Resonant transmittance through metal films with fabricated and light-induced modulation

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It is shown that the optical transmittance through a periodically modulated metal film is strongly enhanced when an incident wave is in resonance with surface plasmon-polaritons in the film. Analytical equations describing the resonance transmittance, reflectance, and absorptance are derived. The explicit dependence of the transmittance, reflectance, and absorptance on the dielectric permittivity of the film, its thickness, and modulation is obtained and analyzed. The developed approach includes nonlinear effects and describes the case when the film properties depend on the intensity of incident light. A means of inducing and controlling the extraordinary optical transmittance with light itself is proposed and discussed. It is predicted that an optical bistability can occur in a modulated metal film.

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I. INTRODUCTION

A flat metal surface is almost a perfect reflector for electromagnetic waves in the visible region, and the application of metal films as mirrors has a long history. In this paper we show that even a small periodic corrugation could make a metal film semitransparent. A metal film becomes semitransparent at resonant wavelengths, allowing the excitation of electromagnetic waves propagating on the surface of the film.

We begin our consideration with a flat metal surface. In the optical and infrared spectral ranges, the collective excitation of the electron density coupled to the electromagnetic field results in surface plasmon polaritons (SPPs) traveling on the metal surface (see, e.g., Refs. 1–3). The SPPs can be excited when the real part of the metal permittivity, $\varepsilon_m = \varepsilon_m' + i\varepsilon_m''$, is negative, $\varepsilon_m' < 0$, and losses are relatively small, $\kappa = \varepsilon_m''/|\varepsilon_m'| \ll 1$, which is typical for metals in the optical range. First, for simplicity, we set $\kappa = 0$, and denote the negative metal permittivity ε_m as $-n^2$, where n is the magnitude of the refractive index; the role of losses will be considered later.

At the metal-dielectric interface, the SPP is an H wave, with the direction of the magnetic field **H** parallel to the metal surface.[1] In the direction perpendicular to the interface, SPPs exponentially decay in both media. The relation between the frequency ω and wave vector k_p of SPP can be found from the following consideration. We set the metal-dielectric interface as the xy plane, and assume that the SPP propagates in the x direction, with the field **H** directed in the y direction: $\mathbf{H}=\{0,H,0\}$. For simplicity, we also assume that the $z<0$ half-space is a vacuum, with the dielectric constant $\varepsilon = 1$, and neglect, as mentioned, losses for metal in the $z>0$ half-space. We seek solutions for the magnetic field H in the forms

$$H = H_1 = H_0 \exp(ik_p x + \Lambda_0 z), \quad \Lambda_0 = \sqrt{k_p^2 - k^2} \quad \text{for } z<0,$$

$$H = H_2 = H_0 \exp(ik_p x - \Lambda_1 z), \quad \Lambda_1 = \sqrt{k_p^2 + (kn)^2} \quad \text{for } z>0, \tag{1}$$

where $k = \omega/c$ is the wave vector. Thus the boundary conditions, requiring that the tangential components of the magnetic field are continuous at $z=0$, are satisfied: $H_1(x,z=0) = H_2(x,z=0)$. The electric field **E**, found from the Maxwell equation curl $\mathbf{H} = -ik\varepsilon\mathbf{E}$, with $\varepsilon = 1$ for $z<0$ and $\varepsilon = \varepsilon_m = -n^2$ for $z>0$, has components E_x and E_z. The continuity requirement for the tangential component E_x results in the condition

$$\frac{\partial H_1}{\partial z} = -\frac{1}{n^2}\frac{\partial H_2}{\partial z} \tag{2}$$

for $z=0$. At $n>1$, this equation can be satisfied, and it yields the dispersion equation

$$k_p = \frac{kn}{\sqrt{n^2-1}} \tag{3}$$

for the wave vector k_p of the SPP.[1] Note that, for $n>1$, the wave vector k_p is real and $k_p > k$, so that the field [see Eq. (1)] decays exponentially in the metal and vacuum.

The component E_z (perpendicular to the propagation plane) of the electric field in the SPP takes the following values on the metal interface: E_z is equal to $E_z(-0) = -(k_p/k)H_0\exp(ik_p x)$ in the vacuum side of the interface and $E_z(+0) = (k_p/n^2 k)H_0\exp(ik_p x) \neq E_z(-0)$ in the opposite (metal) side. The discontinuity in the electric field is due to the surface charge density

$$\rho(x) = \frac{1}{4\pi}[E_z(+0) - E_z(-0)] = \frac{(1+n^2)}{4\pi n\sqrt{n^2-1}} H_0 \exp(ik_p x), \tag{4}$$

which propagates together with the electric and magnetic fields along the interface.

Thus the SPP is a wave that consists of an electromagnetic field and surface charges coupled together. Since the SPP propagation includes a rearrangement of the electron density,

it is not surprising that its speed $c_p=\omega/k_p=c\sqrt{n^2-1}/n$ is less than the speed of light c. As a result, the SPP cannot be excited by an external electromagnetic wave on a perfectly flat metal surface. When the refractive index n approaches 1 from above (that is the metal dielectric constant $\varepsilon_m\to-1-0$), the SPP velocity c_p vanishes, so that the SPP ''stops'' on the metal surface. In this case, the surface charge diverges as $(n^2-1)^{-1/2}$; so does the normal component of the electric field. This phenomenon is known as plasmon resonance in a thin metal plate. We note that the SPPs can propagate not only on a metal surface but also on surfaces of artificial electromagnetic crystals, for example, wire-mesh crystals[4–7] or specially organized metal-dielectric layers.[8] This can occur because the real part of the *effective* dielectric constant can be negative in these mesastructures. Such excitation of SPPs was observed in two-dimensional superconducting wire networks deposited onto a dielectric substrate.[9]

We now consider SPPs in a metal film with a finite thickness d placed in the $z=0$ plane. There are two kinds of SPPs in a finite film that correspond to symmetric and antisymmetric solutions to the Maxwell equations (with respect to the reflection in the film's principal plane $z=0$). Hereafter, we still use the ''refraction index'' defined as $n=\sqrt{-\varepsilon_m'}$ and neglect losses; it is also supposed that $n>1$. We are interested in the case of a strong skin effect when $\exp(-dkn)\ll1$, so that the field decays exponentially in the film. Then, by applying the same approach for a film of a finite thickness, we find that the propagation of SPPs is determined by the equation

$$k_{1,2}=k_p\left[1\pm\frac{2n^2}{n^4-1}\exp(-dk_pn)\right], \tag{5}$$

where the wave vectors k_1 and k_2 correspond to symmetric and antisymmetric SPPs, respectively, and k_p is defined by Eq. (3). For further consideration it is important to note that the symmetric and antisymmetric SPPs propagate on both sides of the film. Moreover, since both SPPs represent the eigenmodes of the film, the magnitudes of the electric and magnetic fields are the same on the both interfaces. This consideration holds for an arbitrarily thick film, although the difference in speeds for the two SPPs becomes exponentially small for the optically thick films $dk_pn\gg1$. The velocities of symmetric and antisymmetric SPPs are both less than the speed of light, and these SPPs cannot be excited by an external electromagnetic wave incident from a vacuum, because this would violate the momentum conservation.

The situation changes dramatically when the film is periodically modulated. In this case, the EM field inside the film is also modulated. When the spatial period of the modulation coincides with the wavelength of a SPP, it can be excited by an incident em wave. An example of such spatial modulation is a square array of nanoholes punched in a film as in Refs. 10–15. Another example of a regular modulation of the refractive index (which we propose and study in this paper) is the light-induced modulation in the refractive index n, occurring, for example, because of the optical Kerr nonlinearity.

