Josephson current in the presence of a precessing spin

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The Josephson current in the presence of a precessing spin between various types of superconductors is studied. It is shown that the Josephson current flowing between two spin-singlet pairing superconductors is not modulated by the precession of the spin. When both superconductors have equal-spin-triplet pairing state, the flowing Josephson current is modulated with twice the Larmor frequency by the precessing spin. It was also found that up to the second tunneling matrix elements, no Josephson current can occur with only a direct exchange interaction between the localized spin and the conduction electrons, if the two superconductors have different spin-parity pairing states.

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There is an intense interest in a number of techniques that allow one to detect and manipulate a single spin in the solid state. Partial list includes optical detection of electron-spin resonance (ESR) in a single molecule,[1] tunneling through a quantum dot,[2] and more recently electron-spin-resonance–scanning tunneling microscopy (ESR-STM) technique.[3–6] It has also been recognized that the ESR-STM technique is capable of detecting the precession of a single spin through the modulation of the tunnel current. Interest in ESR-STM lies in the possibility to detect and manipulate a single spin,[6] which is crucial in spintronics and quantum information processing.

Several proposals have been made for the mechanism of the spin detection with the ESR-STM. One is the effective spin-orbit interaction of the conduction electrons in the two-dimensional surface coupling the injected unpolarized current to the precessing spin.[7] Another one is the interference between two resonant tunneling components through the magnetic field split Zeeman levels.[8] Both these mechanisms rely on a spin-orbit coupling to couple the local spin to the conduction electrons and have assumed no spin polarization of tunneling electrons.[9] On the other hand, one can perform ESR-STM measurements on samples with much smaller spin-orbit coupling.[5] Theoretically it is also important to investigate the role of direct exchange in ESR-STM measurements, without any spin-orbit coupling.[10] Exchange interaction has a tremendous effect on the physics of conducting substances when magnetic impurities are present.[11,12]

The above-mentioned experimental and theoretical studies are concentrated on the tunneling between two normal metals. A natural extension is a question of the role of a precessing spin localized inside a tunneling barrier on the Josephson current between two weakly coupled superconductors. This is the problem we address in this paper.

Previously, the Josephson effect between superconductors with nontrivial pairing symmetry has been extensively studied, see, e.g., Ref. 13. There are two main aspects of the current study that differ from the previous work.

(1) We will consider the effect of the precessing localized spin in the junction on the Josephson current. This effect, to our knowledge, has not been addressed before.

(2) We will assume *no spin-orbit coupling* between the superconductors. The role of the spin-orbit coupling will be addressed elsewhere.

The model system under consideration is illustrated in Fig. 1. It consists of two ideal superconducting leads coupled to each other by a single magnetic spin. In the presence of a magnetic field, the spin precesses around the field direction. We neglect the interaction of the spin with two superconducting leads. The Hamiltonian for the Josephson junction can then be generally written as[13]

$$H = H_L + H_R + H_T . \tag{1}$$

The first two terms are, respectively, the Hamiltonian for electrons in the left and right superconducting leads of the tunneling junction:

$$H_{L(R)} = \sum_{\mathbf{k}(\mathbf{p});\sigma} \epsilon_{\mathbf{k}(\mathbf{p})} c^{\dagger}_{\mathbf{k}(\mathbf{p}),\sigma} c_{\mathbf{k}(\mathbf{p}),\sigma} + \frac{1}{2} \sum_{\mathbf{k}(\mathbf{p});\sigma,\sigma'} [\Delta_{\sigma\sigma'}(\mathbf{k}(\mathbf{p})) c^{\dagger}_{\mathbf{k}(\mathbf{p}),\sigma} c^{\dagger}_{-\mathbf{k}(-\mathbf{p}),\sigma'} + \text{H.c.}], \tag{2}$$

where we have denoted the electron creation (annihilation) operators in the left (L) lead by $c^{\dagger}_{\mathbf{k}\sigma}$ ($c_{\mathbf{k}\sigma}$) while those in the right (R) lead by $c^{\dagger}_{\mathbf{p}\sigma}$ ($c_{\mathbf{p}\sigma}$). The quantities $\mathbf{k}$ ($\mathbf{p}$) are momenta and σ is the spin index. The quantities $\epsilon_{\mathbf{k}(\mathbf{p}),\sigma}$, $\Delta_{\sigma\sigma'}(\mathbf{k}(\mathbf{p}))$ are, respectively, the single-particle energies of conduction electrons, and the pair potential (also called gap function) in the leads. For the purpose of this work, the physical origin for the superconducting instability is beyond the scope of our discussion. The two leads are weakly coupled with the tunneling Hamiltonian:

