Dephasing due to background charge fluctuations

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(Received 14 March 2002; revised manuscript received 6 August 2002; published 16 May 2003)

In quantum computation, quantum coherence must be maintained during gate operation. However, in physical implementations, various couplings with the environment are unavoidable and can lead to a dephasing of a quantum bit (qubit). The background charge fluctuations are an important dephasing process, especially in a charge qubit system. We examined the dephasing rate of a qubit due to random telegraph noise. Solving stochastic differential equations, we obtained the dephasing rate of a qubit constructed of a coupled-dot system; we applied our results to the charge Josephson qubit system. We examined the dephasing rates due to two types of couplings between the coupled-dot system and the background charge, namely, fluctuation in the tunnel coupling constant and fluctuation in the asymmetric bias. For a strong-coupling condition, the dephasing rate was inversely proportional to the time constant of the telegraph noise. When there is fluctuation in the tunnel coupling constant, Gaussian decay occurs in the initial regime. We also examined the rate of dephasing due to many impurity sites. For a weak-coupling condition with fluctuation in the asymmetric bias, the obtained dephasing rate coincided with that obtained by the perturbation method using the spectral weight of a boson thermal bath, which is proportional to the inverse of the frequency.

DOI: 10.1103/PhysRevB.67.195320 PACS number(s): 73.20.Hb, 73.20.Mf, 73.20.Jc

I. INTRODUCTION

Efforts to implement quantum computation have recently intensified. The application of the quantum bit (qubit) to solid-state materials, such as superconducting Josephson junctions[1] and quantum dots,[2–4] is particularly promising, because these implementations have the advantage of scalability. In a coupled-dot system, for example, the localized states in the left and right dots are treated as a basic two-level system, in which the tunnel coupling between the two dots constructs a quantum superposition of the dots. This superposition manifests itself in coherent quantum oscillation (Rabi oscillation), and a transition can be induced between the superposed states.[5] A nanometer-scale superconducting electrode connected to a reservoir via a Josephson junction constitutes an artificial two-level system in which the charge states, coupled by tunneling, differ by $2e$, where e is the electronic charge. This system has shown clear Rabi oscillation. [1]

Quantum coherence must be maintained during quantum gate operation. Dephasing, characterized by the dephasing time, originates from various couplings between the qubit and environment. When the qubit is implemented in a solid-state system, the effects of phonons and electromagnetic and background charge fluctuations (BCF's) are important in the dephsing process. The effect of phonons has been examined in semiconductor quantum dots as the source of the dephasing accompanying dissipation; [6] the effect of electromagnetic fluctuation in Josephson junction qubits has been extensively studied. [7] However, BCF's have not yet been examined systematically, in spite of their importance in the dephasing process.

BCF's have been observed in many systems.[8–11] In nanoscale systems, they are the electrostatic potential fluctuations due to the dynamics of electrons or holes trapped at impurity sites. In particular, the charge of a single impurity fluctuates with the Lorentzian spectrum form, which is called ''random telegraph noise'' in the time domain.[10,12] The random distribution of the positions of such impurities and their time constants lead to BCF's or $1/f$ noise.[13] In solid-state charge qubits, these BCF's lead to a dynamical electrostatic disturbance and hence the dephasing. The effect of $1/f$ noise on a charge Josephson qubit has been examined theoretically—the interaction between the qubit and environment has been treated by the perturbation method, [14,15] by Gaussian approximation,[16] and by the path integral method within a spin-boson model. [14,15] When fluctuating impurities exist in the substrate, not in the junction,[11] the coupling between the qubit and BCF's is weak, and the perturbation method is sufficient. However, when the interaction between the qubit and environment is strong, methods that go beyond perturbation are needed.

In this study, we investigated how the electrostatic disturbance of time constant τ_0 coming from a single impurity affects the quantum coherence of a qubit irrespective of the strength of the qubit-impurity coupling. We also examine the effect of many impurity sites. This approach is in clear contrast with previous ones, [16,15] in which the phenomenological spectral weight of the boson thermal bath was used to characterize the effect of BCF's.

We consider two types of couplings between the qubit and environment: pure dephasing and dephasing accompanied with relaxation of the population. [17] In symmetrical coupled-dot systems, the former corresponds to a fluctuation in the tunnel coupling constant and the latter to that in the asymmetric bias.[18,19] The mapping from a coupled-dot system to the Josephson charge qubit is discussed in Sec. VI. By using the method of stochastic differential equations, we obtain analytically the dephasing rate, which is shown to be always smaller than τ_0^{-1}. It should be noted that this dephasing process does not mean the qubit becomes entangled with the environment, but rather it means the stochastical evolution of an external classical field, suppressing the off-diagonal density matrix elements of the qubit after being averaged over

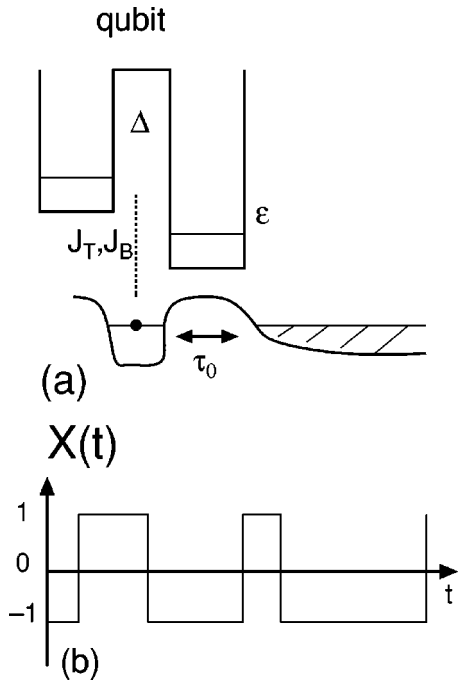

FIG. 1. (a) Schematic diagram of coupled dots and environment constituting an impurity site and the electron reservoir. (b) Example time sequence of random telegraph noise.

statistically distributed samples.

Section II defines the Hamiltonian of the system. Section III explains the method of stochastic differential equations. The fluctuations in tunnel coupling and asymmetric bias are examined in Secs. IV and V, respectively. Section VI is devoted to a discussion, including the effect of many impurities. Section VII summarizes the paper.

