

## Vortons in the SO(5) model of high-temperature superconductivity

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It has been shown that superconducting vortices with antiferromagnetic cores arise within Zhang's SO(5) model of high-temperature superconductivity. Similar phenomena where the symmetry is not restored in the core of the vortex were discussed by Witten in the case of cosmic strings. It was also suggested that such strings can form stable vortons, which are closed loops of such vortices. Motivated by this analogy, in following we will show that loops of such vortices in the SO(5) model of high- $T_c$  superconductivity can exist as classically stable objects, stabilized by the presence of conserved charges trapped on the vortex core. These objects carry an angular momentum which counteracts the effect of the string tension that causes the loops to shrink. The existence of such quasiparticles, which are called vortons, could be interesting for the physics of high-temperature superconductors. We also speculate that the phase transition between superconducting and antiferromagnetic phases at zero external magnetic field when the doping parameter changes is associated with vortons.

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### I. INTRODUCTION

In the pursuit of a unified theory of high temperature superconductivity and antiferromagnetism, Zhang proposed the SO(5) theory of antiferromagnetism (AF) and  $d$ -wave superconductivity (dSC) in the high- $T_c$  cuprates.<sup>1</sup> The order parameter for antiferromagnetism is the Neel vector  $\vec{m}$  which is a vector under the action of the SO(3), the group of 3- $d$  spatial rotations. On the superconducting side, the relevant order parameter is the complex superconducting order parameter  $\psi$ , which describes the gap in the electron spectrum. The effective Lagrangian for  $\psi$  is invariant under the group U(1). The big step that Zhang originally proposed is that the two symmetry groups can be combined within a larger symmetry group, namely, SO(5). This means that the three component vector  $\vec{m}$  and the complex order parameter  $\psi$  can be combined to form a "superspin" vector  $\vec{n} = (\psi, m_1, m_2, m_3, \psi)$  which transforms under the group SO(5). The presence of doping in the cuprates actually breaks this symmetry down to SO(3)  $\times$  U(1). At low doping, the AF phase is favored, corresponding to nonzero expectation value for  $|\vec{m}|$ , ( $\langle |\psi| \rangle = 0$  and  $\langle |\vec{m}| \rangle \neq 0$ ). As the doping is increased eventually the dSC phase becomes energetically favorable with  $\langle |\psi| \rangle \neq 0$  and  $\langle |\vec{m}| \rangle = 0$ . As Zhang originally discussed in Ref. 1, the region of intermediate doping (near the AF-dSC phase boundary) should be characterized by conventional superconducting vortices, but possessing antiferromagnetic cores. This suggestion was verified by various groups who looked for numerical solutions of the classical equations of motion for different parameters.<sup>2-4</sup> Furthermore, there has been recent experimental evidence that suggests this theoretical picture may be correct.<sup>5-9</sup>

Our interest in this topic arises from recent work<sup>10-13</sup> within a completely different context, namely the theory of the strong interaction, QCD, at high baryon density<sup>14,15</sup> (see Ref. 16 for a good review of high density QCD and a long list of references). In Ref. 12 we have shown that similar vortices with nontrivial core structure are present within

QCD at high baryon density for physical values of the parameters. In this case, the symmetry breaking parameter responsible for the anisotropy is the difference between the up and down quark masses. The effective Lagrangian which describes the SO(5) theory of high- $T_c$  superconductivity is very similar to the one used in Ref. 12, aside from numerical constants of course. This will prove to be useful in making analogies between condensed matter physics and particle physics throughout the course of this work. We will make use of the results given in Refs. 12 and 13 throughout this paper.

In the present paper we will show that it is possible to have loops of dSC vortices with AF cores that are classically stable objects. The source of this stability is the presence of conserved charges trapped on the vortex core, leading to nonzero angular momentum. The conservation of angular momentum prevents the vortex loops from shrinking and eventually disappearing. This class of quasiparticles, which generally possess nonzero angular momentum and charge, is called vortons. The presence of the AF condensate is crucial, as it is what allows the vortons to carry angular momentum and become classically stable quasiparticles.

The phenomenon where a condensate forms in the core of a vortex, such that the vortex can form a spinning loop leading to classical stability, is not by any means a new phenomenon. This idea was considered long ago in the context of cosmology and cosmic strings.<sup>17-27</sup> Our contribution here is the application of the previously developed (for cosmic strings and high density QCD) technique to the high- $T_c$  superconductors.

This paper is organized as follows. In Sec. II we will review the work of Refs. 1-4, where dSC vortices with AF cores were originally presented. A comparison will be made between these vortices and other vortices with nontrivial core structure present in high density QCD.<sup>12</sup> Although no new results are obtained in this section, we believe that making a correspondence between two very different fields of physics is quite a useful exercise. In particular, applying the topological (and some analytical) arguments developed in

Ref. 12 we reproduce the result<sup>1-4</sup> that there is a critical value for the coupling constant above which the AF core is not developed. In a sense to our knowledge this is a new explanation of the phenomenon based on analytical (rather than numerical) calculations. Section III will contain our results, where we will show that classically stable quasiparticles called vortons are present within the SO(5) theory of high temperature superconductivity. Section IV will end with concluding remarks and possible experimental signatures of the quasiparticles. We also formulate a conjecture that the vortons are responsible for the phase transition between AF and dSC phases at zero external magnetic field, when the doping parameter  $\mu$  is changed.

## II. VORTICES WITH NONTRIVIAL CORE STRUCTURE-QCD VS HIGH TEMPERATURE SUPERCONDUCTIVITY

We will begin by briefly describing the work of Ref. 12 where vortices with nontrivial core structure were described within the context of QCD at large baryon chemical potential. This will be beneficial in order to make analogies between condensed matter physics and particle physics. We should note that this is only a review section and no new material will be presented here. However, the analogy discussed below will prove to be useful for the analysis which follows. We will then continue with a review and comparison of the vortices with nontrivial core structure which appear within the SO(5) theory of high  $T_c$  superconductivity.<sup>1-4</sup>

### A. Vortices in high density QCD

There has been a large amount of interest within the particle physics community on the subject of QCD, the theory of the strong interaction, at large baryon density<sup>14,15</sup> (for many more references and a nice review, see Ref. 16). At zero baryon density, QCD is a theory of quarks and gluons which are strongly coupled, such that confinement takes place and the observable particles are colorless hadrons rather than quarks and gluons. As one increases the baryon chemical potential the new superconducting phase when the baryon symmetry is spontaneously broken occurs. To explain this phenomenon, let us recall that, in QED, the electron-electron interaction is in general repulsive, and superconductivity is a very subtle effect. In non-Abelian theory, QCD, simple one gluon exchange is always attractive in the color  $\bar{3}$  channel. As is well known from conventional BCS theory of superconductivity, an arbitrarily small attractive interaction will lead to the formation of condensate of Cooper pairs near the Fermi surface. This is in fact what happens in QCD at large baryon density. The ground state of the high density phase of QCD is characterized by a diquark condensate<sup>14,15</sup> analogous to the condensate of electron Cooper pairs present in a conventional superconductor. This phase of QCD is referred to as a color superconducting phase. The typical chemical potential where this phase is thought to occur ( $\mu \sim 500$  MeV,  $\Delta \approx 100$  MeV, temp  $T_c \sim 0.6\Delta$ , where  $\Delta$  is superconducting gap) cannot be realized on Earth. The interest in this region of the QCD phase diagram is motivated by the fact that such densities may be realized within the core of compact stars, such as neutron stars.<sup>28</sup>