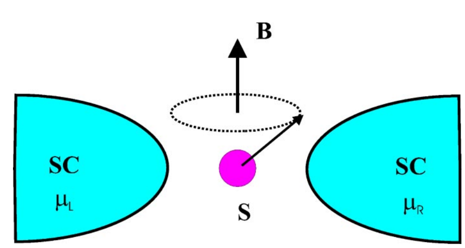

FIG. 1. Magnetic spin coupled to two superconducting leads. In the presence of a magnetic field **B**, the spin precesses around the field direction.

$$H_T = \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} [T_{\sigma\sigma'}(\mathbf{k},\mathbf{p}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma'} + \text{H.c.}], \tag{3}$$

where the tunneling matrix element $T_{\sigma\sigma'}(\mathbf{k},\mathbf{p})$ transfer electrons through an insulating barrier. When a local spin is embedded into the tunneling barrier, the tunneling matrix can be written in the spin space as[10]

$$\hat{T} = T_0 \exp\left[-\sqrt{\frac{\Phi - J\mathbf{S}\cdot\hat{\boldsymbol{\sigma}}}{\Phi_0}} \right], \tag{4}$$

where Φ is the spin-independent potential barrier, $\Phi_0 = \hbar^2/2m_e d^2$ is the characteristic energy scale for the barrier width d, J is the exchange interaction between the local spin $\mathbf{S}$ and the tunneling electrons denoted by the Pauli matrix $\hat{\boldsymbol{\sigma}}$. In an external magnetic field $\mathbf{B}$, a torque will act on the magnetic moment $\boldsymbol{\mu}$ of amount $\boldsymbol{\mu}\times\mathbf{B}$, where $\boldsymbol{\mu} = \gamma\mathbf{S}$ with γ being the gyromagnetic ratio. The equation of motion of the local spin is given by $d\boldsymbol{\mu}/dt = \boldsymbol{\mu}\times(\gamma\mathbf{B})$. For a static magnetic field applied along the z direction, we shall see that the local spin would precess about the field at the Larmor frequency $\omega_L = \gamma B$, i.e., $\mathbf{S} = \mathbf{n}(t)S$, where S is the magnitude of the local spin and $\mathbf{n}(t) = (n_x, n_y, n_z) = (n_\perp\cos(\omega_L t), -n_\perp\sin(\omega_L t), n_\parallel)$ the unit vector for the "instantaneous" spin orientation. Here $n_\parallel$ and $n_\perp$ are the magnitudes of the longitudinal and transverse components of $\mathbf{S}$ to the field direction. They obey the sum rule $n_\parallel^2 + n_\perp^2 = 1$. We note that the expression for $\mathbf{n}(t)$ shows the constant left-handed precession, and the z component of $\mathbf{S}$ is time independent. The precession of the spin can also be obtained quantum mechanically by replacing the local spin operator with its average value. The exchange term in the exponent of the tunneling matrix element is very small as compared with the barrier height Φ. We then perform the Taylor expansion in JS and arrive at

$$\hat{T} = T_0 \exp\left(-\sqrt{\frac{\Phi}{\Phi_0}} \right) \left[\cosh\left(\frac{JS}{2\Phi}\sqrt{\frac{\Phi}{\Phi_0}} \right) + \mathbf{n}(t)\cdot\hat{\boldsymbol{\sigma}} \sinh\left(\frac{JS}{2\Phi}\sqrt{\frac{\Phi}{\Phi_0}} \right) \right]. \tag{5}$$

Since the energy associated with the spin precession $\hbar\omega_L \sim 10^{-6}$ eV is much smaller than the typical electronic energy on the order of 1 eV, the spin precession is very slow as compared to the time scale of all conduction electron process. This fact allows us to treat the electronic problem adiabatically as if the local spin is static for every instantaneous spin orientation.[14] Our remaining task is to calculate the Josephson current in the presence of the spin. The current operator is given by

$$\hat{I} = ie \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} [T_{\sigma\sigma'}(\mathbf{k},\mathbf{p};t) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma'} - \text{H.c.}]. \tag{6}$$