II. HAMILTONIAN

The qubit and effect of a single impurity are examined in terms of the following Hamiltonian:

$$\mathcal{H}=H_{qb}+H_{qb-imp}, \tag{1}$$

$$H_{qb}=\frac{\hbar\Delta}{2}(c_L^\dagger c_R+c_R^\dagger c_L)+\frac{\hbar\epsilon}{2}(c_L^\dagger c_L-c_R^\dagger c_R), \tag{2}$$

where $c_{L,R}^\dagger$ and $c_{L,R}$ are the creation and annihilation operators of the left and right dots, assuming a single level for each dot, as shown in Fig. 1(a). The Δ is the tunnel coupling between the dots, and ϵ is the asymmetric bias between them. The interaction between a qubit and the charge at the impurity site is described by the following Hamiltonian:

$$H_{qb-imp}=\frac{\hbar J_T}{2}(c_L^\dagger c_R+c_R^\dagger c_L)2(d^\dagger d-1/2)$$

$$+\frac{\hbar J_B}{2}(c_L^\dagger c_L-c_R^\dagger c_R)2(d^\dagger d-1/2), \tag{3}$$

where J_T is the magnitude of the fluctuation in the tunnel coupling, J_B is the magnitude of the fluctuation in the asymmetrical bias, and $d^\dagger$ and d are the creation and annihilation operators of the charge at the impurity site, respectively.

We assume that the time evolution of statistical variable $X(t)$ $(=2\langle d^\dagger d-1/2\rangle)$ is a Poisson process. Assuming a strong coupling between the charge at the impurity site and the nearby electron reservoir, the dynamics of the charge induces not Gaussian white noise, but random telegraph noise.[12,18,19] We therefore consider the effect of random telegraph noise with characteristic time constant τ_0, where the statistical variable $X(t)$ takes the value 1 or -1 [Fig. 1(b)].

The time constant is determined by the barrier height of the electron or hole trap and the temperature, like $\tau_0=Ae^{W/k_BT}$, where W, k_B, T, and A are the activation energy of the impurity potential, Boltzmann's constant, the temperature, and the temperature-independent prefactor, respectively.[20] When the temperature decreases, τ_0 becomes longer. The telegraph noise has been experimentally observed to take a value of 1 or -1 with asymmetric probabilities, which arises from the difference between the Fermi energy of the electron reservoir and the energy level of the impurity sites.[12] To include this asymmetric weight, we introduce asymmetric probabilities p_u and p_d, which relate the asymmetric transition rates for the process from -1 to 1 $[\tau_u^{-1}=(p_u\tau_0)^{-1}]$ to those of the opposite process $[\tau_d^{-1}=(p_d\tau_0)^{-1}]$.

We neglect the back action from the qubit to the charge at the impurity site, so this environment reduces to a classical stochastic external field. We also assume that the temperature is high enough for the effect of the quantum fluctuation of the charge between the impurity site and electron reservoir to be neglected.

For compactness, we rewrite the Hamiltonian in terms of the Pauli matrices while rotating the basis $\pi/2$ from the basis of localized states in the left and right dots to the bonding-antibonding basis:

$$\mathcal{H}=\frac{\hbar\Delta}{2}\sigma_z+\frac{\hbar\epsilon}{2}\sigma_x+\frac{\hbar J_T}{2}\sigma_z X(t)+\frac{\hbar J_B}{2}\sigma_x X(t). \tag{4}$$

In the following, we consider only the case of a symmetrical coupled-dot system $\epsilon=0$, in which the effect of the bias fluctuation due to the dephasing starts only from the second order J_B^2, as shown in the following, and is less effective in the perturbation regime. As the reduced Hamiltonian, Eq. (4), suggests, the present results can also be applied to other quantum two-level systems in which telegraph-type fluctuation exists (see Sec. VI).

III. METHOD

We are interested in the time evolution of the qubit's two-by-two density matrix, $\rho(t)$, with an arbitrary initial condition at $t=0$, $\rho(0)$. If BCF is absent, starting with the initial condition that the left dot be occupied, for example, the density matrix at $t=0$ is given by

$$\rho(t=0)=W\left(\frac{\pi}{2}\right)\begin{pmatrix}1&0\\0&0\end{pmatrix}W^\dagger\left(\frac{\pi}{2}\right), \tag{5}$$

where $W(\pi/2)$ is the matrix of rotation from the localized basis to the bonding-antibonding basis. At time t,

$$\rho(t)=e^{-it\Delta\sigma_z/2}W\left(\frac{\pi}{2}\right)\begin{pmatrix}1&0\\0&0\end{pmatrix}W^{\dagger}\left(\frac{\pi}{2}\right)e^{it\Delta\sigma_z/2} \tag{6}$$

$$=W\left(\frac{\pi}{2}\right)\begin{pmatrix}\dfrac{1+\cos\Delta t}{2} & i\sin\Delta t\\ -i\sin\Delta t & \dfrac{1-\cos\Delta t}{2}\end{pmatrix}W^{\dagger}\left(\frac{\pi}{2}\right), \tag{7}$$

where the bases of the inner matrix are the left and right occupancy states. Therefore, the density matrix shows Rabi oscillation with frequency Δ.

In the following, we keep the matrix indices in the bonding-antibonding basis. To examine the instantaneous potential change, we use the method of stochastic differential equations.[21] The density matrix averaged over all possible sequences of telegraph noise can be represented as a series:

$$\rho(t)e^{t/\tau_0}=\sum_{k=0}^{\infty}\frac{1}{\tau_0^k}\int_0^t dt_k\int_0^{t_k}dt_{k-1}\cdots\int_0^{t_2}dt_1\int_X dW(X_k)\times\int_X dW(X_{k-1})\cdots\int_X dW(X_0)\rho(t,\{t_k\}), \tag{8}$$

where $dW(X)$ is the distribution of X with the probability of p_u for $X=1$ and that of p_d for $X=-1$, with the constraint $p_u+p_d=1$. The density matrix before the ensemble average, $\rho(t,\{t_k\})$, is given by

$$\rho(t,\{t_k\})=S(X_k;t,t_k)\cdots S(X_1;t_2,t_1)S(X_0;t_1,0)\rho(0)\times S^{-1}(X_0;t_1,0)S^{-1}(X_1;t_2,t_1)\cdots S^{-1}(X_k;t,t_k), \tag{9}$$

where $S(X;t,t')=S(X,t-t')$ is the unitary time evolution operator, which is determined by