We will not go into specific details, only state a few of the main features of the color superconducting phase of high density QCD. For the number of quark flavors  $N_f=3$  (up, down, and strange) and the number of colors  $N_c=3$ , the dominant parts of the diquark condensate take the following forms:<sup>15</sup>

$$\begin{aligned} \langle q_{L\alpha}^{ia} q_{L\beta}^{jb} \rangle^* &\sim \epsilon_{\alpha\beta\gamma} \epsilon^{ij} \epsilon^{abc} X_c^\gamma, \\ \langle q_{R\alpha}^{ia} q_{R\beta}^{jb} \rangle^* &\sim \epsilon_{\alpha\beta\gamma} \epsilon^{ij} \epsilon^{abc} Y_c^\gamma, \end{aligned} \quad (1)$$

where  $L$  and  $R$  represent left and right handed quarks,  $\alpha$ ,  $\beta$ , and  $\gamma$  are the flavor indices,  $i$  and  $j$  are spinor indices,  $a$ ,  $b$ , and  $c$  are color indices, and  $X_c^\gamma$  and  $Y_c^\gamma$  are complex color-flavor matrices describing the Goldstone bosons. This diquark condensate breaks the original symmetry group  $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$  [color gauge symmetry, left and right flavor symmetries, baryonic symmetry, and axial  $U(1)_A$  symmetry] down to the diagonal subgroup  $SU(3)_{c+L(R)}$ . This diagonal subgroup tells us that whenever we perform an  $SU(3)$  color rotation, we must simultaneously perform a left (right) handed flavor rotation. Since color rotations are now linked with flavor rotations, this phase of high density QCD with  $N_f=N_c=3$  is referred to as the color-flavor locked phase (CFL).<sup>15</sup> Counting the number of broken generators, we see that there should be 18 Goldstone bosons (GBs). Of these 18 GBs, eight of them are eaten by the Higgs mechanism resulting in all eight gluons acquiring a mass. This leaves ten GBs, an octet related to the breaking of  $SU(3)$  and two singlets related to  $U(1)_B$  and  $U(1)_A$ . All of these bosons [except the one related to  $U(1)_B$ ] are actually pseudo-Goldstone bosons due to the small explicit violation of the symmetry. In order to describe the low energy degrees of freedom, namely the octet of Goldstone bosons, one can construct the following gauge invariant field:

$$\Sigma_\gamma^\beta = \sum_c X_c^\beta Y_c^{\gamma*} = \exp(i\pi^a \lambda^a / f_\pi), \quad (2)$$

with the  $SU(3)$  generators  $\lambda^a$  normalized as  $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$  and  $f_\pi^2 \sim \mu^2 / (2\pi^2)$  being the pion decay constant which can be calculated in the large  $\mu$  limit. Prior to the work of Refs. 29-31 it was believed that the ground state of the CFL phase was given by  $\Sigma_o = \text{diag}(1,1,1)$ . However, it was noticed that for a physical value of the strange quark mass ( $m_s \gg m_u, m_d$ ) this may not necessarily be the case. In particular, it was argued in that for  $m_s > 60$  MeV along with the diquark condensate (1) a new  $K^0$  condensation would occur and that  $\Sigma_o = \text{diag}(1,1,1)$  as given above would no longer represent the true ground state of the CFL phase, but rather the vacuum expectation value of the nondiagonal elements of  $\sim \langle \Sigma_2^1 \rangle$  representing the  $K^0$  GB would get a non-zero magnitude.<sup>30</sup> Therefore,  $\Sigma_o$  would be rotated in some different direction in flavor space. In the physical case where the isospin symmetry is not exact (i.e., the up and down quarks have different masses,  $m_d > m_u$ ) and we have overall electric charge neutrality,  $K^0$  condensation occurs. The ap-

appropriate expression for  $\Sigma_o$  describing the  $K^0$  condensed ground state can be parametrized as

$$\Sigma_o = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{K^0} & \sin \theta_{K^0} e^{-i\phi} \\ 0 & -\sin \theta_{K^0} e^{i\phi} & \cos \theta_{K^0} \end{pmatrix}, \quad (3)$$

where  $\phi$  describes the new Goldstone mode associated with  $K^0$  condensation along with the diquark condensate discussed above and  $\theta_{K^0} \equiv \sqrt{2} \langle |K^0| \rangle / f_\pi$  describes the strength of the kaon condensation with<sup>30</sup>

$$\cos \theta_{K^0} = \frac{m_0^2}{\mu_{\text{eff}}}, \quad \mu_{\text{eff}} = \frac{m_s^2}{2\mu},$$

$$m_0^2 = a m_u (m_d + m_s), \quad a = \frac{3\Delta^2}{\pi^2 f_\pi^2}. \quad (4)$$

where  $m_u, m_d$ , and  $m_s$  are the masses of the up, down, and strange quarks, respectively,  $\mu$  is the chemical potential, and  $\Delta$  is the size of the color superconducting gap ( $\sim 100$  MeV). In order for kaon condensation ( $\theta_{K^0} \neq 0$ ) to occur, we must have  $m_0 < \mu_{\text{eff}}$ . This leads to the breaking of the hypercharge  $U(1)_Y$  symmetry. As discussed in Refs. 30 and 12, the lightest degrees of freedom in the CFL+ $K^0$  phase are the  $K^0$  and  $K^+$  mesons. The essential physics of these mesons can be captured with the following effective Lagrangian:

$$\mathcal{L} = |\partial_0 \Phi|^2 - v^2 |\partial_i \Phi|^2 - \lambda \left( |\Phi|^2 - \frac{\eta^2}{2} \right)^2 - \delta m^2 \Phi^\dagger \tau_3 \Phi, \quad (5)$$

where  $\Phi = (K^+, K^0)$  is a complex doublet describing the  $K^0$  and  $K^+$  mesons,  $\tau_3$  is the third Pauli matrix. The constants  $f_\pi$  and  $v$  have been calculated in the leading perturbative approximation and are given by<sup>32,33</sup>

$$f_\pi^2 = \frac{21 - 8 \ln 2}{18} \frac{\mu^2}{2\pi^2}, \quad v^2 = \frac{1}{3}. \quad (6)$$