When a voltage bias $eV = \mu_L - \mu_R$ is applied across the junction, following the standard procedure,[15] we can write the phase dependent contribution, i.e., the Josephson current as

$$I_J(t) = e\int_{-\infty}^{t} dt' [e^{ieV(t+t')}\langle [A(t),A(t')]_-\rangle - e^{-ieV(t+t')}\langle [A^\dagger(t),A^\dagger(t')]_-\rangle], \tag{7}$$

where the operator

$$A(t) = \sum_{\mathbf{k},\mathbf{p};\sigma,\sigma'} T_{\sigma\sigma'}(\mathbf{k},\mathbf{p};t) \tilde{c}^\dagger_{\mathbf{k}\sigma}(t) \tilde{c}_{\mathbf{p}\sigma'}(t).$$

Here the operators $\tilde{c}_{\mathbf{k}(\mathbf{p})\sigma}(t) = e^{iK_{L(R)}t} c_{\mathbf{k}(\mathbf{p})\sigma} e^{-iK_{L(R)}t}$ with

$$K_{L(R)} = H_{L(R)} - \mu_{L(R)} N_{L(R)}$$

and

$$N_{L(R)} = \sum_{\mathbf{k}(\mathbf{p}),\sigma c^\dagger_{\mathbf{k}(\mathbf{p})\sigma} c_{\mathbf{k}(\mathbf{p})\sigma}}.$$

For either spin-singlet or spin-triplet superconductors, we can perform the Bogoliubov transformation to express the electron operators in terms of quasiparticle operators:

$$c_{\mathbf{k}\sigma} = \sum_{\sigma'} (u_{\mathbf{k}\sigma\sigma'}\gamma_{\mathbf{k}\sigma'} - \sigma v^*_{-\mathbf{k}\sigma\sigma'}\gamma^\dagger_{-\mathbf{k}\sigma'}) \tag{8}$$

to diagonalize the unperturbed Hamiltonian, where $(u_{\mathbf{k}\sigma\sigma'}, v_{\mathbf{k}\sigma\sigma'})^T$ is the Bogoliubov quasiparticle wave function. For a spin singlet superconductor, the order-parameter matrix can be written as $\hat{\Delta}(\mathbf{k}) = (i\hat{\sigma}_y)\psi(\mathbf{k})$, where $\psi(\mathbf{k})$ is an even function of $\mathbf{k}$. The quasiparticle wave function is then given by

$$\begin{pmatrix} u_{\mathbf{k}\sigma\sigma'} \\ v_{\mathbf{k}\sigma\sigma'} \end{pmatrix} = \begin{pmatrix} u_{\mathbf{k}} e^{i(\varphi_{\mathbf{k}}+\varphi)}\delta_{\sigma\sigma'} \\ v_{\mathbf{k}}\delta_{\sigma,-\sigma'} \end{pmatrix}, \tag{9}$$

with

$$\begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}\left(1+\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right)} \\ \sqrt{\frac{1}{2}\left(1-\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right)} \end{pmatrix}, \tag{10}$$

where we have introduced $\psi(\mathbf{k}) = |\psi(\mathbf{k})| e^{i(\varphi_{\mathbf{k}}+\varphi)}$ with $\varphi_{\mathbf{k}}$ and φ being the internal and global phases, and $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$, and $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\psi(\mathbf{k})|^2}$. For the spin-triplet pairing state, the order parameter can be written as: $\hat{\Delta}(\mathbf{k}) = i(\mathbf{d}(\mathbf{k})\cdot\hat{\boldsymbol{\sigma}})\hat{\sigma}_y$, where $\mathbf{d} = (d_u, d_v, d_w)$ is an odd vectorial function of $\mathbf{k}$ defined in a three-dimensional spin space spanned by (u,v,w). We shall be typically concerned with two types of triplet pairing states—nonequal-spin pairing, where the Cooper pairs are formed by electrons with antiparallel spins, and equal-spin pairing, where the Cooper pairs are formed by electrons with parallel spins. The non-equal spin pairing state has the form

$$\hat{\Delta}(\mathbf{k}) = \begin{pmatrix} 0 & d_I(\mathbf{k}) \\ d_I(\mathbf{k}) & 0 \end{pmatrix}, \tag{11}$$