$$i\hbar\frac{dS(X,t)}{dt}=\mathcal{H}(X)S(X,t). \tag{10}$$

The explicit form of $S(X,t)$ is given by

$$S(X,t)=\begin{pmatrix}\cos\dfrac{1}{2}at-i\left(\dfrac{\Delta+J_TX}{a}\right)\sin\dfrac{1}{2}at & -i\dfrac{J_BX}{a}\sin\dfrac{1}{2}at\\ -i\dfrac{J_BX}{a}\sin\dfrac{1}{2}at & \cos\dfrac{1}{2}at+i\dfrac{\Delta+J_TX}{a}\sin\dfrac{1}{2}at\end{pmatrix}, \tag{11}$$

where $a=\sqrt{(\Delta+J_TX)^2+J_B^2}$. Equation (8) can be rewritten in terms of the integral equation

$$\rho(t)e^{\tau/\tau_0}=\int_X S(X;\tau,0)\rho(0)S^{-1}(X;\tau,0)dW(X)+\frac{1}{\tau_0}\int_0^{\tau}e^{t/\tau_0}\times\int_X S(X;\tau,t)\rho(t)S^{-1}(X;\tau,t)dW(X)dt. \tag{12}$$

Using $S(X;\tau,t)$, we define matrix $R^{im}(\tau,t)$ as follows:

$$R_{lk}^{im}(\tau,t)=\int_X S_{ik}(X;\tau,t)S_{lm}^{-1}(X;\tau,t)dW(X), \tag{13}$$

$$R_{lk}^{im}(\tau,t)=(R_{kl}^{mi})^*(\tau,t). \tag{14}$$

We can then reduce Eq. (12) to the following compact form:

$$\rho_{im}(\tau)=e^{-\tau/\tau_0}\,\mathrm{Tr}[R^{im}(\tau,0)\rho(0)]+\frac{1}{\tau_0}\int_0^{\tau}dt\times\exp\left[-\frac{\tau-t}{\tau_0}\right]\mathrm{Tr}[R^{im}(\tau,t)\rho(t)]. \tag{15}$$

IV. FLUCTUATION IN TUNNEL COUPLING

First, we consider the case of fluctuation in tunnel coupling ($J_T\neq 0, J_B=0$). Since the interaction Hamiltonian commutes with H_{qb}, the environment leads to pure dephasing without energy dissipation. We derive $S(X,t)$ and $R_{lk}^{im}(t)$ from Eqs. (11) and (13) as follows:

$$S_{kl}(X,\tau-t)=\exp\left[\frac{i}{2}[\Delta(\tau-t)-J_TX(\tau-t)](-1)^k\right]\delta_{kl}, \tag{16}$$

$$R_{lk}^{nm}(\tau-t)=\int_X \exp[i\Delta(\tau-t)+iJ_T X(\tau-t)]\delta_{nk}\delta_{lm}dW(X). \tag{17}$$

As a result, the off-diagonal element of the density matrix obeys the following integral equation:

$$\sigma(\tau)=e^{-\tau/\tau_0}R(\tau)+\frac{1}{\tau_0}\int_0^\tau R(\tau-t)\exp\left[-\frac{\tau-t}{\tau_0}\right]\sigma(t)dt, \tag{18}$$

where $\sigma(\tau)=e^{-i\Delta\tau}\rho_{12}(\tau)/|\rho_{12}(0)|$ is a normalized off-diagonal element of the qubit density matrix measured in a Rabi oscillation frame and

$$R(\tau-t)=p_u\exp[iJ_T(\tau-t)]+p_d\exp[-iJ_T(\tau-t)]. \tag{19}$$

Equation (18) can be rewritten as a differential equation

$$\sigma''(\tau)+\frac{1}{\tau_0}\sigma'(\tau)+\left[J_T^2+i\left(\frac{p_u-p_d}{\tau_0}\right)J_T\right]\sigma(\tau)=0. \tag{20}$$

The initial conditions are $\sigma(0)=e^{i\phi}=\rho_{12}(0)/|\rho_{12}(0)|$ and $\sigma'(0)=0$. As a result of coupling with the environment, the off-diagonal element of the density matrix decays as a function of time. When the real part of the two roots of the characteristic equation (20) almost completely degenerates, the short-time behavior for $t<\min(\sqrt{2}/J_T,3\tau_0)$ (initial regime) is not a simple exponential decay. In this initial regime, the off-diagonal element of the density matrix shows Gaussian decay, $\sigma(t)\sim\sigma(0)(1-J_T^2t^2/2+\cdots)\simeq\sigma(0)e^{-J_T^2t^2/2}$, irrespective of the asymmetric probabilities, p_u and p_d. The decay of the off-diagonal element of the density matrix becomes exponential for the asymptotic regime, $t\gg(1/(\sqrt{|1/\tau_0^2-4J_T^2|})$, when $p_u=p_d$. For $J_T\tau_0\ll 1$, this criterion is obtained when one of the two exponential decay terms becomes negligibly small. For $J_T\tau_0\gg 1$, the time constants of the envelope of the two dumped oscillating terms are the same, $1/(2\tau_0)$. Exponential decay appears after the inverse of the oscillation frequency: $1/\sqrt{|4J_T^2-1/\tau_0^2|}$. When $p_u=p_d$ and $J_T\tau_0=\frac{1}{2}$, one obtains, $\sigma(t)=e^{-t/4\tau_0}(1+t/2\tau_0)$, where the dephasing can never be a simple exponential decay.

The time constant of this exponential decay corresponds to the dephasing time T_2. For $p_u\neq p_d$,[18]

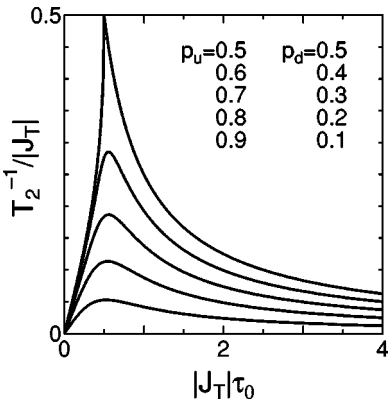

FIG. 2. Dependence of dephasing rate $T_2^{-1}/|J_T|$ on tunnel coupling constant $|J_T|\tau_0$ for various values of p_u and p_d with $J_T\neq 0$ and $J_B=0$.

$$T_2^{-1}=\frac{1}{2}\mathrm{Re}\left[\frac{1}{\tau_0}-\sqrt{\left(\frac{1}{\tau_0}\right)^2-4J_T^2-4i\left(\frac{p_u-p_d}{\tau_0}\right)J_T}\right]. \tag{21}$$

Figure 2 shows the $|J_T|\tau_0$ dependence of dephasing rate T_2^{-1} for $p_u=0.5$, 0.6, 0.7, 0.8, and 0.9. In the two limits of weak and strong coupling, we have

$$T_2^{-1}=\begin{cases}(1-(p_u-p_d)^2)J_T^2\tau_0, & 1/\tau_0\gg|J_T|.\\ (1-|p_u-p_d|)/2\tau_0, & 1/\tau_0\ll|J_T|.\end{cases} \tag{22}$$

Namely, for a fixed $|J_T|$, when τ_0 increases from 0, T_2^{-1} first increases and then decreases. It has a single maximum of $(1-\sqrt{|p_u-p_d|})|J_T|$ when $\tau_0=1/(2|J_T|)$; therefore, for any parameters, $T_2>2\tau_0$. Changing weight p_u to make it more asymmetric reduces T_2^{-1}. In the limit of $p_u\to 0$ or 1, the environment is nearly frozen, so the dephasing time becomes infinity.