The problem of light interaction with periodically modulated metal surfaces and films has a long history, starting with a discovery of the Wood anomalies. In 1902 Wood reported[16] that the reflection spectrum from a metal diffraction grating drastically changes at some frequencies, which are now known to correspond to the excitation of SPPs.[17–19] Earlier experimental studies, as well as theories based on the perturbation approach, were reviewed in Ref. 2 The first full-scale computer simulation of SPPs that propagate on a metal surface with sinusoidal and sawtooth profiles was performed in Ref. 20. The grating amplitude in this work was relatively small, yet the authors obtained almost a flat dispersion curve $\omega(k_p)$ for the SPP which, as we understand now, could indicate an electromagnetic field concentration in the grooves. The problem of the electromagnetic wave interaction with corrugated metal surfaces was extensively studied in the ''microwave'' literature. In particular, ribbed metal surfaces and waveguides (called also septate waveguides) are shown to support slow electromagnetic modes with the properties similar to SPPs in optics.[21–23] The possibility to decrease losses in waveguides by corrugating their walls (gliding effect), was also considered in Ref. 24 (also see Refs. 25–27).

Recently, a long-standing problem of the existence of modes localized in the subwavelength grooves has attracted a lot of attention (see, e.g., Refs. 28–37 and references therein). Experiments reported in[37] show well-pronounced minima in the specular reflection that can be explained by localization of SPPs in the groves, whose widths are much smaller than the radiation wavelength. The plasmon localization could also occur in the intergrain gaps in metal films, in random metal-dielectric films, and in metal nanocavities, as reported in Refs. 38–40, and Ref. 41, respectively. In Ref. 42 it was predicted that, in the optical frequency range, the local magnetic fields can also be strongly enhanced, along with the electrical fields, in a random array of nanoparticles. It is worth mentioning that the electric and magnetic fields are typically localized in different places of the array. The excitation of SPPs in a long chain of gold nanoparticles was observed in Ref. 43. It is interesting to note that the plasmon excitations were localized in the interparticle gaps in this experiment; this is in contrast with the case of a single particle, where the electric field associated with the plasmon mode has a maximum in the particle. Localization of SPPs in a subwavelength cavity was observed in near-field experiments[44] (also see Ref. 45 and references therein); the waveguiding of SPPs in an array of metal nanostructures was investigated in Ref. 46.

The electromagnetic surface-plasmon resonances attracted a great deal of attention because the enhanced local fields associated with these resonances play an important role in the surface-enhanced Raman scattering and nonlinear optical processes on rough metal surfaces and semicontinuous metal films (see, e.g., Ref. 31 and 7). Another motivation for studying the electromagnetic properties of nanosized features is the possibility to use them for a near-field superresolution readout of the optical disks and subwavelength lithography (see, e.g., Refs. 47–49).

The growing interest in the subwavelength optics of metal films was further boosted by Ebbesen, Lezec, Ghaemi, Thio, and Wolff when they discovered the extraordinary optical transmission through nanohole arrays in optically thick metal

films.[10–15] A review of earlier studies of light interaction with subwavelength holes can be found in Ref. 50. The possibility that an opaque metallic film can be transparent, provided that an incident electromagnetic wave excites the coupled surface plasmons, was demonstrated in early work in Ref. 51. (It is worth noting that in the beginning of the last century Lord Rayleigh pointed out that a flat rigid surface with cylindrical holes can have acoustic resonances at some special depths of the holes.[52])

The electromagnetic field in subwavelength holes, grooves, or slits in a metal film was simulated in many computer experiments after discovering the extraordinary optical transmittance (see, e.g., Refs. 50 and 53–62). In nearly all simulations, the local electromagnetic (EM) field is strongly enhanced in subwavelength apertures, for certain frequencies. This enhancement is considered to be the reason for the enhanced resonant transmittance. Yet distributions for the local EM fields are rather different in simulations performed by different authors. Thus the resonant field concentrates exactly inside deep grooves and slits, according to simulations of Refs. 31,34,36,56 and 57; on the other hand, simulations in Refs. 33 and 53 predict that the resonant field is strongly enhanced in a close vicinity of but outside the subwavelength grooves. According to Ref. 54, the field is concentrated in deep slits but distributed all over the film for more shallow slits. Simulations of Refs. 59 and 62 predicted that the EM field is localized at the edges of subwavelength holes, in the case of the extraordinary optical transmittance. Computer simulations in Ref. 50 predicted a rather broad maximum for the local-field intensity that centers at a nanohole but extends over distances much greater than the hole diameter. According to Ref. 55, where a so-called dynamic diffraction was studied in computer simulations for thin metallic gratings, the local magnetic field is strongly enhanced in some regions outside slits in a metal film; in the slits themselves, the local field has a clear minimum, according to these calculations. Also note that it was stressed in Ref. 55 that the field maxima obtained are different from the SPP. An Analysis of Ebbesen and co-workers experiments[10–15] was performed in Ref. 66, in terms of the diffraction theory, and it also did not invoke the SPP excitation. In the near-field experiments of Ref. 67 the EM field intensity (at $\lambda = 0.9\mu$ m) was measured around a single nanohole and a pair of nanoholes, having a diameter $a = 0.3\,\mu$m and separated by a 2-μm distance from each other. For a single hole, the field was concentrated inside the hole, whereas for the pair of holes a well-defined trace of the EM field was observed between the holes, which was interpreted as a SPP.

Here we also mention a theory of the extraordinary optical transmittance presented in Ref. 15. This theory considers SPPs on both sides of an optically thick metal film, which are connected through the evanescent modes in the nanoholes treated as subwavelength waveguides. The theory qualitatively reproduces the long-wave peak in the extraordinary optical transmittance obtained in one of the Ebbesen *et al.*'s experiments.[14] The theory of Ref. 15 also predicted that the transmittance of a thin lossless metal film has two asymmetric maxima that merge together with an increase of the film thickness; eventually, there is a single peak that becomes progressively smaller with a further increase of the film thickness. Results of the present paper are is in qualitative agreement with this behavior of the transmittance.

Yet another theory for light transmission through a subwavelength periodic hole array was developed in Ref. 58. This theory also attributed the resonant transmittance to the coupling of SPPs through the nanoholes; it predicted that this coupling exists only for a metal with a finite conductivity. Therefore, according to theory,[58] the transmittance should vanish when all parameters of the system are fixed and the metal conductivity goes to infinity. This result is in contradiction with results of theory.[15]

Theories of Refs. 15 and 58 not only contradict one another, they both disagree with recent computer simulations and qualitative considerations performed in Refs. 60 and 61 (where the extraordinary transmittance through a periodical array of subwavelength slits was considered). The authors of Refs. 60 and 61 attributed the transmittance to the *internal* plasmon modes of the slits, and arrived at the conclusion that the SPP excitation ''is not the prime mechanism responsible for the extraordinary transmission of subwavelength metallic gratings with very narrow slits.'' Moreover, according to Refs. 60 and 61, the SPP may play a negative role in the transmission at some special conditions.

In Refs. 42 and 63 it was concluded that the extraordinary transmittance could result from the excitation of internal modes in holes punched in a metal film. Then this effect would not depend on the periodicity of the hole array. However, the radiative dumping of the internal modes was not taken into account in this consideration. The light emission and the excitation of SPPs can damp the resonances of the internal modes. Therefore, results of Refs. 42 and 63 for the transmittance through randomly distributed holes should be considered as the upper limit. Note that in a regular array of holes, the emission of SPPs can be a reversible process since SPPs can be converted back to the internal modes, when the proper phase conditions are fulfilled. The importance of the SPP excitation around a hole was demonstrated in recent experiments,[64,65] where the light transmittance through a hole surrounded by a system of periodic ring grooves was investigated. When the period of these rings is approximately equal to the wavelength of SPP, the electric field in a vicinity of the hole increases significantly. We speculate that the system of ring grooves operates as a band-gap material for the SPP radiated from the hole. The rings effectively confine the SPP near the hole. This confinement results in an enhancement of the light transmittance through the hole by more than an order of magnitude in comparison with a ''bare'' hole.[64] It is interesting to note that direct experimental measurements[65] show that, at resonance, the local EM field concentrates in a close proximity to the hole when it is surrounded by the system of rings.

As mentioned above, Ref. 42 considered the transmittance in a system of holes punched in an otherwise impenetrable light film. In Ref. 42, we also reported preliminary results for the extraordinary transmittance though a metal film with a periodically modulated refractive index. In this case, the enhancement in the transmittance was attributed to the excitation of SPPs that propagate on both interfaces of the film.