The remaining parameters in the effective Lagrangian (5) have been obtained from a more complete description of the octet of Goldstone bosons:<sup>30,12</sup>

$$\delta m^2 = \frac{a}{2} m_s (m_d - m_u),$$

$$\eta^2 = \frac{\mu_{\text{eff}}^2 - m_0^2}{\lambda}. \quad (7)$$

In the case that the parameter  $\delta m^2$  in Eq. (5) is zero, the Lagrangian is invariant under the symmetry group  $SU(2)_I \times U(1)_Y \rightarrow U(1)$  [broken down to  $U(1)$ ]. From topological arguments we know that such a Lagrangian does not possess vortices since the vacuum manifold is that of a 3 sphere and therefore does not have noncontractible loops. In the case that  $\delta m^2$  is relatively large, then the residual symmetry group is  $U(1) \times U(1) \rightarrow U(1)$  and the vacuum manifold is

that of a circle, leading to the formation of classically stable global vortex solutions. Since  $\delta m^2 > 0$ , then it is  $K^0$  which forms the normal global strings with  $|K^0(r=0)|=0$  and  $|K^0(r=\infty)| = \eta/\sqrt{2}$ , where the phase varies from 0 to  $2\pi$  as one encircles the core of the vortex. From these two limiting cases, it is clear that there should be some intermediate region that somehow interpolates (as a function of  $\delta m^2$ ) between the two cases. At some finite magnitude of  $\delta m^2$ , an instability arises through the condensation of the  $K^+$  field inside the core of the vortex. As the magnitude of  $m_d - m_u$  decreases, the size of the core becomes larger and larger with nonzero values of both  $K^0$  and  $K^+$  condensates inside the core. Finally, at  $m_d = m_u$  the core of the string (with nonzero condensates  $K^0$  and  $K^+$ ) fills the entire space, in which case the meaning of the string is completely lost, and we are left with the situation when  $SU(2)$  symmetry is exact: no stable strings are possible. As discussed in Ref. 12, the  $K^0$  vortex with a  $K^+$  condensate on the core can be approximately described by the following ansatz:

$$K^0 = \frac{\eta}{\sqrt{2}} f(r) e^{i\phi}, \quad (8)$$

$$K^+ = \frac{\sigma}{\sqrt{2}} g(r). \quad (9)$$

where  $\phi$  is the azimuthal angle in cylindrical coordinates,  $f(r)$  and  $g(r)$  are solutions to the classical equations of motion obeying the boundary condition  $f(0)=0, f(\infty)=1, g'(0)=0, g(\infty)=0, g(0)=1$ , and  $\sigma$  gives the size of the condensate at the string core ( $r=0$ ) determined by the parameters of the Lagrangian. The width of the string when symmetry is restored in the core, is given by  $1/\kappa$ , where  $\kappa^2 \sim (\mu_{\text{eff}}^2 - m_0^2 + \delta m^2)$  (the mass scale for  $K^0$ ). The width of the  $K^+$  condensate when symmetry is not restored can be estimated as  $1/\beta$ , where  $\beta^2 \sim 2\delta m^2$  is the mass difference between  $K^+$  and  $K^0$  off of the vortex. From these estimates we see that, as  $\delta m^2 \rightarrow 0$ , the width of the vortex core increases as explained qualitatively above. In order to estimate the critical point where  $K^+$  condensation occurs, we have studied the dynamics of  $K^+$  in the background of a  $K^0$  global vortex solution. This is done by substituting the string solution (8) into the energy expression derived from Lagrangian (5) and keeping only terms which are quadratic order in  $K^+$ . The shift in the energy (per unit length) in the background of a  $K^0$  vortex is then given in dimensionless variables as follows<sup>12</sup>:

$$\delta E = \frac{\eta^2 v^2}{2} \int d^2 \tilde{r} g(\tilde{r}) [\hat{O} + \epsilon] g(\tilde{r}), \quad (10)$$

where

$$\hat{O} = -\frac{1}{\tilde{r}} \frac{d}{d\tilde{r}} \left( \tilde{r} \frac{d}{d\tilde{r}} \right) - (1 - \cos \theta_{K^0}) [1 - f^2(\tilde{r})], \quad (11)$$

$$\epsilon = \frac{a}{2} \frac{m_s(m_d - m_u)}{\mu_{\text{eff}}^2}. \quad (12)$$

The problem is reduced to the analysis of the two-dimensional Schrodinger equation for a particle in an attractive potential  $V(\tilde{r}) = -(1 - \cos \theta_{K^0})[1 - f^2(\tilde{r})]$  with  $f(\tilde{r})$  being the solution of the classical equation of motion for  $K^0$  with the boundary conditions  $f(0) = 0$  and  $f(\infty) = 1$ . Such a potential is negative everywhere and approaches zero at infinity. As is known from standard course in quantum mechanics,<sup>34</sup> for an arbitrarily weak potential well there is always a negative energy bound state in one and two spatial dimensions; in three dimensions a negative energy bound state may not exist. For the two-dimensional case (the relevant problem in our case) the lowest energy level of the bound state is always negative and exponentially small for small  $\lambda'$ . One should note that our specific potential  $V(\tilde{r}) = -(1 - \cos \theta_{K^0})[1 - f^2(\tilde{r})]$ , which enters Eq. (10) is not literally the potential well, however one can always construct the potential well  $V'$  such that its absolute value is smaller than  $|V(\tilde{r})|$  everywhere, i.e.,  $|V'| < |V(\tilde{r})|$  for all  $\tilde{r}$ . For the potential well  $V'$  we know that the negative energy bound state always exists; when  $V'$  is replaced by  $V$  it makes the energy eigenvalue even lower. Therefore, operator (11) always has a negative mode irrespective of the local properties of function  $f(r)$ . As a consequence, if  $\epsilon = 0$  then string (8) is an unstable solution of the classical equation of motion, the result we expected from the beginning from topological arguments. The instability manifests itself in the form of a negative energy bound state solution of the corresponding two-dimensional Schrodinger equation (10) irrespective of the magnitudes of the parameters. The problem of determining when  $K^+$ -condensation occurs is now reduced to solving the two-dimensional Schrodinger type equation  $\hat{O}g = \hat{E}g$ . From the previous discussions we know that  $\hat{E}$  for the ground state is always negative. However, to insure the instability with respect to  $K^+$  condensation one should require a relatively large negative value, i.e.,  $\hat{E} + \epsilon < 0$ . This cannot happen for an arbitrary weak coupling constant  $\sim (1 - \cos \theta_{K^0})$  when  $\theta_{K^0}$  is small. However, it does happen for relatively large  $\theta_{K^0}$ . To calculate the minimal critical value  $\theta_{\text{crit}}$  when  $K^+$  condensation develops, one should calculate the eigenvalue  $\hat{E}$  as a function of parameter  $\theta_{K^0}$  and solve the equation  $\hat{E}(\theta_{\text{crit}}) + \epsilon = 0$ . For a very small coupling constant  $\lambda' = (1 - \cos \theta_{K^0}) \rightarrow 0$  the bound state energy is negative and exponentially small,  $\hat{E} \sim -e^{-1/\lambda'}$ . However, for realistic parameters of  $\mu$ ,  $\Delta$ ,  $m_s$ ,  $m_u$ ,  $m_d$  the parameter  $\epsilon$  is not very small and we expect that in the region relevant for us the bound state energy  $\hat{E}$  is the same order of magnitude as the potential energy  $\sim \lambda'$ . In this case we estimate  $\theta_{\text{crit}}$  from the following conditions  $-\hat{E}(\theta_{\text{crit}}) \sim \lambda' \sim (1 - \cos \theta_{\text{crit}}) \sim \epsilon$ , with a result which can be parametrically represented as

$$\sin \frac{\theta_{\text{crit}}}{2} \sim \text{const} \frac{\Delta}{m_s} \sqrt{\frac{(m_d - m_u)}{m_s}}, \quad (13)$$

where we have neglected all numerical factors in order to explicitly demonstrate the dependence of  $\theta_{\text{crit}}$  on the external parameters. The limit of exact isospin symmetry, which corresponds to  $m_d \rightarrow m_u$  when the string becomes unstable, can be easily understood from expression (13). Indeed, in the case that the critical parameter  $\theta_{\text{crit}} \rightarrow 0$  becomes an arbitrarily small number the  $K^+$  instability would develop for arbitrarily small  $\theta_{K^0} > 0$ . The region occupied by the  $K^+$  condensate at this point is determined by the behavior of lowest energy mode  $g$  at large distances,  $g(\tilde{r} \rightarrow \infty) \sim \exp(-\hat{E}\tilde{r})$  such that a typical  $\tilde{r} \sim (m_d - m_u)^{-1} \rightarrow \infty$  as expected.