V. FLUCTUATION IN ASYMMETRIC BIAS

Next we examine the effect of bias fluctuation ($J_T=0, J_B\neq 0$). We consider only the case of symmetrically weighted telegraph noise ($p_u=p_d=1/2$) for simplicity. In this model, H_{qb} and the interaction Hamiltonian do not commute and the dephasing process is accompanied by a relaxation of the population. The unitary operator is thus

$$S(X,t)=\begin{pmatrix}\cos\frac{1}{2}\Omega t-i\frac{\Delta}{\Omega}\sin\frac{1}{2}\Omega t & -i\frac{J_BX}{\Omega}\sin\frac{1}{2}\Omega t\\ -i\frac{J_BX}{\Omega}\sin\frac{1}{2}\Omega t & \cos\frac{1}{2}\Omega t+i\frac{\Delta}{\Omega}\sin\frac{1}{2}\Omega t\end{pmatrix}, \tag{23}$$

where $\Omega=\sqrt{\Delta^2+J_B^2}$ is the nutation frequency. After averaging, the few nonvanishing elements of matrix $R_{lk}^{im}(t)$ have the form

$$R_{11}^{11}(\tau-t)=R_{22}^{22}(\tau-t)=1-P_{12}(\tau-t),$$

$$R_{22}^{11}(\tau-t)=R_{11}^{22}(\tau-t)=R_{12}^{12}(\tau-t)=R_{21}^{21}(\tau-t)=P_{12}(\tau-t),$$

$$R_{21}^{12}(\tau-t)=R_{12}^{21*}(\tau-t)=\left[P_{12}(\tau-t)+\cos\Omega(\tau-t)+\frac{\Delta}{\Omega}\sin\Omega(\tau-t)\right], \quad (24)$$

where

$$P_{12}(\tau-t)=\frac{J_B^2}{\Omega^2}\sin^2\frac{\Omega(\tau-t)}{2}. \quad (25)$$

The time evolution of the diagonal element of this matrix is determined by the following differential equation: [21,22]

$$n'''(\tau)+\frac{2}{\tau_0}n''(\tau)+\left(\frac{1}{\tau_0^2}+\Delta^2+J_B^2\right)n'(\tau)+\frac{J_B^2}{\tau_0}n(\tau)=0. \quad (26)$$

Here, $n(\tau)=\rho_{11}-\rho_{22}$, and the initial conditions are $n''(0)=-J_B^2 n(0)$ and $n'(0)=0$, and $n(0)$ depends on the initial population having an arbitrary value between 1 and -1. The time constant of the exponential decay of $n(\tau)$ is the relaxation time of the population, T_1. When $|J_B|\ll\Delta,1/\tau_0$ (weak coupling), $T_1^{-1}\sim J_B^2\tau_0$. When $|J_B|\gg\Delta,1/\tau_0$ (strong coupling), $T_1^{-1}\sim 1/2\tau_0$.

The obtained differential equation for the off-diagonal element is

$$\sigma_{12}'''(\tau)+\frac{2}{\tau_0}\sigma_{12}''(\tau)+\left(\frac{1}{\tau_0^2}+\Delta^2+J_B^2+i\frac{\Delta}{\tau_0}\right)\sigma_{12}'(\tau)+\left(i\frac{\Delta}{\tau_0^2}+\frac{\Delta^2}{\tau_0}+\frac{J_B^2}{2\tau_0}\right)\sigma_{12}(\tau)=\frac{J_B^2}{2\tau_0}\sigma_{21}(\tau), \quad (27)$$

where $\sigma_{12}(\tau)=\rho_{12}(\tau)/|\rho_{12}(0)|$ is the normalized off-diagonal element of the qubit density matrix. The initial conditions are

$$\sigma_{12}''(0)=\left(-\Delta^2-\frac{J_B^2}{2}\right)\sigma_{12}(0)+\frac{J_B^2}{2}\sigma_{21}(0), \quad (28)$$

$$\sigma_{12}'(0)=-i\Delta\sigma_{12}(0), \quad (29)$$

$$\sigma_{12}(0)=e^{i\phi}, \quad (30)$$

where ϕ is the initial phase of the off-diagonal density matrix element.

A. Analytic solutions

The differential equation (27) with initial condition Eqs. (28)–(30) has explicit solutions

$$\sigma_{12}(u)=\sum_{i=1}^{3}\left[e^{\lambda_i u}+\frac{1}{\alpha}\lambda_i C_{ic}(e^{\lambda_i u}-e^{-u})\right]\times(C_{ic}\cos\phi+iC_{is}\sin\phi)+ie^{-u}\sin\phi, \quad (31)$$

where $u=\tau/\tau_0$. For $i=1$–3, the coefficients are given by

$$C_{ic}=\frac{1}{\Delta_i}[(1+\beta^2)\lambda_i^2+(1+\beta^2-2\alpha^2)\lambda_i+(1+\beta^2)\gamma^2-3\alpha^2], \quad (32)$$

$$C_{is}=\frac{1}{\Delta_i}\alpha(2\lambda_i^2+3\lambda_i+1+\beta^2), \quad (33)$$

$$\Delta_i=(\lambda_i^2+2\lambda_i+1+\gamma^2)(3\lambda_i^2+2\lambda_i+\gamma^2), \quad (34)$$

and the λ_i's are the three solutions of

$$\lambda^3+\lambda^2+\gamma^2\lambda+\alpha^2=0, \quad (35)$$

where $\alpha=\tau_0\Delta$, $\beta=\tau_0 J_B$, and $\gamma^2=\alpha^2+\beta^2$.