Among most recent publications describing a system similar, to some extent, to that considered in the present paper, we mention computer simulations of Refs. 57 and 68. The authors considered an optically thick metal film with a periodical set of symmetric conical[57] and sinusoidal[68] grooves on both side of the film. The grooves were placed in such a way that their bottoms were opposite to each other (in Ref. 68, the gratings on the two sides shifted for a half period with respect to each other were also considered). At certain frequencies, the electromagnetic wave incident on the film causes the resonant enhancement for the field inside the grooves. The excitation, which the authors of Ref. 57 treated as standing (localized) SPPs, tunnels through the metal film from the bottom of the illuminated grooves to the opposite side of the film and excites there the standing SPPs. Thus resonance transmittance through the otherwise optically thick metal film can occur. The transmittance vanishes exponentially fast when the distance between the bottoms of the grooves in the metal film exceeds 100 nm. Although results of simulations in Refs. 57 and 68 gave a qualitatively similar physical picture, the actual transmittance spectra obtained are rather different. In Ref. 57 a double-peak maximum in the transmittance was predicted for a sufficiently small distance between the grooves on the two sides, whereas simulations of Ref. 68 predicted one maximum in the transmittance, even when the bottoms of the grooves touch each other. Experiments performed in Ref. 68 for corrugated gold films also showed only one maximum in the transmittance.

Finally, we mention experimental results by Schroter and Heitmann[69] and their computer simulations. The authors investigated the optical transmittance and reflectance for a modulated silver film with the average thickness 50 nm and a modulation amplitude of 30–35 nm. The transmittance (reflectance) observed has a double-peak maximum (minimum) structure corresponding to the excitation of the SPP on the surface of the film. We note that theoretical results reported in the present paper are in good qualitative agreement with experiments of Ref. 69.

In this extended literature survey, we did not try to analyze all numerous papers considering the exciting phenomenon of the extraordinary optical transmittance. Still, it can be concluded from the examples presented above that there is as yet no generally accepted quantitative theory for this important phenomenon. Therefore, a simple model, which (a) reproduces main features of the extraordinary transmittance and (b) provides us with an explicit analytical solution, can serve as a starting point in considering behaviors of more complicated systems. In this paper, we suggest such a model, and find its analytical solution. The obtained extraordinary transmittance has a behavior, which is in qualitative agreement with results of Refs. 15 and 57.

In our paper, we adopt an approach where the film modulation is assumed to be small. This approach allows us to develop an *analytical theory* for the extraordinary optical transmittance that has certain advantages over numerical simulations because of its generic nature. A detailed analysis performed allows us to find the resonance conditions under which a modulated film becomes semitransparent. We also extend our consideration to the case when nonlinear optical effects are essential. Specifically, we consider a dependence of the metal dielectric function on the intensity of the SPP (which is strongly enhanced in the resonance) so that the SPP can further increase an initial (''seed'') modulation in the film's refractive index and by this means the film transmittance. We also suggest the possibility of the *light-induced* extraordinary optical transmittance through a thick metal film.

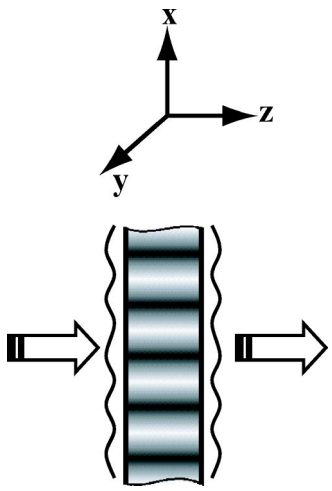

FIG. 1. Light incident on a modulated film. The light first excites the surface plasmon-polariton (SPP) on the front interface of the film, which then couples to the SPP on the back interface; the backside SPP is eventually converted into the transmitted light.

When considering the transmittance we suppose, for simplicity, that the electromagnetic wave is incident normally to the film, as sketched in Fig. 1. The refractive index of the film is spatially profiled, with the period $a=2\pi/q$, where q is the spatial wave vector of the film modulation. The modulation can be either prefabricated or induced by the impinged light due to the optical nonlinearity in the film material. When the frequency of an incident wave is such that SPP wavelengths $\lambda_{1,2}=2\pi/k_{1,2}$ [where $k_{1,2}$ are given by Eq. (5)] coincide with the period of the modulation a, the SPPs are excited in the film. Since the film is optically thick the SPP is excited first on the front interface of the film. Yet, eventually, it ''spreads'' over the other side of the film, so that SPPs on both front and back interfaces of the film are eventually excited. There is a straightforward analogy between the two SPPs on the opposite sides of the film, and two identical coupled oscillators. The coupling between two oscillators can be arbitrary weak; nevertheless, if we push the first oscillator then, in some time (which depends on the coupling strength), the second oscillator starts to oscillate with the same amplitude as the first one. By the same token, the two SPPs on different sides of the film will eventually have the same amplitude. When a SPP propagates on the back side of the film, it interacts with the film modulation and, as a result, converts its energy back to the plane wave re-emitted from the film. Therefore, at the resonance, the film becomes almost transparent, regardless of its thickness; however, the width of the transmittance resonance shrinks when the film thickness d increases.[42] Note that in the speculations above the amplitude $g\simeq\Delta n/n$ of the film modulation does not play

any role. The modulation g can be arbitrary small, yet the SPPs are excited and the film may become transparent.

Moreover, we do not even need the presence of holes, for the resonant transmittance to occur. The only thing needed is that both sides of a metal film are modulated with the same spatial period. The minimum of the modulation required for the extraordinary transmittance depends on losses in metal, which can be relatively small (in particular, when the skin effect is strong). Note that the transmittance maxima have typically a double-peak structure corresponding to the excitation of symmetric and antisymmetric SPPs. In the next two sections we consider in detail a theory for the resonant light transmittance through linear and nonlinear metal films.

II. RESONANT TRANSMITTANCE THROUGH THIN FILMS

A periodical modulation in a film can be represented by the Fourier series. The resonant transmittance takes place when the frequency of an incident wave is such that one of SPP wave vectors $k_{1,2}$ is equal to the wave vector q of a spatial harmonic of the decomposition. The resonance interaction of a SPP with the q-th spatial harmonic results in the enhanced transmittance. Since other spatial harmonics are off resonance, their amplitudes are small, so that we can consider the interaction of the incident wave with the resonant spatial mode only. As mentioned, we suppose that the magnetic field **H** in the incident wave has only a y component $\mathbf{H}=\{0,H,0\}$, and consider the interaction of the incident wave with a metal film having a dielectric constant that varies as $\varepsilon(\mathbf{r})=-n^2[1+g\cos(qx)]$, where the amplitude of the modulation is small, $g\ll 1$ (see Fig. 1). In actual calculations it is convenient to use a slightly different equation for the modulated dielectric constant

$$\varepsilon(\mathbf{r})=-n^2[1-g\cos(qx)]^{-1}, \tag{6}$$

which is equivalent with the one above provided that $g\ll 1$. For a plane electromagnetic wave propagating normal to the film (along the z axis), the amplitude depends on z only. In the course of the interaction with the film modulation [Eq. (6)], an electromagnetic harmonic is generated, which is proportional to $\cos(qx)$. The amplitude of this harmonic is proportional to the small film modulation g. This harmonic, in turn, also interacts with the film modulation and converts into $\cos(2qx)$ harmonic, etc. In this cascade process, the whole spectrum of the electromagnetic waves is excited in the film, when the incident plane wave interacts with the film modulation. The amplitudes of the $\cos(2qx)$ harmonic are proportional to g^2; the $\cos(3qx)$ harmonics are proportional to g^3, etc. The resonant transmittance occurs when these harmonics are converted back to the plane wave transmitted through the film. We are interested in the electromagnetic harmonics that can be converted back to the plane wave in such a way that this optical process is proportional to the lowest possible order in the decomposition over the modulation g. Therefore, we restrict our consideration to the $\cos(qx)$ harmonic and consider the magnetic field in the following form $H_y(x,z)=H(z)+H_q(z)\cos(qx)$, where $H(z)$ and $H_q(z)$ are two unknown functions that determine the electromagnetic field inside and outside the film. Thus we neglect the generation of $\cos(lq)$ harmonics with l higher than one.