corresponding to $(d_u, d_v, d_w) = (0,0,d_I(\mathbf{k}))$. This type of pairing state may be realized in the recently discovered superconducting Sr_2RuO_4.[16,17] The equal-spin pairing state has the form

$$\hat{\Delta}(\mathbf{k}) = \begin{pmatrix} 2d_{\mathrm{II}}(\mathbf{k}) & 0 \\ 0 & 2d_{\mathrm{II}}(\mathbf{k}) \end{pmatrix}, \tag{12}$$

corresponding to $(d_u, d_v, d_w) = (0, -i2d_{\mathrm{II}}(\mathbf{k}), 0)$. This state may be relevant to the A phase of superfluid ^{3}He (Ref. 18) and of heavy fermion UPt_3.[19] A little algebra yields the quasiparticle wave function

$$\begin{pmatrix} u_{\mathbf{k}\sigma\sigma'} \\ v_{\mathbf{k}\sigma\sigma'} \end{pmatrix} = \begin{pmatrix} \sigma u_{\mathrm{I},\mathbf{k}} e^{i(\varphi_{\mathbf{k}}+\varphi)} \delta_{\sigma\sigma'} \\ v_{\mathrm{I},\mathbf{k}} \delta_{\sigma,-\sigma'} \end{pmatrix}, \tag{13}$$

for the nonequal-spin-triplet pairing state; while

$$\begin{pmatrix} u_{\mathbf{k}\sigma\sigma'} \\ v_{\mathbf{k}\sigma\sigma'} \end{pmatrix} = \begin{pmatrix} \sigma u_{\mathrm{II},\mathbf{k}} e^{i(\varphi_{\mathbf{k}}+\varphi+\pi)} \delta_{\sigma\sigma'} \\ v_{\mathrm{II},\mathbf{k}} \delta_{\sigma\sigma'} \end{pmatrix} \tag{14}$$

for the equal-spin-triplet pairing state. Here $(u_{\mathrm{I(II)},\mathbf{k}}, v_{\mathrm{I(II)},\mathbf{k}})^T$ has the same form as that given by Eq. (10) except $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |d_{\mathrm{I,II}}(\mathbf{k})|^2}$ and $d_{\mathrm{I,II}}(\mathbf{k}) = |d_{\mathrm{I,II}}(\mathbf{k})| e^{i(\varphi_{\mathbf{k}}+\varphi)}$, respectively. Due to the opposite parity of the triplet pairing state as compared with the singlet counterpart, there appears an additional factor σ $(=\pm 1)$ in Eqs. (13) and (14), which will crucially influence the Josephson current between two superconductors of dissimilar spin parity. We shall also note that the electron component of the eigenfunction is an even function of $\mathbf{k}$ (i.e., $u_{-\mathbf{k}} = u_{\mathbf{k}}$ arising from $\varphi_{-\mathbf{k}} = \varphi_{\mathbf{k}}$) for the spin-singlet pairing state while is an odd function of $\mathbf{k}$ (i.e., $u_{\mathrm{I(II)},-\mathbf{k}} = -u_{\mathrm{I(II)},\mathbf{k}}$ arising from $\varphi_{-\mathbf{k}} = \varphi_{\mathbf{k}} + \pi$) for the triplet pairing state.

The Josephson current I_J originates from a number of terms in the perturbation expression Eq. (7) in which the expectation value of two creation operators in one superconductor is combined with the expectation value of two annihilation operators in the other superconductor, that is, $\langle \tilde{c}^\dagger_{\mathbf{k}\sigma_1}(t) \tilde{c}^\dagger_{-\mathbf{k}\sigma_2}(t') \rangle \langle \tilde{c}_{\mathbf{p}\sigma_1'}(t) \tilde{c}_{-\mathbf{p}\sigma_2'}(t') \rangle$. Using the above symmetry properties, one can find the expectation values for superconductors with a spin-singlet, nonequal-spin-triplet, and equal-spin-triplet pairing state:

$$\langle \tilde{c}^\dagger_{\mathbf{k}\sigma}(t) \tilde{c}^\dagger_{-\mathbf{k}\sigma'}(t') \rangle = \begin{pmatrix} \sigma \delta_{\sigma,-\sigma'} u^*_{\mathbf{k}} v_{\mathbf{k}} \\ \delta_{\sigma,-\sigma'} u^*_{\mathrm{I},\mathbf{k}} v_{\mathrm{I},\mathbf{k}} \\ -\delta_{\sigma,\sigma'} u^*_{\mathrm{II},\mathbf{k}} v_{\mathrm{II},\mathbf{k}} \end{pmatrix} [e^{iE_{\mathbf{k}}(t-t')} f(E_{\mathbf{k}}) - e^{-iE_{\mathbf{k}}(t-t')} f(-E_{\mathbf{k}})], \tag{15a}$$

and

$$\langle \tilde{c}_{\mathbf{p}\sigma}(t) \tilde{c}_{-\mathbf{p}\sigma'}(t') \rangle = \begin{pmatrix} \sigma \delta_{\sigma,-\sigma'} u_{\mathbf{p}} v^*_{\mathbf{p}} \\ \delta_{\sigma,-\sigma'} u_{\mathrm{I},\mathbf{p}} v^*_{\mathrm{I},\mathbf{p}} \\ -\delta_{\sigma,\sigma'} u_{\mathrm{II},\mathbf{p}} v^*_{\mathrm{II},\mathbf{p}} \end{pmatrix} [e^{-iE_{\mathbf{p}}(t-t')} f(-E_{\mathbf{p}}) - e^{iE_{\mathbf{p}}(t-t')} f(E_{\mathbf{p}})], \tag{15b}$$

where the Fermi distribution function $f(E) = 1/[\exp(E/T)+1]$. We evaluate the Josephson current in various types of superconducting junctions. First let us consider that both the left and right superconductors are of spin-singlet pairing symmetry, one can arrive at the Josephson current as

$$I_J = e \sum_{\mathbf{k},\mathbf{p}} \sum_{\sigma\sigma'} (\sigma\sigma') \mathrm{Im}[T_{\sigma\sigma'}(t) T_{-\sigma,-\sigma'}(t) e^{i(2eVt+\delta\varphi)}] \times \frac{|\psi_{\mathbf{k}}||\psi_{\mathbf{p}}| \Omega_{\mathbf{k},\mathbf{p}}(eV)}{2E_{\mathbf{k}}E_{\mathbf{p}}}, \tag{16}$$

where the phase difference $\delta\varphi = (\varphi_R - \varphi_L) + (\varphi_{\mathbf{p}} - \varphi_{\mathbf{k}})$, and

$$\Omega_{\mathbf{k},\mathbf{p}}(eV) = \left[\frac{1}{eV+E_{\mathbf{k}}-E_{\mathbf{p}}} - \frac{1}{eV-E_{\mathbf{k}}+E_{\mathbf{p}}}\right][f(E_{\mathbf{k}}) - f(E_{\mathbf{p}})] + \left[\frac{1}{eV+E_{\mathbf{k}}+E_{\mathbf{p}}} - \frac{1}{eV-E_{\mathbf{k}}-E_{\mathbf{p}}}\right] \times [1 - f(E_{\mathbf{k}}) - f(E_{\mathbf{p}})]. \tag{17}$$

The summation over spin indices involves terms, $T_{\uparrow\uparrow}T_{\downarrow\downarrow}$ and $T_{\uparrow\downarrow}T_{\downarrow\uparrow}$. It then follows from the structure of the tunneling matrix as given by Eq. (5), which has the property $T_{\downarrow\uparrow} = T^*_{\uparrow\downarrow}$, that the flowing Josephson current is not modulated with time by the precessing spin. Similarly, one can find that this conclusion is also true for the Josephson current between two superconductors both of nonequal-spin-triplet pairing symmetry. However, when each side of the junction is a superconductor having equal-spin-triplet pairing symmetry, the Josephson current becomes

$$I_J = -e \sum_{\mathbf{k},\mathbf{p}} \sum_{\sigma\sigma'} \mathrm{Im}[T_{\sigma\sigma'}(t) T_{\sigma\sigma'}(t) e^{i(2eVt+\delta\varphi)}] \times \frac{|d_{\mathrm{II},\mathbf{k}}||d_{\mathrm{II},\mathbf{p}}| \Omega_{\mathbf{k},\mathbf{p}}(eV)}{2E_{\mathbf{k}}E_{\mathbf{p}}}, \tag{18}$$