We will show one special case and three asymptotes.

$\beta=0$ *isolated system*. Since Eq. (35) has solutions $\lambda_1=-1$ and $\lambda_{2,3}=\pm i\alpha$, the coefficients are determined as $C_{1c}=C_{1s}=0$, $C_{2c}=C_{3c}=\frac{1}{2}$, and $C_{2s}=C_{3s}=1/2i$. Therefore, we get $\sigma_{12}(u)=e^{-i\alpha u+i\phi}$, which is simply a natural rotation of the off-diagonal element of the density matrix.

$\alpha,\beta\ll 1$ *fast modulation*. In this asymptotic case, the roots of Eq. (35) are $\lambda_1\sim-1+\beta^2$ and $\lambda_{2,3}\sim-\frac{1}{2}\beta^2\pm i\alpha$. After determining coefficients C_i, we obtain

$$\sigma_{12}(u)\sim e^{-i\alpha u-\beta^2 u+i\phi/2}. \quad (36)$$

Therefore, the off-diagonal element of the qubit density matrix decays exponentially at a rate of $T_2^{-1}\sim\frac{1}{2}J_B^2\tau_0$.

$\alpha,\beta\gg 1$ *slow modulation*. The roots are $\lambda_1\sim-\alpha^2/\gamma^2$ and $\lambda_{2,3}\sim-\beta^2/2\gamma^2\pm\gamma i$, so $\mathrm{Re}\,\sigma_{12}(u)\sim(1/\gamma^2)\times(\beta^2 e^{(-\alpha^2/\gamma^2)u}+\alpha^2 e^{(-\beta^2/2\gamma^2)u}\cos\gamma u)\cos\phi+(\alpha/\gamma)\times e^{(-\beta^2/2\gamma^2)u}\sin\gamma u\sin\phi$ and $\mathrm{Im}\,\sigma_{12}(u)\sim e^{(-\beta^2/2\gamma^2)u}[(-\alpha/\gamma)\sin\gamma u\cos\phi+\cos\gamma u\sin\phi)$. In particular, for $1\ll\alpha\ll\beta$ (strong-coupling limit),

$$\sigma_{12}(\tau)\sim e^{(-\alpha^2/\beta^2)u}\cos\phi+ie^{-1u/2}\cos\beta u\sin\phi. \quad (37)$$

These apparent different time dependences between the real and imaginary parts stem from the choice of coupling in the form $J_B\sigma_x X/2$. If we choose the form $J_B\sigma_y X/2$ instead, the time dependences of the real and imaginary parts are interchanged. In this strong-coupling limit, the time evolution of $\sigma_{12}(\tau)$ explicitly depends on its initial phase, ϕ, not the simply like $e^{i\phi}$. Therefore, as will be discussed later, if there are several such impurities, the total time evolution of $\sigma_{12}(\tau)$ is not the simple product of each impurity's contribution. For $1\ll\beta\ll\alpha$ (weak coupling), we have

$$\sigma(u) \sim e^{(-\beta^2/2\alpha^2)u + i\phi}. \tag{38}$$

In this case, we have exponential decay with $T_2^{-1} \sim (J_B^2/2\Delta^2)(1/\tau_0)$.

For $\alpha, 1 \gg \beta$ *weak coupling or preservative regime*, we have

$$\lambda_1 \sim -1 - \frac{\beta}{\alpha^2+1} i$$

and

$$\lambda_{2,3} \sim -\frac{\beta^2}{2(\alpha^2+1)} \pm \left(1 + \frac{\beta^2}{2(\alpha^2+1)}\right) i,$$

so

$$\sigma_{12} \sim \exp\left[-i\alpha\left(1 + \frac{\beta^2}{2(\alpha^2+1)}\right)u - \frac{\beta^2}{2(\alpha^2+1)}u + i\phi\right].$$

Therefore, we again have exponential decay with $T_2^{-1} \sim J_B^2 \tau_0/2(1+\Delta^2\tau_0^2)$. This coincides with the Redfield result, which was obtained by perturbation theory and is justified in the weak-coupling case, $|J_B| \ll 1/\tau_0$. [23,24] Taking the limit $\alpha \ll 1$ further, we restore the result for $\alpha, \beta \ll 1$, fast modulation.

To summarize, the dephasing rate is given by

$$T_2^{-1} = \begin{cases} J_B^2 \tau_0/2 \quad \text{for} \quad \Delta, |J_B| \ll 1/\tau_0, \\ \Delta^2/J_B^2\tau_0 \quad \text{for the real part and} \quad 1/\tau_0 \ll \Delta \ll |J_B|, \\ 1/2\tau_0 \quad \text{for the imaginary part and} \quad 1/\tau_0 \ll \Delta \ll |J_B|, \\ J_B^2/2\Delta^2\tau_0 \quad \text{for} \quad 1/\tau_0 \ll |J_B| \ll \Delta, \\ J_B^2\tau_0/2(1+\Delta^2\tau_0^2) \quad \text{for} \quad |J_B| \ll \dfrac{1}{\tau_0}, \Delta. \end{cases} \tag{39}$$

In all regimes, $T_2 > 2\tau_0$.

B. Numerical results

Here we show the results of solving Eqs. (26) and (27) numerically.

Figure 3 shows the τ/τ_0 dependence of $n(\tau)$ and $|\sigma_{12}(\tau)|$ when $|J_B| \ll \Delta, 1/\tau_0$ (weak coupling) along with the asymptotic curves obtained analytically.

Figure 4 shows the τ/τ_0 dependence of $n(\tau)$ and $|\sigma_{12}(\tau)|$ in the case of strong coupling. It also shows the asymptotic envelope for $n(\tau)$.

Figure 5 shows the τ/τ_0 dependence of $\mathrm{Re}\,\sigma_{12}(\tau)$ in the case of strong coupling. It also shows the asymptotic curve. As shown in Figs. 3, 4, and 5, in the two contrasting limits, the numerical and analytical results coincide very well. It should be noted that we do not find Gaussian decay of the off-diagonal element of the density matrix for the initial regime, in contrast to the fluctuation in the tunneling coupling constant.