To find electric **E** and magnetic **H** fields inside the film we substitute the magnetic field

$$\mathbf{H}=\{0,\ H(z)+H_q(z)\cos qx,\ 0\} \tag{7}$$

and modulated dielectric constant [Eq. (6)] in the Maxwell equations, and equate the terms that have the same dependence on the "x" coordinate.[70] Neglecting the generation of higher harmonics, we find a system of two differential equations,

$$H(z)''-(kn)^2H(z)-\frac{g}{2}H_q(z)''=0,$$

$$H_q(z)''-[(kn)^2+q^2]H_q(z)-gH(z)''=0, \tag{8}$$

that determines the fields inside the film. The solution to these equations has the form

$$\begin{Bmatrix} H(z) \\ H_q(z) \end{Bmatrix} = A\ \mathrm{sech}\left(\frac{dkn}{2}\right)[X_1\cosh(knz\Lambda_1) - X_2\sinh(knz\Lambda_1)] + B\ \mathrm{sech}\left(\frac{dnq}{2}\right) \times[Y_1\cosh(knz\Lambda_2)-Y_2\sinh(knz\Lambda_2)], \tag{9}$$

where Λ_1 and Λ_2 are are dimensionless eigenvalues,

$$\Lambda_1=\sqrt{\frac{2-Q+q_1^2}{2-g^2}}, \qquad \Lambda_2=\sqrt{\frac{2+Q+q_1^2}{2-g^2}}, \tag{10}$$

$$Q=\sqrt{q_1^4+2g^2(1+q_1^2)}, \qquad q_1=\frac{q}{kn}, \tag{11}$$

and A and B in Eq. (9) are eigenvectors for Eqs. (8) that are equal to

$$A=\begin{Bmatrix} \dfrac{Q+q_1^2-g^2(1+q_1^2)}{(2-g^2)q_1^2} \\ g\dfrac{2-Q+q_1^2}{(2-g^2)q_1^2} \end{Bmatrix}, \qquad B=\begin{Bmatrix} -g\dfrac{2+Q+q_1^2}{2(2-g^2)q_1^2} \\ \dfrac{g^2+Q+q_1^2}{(2-g^2)q_1^2} \end{Bmatrix}, \tag{12}$$

with Q and q_1 given by Eqs. (11). The electric field **E** inside the film is obtained by substituting the magnetic field given by Eqs. (7) and (9) into Maxwell equations and neglecting the spatial harmonics $\sim\cos(lqx)$ and $\sim\sin(lqx)$ with $l>1$. Then it acquires the following form:

$$\mathbf{E}=\{E(z)+E_q(z)\cos qx,\ 0,\ E_z(z)\sin qx\}. \tag{13}$$

For further consideration, we need an explicit form of $E(z)$, and $E_q(z)$, which is given by

$$\begin{Bmatrix} E(z) \\ E_q(z) \end{Bmatrix} = U \cdot A\Lambda_1 \mathrm{sech}\left(\frac{dkn}{2}\right)[X_1 \sinh(knz\Lambda_1) - X_2 \cosh(knz\Lambda_1)] + U \cdot B\Lambda_2 \mathrm{sech}\left(\frac{dnq}{2}\right) \times [Y_1 \sinh(knz\Lambda_2) - Y_2 \cosh(knz\Lambda_2)], \quad (14)$$

with the matrix U defined by

$$U = \frac{i}{n}\begin{Bmatrix} 1 & g/2 \\ g & 1 \end{Bmatrix}, \quad (15)$$

and vectors A and B given by Eq. (12).

It follows from Eqs. (7), (9), (13), and (14) that the fields inside the film are fully determined by the two pairs of constants, namely, $\{X_1, X_2\}$ and $\{Y_1, Y_2\}$. For a flat film, $g=0$, so that the eigenvalues Λ_1 and Λ_2 in Eqs. (9) and (14) are equal to $\Lambda_1 = 1$ and $\Lambda_2 = (kn)^{-1}\sqrt{k^2 n^2 + q^2}$, while the eigenvectors A and B [see Eq. (12)], and the matrix U [see Eq. (15)] acquire the following values $A=\{1,0\}$, $B=\{0,1\}$, and $U = i\{\{1,0\},\{0,1\}\}/n$. Therefore, the constants $\{X_1, X_2\}$ in Eqs. (9) and (14) correspond to the fundamental beam, whereas the other two constants, $\{Y_1, Y_2\}$, describe the $\cos(qx)$ mode.

The magnetic field in the incident and reflected waves can be represented as $H(z) = H_0 \exp(ikz) + rH_0 \exp(-ikz)$, $z \leqslant -d/2$, where H_0 is the amplitude of the incident wave, r is the reflection coefficient and $R = |r|^2$ is the reflectance of the film. The field in the transmitted wave has the form $H(z) = tH_0 \exp(ikz)$, $(z \geqslant d/2)$, where $T = |t|^2$ is the transmittance. For the q mode we use the radiation boundary conditions, namely, $H_q(z) = Y_3 H_0 \exp[-i\sqrt{k^2 - q^2}(z + d/2)]$, for $z \leqslant -d/2$, and $H_q(z) = Y_4 H_0 \exp[i\sqrt{k^2 - q^2}(z - d/2)]$ for $z \geqslant d/2$, where Y_3 and Y_4 are some constants. Note that for the resonant transmittance the wave vector k is less than the modulation vector q, which is given by one of the two SPP wave vectors $k_{1,2} > k$, so that the field H_q decays exponentially outside the film.

Thus we obtain that the electromagnetic field in the whole space is completely determined by vector $X = \{X_1, X_2, X_3, X_4\}$, where we choose

$$X_3 \equiv r \exp(ikd/2), \quad X_4 \equiv t \exp(ikd/2), \quad (16)$$

and vector $Y = \{Y_1, Y_2, Y_3, Y_4\}$. We set the amplitude of the incident wave $H_0 = \exp(ikd/2)$ so that the electric, $\mathbf{E}_{rf}$, and magnetic, $\mathbf{H}_{rf}$, fields before the film, take the following forms

$$\mathbf{E}_{rf} = \begin{Bmatrix} e^{ik(z+d/2)} - X_3 e^{-ik(z+d/2)} - Y_3 \dfrac{\sqrt{k^2 - q^2}}{k} \cos(qx) e^{-i\sqrt{k^2 - q^2}(z+d/2)} \\ 0 \\ -iY_3 \dfrac{q}{k} \sin(qx) e^{-i\sqrt{k^2 - q^2}(z+d/2)} \end{Bmatrix},$$

$$\mathbf{H}_{rf} = \begin{Bmatrix} 0 \\ e^{ik(z+d/2)} + X_3 e^{-ik(z+d/2)} + Y_3 \cos(qx) e^{-i\sqrt{k^2 - q^2}(z+d/2)} \\ 0 \end{Bmatrix}, \quad z \leqslant -d/2. \quad (17)$$

The electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ inside the film $(-d/2 \leqslant z \leqslant d/2)$ are given by Eqs. (13), and (14), and (7) and (9), respectively. The fields behind the film, i.e., transmitted fields, are equal to

$$\mathbf{E}_{tr} = \begin{Bmatrix} X_4 e^{ik(z-d/2)} + Y_4 \dfrac{\sqrt{k^2 - q^2}}{k} \cos(qx) e^{i\sqrt{k^2 - q^2}(z-d/2)} \\ 0 \\ -iY_4 \dfrac{q}{k} \sin(qx) e^{i\sqrt{k^2 - q^2}(z-d/2)} \end{Bmatrix},$$

$$\mathbf{H}_{tr} = \begin{Bmatrix} 0 \\ X_4 e^{ik(z-d/2)} + Y_4 \cos(qx) e^{i\sqrt{k^2 - q^2}(z-d/2)} \\ 0 \end{Bmatrix}, \quad z \geqslant d/2. \quad (18)$$

We now match the x (y) component of the electric (magnetic) fields $\mathbf{E}_{rf}$ ($\mathbf{H}_{rf}$) and the corresponding components of the inside fields $\mathbf{E}$ ($\mathbf{H}$) at $z = -d/2$. Then we equate the terms with the same dependence on the x coordinate. Thus we obtain four linear equations connecting X_1, X_2, Y_1, Y_2, and X_3, Y_3. We repeat the matching for fields $\mathbf{E}_{tr}$, $\mathbf{H}_{tr}$ and $\mathbf{E}$, $\mathbf{H}$ at the plane $z = d/2$ and obtain other four linear equations connecting, this time, X_1, X_2, Y_1, Y_2, and X_4, Y_4. Thus the eight equations obtained for the components of vectors X and Y can be written in matrix form as

$$\hat{H} \cdot X + g\ \hat{G}_1 \cdot Y = Z, \quad \hat{H}_q \cdot Y + g\ \hat{G}_2 \cdot X = 0, \quad (19)$$

where vector Z is proportional to the amplitude of the incident wave H_0, which we chose above to be equal to $H_0 = \exp(ikd/2)$ so that

$$Z = \{1,1,0,0\}; \quad (20)$$

the matrices $\hat{H}$, $\hat{H}_q$, $\hat{G}_1$, and $\hat{G}_2$ are 4×4 matrices, whose explicit forms will be discussed below. At this point we note that all the matrices remain finite at $g\to0$.