### B. Vortices in the SO(5) theory of high-temperature superconductivity

We will now review the work of Refs. 1–4, where it was shown that vortices with nontrivial core structure similar to the ones discussed above for high density QCD are present within the SO(5) theory of high- $T_c$  superconductivity. The effective Lagrangian which describes the Neel vector  $\vec{m}$  and dSC order parameter  $\psi$  in the presence of a zero external electromagnetic field is given by<sup>1</sup>

$$\mathcal{L} = \frac{\chi}{2} (|\partial_t \vec{m}|^2 + |\partial_t \psi|^2) - \frac{\rho}{2} (|\nabla \vec{m}|^2 + |\nabla \psi|^2) + (\tilde{g} - a) |\vec{m}|^2 - a |\psi|^2 - \frac{1}{2} b |\vec{m}|^4 - \frac{1}{2} b |\psi|^4 - b |\vec{m}|^2 |\psi|^2, \quad (14)$$

where we neglected the electromagnetic contribution to the vortex structure. Actually, one can show that the electromagnetic field does not change the qualitative effects which are the subject of the present paper, and therefore, will be ignored in what follows. Here  $\chi$  is the susceptibility and  $\rho = \hbar^2/m^*$  is the stiffness parameter. In reality, we know that the properties of  $\chi$  and  $\rho$  are different in different directions. However, when we discuss the topological properties of the configuration this difference can change the quantitative results but cannot change the qualitative picture. The Neel vector has three spatial components  $\vec{m} = (m_1, m_2, m_3)$  and the superconducting order parameter is a complex field  $\psi = \psi_1 + i\psi_2$ . The parameters of the above effective Lagrangian are given by

$$a < 0, \quad b > 0, \quad (15)$$

$$\tilde{g} = 4\chi(\mu_c^2 - \mu^2). \quad (16)$$

$$\xi = \sqrt{\frac{\rho}{2|a|}} \quad (17)$$

where  $\mu$  is the chemical potential (or doping, not to be confused with the chemical potential for QCD in Sec. II A),  $\mu_c$  is the critical chemical potential which defines the AF-dSC phase boundary, and  $\xi$  is the coherence length. The anisotropy  $\tilde{g}$  is included which explicitly breaks the SO(5) symmetry in the following fashion,  $\text{SO}(5) \rightarrow \text{SO}(3) \times \text{U}(1)$ . If

$\tilde{g}=0$  then the SO(5) symmetry is restored and the order parameters  $\vec{m}$  and  $\psi$  can be organized into a superspin order parameter  $\vec{n}=(\psi_1, m_1, m_2, m_3, \psi_2)$  which transforms in the vector representation of SO(5), as Zhang originally proposed in Ref. 1. In the following we will consider  $\mu > \mu_c$  and  $\tilde{g} < 0$  so that we are in the dSC phase and  $\langle |\psi| \rangle = \sqrt{|a|/b}$  and  $\langle |\vec{m}| \rangle = 0$  in the bulk. One immediately notices that the form of the Lagrangian for the SO(5) theory is very similar to the Lagrangian used to describe  $K$  strings in high density QCD [Eq. (5)] in the previous section. In particular, the key element in construction of the vortices with a nonzero condensate in the core, the asymmetry parameter, is determined by the magnitude of  $\delta m^2$  in Eq. (5). For the SO(5) theory it is replaced by the parameter of anisotropy  $\tilde{g}$  in Eq. (16).

A nonzero vacuum expectation value for  $\psi$  signals the onset of superconductivity and the breaking of the U(1) symmetry. It is well known that stable vortices can form since the topology of the vacuum manifold is that of a circle. Analogous to the situation for high density QCD, for a certain range of the anisotropy parameter  $\tilde{g}$  these vortices should have an antiferromagnetic core ( $\langle |\vec{m}(\vec{r}=0)| \rangle \neq 0$ ). This was initially pointed out by Zhang<sup>1</sup> when he introduced the SO(5) model and further studied in Refs. 2–4. Similar to the  $K$  vortex/condensate solution given by Eqs. (8) and (9), these vortices are described by the following static field configurations:

$$\psi = \sqrt{\frac{|a|}{b}} f(r) e^{i\phi}, \quad (18)$$

$$\vec{m} = \sigma \sqrt{\frac{|a|}{b}} g(r) \hat{m}, \quad (19)$$

where  $\phi$  is the azimuthal angle in cylindrical coordinates,  $\sigma$  is the parameter obeying the relation  $0 \leq \sigma < 1$ , and  $\hat{m}$  is an arbitrary unit vector. As before,  $f(r)$  and  $g(r)$  are solutions to the classical equations of motion satisfying the boundary conditions  $f(0)=0, f(\infty)=1$  and  $g'(0)=0, g(\infty)=0$ , and  $g(0)=1$ . The width of the vortex determined by the profile function  $f$  is approximately given by coherence length (17),  $\delta_\psi \approx \xi$ . The width of the condensate in the core (if it forms) is estimated to be of order of  $\delta_m \approx 1/\sqrt{|\tilde{g}|} \sim 1/\sqrt{(\mu^2 - \mu_c^2)}$  and becomes very large at the phase boundary.

Using what we have already learned from QCD and the results from earlier work on these vortices,<sup>2–4</sup> we can immediately summarize the main features of these objects. Numerical calculations in Ref. 3 confirm that as the anisotropy parameter  $|\tilde{g}|$  is decreased, the size and width of the condensate in the core increases. We will support these numerical calculations using some analytical arguments given below.

The free energy (per unit length) obtained from Lagrangian (14) is

$$\begin{aligned} \frac{\mathcal{F}}{l} = \int d^2r & \left[ \frac{\chi}{2} (|\partial_t \vec{m}|^2 + |\partial_t \psi|^2) + \frac{\rho}{2} (|\nabla \vec{m}|^2 + |\nabla \psi|^2) \right. \\ & - (|a| + \tilde{g}) |\vec{m}|^2 - |a| |\psi|^2 + \frac{1}{2} b |\vec{m}|^4 + \frac{1}{2} b |\psi|^4 \\ & \left. + b |\vec{m}|^2 |\psi|^2 \right]. \end{aligned} \quad (20)$$

If anisotropy  $\tilde{g} \equiv 0$  we know from topological arguments that this theory does not possess any vortices since the vacuum manifold is that of a 4 sphere and therefore does not have noncontractible loops. If  $|\tilde{g}|$  is relatively large, then the residual symmetry group has a subgroup U(1) and the vacuum manifold is that of circle, leading to the formation of classically stable global vortex solution described in terms of  $\psi$  field with a typical profile function when  $|\psi(r=0)|=0$  and  $|\psi(r=\infty)|=\sqrt{|a|/b}$ . From these two limiting cases, as discussed in Sec. II A, it is clear that there should be some intermediate region that somehow interpolates (as a function of  $\tilde{g}$ ) between the two cases. The way this interpolation works is as follows (see Sec. II A, where the physical picture is quite analogous to the present case). At some finite magnitude of  $\tilde{g}$ , an instability arises through the condensation of the  $\vec{m}$  field inside the core of the vortex. As the magnitude of  $\tilde{g}$  decreases, the size of the core becomes larger and larger with nonzero values of both  $\vec{m}$  and  $\psi$  condensates inside the core. Finally, at  $\tilde{g}=0$  the core of the string (with nonzero condensates  $\vec{m}$  and  $\psi$ ) fills the entire space, in which case the meaning of the string is completely lost, and we are left with the situation when the SO(5) symmetry is exact: no stable strings are possible.