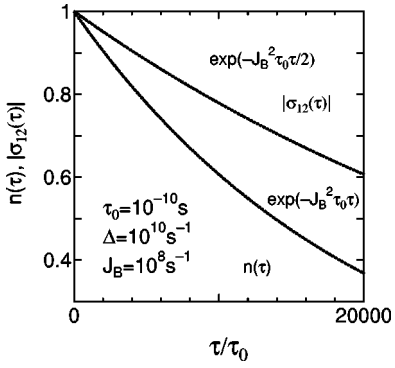

FIG. 3. The τ/τ_0 dependence of the density matrix of $n(\tau)$ and $|\sigma_{12}|(\tau)$ (solid curve) when $J_T = 0$ with $J_B = 10^8\ \mathrm{s}^{-1}$ and $\Delta = 10^{10}\ \mathrm{s}^{-1}$. Dotted lines are analytically obtained asymptotic curves, which are almost identical to the solid curves.

Next, we examine the τ_0 dependence of the relaxation rate, particularly for $|J_B| < \Delta$. Figure 6 shows the τ_0 dependences of T_1^{-1} and T_2^{-1} when $\Delta = 10^{10}\ \mathrm{s}^{-1}$ and $J_B = 10^8\ \mathrm{s}^{-1}$. The limits of the long and short τ_0 fit well with the analytical asymptotic given by Eq. (39). The τ_0 dependence of the relaxation time is fitted as

$$T_1^{-1} = 2T_2^{-1} = \frac{J_B^2\tau_0}{1+\Delta^2\tau_0^2}. \tag{40}$$

When $\tau_0 < \Delta^{-1}$, the relaxation rates increase with τ_0. When $\tau_0 > \Delta^{-1}$, the rates decrease with an increasing τ_0. The shape of the dephasing rate as a function of τ_0 is explained as follows. When $\tau_0 < \Delta^{-1}$, many dephasing events occur during one Rabi oscillation cycle, each event leads to collective disturbance. Because the long-time constant of telegraph noise leads to large fluctuations in the variance of the rotating angle in the Bloch sphere during Rabi oscillation, the dephasing time decreases with an increasing τ_0. When $\tau_0 > \Delta^{-1}$, Rabi oscillation occurs over more than one cycle in time τ_0 and, in this regime, each dephasing event is independent. Hence, the dephasing time increases with τ_0. The maximum around $\tau_0 \sim \Delta^{-1}$ is a kind of resonance.

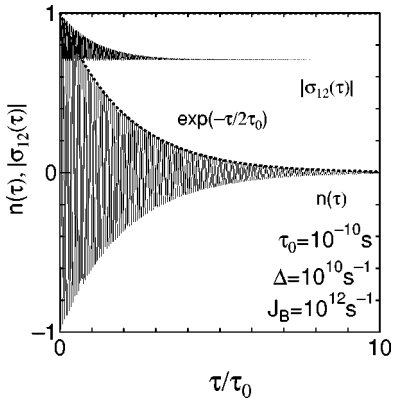

FIG. 4. The τ/τ_0 dependency of the density matrix of $n(\tau)$ and $|\sigma_{12}|(\tau)$ (solid curve) when $J_T = 0$ with $J_B = 10^{12}\ \mathrm{s}^{-1}$ and $\Delta = 10^{10}\ \mathrm{s}^{-1}$. Dotted line is the analytically obtained asymptotic envelope curve.

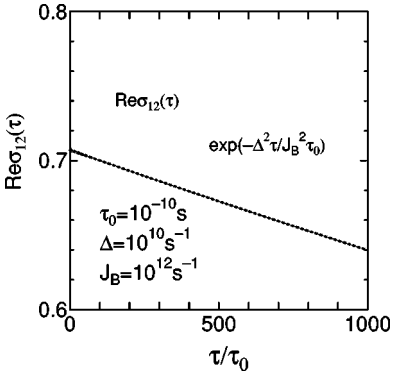

FIG. 5. The τ/τ_0 dependence of the real part of the density matrix $\sigma_{12}(\tau)$ (solid line) when $J_T=0$ with $J_B=10^{12}\ \mathrm{s}^{-1}$ and $\Delta=10^{10}\ \mathrm{s}^{-1}$. Dotted line is the analytically obtained asymptotic curve.

VI. DISCUSSION

We considered the effect of electrostatic disturbance due to background charge fluctuations. To summarize our findings, for pure dephasing ($J_T\neq 0, J_B=0$), $2\tau_0<T_2$, and $T_1=\infty$. For dephasing with relaxation of the population ($J_T=0, J_B\neq 0$), $2\tau_0<T_2=2T_1$.

Next, we discuss the relationship between the experimentally observed T_2 and our results. In the present study, the dephasing time with a single background charge was found to be longer than the time constant of the telegraph noise for both tunneling and bias fluctuations. The observed time constant of a dominant random telegraph noise is about 30 μs or longer,[10,12] so a rather long dephasing time is expected. However, in another experiment, the dephasing time was about 1 ns.[25] Therefore, a single telegraph noise source may not be enough to explain the experimental results; we should thus consider the effect of many impurity sites or other additional effects.

We consider the effect of many impurities for the case in which there is no correlation between background charges. Flucuation in the tunnel coupling arises from the modulation of the wave function in the coupled dots. The gradient of the electrostatic potential around the tunneling barrier, which comes from an electron or hole located at an impurity site, leads to a change in the tunnel coupling as well as a change in the asymmetric bias. [26] For a charge state that couples with a coupled-dot system symmetrically, the pure dephasing event is critical. For dephasing accompanied with relaxation of the population, the unitary operators of each impurity, which lead to dephasing of the qubit, are not commuting. However, by neglecting the higher-order $J_B\tau_0$'s in the dephasing rate, we can take the ensemble sum of the effect of each charge state. In such a weak-coupling case ($J_B\tau_0\ll 1$), we can use Eq. (40), and the simple summation of the dephasing rate is expressed by [20]

$$T_{2\Sigma}^{-1}=\int dJ_B P_\Sigma(J_B)\int dW P(W) T_2^{-1}(J_B,W)$$
$$=\frac{k_BT}{W_0}\frac{\pi\langle J_B^2\rangle_\Sigma}{4\Delta}, \tag{41}$$

where W_0 is the distribution width of the thermal activation energy of the charge states, $P_\Sigma(J_B)$ is the distribution function of J_B, which depends on the relative position between the qubit and impurity site, and $\langle J_B^2\rangle_\Sigma$ [$=\int dJ_B P_\Sigma(J_B) J_B^2$] is the sum over the random impurities. In the second equation, we assume uniform distribution of the activation energies of the background charges, $P(W)=1/W_0$; for typical cases, W_0/k_BT is approximately 23. [20] Use of the perturbation method showed that the dephasing rate of a Josephson charge qubit in terms of $J_B\tau_0$ is proportional to the inverse of E_J in the limit of $E_C=0$, [14,15] where E_J and E_C are the Josephson coupling constant and charging energy, respectively. This is similar to the estimate of $T_{2\Sigma}^{-1}$ in Eq. (41), where $E_J/\hbar$ corresponds to Δ. With a larger Δ and lower temperature, the dephasing rate is lower.