In the limit of vanishing modulation g the matrix $\hat{H}$ is represented by

$$\hat{H}=\begin{Bmatrix} 1 & \tanh\left(\frac{dkn}{2}\right) & -1 & 0 \\ -\frac{i}{n}\tanh\left(\frac{dkn}{2}\right) & -\frac{i}{n} & 1 & 0 \\ 1 & -\tanh\left(\frac{dkn}{2}\right) & 0 & -1 \\ \frac{i}{n}\tanh\left(\frac{dkn}{2}\right) & -\frac{i}{n} & 0 & -1 \end{Bmatrix}. \tag{21}$$

When $g=0$, the first of Eqs. (19) reduces to $\hat{H}\cdot X=Z$ and it gives the well known results for the reflectance, $R=|X_3|^2$, and for the transmittance, $T=|X_4|^2$, of uniform metal films,

$$R=\left|\frac{1+n^2}{m^2-2in\coth(dkn)}\right|^2,$$

$$T=\left|\frac{2n}{2n\cosh(dkn)+im^2\sinh(dkn)}\right|^2, \tag{22}$$

where

$$m=\sqrt{n^2-1} \tag{23}$$

(see, e.g., Ref. 1). When $g=0$, the second of Eqs. (19) reduces to $\hat{H}_q^{(0)}(k)\cdot Y=0$, where $\hat{H}_q^{(0)}(k)=\hat{H}_q(k,\ g=0)$ and we show explicitly the dependence of matrix $\hat{H}_q^{(0)}$ on the wave vector k. The equation $\hat{H}_q^{(0)}(k)\cdot Y=0$ has a nontrivial solution when the determinant of $\hat{H}_q^{(0)}(k)$ is zero. The condition $\det[\hat{H}_q^{(0)}(k)]=0$ gives the dispersion equation $q(k)$ for the SPP in a flat metal film that coincides with Eq. (5).

For a nonzero film modulation g, the solution to Eqs. (19) can be written in the following form

$$X=(\hat{H}-g^2\ \hat{G}_1\cdot\hat{H}_q^{-1}\cdot\hat{G}_2)^{-1}Z. \tag{24}$$

Although the second term in the brackets $\propto g^2$ it cannot be neglected, even for $g\to0$; this is because $\hat{H}_q^{-1}$ is a singular matrix which can be very large at the resonance. In the considered case of a strong skin effect, when $\zeta=\exp(-dkn)\ll1$, the matrix $\hat{H}_q^{-1}$ can be written in the following form

$$\hat{H}_q^{-1}=\frac{1}{D_1}\begin{Bmatrix} -1 & im & -1 & -im \\ 0 & 0 & 0 & 0 \\ -1 & im & -1 & -im \\ -1 & im & -1 & -im \end{Bmatrix} + \frac{1}{D_2}\begin{Bmatrix} 0 & 0 & 0 & 0 \\ -1 & im & 1 & im \\ -1 & im & 1 & im \\ 1 & -im & -1 & -im \end{Bmatrix} + \hat{H}_{q,reg}^{-1}, \tag{25}$$

where

$$D_{1,2}=\frac{2m(1+n^2)\Delta}{n}\pm4\zeta-\frac{g^2n^2}{2}, \tag{26}$$

$$\Delta=k/q-m/n,\qquad \zeta=\exp(-dnq). \tag{27}$$

Equations $D_{1,2}=0$ for the singularities of matrix $\hat{H}_q^{-1}$ coincide with the dispersion equations (5) for the flat metal film where $g=0$. Note, that Eq. (27) for the dimensionless detuning from the resonance Δ can be written in the form

$$\Delta=\frac{a}{\lambda}-\frac{\sqrt{n^2-1}}{n}, \tag{28}$$

where λ is the wavelength of the incident light and, a is the period of the film modulation.

Below we neglect, for simplicity, the regular part $\hat{H}_{q,reg}^{-1}$ of matrix $\hat{H}_q^{-1}$ since it is proportional to g^2. We also neglect terms proportional to g^2 in matrices $\hat{H}$, $\hat{G}_1$ and $\hat{G}_2$, so that they are given by Eq. (21) and by the equations

$$\hat{G}_1=\begin{Bmatrix} -n^2/2 & -n^2/2 & 0 & 0 \\ im/2 & im/2 & 0 & 0 \\ -n^2/2 & n^2/2 & 0 & 0 \\ -im/2 & im/2 & 0 & 0 \end{Bmatrix},$$

$$\hat{G}_2=\begin{Bmatrix} m^2 & m^2 & 0 & 0 \\ -in & -in & 0 & 0 \\ m^2 & m^2 & 0 & 0 \\ in & -in & 0 & 0 \end{Bmatrix}, \tag{29}$$

where m is given by Eq. (23). Note that in derivation of Eqs. (29) we still suppose that $\zeta\ll1$.

Substitution the explicit forms of the vector Z and matrixes $\hat{H}$, $\hat{H}_q^{-1}$, $\hat{G}_1$, and $\hat{G}_2$, given by Eqs. (20), (21), (25), and (29) correspondingly, in Eq. (24) gives the vector X and, therefore, transmittance $T=|X_3|^2$ and reflectance $R=|X_4|^2$ of the film [see Eqs. (16)].

To simplify consideration we neglect the off-resonant (direct) transmittance $\sim\zeta^2$, i.e., we set $\tanh(dkn/2)=1$ in matrix $\hat{H}$ [see Eq. (21)]. Thus we obtain simple formulas for the resonant transmittance,

$$T(\tilde{\Delta})=\frac{4\tilde{g}^4}{[(\tilde{\Delta}-1)^2+(\tilde{g}^2+\tilde{\kappa})^2][(\tilde{\Delta}+1)^2+(\tilde{g}^2+\tilde{\kappa})^2]}, \tag{30}$$

the reflectance,

$$R(\tilde{\Delta})=\frac{(\tilde{g}^4+\tilde{\Delta}^2-1)^2-2(\tilde{g}^4-\tilde{\Delta}^2-1)\tilde{\kappa}^2+\tilde{\kappa}^4}{[(\tilde{\Delta}-1)^2+(\tilde{g}^2+\tilde{\kappa})^2][(\tilde{\Delta}+1)^2+(\tilde{g}^2+\tilde{\kappa})^2]}, \tag{31}$$

and the absorptance,

$$A(\tilde{\Delta})=1-T(\tilde{\Delta})-R(\tilde{\Delta})$$

$$=\frac{4\tilde{g}^2\tilde{\kappa}(1+\tilde{\Delta}^2+(\tilde{g}^2+\tilde{\kappa})^2)}{[(\tilde{\Delta}-1)^2+(\tilde{g}^2+\tilde{\kappa})^2][(\tilde{\Delta}+1)^2+(\tilde{g}^2+\tilde{\kappa})^2]}. \tag{32}$$

The obtained quantities depend on the renormalized detuning from the SPP frequency,

$$\tilde{\Delta}=g^2\frac{n(n-m)^2(n+n^3+2m)}{8(1+n^2)\zeta}-\frac{\Delta}{\zeta}\frac{m(1+n^2)}{2n}, \tag{33}$$

the renormalized modulation,

$$\tilde{g}=\frac{gn\sqrt{m}(n-m)}{2\sqrt{1+n^2}\sqrt{\zeta}}, \tag{34}$$

and the renormalized losses in the system,

$$\tilde{\kappa}=\frac{(1+n^2)\kappa}{4n^2\zeta}, \tag{35}$$

where $\kappa=\varepsilon_m''/|\varepsilon_m|$ and parameters m, Δ, and ζ are given by Eqs. (23), (27), and (28). Note that in Eqs. (30)–(35) we do take losses in the system into account, by writing the metal dielectric constant ε_m in the form $\varepsilon_m=-n^2(1-i\kappa)$, where n is positive and larger than one, while $\kappa\ll 1$. Note also that the transmittance T, reflectance R, and absorptance A depend on the frequency ω of the incident wave through the parameter Δ, which is proportional to the detuning $\Delta\propto\omega-\omega_p$, where $\omega_p=qcm/n$. The frequency ω_p is the frequency at which the SPP (excited on a flat metal surface) has a wave vector $k_p=\omega_p/c$ equal to the modulation wave vector q.