In order to estimate that critical value of the parameter  $\tilde{g} = \tilde{g}_{\text{crit}}$  where an AF core forms inside the vortex, the same method can be applied as described for the QCD color superconductor in Sec. II A. We will use the following change of variables in order to express the free energy in terms of dimensionless variables only:

$$\psi = \sqrt{\frac{|a|}{b}} \psi', \quad \vec{m} = \sqrt{\frac{|a|}{b}} \vec{m}', \quad \vec{r} = \xi \vec{r}'. \quad (21)$$

Expanding the expression for the change in free energy (20) in the background of a  $\psi$  vortex solution given by Eq. (18) and keeping only quadratic terms in  $\vec{m}$ , we have

$$\frac{\delta \mathcal{F}}{l} = \sigma^2 \frac{\rho |a|}{2b} \int d^2r' g(r') [\hat{H} + \epsilon] g(r'), \quad (22)$$

where

$$\hat{H} = -\frac{1}{r'} \frac{d}{dr'} \left( r' \frac{d}{dr'} \right) - [1 - f^2(r')], \quad (23)$$

$$\epsilon = \frac{|\tilde{g}|}{|a|} = 4 \frac{\chi(\mu^2 - \mu_c^2)}{|a|}. \quad (24)$$

Since we are working in the dSC phase  $\tilde{g} < 0$  the perturbation  $\epsilon > 0$ . We have now cast the change in the free energy in the exact same form as we did for the QCD vortices in Sec. II A. The problem is now reduced to the analysis of the two-dimensional Schrödinger equation for a particle in an attractive potential  $V(r') = -[1 - f^2(r')]$ . As before, this potential is negative everywhere and approaches zero at infinity. This means that the ground state eigenfunction  $\hat{H}g_0 = \hat{E}g_0$  has a negative eigenvalue  $\hat{E}$ . The instability with respect to the formation of the AF condensate in the core occurs not for an arbitrarily small negative eigenvalue  $\hat{E}$ , but when the absolute value of  $|\hat{E}|$  is large enough to overcome the positive contribution due to  $\epsilon$ . Therefore, we immediately see that an AF core forms if  $\hat{E} + \epsilon \leq 0$ . If  $|\tilde{g}|$  is greater than some critical value  $\tilde{g}_{\text{crit}}$  then it is not energetically favorable for an antiferromagnetic core to form, and dSC vortices will possess a normal core where the symmetry is restored. Following the same procedure as in the QCD case, we have

$$\frac{|\tilde{g}_{\text{crit}}|}{|a|} = 4 \frac{\chi(\mu^2 - \mu_c^2)}{|a|} \approx 0.2. \quad (25)$$

where for numerical estimates we used the variational approach developed in Ref. 19.

Above we have reviewed the basic properties of superconducting vortices with an antiferromagnetic core within the SO(5) theory of superconductivity. We should emphasize once more that all results presented above are not new and have been discussed previously from a different perspective. Let us repeat the main results of this section once again: If  $\tilde{g} = 0$  then the dSC vortices are unstable. If  $0 < |\tilde{g}| < \tilde{g}_{\text{crit}}$  then an AF core will form inside the dSC vortices. The width of the AF core in this case becomes larger and larger when we approach the phase transition line, i.e.,  $\tilde{g} \rightarrow 0$ . Finally, if  $|\tilde{g}| > \tilde{g}_{\text{crit}}$  then the dSC vortices will have a normal core when symmetry is restored and  $|\vec{m}|(r=0) = |\psi|(r=0) = 0$ . In what follows we will always be working in the region of the phase diagram where  $0 < |\tilde{g}| < \tilde{g}_{\text{crit}}$  and dSC vortices have an AF core (which will be referred to as dSC/AF vortices). Now we will proceed to Sec. III and introduce vortons, loops of dSC/AF vortices which are stabilized by angular momentum.

### III. VORTONS IN HIGH-TEMPERATURE SUPERCONDUCTIVITY

We will now consider the interesting possibility that loops of the dSC/AF vortices can exist as classically stable objects (and at least metastable quantum mechanically). This stability arises through a mechanism where topological and Noether charges can be trapped on the core of the vortex. Such objects, called vortons, have been studied extensively in the context of cosmology where cosmic strings have a nontrivial core structure.<sup>17-24</sup> Such vortons are also present within high density QCD where vortices with a condensate trapped on the core are realized.<sup>13</sup>

As Davis and Shellard originally pointed out in Refs. 20, 22, and 23, if one has a theory which contains vortices

with a condensate trapped on the core then loops of such vortices can form which are stabilized by angular momentum alone. We will consider a large loop of string of radius  $R \gg \delta$ , where  $\delta$  is the vortex thickness, so that curvature effects can be neglected. The  $z$  axis is defined along the length of the string, varying from 0 to  $L = 2\pi R$  as one goes around the loop. Although we are considering a circular loop for simplicity at the moment, we realize that this is probably not the relevant physical case. The results we will discuss in this section should not depend on the geometry of the loop; the important point is the presence of conserved charges which are trapped on the vortex leading to stability. In reality, the final stable configuration of these vortex loops is probably a more complicated shape because of the quasi-two-dimensional nature of the high temperature superconductors. In particular, we have neglected the difference between the transverse and tangential spatial directions in our treatment of the problem. The appropriate calculations would include this difference and lead to a nonsymmetric shape. However, we neglect these complications at this stage.

In order to make an analogy with the QCD case where the condensate on the core is described by a complex field, we are free to represent two degrees of freedom represented by a unit Neel vector  $\hat{m}$ ,  $\hat{m}^2 = 1$  defined by Eq. (19) in terms of a single complex field  $\Phi$  as

$$\hat{m} = \left( \frac{\Phi + \Phi^*}{1 + |\Phi|^2}, \frac{\Phi - \Phi^*}{i(1 + |\Phi|^2)}, \frac{1 - |\Phi|^2}{1 + |\Phi|^2} \right), \quad (26)$$

where  $\Phi = |\Phi|e^{i\alpha}$  (this is simply the projection of the unit sphere onto the complex plane). At this point we are free to pick the direction of the Neel vector. For a background classical field describing a vortex defined along the  $z$  direction, we will pick  $\vec{m}$  to lie in the  $xy$  plane, so that  $m_z = 0$ . We should note that all calculations and results which follow do not depend on the particular choice of  $\vec{m}$  that we have made above. However, we do expect that this will turn out to be the lowest energy configuration when higher order derivative terms are included in the free energy (see below). We neglect fluctuations of the absolute value  $|\Phi|$  for description of the classical background and consider variation of its phase  $\alpha$ . In this case, we have  $|\Phi| = 1$  for the classical background field as it follows from transformation (26), and  $\vec{m}$  simplifies to