We next estimate the magnitude of the fluctuations, J_B. The asymmetric bias fluctuation comes from asymmetric coupling between the two dots and the background charge, which is in the form of a dipole interaction, $J_B\propto e^2 d\cos\theta/r^2$ for $d\ll r$, where d is the distance between the two dots, r is the distance between the coupled-dot system and the background charge, and θ is the angle between them. [27] Therefore, for a smaller qubit or a charge located far from the qubit, the effect of bias fluctuation should be less important. The dephasing rate is proportional to $\langle J_B^2\rangle_\Sigma$, which is estimated as

$$\sum_i\left(\frac{e^2 d}{4\pi\epsilon_r\epsilon_0\hbar r_i^2}\cos\theta_i\right)^2$$
$$\sim\left(\frac{e^2 d}{4\pi\epsilon_r\epsilon_0\hbar}\right)^2 N_i\int_{r_m(d)}^{\infty} r^2 dr\int_0^{\pi}\sin\theta\frac{\cos^2\theta}{r^4}d\theta 2\pi$$
$$=\left(\frac{e^2 d}{4\epsilon_r\epsilon_0\hbar}\right)^2\frac{4\pi}{3}\frac{N_i}{r_m(d)} \tag{42}$$

for the impurity sites where the dipole approximation is appropriate, where $r_m(d)$ is the radius beyond which the dipole approximation is valid, which depends on d, N_i is the density of impurity sites, and ϵ_r is the relative dielectric constant.

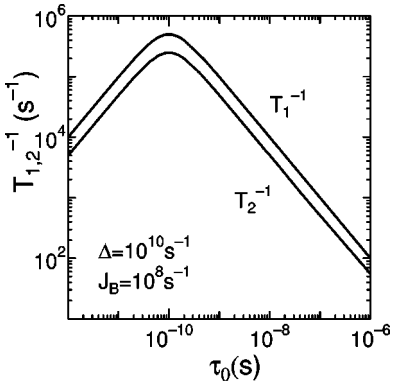

FIG. 6. The τ_0 dependence of T_1^{-1} and T_2^{-1} when $J_T=0$ with $J_B=10^8\ \mathrm{s}^{-1}$ and $\Delta=10^{10}\ \mathrm{s}^{-1}$.

Therefore, the total dephasing time is well defined. The quality factor of a quantum logic gate is defined by the ratio of Δ to $T_{2\Sigma}^{-1}$:

$$Q=\frac{\Delta}{T_{2\Sigma}^{-1}}=\frac{W_0}{k_BT}\frac{4\Delta^2}{\pi\langle J_B^2\rangle_\Sigma}, \tag{43}$$

which represents how many gate operations can be done before the quantum coherence vanishes. From Eq. (43), we conclude that a large $\Delta^2/\langle J_B^2\rangle_\Sigma$ is needed for quantum computation. We estimated Q using $d=0.3\ \mu$m, $r_m=1\ \mu$m, $\Delta=200\ \mu$eV (characteristic parameter values for an experiment in which the quantum mechanical coupling of the dots was observed in the frequency domain [5]), and $\epsilon_r=12.5$ (for GaAs). To enable quantum error correction, the lower bound of the necessary gate quality factor was roughly estimated as $Q>10^6$. [28] Thus, the density of charge states should be less than 5×10^6 cm^{-3} for fabrication. If there is a correlation between impurities (a screening effect), dephasing will be suppressed in general.[29] It should be noted that there might be strong dephasing from the nearby impurities for which the dipole approximation is not adequate, even if these are only a few impurities (a few in this case). The noncommutativity between the qubit Hamiltonian and environment Hamiltonian and the qubit back action makes it difficult to evaluate the dephasing rate for strongly coupled background charge fluctuations in the asymmetric bias case.[14]

Finally, we discuss the Josephson charge qubit system.[1] Under an appropriate condition (single-electron charging energy E_C much larger than Josephson coupling energy E_J and temperature $k_BT\ll E_J$) only two charge states in the Cooper pair box (CPB) are important, and the Hamiltonian is given by

$$H=\frac{E_J}{2}\sigma_x+\frac{\delta E_C}{2}\sigma_z+\frac{\hbar\sum_i J_{Ci}X_i}{2}\sigma_z, \tag{44}$$

where $\delta E_C=4E_C(Q_t/e-1)$ is the energy difference between the two charge states, and Q_t is the total gate-induced charge in the box. The two-charge-state basis is expressed using Pauli matrices, and $\hbar J_C$ is the coupling strength between the qubit and background charge, which induces fluctuation in the charging energy. The $E_J/\hbar$ corresponds to the asymmetric bias ϵ, and $E_C/\hbar$ corresponds to Δ. Here, $E_C\simeq122\ \mu$eV, and $E_J\simeq34\ \mu$eV; (Ref. 16) if we can neglect E_J, the pure dephasing event is critical. In pure dephasing, the effect of a large number of impurities is obtained by simply summing the dephasing rates, because H_{qb} and interaction Hamiltonian commute. When the background charge and CPB interact, the charging energy in the CPB fluctuates. The spectrum of the fluctuation is given by

$$\begin{aligned}S_{\Delta E}(\omega)&=\int dte^{i\omega t}\sum_{ij}\hbar^2\langle J_{Ci}X_i(t)J_{Cj}X_j(0)\rangle\\&=\sum_i\frac{\hbar^2J_{Ci}^2\tau_{0i}}{1+\omega^2\tau_{0i}^2}=\int dJ_CP(J_C)\int d\tau_0P(\tau_0)\\&\quad\times\frac{\hbar^2J_C^2\tau_0}{1+\omega^2\tau_0^2}\simeq\frac{k_BT}{W_0}\frac{\pi\hbar^2\langle J_C^2\rangle}{2\omega},\end{aligned} \tag{45}$$