To analyze the resonant transmittance we first ignore, for simplicity, losses, i.e. we set $\tilde{\kappa}=0$ in Eq. (30), which then simplifies to

$$T(\tilde{\Delta})=\frac{4\tilde{g}^4}{[(\tilde{\Delta}-1)^2+\tilde{g}^4][(\tilde{\Delta}+1)^2+\tilde{g}^4]}. \tag{36}$$

For the renormalized film modulation $\tilde{g}<1$, as follows from Eq. (36), the resonance transmittance $T(\tilde{\Delta})$ has two maxima as a function of $\tilde{\Delta}$, namely, $T(\tilde{\Delta}_1)=T(\tilde{\Delta}_2)=1$ at $\tilde{\Delta}_{1,2}=\pm\sqrt{1-\tilde{g}^4}$. Therefore, a lossless, optically thick metal film becomes absolutely transparent at the resonance, regardless of its thickness. It is instructive to consider how the transmittance changes when the modulation $\tilde{g}$ increases. The distance between the two maxima, $\tilde{\Delta}_1-\tilde{\Delta}_2=2\sqrt{1-\tilde{g}^4}$, decreases with an increase of the amplitude of the renormalized modulation $\tilde{g}$. The film remains completely transparent exactly at the resonances. Finally, when $\tilde{g}$ becomes larger than one, the two maxima merge together. Now the transmittance has one maximum, with the amplitude $T_m=4\tilde{g}^4/(1+\tilde{g}^4)^2<1$ that *decreases* at a further increase of $\tilde{g}$ (see Fig. 2). This result can be understood if we recall that the interaction with the film modulation results in a radiation decay of SPP and its conversion into plane-wave radiation. The radiative losses [term $\sim\tilde{g}^4$ in the denominator of Eq. (36)] lead to the damping of SPPs. As a result, the resonant transmittance decreases with an increase of the renormalized modulation $\tilde{g}$.

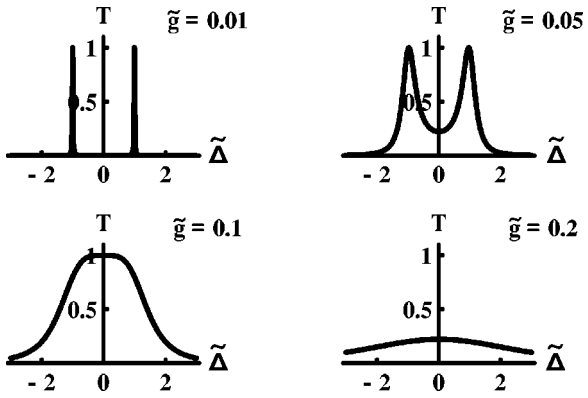

FIG. 2. Resonance transmittance as a function of the normalized detuning $\tilde{\Delta}$ from the resonance; the different graphs correspond to different film modulations $\tilde{g}$.

Note that the renormalized modulation $\tilde{g}$, given by Eq. (34), exponentially increases with the film thickness d, namely, $\tilde{g}\sim g\exp(dnq/2)$. Therefore, as follows from discussion above, the transmittance maxima merge when the thickness d increases. When d becomes larger than $d>d_c\sim 2\ln(1/g)/(nq)$ there is only one maximum in the transmittance that decays exponentially with a further increase of the film thickness d. This behavior of the resonance transmittance as a function of d is in qualitative agreement with results of Refs. 15 and 57.

In a real metal film, losses decrease the resonant transmittance. Yet the effect may remain rather profound. As follows from Eq. (30) the transmittance reaches its extremum when the renormalized detuning $\tilde{\Delta}$ is equal to

$$\tilde{\Delta}_{0,1,2}=\{0,-\sqrt{1-(\tilde{g}^2+\tilde{\kappa})^2},\sqrt{1-(\tilde{g}^2+\tilde{\kappa})^2}\}. \tag{37}$$

Thus, for $\tilde{g}^2+\tilde{\kappa}<1$, the transmittance has two maxima at $\tilde{\Delta}_{1,2}=\pm\sqrt{1-(\tilde{g}^2+\tilde{\kappa})^2}$, where $T_{\rm res}$ is given by $T_{\max}=\tilde{g}^4/(\tilde{g}^2+\tilde{\kappa})^2$. By substituting in this equation the renormalized modulation $\tilde{g}$ [given by Eq. (34)] and the renormalized losses $\tilde{\kappa}$ [Eq. (35)], we obtain an equation for the maximum transmittance,

$$T_{\rm res}=\left(1+\frac{(m+n)^2(1+n^2)^2\kappa}{g^2mn^4}\right)^{-2}, \tag{38}$$

which does not depend on the film thickness d. Thus, in the resonance, the metal film becomes semitransparent, even for a rather large thickness, provided that

$$\tilde{g}^2+\tilde{\kappa}=\frac{e^{dnq}(g^2mn^4(n-m)^2+(1+n^2)^2\kappa)}{4n^2(1+n^2)}<1. \tag{39}$$

For a film thickness $d>d_c$, where the critical thickness d_c is obtained from the condition $\tilde{g}^2+\tilde{\kappa}=1$ [see Eq. (39)], the transmittance decays exponentially with increasing d.

The resonance transmittance $T_{\rm res}$ in Eq. (38) depends on n, and it reaches its maximum, as a function of n, at $n=n^*=1.78$. For $n=n^*$, $T_{\rm max}$ takes the following simple form $T_{\rm res}\simeq 1/(1+12.4\kappa/g^2)^2$. It is interesting to note that the value of the optimal ''refractive index'' n^* depends neither on the film nodulation g nor on losses κ. Actually, $n^*=1.78$ is a constant, which is the same for different metal films.

The resonant transmittance is accompanied by large internal fields due to the excitation of SPPs. These fields (which will be discussed in more detail in Sec. III) are in charge of the anomalous light absorption

$$A_{\rm res}=1-T_{\rm res}-R_{\rm res}=\frac{2(m+n)^2(1+n^2)^2\kappa}{g^2mn^4}\times\left(1+\frac{(m+n)^2(1+n^2)^2\kappa}{g^2mn^4}\right)^{-2}, \tag{40}$$

that occurs at the resonance [$\tilde{\Delta}=\tilde{\Delta}_{1,2}$, see Eq. (37)], when the film becomes semitransparent. It follows from Eqs. (32)–(35) that for the optimal value $n^*=1.78$ the absorptance $A_{\rm res}$ is given by $A_{\rm res}\simeq(\kappa/g^2)/(0.2+2.5\kappa/g^2)^2$ and it depends on the ratio of the loss factor κ and the square of the film modulation, g^2. As a function of the ratio κ/g^2, the absorptance $A_{\rm res}$ reaches the maximum $A_{\rm max}=1/2$ for the modulation $g=3.53\sqrt{\kappa}$. The magnitude of the anomalous absorptance $A_{\rm max}=1/2$ remains the same, even when losses in the metal vanish, i.e., when $\kappa\to 0$. In this case, the amplitudes of the EM fields increase up to infinity and, as a result, the absorptance remains finite despite $\kappa\to 0$. It is not surprising that the anomalous absorption for almost lossless films ($\kappa\to 0$) requires that the modulation g also vanishes, $g=3.53\sqrt{\kappa}\to 0$; this is because the radiative damping of SPPs, which is proportional to the modulation g, should decrease with a decreasing loss factor κ, to keep the absorptance at the same level.