$$\hat{m} = \frac{1}{2} [\Phi + \Phi^*, i(\Phi^* - \Phi), 0]. \quad (27)$$

with  $|\Phi| = 1$  to be fixed. The condensate  $|\Phi| \neq 0$  can carry currents and charges along the string so we will represent it by the following ansatz which describes the dependence of these excitations on  $z$  and  $t$ :

$$\Phi = |\Phi| e^{i\alpha(z,t)} = |\Phi| e^{i(kz - \omega t)}. \quad (28)$$

With this redefinition of the fields, the kinetic term esquires two additional terms due to the  $(z, t)$  dependence of the phase in the core:

$$\frac{\chi}{2} [|\partial_0 \vec{m}|^2 - v_s^2 |\partial_z \vec{m}|^2] \rightarrow \frac{\chi}{2} m^2(r) \{ [\partial_0 \alpha(z,t)]^2 - v_s^2 (\partial_z \alpha(z,t))^2 \}, \quad (29)$$

where  $v_s^2 \equiv \rho/\chi$ . The key point of the time dependent ansatz (28) is as follows. Naively, one could think that the time dependence in a classical solution brings an additional energy into the system which usually does not help to stabilize the configuration. However, as Witten noticed in Ref. 17, if there is a conserved charge in the system the configuration could be stable due to the conservation of the corresponding charge. In a sense, the time-dependent configuration becomes the lowest energy state in the sector with a given nonzero charge. A similar time-dependent ansatz for a different problem was also discussed by Coleman in Ref. 35, where he introduced so-called Q balls, macroscopically large stable objects with a time dependent wave function. We follow Ref. 17 and define a charge  $N$  which is topologically conserved:

$$N = \oint_C \frac{dz}{2\pi} \left( \frac{d\alpha}{dz} \right) = kR, \quad (30)$$

where the path  $C$  is defined along the vortex loop and we assume that  $\omega$  and  $k$  are some constants along the loop. Since  $\alpha$  can change by multiple of  $2\pi$  in circling the vortex loop,  $N$  must be an integer. This is required in order for the condensate  $m$  to remain single valued.

In addition to the topologically conserved winding number  $N$ , there also exist the standard Noether charges and currents which can be trapped on the vortex core associated with the parameter  $\omega, k$  included in the phase  $\alpha$  above. In our case, the relevant symmetry, SO(3), implies a conservation of three Noether charges:

$$Q_k = \int d^3r j_k^0 = i\chi \int d^3r [(\partial_0 m_a)(S_k)_{ab} m_b], \quad (31)$$

while the corresponding three currents are

$$J_k^z = \int d^3r j_k^z = -i\rho \int d^3r [(\partial_z m_a)(S_k)_{ab} m_b], \quad (32)$$

where  $S_k$  are the three generators of SO(3).

A vortex loop with nonzero Noether charges  $Q_k$  and topological charge  $N$  trapped on the core is described by an ansatz of the following form which depends on the position  $\vec{r} = (r, \phi, z)$  and time  $t$  as [using Eqs. (18), (19), (26), and (28)]:

$$\psi = \sqrt{\frac{|a|}{b}} f(r) e^{i\phi}, \quad (33)$$

$$\vec{m} = \sigma \sqrt{\frac{|a|}{b}} g(r) [\cos(kz - \omega t), \sin(kz - \omega t), 0], \quad (34)$$

where as in Sec. II  $f(r), g(r)$  are solutions to the classical equations of motion obeying the appropriate boundary conditions.

For the solution given by Eq. (34) we have a nonzero Noether charge  $Q_z$  which is trapped on the vortex core:

$$Q_z = \chi L \omega \Sigma, \quad (35)$$

where  $\Sigma$  is defined as the integral of  $|\vec{m}|^2$  over the vortex cross section:

$$\Sigma = \int_{\times} d^2r |\vec{m}|^2. \quad (36)$$

We should note that for a different choice of the direction of Neel vector  $\hat{m}$ , the conserved charge which is nonzero would be different. The important point is that a nonzero charge will always be present independent of the of the Neel direction  $\hat{m}$ .

As we mentioned at the beginning of this section, these vortons are spinning and carry a nonzero angular momentum. The vortons are stable against shrinking due to the conservation of the angular momentum. To calculate the angular momentum of a vorton with nonzero charges  $N$  and  $Q_z$  trapped on the core, we use the standard formula for the angular momentum expressed in terms of the energy-momentum tensor:

$$M_{ij} = \int d^3r (T_{0i} x_j - T_{0j} x_i), \quad (37)$$

which can be approximated for a large vorton in the plane as

$$M \approx 2\pi\chi R^2 \sigma^2 \frac{|a|}{b} \int d^2r g(r)^2 \omega k = 2\pi\chi R^2 \omega k \Sigma. \quad (38)$$

The angular momentum points in the direction normal to the surface formed by the vorton. We now see that the reason we would expect such configurations to be classically stable is simple; it is just because these vortons are spinning and angular momentum is conserved. One can say that the vorton is stable because it is the lowest energy configuration in the given sector with nonzero conserved charges  $N, Q_z$ . Angular momentum  $M$  [Eq. (38)], which is essentially the product of two charges  $N$  and  $Q_z$ , is also nonzero when both charges  $N$  and  $Q_z$  are nonzero. In the discussion above we neglected the higher order derivative terms. In particular, there will be some correlation between the charge  $Q_i$  [Eq. (31)] and the momentum along the vorton  $P_i = T_{0i}$  in the expression for the free energy ( $\sim T_{0i} Q_i$ ). Such a correlation implies that ansatz (27) will represent the lowest energy configuration if the angular momentum  $\vec{M}$  [Eq. (38)] points in the direction normal to the surface formed by the vorton.

We will assume that we now have a vorton configuration with nonzero values of  $N$  and  $Q_z$ . In order to assign specific numbers for these quantities one must look at the mechanism of formation. We will not address such complex issues in this paper and simply assume that there is some nonzero probability for a vorton to form. For recent work on the issue of vorton formation we refer the reader to Ref. 36. The free energy of a vorton can be obtained by substituting Eqs. (33) and (34) into Eqs. (20):

$$\mathcal{F} = \int d^3r \left[ \frac{\chi}{2} [(\omega^2 + v_s^2 n^2) m^2 + v_s^2 (\nabla_r m)^2 + v_s^2 |\nabla \psi|^2] - (|a| + \tilde{g}) m^2 - |a| |\psi|^2 + \frac{1}{2} b (m^4 + |\psi|^4 + 2m^2 |\psi|^2), \right] \quad (39)$$

where  $m = \sigma \sqrt{|a|/b} g(r)$ . We can simplify this expression further by using the fact that  $m$  is a solution to the equation of motion and represent the free energy in the following way:

$$\mathcal{F} = L \left( \rho \pi \frac{|a|}{b} \ln(\Lambda/\xi) - \frac{b}{2} \Sigma_4 + \chi \omega^2 \Sigma \right), \quad (40)$$

where we have defined the quantity  $\Sigma_4$  for brevity:

$$\Sigma_4 = \int_{\times} d^2r |\vec{m}|^4. \quad (41)$$

The first term in Eq. (40) is simply the energy from the dSC vortex with no condensate present in the core (to a logarithmic accuracy). Here  $\Lambda$  is the long distance cutoff which must be included to regulate the logarithmic divergence of the normal global string. The long distance cutoff is typically the distance between vortices, so in our case we will take  $\Lambda = L$  where  $L$  is the length of the vortex loop. The second term is negative, reflecting the fact that it is energetically favorable to have an AF core. And the third term is the additional contribution to the energy due to nonzero  $Q_z(N)$ .