which will be time dependent even in the absence of the voltage bias when the spin is precessing at ω_L. In some detail, because $T_{\uparrow\downarrow} = T^*_{\downarrow\uparrow} = |T_{\uparrow\downarrow}| e^{i\omega_L t}$, I_J contains a term with a prefactor $\cos(2\omega_L t)$. This implies that the Josephson current flowing between two equal-spin-triplet pairing superconductors is modulated in time at a frequency of $2\omega_L$, i.e., *twice the Larmor frequency*. The relative ratio between the Larmor modulation part δI_J and the constant part I_{J0} is:

$$\frac{\delta I_J}{I_{J0}} = \frac{J^2 S^2}{2\Phi\Phi_0} \sim 10^{-2} - 10^{-3}, \tag{19}$$

for $\Phi = 1$ eV, $\Phi_0 = 0.05$ eV, $JS = 0.1$ eV, which is experimentally detectable. The modulation of a Josephson current by a precessing spin could be used for a single spin detection.

If we suppose that the left superconductor is a spin-singlet superconductor that is weakly coupled to the right superconductor having nonequal-spin-triplet pairing symmetry, the Josephson current is found to be

$$I_J = e\sum_{\mathbf{k},\mathbf{p}}\sum_{\sigma\sigma'}\sigma\,\mathrm{Im}[T_{\sigma\sigma'}(t)T_{-\sigma,-\sigma'}(t)e^{i(2eVt+\delta\varphi)}]\times\frac{|\psi_{\mathbf{k}}||d_{\mathrm{I},\mathbf{p}}|\Omega_{\mathbf{k},\mathbf{p}}(eV)}{2E_{\mathbf{k}}E_{\mathbf{p}}}. \tag{20}$$

Notice that the summation $\Sigma_{\sigma\sigma'}\sigma T_{\sigma\sigma'}(t)T_{-\sigma,-\sigma'}(t) = -\Sigma_{\sigma\sigma'}\sigma T_{\sigma\sigma'}(t)T_{-\sigma,-\sigma'}(t)$. This property mandates that I_J has to be zero. Also the Josephson current cannot occur when a spin-singlet superconductor is weakly coupled to a superconductor of equal-spin-triplet pairing symmetry, due to the summation $\Sigma_{\sigma\sigma'}\sigma T_{\sigma\sigma'}(t)T_{-\sigma,\sigma'}(t)=0$. Therefore, we conclude that the Josephson current cannot flow between two superconductors with the pairing symmetry of different spin parity even if there is a precessing spin located in the tunnel barrier.

Based on the above microscopic analysis, we can establish a simple phenomenological theory for the Josephson effect through the precessing spin. We define $\Psi_\sigma = \sigma\langle c_{\mathbf{k}\sigma}c_{-\mathbf{k},-\sigma}\rangle$, $\langle c_{\mathbf{k}\sigma}c_{-\mathbf{k},-\sigma}\rangle$, $\langle c_{\mathbf{k}\sigma}c_{-\mathbf{k},\sigma}\rangle$ as the macroscopic wave function on each side of the junction with spin-singlet, nonequal-spin and equal-spin triplet, pairing symmetry. The equations of motion can be written as

$$i\frac{\partial\Psi_{L,\sigma}}{\partial t} = eV\Psi_{L,\sigma} + \sum_{\sigma'} K_{\sigma\sigma'}\Psi_{R,\sigma'}, \tag{21a}$$

$$i\frac{\partial\Psi_{R,\sigma}}{\partial t} = -eV\Psi_{R,\sigma} + \sum_{\sigma'} K^*_{\sigma\sigma'}\Psi_{L,\sigma'}, \tag{21b}$$

