2^2+\lambda_6\varphi_6^2+\frac{\gamma_R}{4}\{\varphi_2^4+\varphi_6^4\}+\mu_R\varphi_2^2\varphi_6^2, \quad (5.7)$$

which is written in terms of renormalized interaction constants $[\lambda_2=\overline{v_F}(1+F_2),\lambda_6=\overline{v_F}(1+F_6)]$. This free energy presents the interesting possibility of quantum phase transitions[30] among isotropic ($\varphi_2=\varphi_6=0$), nematic ($\varphi_2\neq0,\varphi_6=0$), and hexatic states ($\varphi_2=0,\varphi_6\neq0$). There is also the possibility of coexisting nematic-hexatic phases ($\varphi_2\neq0,\varphi_6\neq0$).

It is important to note that while the phase transitions are triggered by the values of F_2 and F_6, the structure of the phase diagram depends on the other Landau parameters F_n ($n\neq2,6$) hidden in γ_R and μ_R. For instance, if $|F_n|\ll1$ (for all $n\neq2,6$), then $\mu_R\approx\gamma_R/2$, and the qualitative phase diagram is shown in Fig. 1. This diagram presents a bicritical point at the origin of the λ_1-λ_2 plane. The bold lines represent second-order phase transitions between the isotropic/nematic and the isotropic/hexatic phases. Moreover, the interphase nematic/hexatic (dotted line in Fig. 1) corresponds to a first-order phase transition.

However, if $F_n\approx1$ there is the possibility of having $\gamma_R^2/16<\mu_R$. In this case, the phase diagram changes qualitatively (see Fig. 2). There is a tetracritical point with four second-order phase transitions. For $\lambda_1<0,\lambda_2<0$, there is a region of coexisting phases, which is absent in the preceding case.

In order to study the electronic properties of these phases, we need to consider the dynamics associated with the Lagrangian, Eq. (4.7), and to evaluate quantum fluctuations around the saddle points found in these sections. This discussion will be presented elsewhere.

VI. DISCUSSION AND CONCLUSIONS

In this work, we have constructed the bosonized action for a general fermionic system having a smooth Fermi surface and a nonlinear energy dispersion relation in higher dimensions. We have shown that the effect of the nonlinear terms in the energy dispersion is that of producing interactions in the bosonized theory. To the best of our knowledge, this is the first explicit generalization to higher dimensions of the well-known analogous result in one dimension.[1]

The Hamiltonian, Eq. (3.25), the corresponding action in the coherent-state basis, Eq. (4.7), and the free energy in Eq. (4.9) are the main results of this paper.

From a physical point of view, dispersion effects are irrelevant when the system is in the normal Fermi-liquid regime, however, they are essential when due to some fermion interactions an instability of the Fermi surface occurs, driving the electronic system outside this regime. In the latter case, the induced nonquadratic terms in the bosonized free energy (4.9) will stabilize the electronic system in a new ground state.

In particular, we have concentrated in two quantum liquid-crystal states: nematic and hexatic. The corresponding phase transitions are triggered by negative values of the Landau parameters F_2 and F_6, associated with the fermion interaction.[11] However, the qualitative structure of the phase diagrams (Figs. 1 and 2) depends on the relative values of all the other ''stable'' Landau parameters. When they are small, the phase diagram has a tricritical point where two second-order phase transitions (isotropic/nematic, isotropic/hexatic) and a first-order one (nematic/hexatic) meet together. In the opposite case, there is the possibility of having a coexisting nematic-hexatic phase with a tetracritical point.

These results were obtained by means of a mean-field saddle-point calculation on the bosonized action. It is clear that some of the global features (especially, the first-order phase transition in Fig. 1) could be modified by quantum fluctuations.

In particular, it would be very interesting to investigate the dynamical electronic properties of these phases. In this regard, it was shown[11] that the Goldstone modes of the nematic phase are damped (except for certain Fermi points dictated by symmetry considerations), while the fermion correlation function shows a non-Fermi-liquid behavior. In order to address these points in the nonperturbative bosonization framework, it is necessary to introduce quantum fluctuations around each saddle point, and employ the Lagrangian, Eq. (4.7), and the fermionic operator (3.13) to compute fermion correlators. We hope to present results on this issue soon.

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can substitute the local density of states $N_{\vec{k}_F}$ by the total density of states $N(0)$. The quotient between these two quantities is just the solid angle on the Fermi surface $S_d = \int d\Omega = N(0)/N_{\vec{k}_F}$ (see Ref. 3, p. 4087).
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