There are various cases which must be considered,  $v_s k > \omega$ ,  $v_s k < \omega$ , and  $v_s k = \omega$ . Notice that the effect of adding a having  $k, \omega$  nonzero is the addition of a masslike term for  $\vec{m}$  to the Lagrangian:

$$\delta \mathcal{L} = \frac{\chi}{2} (\omega^2 - v_s^2 k^2) m^2. \quad (42)$$

If  $v_s k > \omega$  then the effect of a nonzero  $v_s k, \omega$  is to add a positive mass term for  $|m|$  to the Lagrangian. This counteracts the effects of the negative mass term in the original Lagrangian [Eq. (14)]. Since  $k \sim 1/L$  quenching occurs and the size of the condensate  $\Sigma$  decreases as the vortex loop gets smaller. Conversely, if  $\omega > v_s k$  one has the opposite situation and anti-quenching occurs. As the vortex loop shrinks, the size of the condensate  $\Sigma$  gets larger as one would expect. The different cases have been examined using numerical calculations in Refs. 20 and 22. Recall Eq. (35), where  $\omega$  is given as

$$\omega = \frac{Q_z}{\chi L} \Sigma^{-1}. \quad (43)$$

As Davis and Shellard point out in Ref. 22, if  $\omega$  starts out less than  $v_s k$  quenching occurs and forces  $\omega$  increase faster than  $Q_z/L$ . In the opposite case where  $\omega > v_s k$  anti-quenching occurs and therefore  $\omega$  increases more slowly than  $Q_z/L$ . The important conclusion that was drawn from this analysis is that  $\omega/(v_s k) \rightarrow 1$  is an attractor.<sup>22</sup> As a loop shrinks  $\omega/(v_s k)$  approaches 1 and the quenching (or anti-quenching) slows and the eventually stops leaving a classi-

cally stable vorton behind. Therefore, for simplicity we will focus on the so called chiral case when  $\omega = v_s k$  which is the most stable configuration. Realistically, we know that the periodic structure of the material of the superconductor breaks the rotational symmetry, leading to non-conservation of angular momentum (it can be transferred to the material). However, the topological charge  $N$  is still a conserved quantity. Therefore, for the chiral vortons,  $Q \sim N$  is also conserved due to relations (30) and (35). For such configurations, stability is ensured. When  $\omega \neq v_s k$ , the transfer of angular momentum to the lattice is possible, eventually settling to the chiral case with  $\omega = v_s k$ .

In the chiral case the size of the condensate  $\Sigma$  is independent of  $N, Q_z, L$  and the free energy [Eq. (40)] can be written as

$$\mathcal{F} = L \alpha_s + (2\pi)^2 \rho \frac{N^2 \Sigma}{L},$$

$$\alpha_s = \rho \pi \frac{|a|}{b} \ln(\Lambda/\xi) - \frac{b}{2} \Sigma_4, \quad (44)$$

where  $\alpha_s$  is the string tension of the bare dSC/AF vortex with  $Q_z = N = 0$  and  $k$  is expressed in terms of the conserved winding number  $N$  according to Eq. (30). Written in this form, it is immediately obvious that for a given nonzero value of  $N$  the free energy has a minimum at  $L = L_0$ :

$$\frac{N}{L_0} = \frac{1}{2\pi} \sqrt{\frac{\alpha_s}{\rho \Sigma}}. \quad (45)$$

We can give a crude estimate of the winding number density,  $n_0 \equiv N/L_0$ , of a stable vorton configuration,

$$\Sigma \sim \sigma^2 \frac{|a|}{b} \delta_m^2, \quad (46)$$

$$\alpha_s \sim \rho \frac{|a|}{b}, \quad (47)$$

where  $\delta_m$  is the width of the condensate. This gives us

$$n_0 \equiv \frac{N}{L_0} \sim \frac{1}{\delta_m} \sim \sqrt{|\tilde{g}|} \sim \sqrt{4\chi(\mu^2 - \mu_c^2)}, \quad (48)$$

which is approximately the inverse width of the condensate. As expected, the winding number density  $n_0$  does not depend on the large number  $N$ , but depends only on the internal structure of the vorton, i.e. on the width of AF condensate in dSC vortex core. Equation (48) tells us that as one goes around a vorton the direction of the Neel vector  $\hat{m}$  varies over a distance scale  $\sim \delta_m$ , the width of the condensate. As the doping is decreased and the AF-dSC phase boundary is approached from above the width of the condensate increases. For a given value of  $N(Q_z)$  (determined at the time of formation) the size of a stable vorton increases as one approaches the AF-dSC phase boundary.

The discussion above has shown that vortons are indeed classically stable. This would imply that on the quantum mechanical level such quasiparticles are at least metastable. The



issue of quantum stability of vortons was addressed in a recent paper.<sup>37</sup> In this paper they calculated the lifetime of a vorton with  $Q=0, N \neq 0$  (pure current case). The mechanism of decay is some quantum mechanical tunneling process where the condensate instantaneously goes to zero on the core, allowing the winding number to decrease by one unit from  $N$  to  $N-1$ .

#### IV. CONCLUSION AND FURTHER SPECULATIONS

In this paper we have reviewed the dSC vortices which have an antiferromagnetic core within the SO(5) theory of high temperature superconductivity.<sup>1-4</sup> We have compared these dSC/AF vortices with similar vortices which arise in a completely different context, high density QCD.<sup>10-12</sup>

The main point that was presented in this paper is that loops of dSC/AF vortices called vortons can exist as classically stable objects in the presence of a zero external magnetic field. The source of the stability of these vortons is the conservation of angular momentum that counteracts the string tension, which ‘‘prefers’’ to minimize the length of the vortex loop. The fact that there is a condensate trapped on the vortex core is crucial for the stability of vortons. It is the condensate which allows nonzero charges to be trapped on the core, leading to the presence of nonzero angular momentum. It remains to be seen if such quasiparticles will be important for the physics of the high- $T_c$  superconductivity. In what follows, we provide arguments supporting the idea that the vortons can play a key role in AF-dSC phase transition. At this point we consider the vorton mechanism driving AF-dSC phase transition as a conjecture.

The first argument goes as follows. As we have shown above, for a given value of  $N$  there exists classically stable vorton configurations with size  $L$  and fixed ratio  $N/L_0 \sim 1/\delta_m$ , where  $\delta_m$  is the width of the condensate that is trapped on the vortex core. As one decreases the doping parameter and approaches the AF-dSC phase boundary, the width of the condensate  $\delta_m$  increases. This is the direct consequence of the fact that the asymmetry parameter  $|\tilde{g}|$  becomes smaller and smaller when the phase boundary is approached. From the relation  $L_0/N \sim \delta_m$  given above, this would imply that  $L_0$ , the length of a classically stable vorton, must increase. Decreasing the asymmetry parameter  $|\tilde{g}| \sim 4\chi(\mu^2 - \mu_c^2)$  further would result in a large vorton with large core size. The volume of the regions filled with the AF state behaves like  $L_0 \delta_m^2 \sim |\tilde{g}|^{-3/2}$ . When the phase transition line is approached, the regions with the AF state fill the entire space. At this point the AF-dSC phase transition occurs. Although our approximations are no longer valid at this point, because our description assumes that the vorton core size is much smaller than  $L$  and the interaction between strings can be neglected (plus many other assumptions not to be mentioned). These assumptions certainly fail in the vicinity of the phase transition. Nevertheless, the fact that the size of the AF regions inside of the dSC phase increases rapidly when the AF-dSC phase transition is approached should be considered as a strong argument in favor of the vorton mechanism driving the AF-dSC phase transition.