In Figs. 3(a) and 3(b) we show the transmittance and absorptance for a rather thick silver film, with $d=0.12$ μm and the modulation given by Eq. (6), with a spatial period $a=0.5$ μm. As seen in the figures, the transmittance and absorptance have resonances at the wavelength of an incident wave $\lambda\simeq 0.53$ μm. In the following estimates we use the Drude approximation for the dielectric constant,

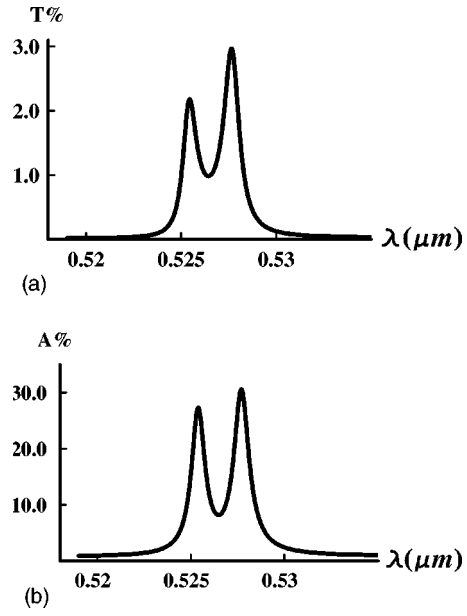

FIG. 3. Transmittance (a) and absorptance (b) for a modulated silver film; the film thickness is $d=0.12$ μm, and the modulation is given by $g=0.2$.

$$\varepsilon_m(\omega)=\varepsilon_b-(\omega_p/\omega)^2/[1+i\omega_\tau/\omega], \tag{41}$$

in silver; this formula well approximates the known experimental data (see, e.g., Ref. 71) for wavelengths larger than $\lambda>0.4$ μm. We use the following values for the constants in the Drude formula: $\varepsilon_b=5$ for the interband transition contribution, $\omega_p=9.1$ eV for the plasma frequency, and $\omega_\tau=0.021$ eV for the relaxation frequency.[7] In the absence of modulation, the film acts as an almost perfect mirror, with reflectance $R=99\%$ and transmittance $T<0.02\%$. The situation changes dramatically when the dielectric constant of the film is modulated. For the spatial period of the modulation $a=2\pi/q=0.5$ μm, we obtain, from a direct solution to Eq. (24), that the absorptance equals almost 30% at the resonant wavelength $\lambda\simeq 0.53$ μm; the transmittance also increases by two orders of magnitude and reaches $T\simeq 4\%$. The absorptance and transmittance have a double-peak structure corresponding to the excitation of symmetric and antisymmetric SPPs.

For real metal films, the transmittance typically increases when losses decrease, as follows from Eq. (38). Let us consider a silver film of high (atomic) quality at cryogenic temperatures, such that the electron mean-free pass is restricted by the film thickness itself. The resonant transmittance and absorptance for such a film with thickness $d=0.18$ μm are shown in Figs. 4(a) and 4(b), respectively. In calculations, we assumed that loss factor κ is ten times smaller than that in Fig. 3. We see that the resonant absorptance increases up to $A_{\rm res}=40\%$, while the resonance transmittance $T_{\rm res}\gtrsim 10\%$. The width of the resonance shrinks to $\sim 10^{-3}$ μm. Note that, without modulation, a film of such thickness has a transmittance $T<10^{-3}\%$ for the wavelength $\lambda>0.4$ μm, in the optical and infrared spectral ranges. Therefore the transmittance increases by four orders of magnitude due to the SSP excitation.

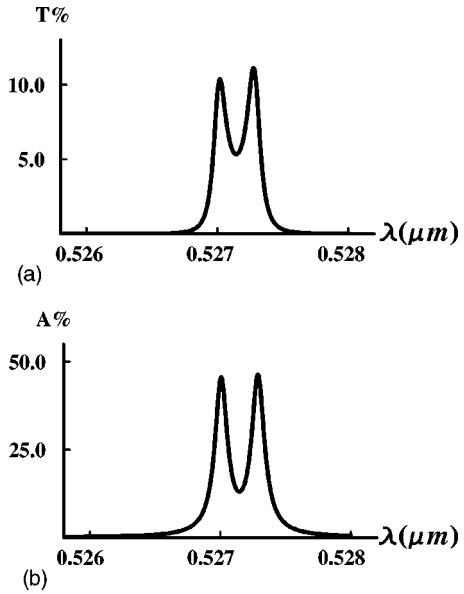

FIG. 4. Transmittance (a) and absorptance (b) for a ''cryogenic'' silver film; the film thickness is $d=0.17$ μm, and the modulation is $g=0.1$.

III. LIGHT-INDUCED AND LIGHT-CONTROLLED RESONANT TRANSMITTANCE

In the consideration above, the film's modulation was supposed to be somehow fabricated. We now consider the case of nonlinear films, when the film's modulation can be induced and controlled by the light itself through, for example, the Kerr optical nonlinearity. We first assume that the dielectric constant of a film has a small ''seed'' modulation given by Eq. (6), where the amplitude of the modulation $g \ll 1$, and find how the modulation increases due to the nonlinearity of the film.

Exactly at resonance, the transmitted intensity $I_t=TI_0$ is of the same order of magnitude as the intensity of the incident wave, $I_0=(c/8\pi)|E_0|^2$, where E_0 is the electric field in the incident wave. The transmitted wave is generated by the SPP, which propagates on the back (output) interface ($z=d/2$); this occurs because the SPP interacts with the film's modulation g and converts back to the light, which emits from the film. The amplitude of such conversion is proportional to the film modulation g. Therefore, the SPP intensity $I_q \equiv |E_q|^2$ can be estimated from the equation $I_t \sim g^2 I_q$, as $I_q \sim I_t/g^2 \sim I_0/g^2 \gg I_0$. At the front (input) interface ($z=-d/2$), the SPP amplitude is of the same order of magnitude, as pointed out in the discussion following Eq. (5). The electric field E_q of the SPP is spatiality modulated, with a resonant wave vector $k_p=q$. Note that the enhanced field E_q is responsible for the anomalous resonance absorption, which could be many orders of magnitude larger than the absorption in flat metal films [see Figs. 3(b) and 4(b)].

Since the electric field E_q of the SSP is strongly enhanced, it makes sense to take into account a possible nonlinearity in the optical response of a metal film. To be specific, we consider the Kerr optical nonlinearity. We assume that the electric displacement $\mathbf{D}_{in}$ in the film equals $\mathbf{D}=\varepsilon_m \mathbf{E}+12\pi\chi^{(3)}\mathbf{E}|\mathbf{E}|^2$, where $\chi^{(3)}$ is the Kerr susceptibility

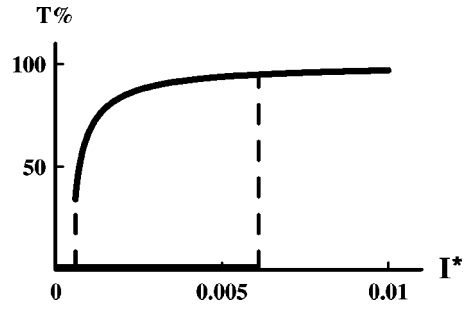

FIG. 5. Transmittance through a ''nonlinear'' silver film as a function of the dimensionless light intensity; the film thickness is $d=0.17$ μm, and the ''seed'' modulation is given by $g=10^{-2}$.

(see, e.g., Ref. 72). We substitute the electric field **E** inside the film given by Eqs. (13) and (14) into the equation for the electric displacement **D**, and obtain that the nonlinear dielectric constant is equal to

$$\varepsilon \cong -n^2[1+g_i(E)\cos qx]^{-1}, \tag{42}$$

where g_i is the induced modulation:

$$g_i = g + \frac{24\pi}{n^2}\chi^{(3)}\mathrm{Re}(E^*E_q). \tag{43}$$

Equation (42) is similar to Eq. (6), but the film modulation depends now on the internal electric field.