where $\langle J_C^2\rangle=\int dJ_CP(J_C)J_C^2$, and we take an ensemble average over the activation energy, as was done in the coupled-dot system. The spectrum of the charging energy fluctuation was experimentally found to be $S_{\Delta E}(\omega)=(4E_C/e)^2\alpha/\omega$, where $\alpha=(1.3\times10^{-3}e)^2$.[16] From this estimation, $\langle J_C^2\rangle\simeq4.6\times10^{23}$ s^{-2} for 20 mK. For an initial regime, the envelope of Rabi oscillation shows Gaussian decay; namely, the off-diagonal element of the density matrix is given by $\rho_{12}(t)\simeq\rho_{12}(0)\exp(-\langle J_C^2\rangle t^2/2)e^{-iE_Jt/\hbar}$. The rate of Gaussian decay is given by $\sqrt{\langle J_C^2\rangle}/2$. For the above value of $\langle J_C^2\rangle$, the time constant of the Gaussian decay is given by 3.6×10^2 ps, which is consistent with the experimental finding of 150 ps. [16] Note that in the Gaussian regime, the time constant does not depend on the temperature. Numerical calculation[14] also suggests this type of Gaussian decay for the pure dephasing case.

At the charge degeneracy point—namely, $\delta E_C=0$—dephasing with relaxation of the population occurs. In this case, the dephasing rate is estimated using the same value of $\langle J_C^2\rangle$:

$$T_2^{-1}=\frac{\pi}{4}\frac{k_BT}{W_0}\frac{\langle J_C^2\rangle}{E_J}, \tag{46}$$

so $T_2\simeq0.28\ \mu$s for $T=20$ mK. In a recent experiment, a longer coherence time of 0.50 μs was found when the saddle point of the ground-state energy was used as a function of Q_t and the flux. [30]

VII. SUMMARY

We examined the effect of the fluctuation of a single charge in an impurity site on a qubit. Using the method of stochastic differential equations, we calculated the time evolution of the ensemble-averaged density matrix of the qubit and obtained analytical results for various conditions. The dephasing time T_2 was always longer than the time constant of the random telegraph noise for both tunneling and bias fluctuations. For bias fluctuation, T_2 was twice the relaxation time of the population in the weak-coupling case. To suppress the bias fluctuation, the coupled dots should be positioned closer together or the tunnel coupling should be made

stronger. We also investigated the case in which many impurity sites are distributed and examined the gate quality factor. For pure dephasing, which corresponds to a Josephson charge qubit experiment, the Gaussian decay of the off-diagonal element of the density matrix dominated. The present results can be applied to other quantum two-level systems in which there is telegraph-type fluctuations.

ACKNOWLEDGMENTS

We thank J. S. Tsai, T. Tanamoto, N. Imoto, M. Koashi, H. Nakano, T. Fujisawa, Y. Hirayama, T. Hayashi, C. Uchiyama, and K. Harmans for stimulating discussions. This work was partly supported by CREST-JST, Interacting Carrier Electronics Project.

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[1] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[2] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[3] T. Tanamoto, Phys. Rev. A **61**, 22 305 (2000).
[4] A. Miranowicz, S. K. Özdemir, M. Koashi, N. Imoto, and Y. Hirayama, Phys. Rev. A **65**, 062321 (2002).
[5] T. H. Oosterkamp, T. Fujisawa, W. G. van der Wiel, K. Ishibashi, R. V. Hijman, S. Tarucha, and L. P. Kouwenhoven, Nature (London) **395**, 873 (1998).
[6] T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science (Washington, DC, U.S.) **282**, 932 (1998).
[7] A. Shnirman, G. Schön, and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997).
[8] P. L. Lafarge, P. Joyez, H. Pothier, A. Cleland, T. Holst, D. Esteve, C. Urbina, and M. H. Devoret, C. R. Acad. Sci. Paris **314**, 883 (1992).
[9] G. Zimmerli, T. M. Eiles, R. L. Kautz, and J. M. Martinis, Appl. Phys. Lett. **61**, 13 (1992).
[10] C. Kurdak, C. J. Chen, D. C. Tsui, S. Parihar, S. Lyon, and G. W. Weimann, Phys. Rev. B **56**, 9813 (1997).
[11] A. B. Zorin, F.-J. Ahlers, J. Niemeyer, T. Weimann, H. Worf, V. A. Krupenin, and S. V. Lotkhov, Phys. Rev. B **53**, 13 682 (1996).
[12] T. Fujisawa and Y. Hirayama, Appl. Phys. Lett. **77**, 543 (2000).
[13] P. Dutta and P. H. Horn, Rev. Mod. Phys. **53**, 497 (1981).
[14] E. Paladino, L. Faoro, G. Falci, and R. Fazio, Phys. Rev. Lett. **88**, 228304 (2002).
[15] A. Shnirman, Y. Makhlin, and G. Schön, Phys. Scr. T **102**, 147 (2002).
[16] Y. Nakamura, Yu. A. Pashkin, T. Yamamoto, and J. S. Tsai, Phys. Rev. Lett. **88**, 047901 (2002).
[17] In the present study, dissipation means the relaxation of the population of bonding-antibonding states. On large time scales, the qubit state becomes mixed with equal populations of bonding and antibonding states; this relaxation is not an energy relaxation.
[18] Y. M. Galperin and K. A. Chao, Phys. Rev. B **52**, 12 126 (1995).
[19] R. Bauernschmitt and Yuli V. Nazarov, Phys. Rev. B **47**, 9997 (1992).
[20] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, 1997).
[21] A. I. Burshtein, Sov. Phys. JETP **22**, 939 (1965).
[22] A. I. Burshtein, Sov. Phys. JETP **21**, 567 (1965).
[23] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[24] C. L. Slichter, *Principles of Magnetic Resonance*, 3th ed. (Springer-Verlag, Berlin, 1997).
[25] T. Fujisawa, W. G. van der Wiel, and L. P. Kouwenhoven, Physica E (Amsterdam) **7**, 413 (2000).
[26] G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B **59**, 2070 (1999).
[27] In the present study, we assume the interaction between the coupled-dot system and charge state to be dipole-monopole coupling. When the electron reservoir screens the charge state effectively, the charge state and reservoir act on the system as a dipole. The interaction type is dipole-dipole, and the interaction strength decreases more rapidly as the distance between the charge state and system increases. A longer dephasing time is thus expected compared with that of a dipole-monopole interaction.
[28] J. Preskill, Proc. R. Soc. London, Ser. A **454**, 385 (1998).
[29] T. Itakura and Y. Tokura (unpublished).
[30] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science (Washington, DC, U.S.) **296**, 886 (2002).