We would also like to make the observation that on the other side of the phase transition boundary, in the AF state, there are quasiparticles whose cores are in dSC phase.<sup>38</sup> Therefore, one can imagine a situation when one type of quasiparticles (dSC vortices with an AF core) becomes a different type of quasiparticles (AF skyrmions with a dSC core) when the doping parameter decreases and the phase transition line is crossed.

The next natural question to ask is as follows: let us assume that vortons are indeed the relevant quasiparticles which drive AF-dSC phase transition at small temperatures. Can the same vortons be an essential part of the dynamics when the temperature (rather than the chemical potential  $\mu$ ) crosses the superconducting phase transition at  $T_c$ ? If the answer is positive, we would have a nice unified picture for two different phase transitions on the  $(T, \mu)$  plane. We believe the answer, indeed, could be positive (see the arguments below).

We start by reminding the reader that the pseudogap phase is characterized by the temperature  $T_c < T < T^*$ , when the Cooper pairs are already formed but the long-range phase coherence sets in only at the much lower temperature  $T_c \ll T^*$ . It is believed that in this regime the phase order is destroyed by fluctuating vortices of the Cooper pair field  $\psi$  above  $T_c$ .<sup>39</sup> It is quite natural to *identify* our vortons (loops of vortices) sliced by a two dimensional plane with vortex-antivortex pairs with distinct experimental signatures from Ref. 39. In this case, since underdoped cuprates are effectively two dimensional, at finite temperature the loss of phase order may be expected to proceed via the Berezinsky-Kosterlitz-Thouless phase transition. In this case, the vortons discussed in the present paper, being sliced by the two-dimensional plane, become the vortex-antivortex pairs analyzed in Ref. 39 and could be responsible for the phase transition at  $T_c$ . However, the picture of the phase transition here is quite different from what we previously discussed regarding the AF-dSC separating line. In the present case, when  $T$  crosses  $T_c$  the transition happens because of the vortex-antivortex interaction which is proportional to  $\rho\pi(|a|/b)\ln(x_1-x_2)$  and not because the seeds of a new phase (the vorton cores) fill the entire space. This is the typical two dimensional form due to the global nature of the vortices (local strings do not possess this feature). The volume occupied by the vortex cores at this point is still much smaller than the volume of the system. It is well known that such a logarithmic interaction is a key element for understanding the Berezinsky-Kosterlitz-Thouless phase transition.

Encouraged by the argument given above, we extend our conjecture and assume that the same quasiparticles, vortons, are responsible not only for the AF-dSC phase transition but also for the phase transition separating the pseudogap and dSC phases at temperature  $T_c$  when  $\mu > \mu_c$ . The natural question to ask is: how does the critical temperature  $T_c(\mu)$  depend on the chemical potential  $\mu$  within this conjecture? To answer this question we recall that the critical temperature  $T_c$  for the Berezinsky-Kosterlitz-Thouless phase transition is proportional to the strength of the logarithmic interaction mentioned above. In our case it is nothing but the

string tension determined (mainly) by the vacuum expectation value of  $\psi$  field, [Eq. (44)], i.e.,  $T_c \sim \alpha_s$ . When  $\mu = \mu_c$  the asymmetry parameter is zero and  $\langle \psi^2 \rangle = |a|/b$  is determined by the unperturbed coefficients  $a, b$ . The simplest way to determine how the expectation value of the field  $\psi$  varies when  $\mu$  increases is to introduce the asymmetry parameter in the ‘‘symmetric’’ manner.<sup>40</sup>  $\mathcal{F}_{asym} = -\tilde{g}(\vec{m}^2 - |\psi|^2)$  such that negative  $\tilde{g}$  corresponds to the condensation of the  $|\psi|^2$  field, and positive  $\tilde{g}$  corresponds to the condensation of the  $|\vec{m}|^2$  field. This asymmetry parameter enters the string tension in dSC phase as follows,  $\alpha_s \sim (|a| - \tilde{g})/b$ ,  $\tilde{g} < 0$ . From this expression one can immediately deduce that  $\alpha_s$  (and therefore,  $T_c$ ) will increase with  $\mu$  when  $\tilde{g}$ , being negative, becomes larger and larger in magnitude. It is interesting to note that the very same conclusion has been reached in the original paper,<sup>1</sup> where the increase of  $T_c$  with chemical potential was explained as a result of mass increase of the  $\pi$  triplet (see Ref. 1 for details).

Having presented our arguments supporting the conjecture that vortons might be the relevant degrees of freedom in the dSC phase at zero external magnetic field, we now conclude with a few remarks on how this picture can be experimentally tested. First, the possible experimental methods (such as  $\mu$ SR and inelastic neutron scattering) for observing AF vortex cores were discussed in Ref. 2 and we shall not repeat their analysis. However, we should mention that the fact that the AF cores do appear in the vortices<sup>5-9</sup> suggests that the SO(5) model of high- $T_c$  superconductivity may be correct. Our original remark here is that the AF vortex core size is on the order of dSC coherence length far away from the point  $\tilde{g}=0$ , and becomes larger when the chemical potential approaches  $\mu_c$  at zero external magnetic field. We expect that the average size  $\langle L \rangle$  of vortons grows with temperature, and therefore, correlations between AF cores will grow with temperature as well. Similar behavior is expected to occur when  $\mu$  approaches  $\mu_c$  at a fixed temperature. In this case the effect is expected to be even more pronounced because the volume occupied by the AF cores grows as  $|\tilde{g}|^{-3/2}$ , as discussed above, and therefore the correlations as well as the magnitude of the local electron magnetic fields should scale accordingly. This picture suggests that the AF

correlation length is proportional to  $L$  and could be very large, much larger than any other scale of the problem. Apparently, such large AF correlation lengths have already been observed in Ref. 6, and we would like to argue that this correlation is related to our vortons. One should remark here that such a large correlation length cannot be simply explained by the interaction between vortices (which have size  $\xi \sim 20 \text{ \AA}$ ) because it would lead to a strong dependence on the Neel temperature as a function of the intervortex spacing controlled by the external magnetic field, while observations suggest that the Neel temperature is field independent<sup>6</sup>.

All the effects previously mentioned require the existence of the AF core in the vortex and they are not specifically sensitive to the existence of vortons, which is the subject of this work. The main feature of the vortons is that they can carry angular momentum [see Eq. (38)] and provide large AF correlation lengths (see discussion above). Therefore, these excitations should be present if a dSC sample is rotated with nonzero angular momentum. The situation is very similar to the <sup>3</sup>He and <sup>4</sup>He systems where superfluid vortices can be studied by rotating liquid helium in a can. In many respects our vortons are similar to rotons, and presumably can be studied in a similar way using the technique developed for these systems.

Finally, if vortons are indeed the relevant degrees of freedom in the high- $T_c$  superconductors, it provides a unique opportunity to study cosmology and astrophysics by doing laboratory experiments in condensed matter physics. Over the last few years several experiments have been done to test ideas drawn from cosmology (see Refs. 41 and 42 for further details).

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