We restrict our consideration to the nonlinearity effect on the film modulation and neglect, for simplicity, small nonlinear corrections in the numerator of Eq. (42). To further simplify the consideration, we replace the field term $\mathrm{Re}(E^*E_q)$ in Eq. (43) by its average over the film thickness $\langle \mathrm{Re}(E^*E_q)\rangle$. We use the average because the fundamental field, as well as the SSP, in the resonance, have nearly the same amplitudes on both interfaces of the film. Exactly in the resonance, i.e., when $\tilde{\Delta}=\tilde{\Delta}_{1,2}$ [see Eq. (37)], the term $\langle \mathrm{Re}(E^*E_q)\rangle$ acquires the form

$$\langle \mathrm{Re}(E^*E_q)\rangle_{\mathrm{res}} = \frac{32\pi}{c}\,\frac{(2n^3+m(1+n^2))}{dgkn^4}\,T_{\mathrm{res}}I_0, \tag{44}$$

where I_0 is the instantaneous intensity of the incident light, T_{res} is the resonant transmittance given by Eq. (38), parameter m is given by Eq. (23), and d and k are the film thickness and wave vector of the incident wave, respectively.

It follows from Eqs. (43) and (44) that the SPP field induces, via the Kerr optical nonlinearity, the refractive-index modulation g_i. The induced modulation, in turn, increases the transmittance and, therefore, the SPP amplitude. This positive feedback may eventually result in a bistability phenomenon, as shown in Fig. 5.

We substitute Eqs. (38) and (44) into Eq. (43) and obtain an equation for the induced refractive-index modulation,

$$g_i = g + \frac{g_i^3(2n^3+(1+n^2)m)}{dkn^4\left(g_i^2+\dfrac{(1+n^2)^2(n+m)^2\kappa}{n^4 m}\right)^2}I^*, \tag{45}$$

which holds in the resonance, when $\tilde{\Delta}=\tilde{\Delta}_{1,2}$ [see Eq. (37)]. Here we introduce the dimensionless intensity of the incident light, $I^*=384\pi\chi^{(3)}I_0/c$. Now we solve Eq. (45) with the same parameters n and κ as we used to obtain Fig. 4; thus we find the light-induced modulation (as a function of dimensionless intensity I^*), which allows us to calculate the transmittance shown in Fig. 5.

When the dimensionless intensify I^* of the incident wave becomes larger than $I_1^*\simeq 6\times 10^{-3}$, the transmittance jumps from a nearly zero value up to $T\simeq 1$, i.e., the film suddenly becomes transparent (see Fig. 5). If we now decrease the intensity of the incident wave, the film still remains transparent, even for $I^*<I_1^*$; this is because the SPP has been already excited in the film. The transmittance decreases steeply for $I^*\sim 10^{-3}<I_1^*$. Thus the optical bistability phenomenon occurs in the modulated metal film.

It is well known that the susceptibility $\chi^{(3)}$ is rather large for noble metals, $\chi^{(3)}>10^{-8}$ esu (see, for example, Ref. 73 and references therein), so that the intensity I_0 required for the bistability can be easily achieved with conventional lasers. [Note that the upper curve in Fig. 5 should be considered as an extrapolation since we restricted the expansion of the nonlinear dielectric constant $\varepsilon(|E|^2)$ in the series over $|E|^2$ to the first two terms only].

We also note that the seed modulation g can be created by two (four) additional, control laser beams, which are incident on the front surface (front and back surfaces) from different sides with respect to the normal. The interfering beams then would result (through the optical nonlinearity) in the film modulation; these ''gate'' beams can control the transmittance of the fundamental beam, propagating normal to the film. To provide a modulation in the refractive index one can also use a thin layer of highly nonlinear (dielectric) material placed on top of the metal film. For example, photorefractive quantum-well structures (see, e.g., Ref. 74), which are known to produce large refractive-index grating at very low intensities (below 1 mW), can be used for this purpose. In this case, the required modulation at the metal-dielectric interface can be accomplished at very low light intensities.

IV. CONCLUSIONS AND DISCUSSION

In summary, in this paper we show that the excitation of surface plasmon-polaritons in modulated metal films can result in an enhanced resonant transmittance, so that an optically thick film can become semitransparent. At resonance, the transmittance can be increased by a factor of 10^4. The maximum in the transmittance has a characteristic double-peak structure due to the splitting of SPPs into symmetric and antisymmetric modes. The resonant transmittance increases with a decrease of losses in the system, which can be accomplished, for example, by cooling the film down to the cryogenic temperature. In the resonance, the amplitude of the SPP field can be larger than the amplitude of an incident wave by several orders of magnitude. Then the optical nonlinearity can become important and result in a significant enhancement of the discussed effect. We predict that at sufficiently large intensities of an incident wave, the film can manifest the optical bistability phenomenon. Films with such a bistable behavior in the resonant transmittance can be used, for example, as optical switches.

Our results are in good qualitative agreement with recent experiments,[69] where the transmittance through a thin modulated silver film was investigated. In this experiment the film was deposited on a quartz substrate, in contrast to a free-standing film studied in the present paper. Therefore, a direct comparison of our theory and experiments of Ref. 69 is difficult. Yet we can speculate that if the film thickness used in the experiments of Ref. 69 was larger; then, even a stronger enhancement of the transmittance could be obtained, provided that the conditions for the resonant excitation of SPPs were fulfilled.

Results of this paper are also similar, to some extent, to those obtained in Ref. 15 for the extraordinary optical transmittance through a square array of subwavelength holes punched in a metal film. The transmittance calculated in Ref. 15 for a lossless film has a resonance behavior with two asymmetric maxima. In the maximum, the transmittance reaches one, i.e., the film becomes transparent at this point. The maxima merge together with increasing the film thickness so that only one maximum remains for the thickness, which is approximately twice as large as the diameter of a hole. This maximum transmittance decreases in absolute value with a further increase of the film thickness. This is similar to the behavior of the transmittance, as a function of film's thickness, obtained in the present paper. The similarity looks even more remarkable when we take into account that the investigated systems are quite different. Thus the authors of Ref. 15 calculated the transmittance for the film, which is impenetrable for electromagnetic waves. Therefore, the SPPs excited on both (front and back) interfaces of the film are connected through holes only. In contrast, we consider the extraordinary transmittance due to the tunneling of SPPs throughout a modulated metal film. Therefore, it is not surprising that the EM field distributions are indeed different for the two systems. In our modulated film the local field is $\sim E+E_q\cos(x/a)$, where a is the period of modulation, while in the system considered in Ref. 15 the em field concentrates at the rims of the holes, as shown in computer simulations.[62]

The tunneling of SPPs, through the deep grooves made on both interfaces of the metal film, was considered in computer simulations of Ref. 57. A typical double peak resonance was obtained in the transmittance. Again, the transmittance maxima merge when the distance between the bottoms of the grooves (in the front and back interfaces) increase. When the metal thickness between the grooves exceeds the depth of a groove by three times, the extraordinary transmittance vanishes. For this system, as in the case of holes, the local field is highly nonuniform in the plane of the film, when the resonant transmittance occurs; specifically, the field concentrates inside deep grooves, in this case.

Thus we see that for three different physical systems the dependence of the extraordinary transmittance on the wavelength and film thickness appears to be quite similar. We speculate that the explicit equations obtained in this paper capture important generic features of the extraordinary optical transmittance. Therefore, we believe that our analytical

equations can be used for estimations of EOT in more complicated systems, where analytical solutions are yet to be found.

Finally, we would like to stress out one more distinct feature of the modulated films considered in this paper. The modulation and, therefore, the extraordinary transmittance, can be tuned dynamically using, for example, auxiliary light beams and employing the optical nonlinearity of the film material. Therefore, such films can be used as active optical elements, in contrast to most of other systems with the extraordinary optical transmittance considered in the literature (see, e.g., Refs. 15, 57 and 62) that typically can be used only as passive optical elements.

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