where $K_{\sigma\sigma'}$ represents the spin-dependent coupling across the barrier. Making the substitutions $\Psi_{L(R),\sigma} = \mathcal{N}^{1/2}_{L(R),\sigma}\exp(i\theta_{L(R)})$ with $\mathcal{N}_{L(R)} = \Sigma_\sigma \mathcal{N}_{L(R),\sigma}$ being the number of Cooper pairs on each side, one can get the Josephson current $I_J = -\mathcal{N}_L^{1/2}\mathcal{N}_R^{1/2}\,\mathrm{Im}[K_{\sigma\sigma'}\exp(i\delta\theta)]$ where $\delta\theta = \theta_R - \theta_L = 2eVt + \varphi_R - \varphi_L$. For the junction formed by two spin-singlet paring superconductors, all $K_{\sigma\sigma'}$ are time independent. For the spin-singlet/spin-triplet junction, the terms contributing to the summation over spin indices cancel each other. However, for the spin-triplet/spin-triplet junction, there are terms proportional to $|K_{\uparrow\downarrow}|\exp(\pm 2i\omega_L t)$. With this characteristic of K, one can then arrive at the same conclusion as from the microscopic analysis. We should stress that in the presence of spin-orbit coupling, one is allowed to have direct coupling of a current produced by spin-singlet superconductors and local spin **S**. This would lead to the time-dependent contribution of a Josephson current regardless of the pairing symmetry.

In summary, we have studied the Josephson current through a precessing spin between various types of superconducting junctions. It is shown that the Josephson current flowing between two spin-singlet pairing superconductors is not modulated by the precession of the spin. When both superconductors have equal-spin-triplet pairing state, the flowing Josephson current is modulated with twice the Larmor frequency by the precessing spin. It was also found that up to the second order in the tunneling matrix elements no Josephson current can occur by the direct exchange interaction between the localized spin and the conduction electrons, if the two superconductors have different spin-parity pairing states. As far as we know, no measurements of Josephson current through a precessing spin between two superconductors have been reported yet. We believe that the observation of our predictions is within the reach of present technology. On one hand, the ESR-Josephson junction spectrometer, using the idea of coupling the magnetic impurities to an radio-frequency (rf) field,[20,21] has been realized[22,23] in an earlier time. In this technique, the effects of magnetic impurities enter via a complex susceptibility, which induces a fluctuating magnetization in the presence of the rf field. The spin-relaxation time is generally governed by the coupling of the spin with its environment. In the situation studied by these earlier authors, relaxation is determined both by the strong correlation between magnetic impurities and by the tunneling single electrons when a voltage bias is applied. In the present work, the spin relaxation is strongly suppressed due to the gapped nature of quasiparticles when a dc Josephson effect is considered. Therefore, we can expect a very long spin-relaxation time when the local single spin is embedded into the tunneling barrier of the junction. This advantage allows us to treat the single spin to precess freely along the direction of the applied static magnetic field. On the other hand, as a possible experiment, we mention results on atomically sharp superconducting tip in low temperature STM in both the quasiparticle tunneling regime[24] and the Josephson tunneling regime[25] [coined as "Josephson STM" or JSTM (Ref. 26)] on conventional superconductors. Therefore, it would be very interesting to extend the JSTM technology by using a superconducting tip to study the Josephson current in the vicinity of an atomic spin on the superconducting surface.

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[1] J. Kohler, J.A.J.M. Disselhorst, M.C.J.M. Donckners, E.J.J. Groenen, J. Schmidt, and W.E. Moerner, Nature (London) **363**, 342 (1993); J. Wrachtrup, C. von Borczyskowski, J. Bernard, M. Orrit, and R. Brown, *ibid.* **363**, 244 (1993); Phys. Rev. Lett. **71**, 3565 (1993).

[2] H.-A. Engel and D. Loss, Phys. Rev. Lett. **86**, 4648 (2001); Phys. Rev. B **65**, 195321 (2002).

[3] Y. Manassen, R.J. Hamers, J.E. Demulth, and A.J. Castellano, Jr., Phys. Rev. Lett. **62**, 2531 (1989); D. Shachal and Y. Manassen, Phys. Rev. B **46**, 4795 (1992); Y. Manassen, J. Magn. Reson. **126**, 133 (1997).

[4] Y. Manassen, I. Mukhopadhyay, and N. Ramesh Rao, Phys. Rev. B **61**, 16 223 (2000).

[5] C. Durkan and M.E. Welland, Appl. Phys. Lett. **80**, 458 (2002).

[6] H. Manoharan, Nature (London) **416**, 24 (2002).

[7] A.V. Balatsky and I. Martin, cond-mat/0112407 (unpublished).

[8] D. Mozyrsky, L. Fedichkin, S.A. Gurvitz, and G.P. Berman, Phys. Rev. B **66**, 161313 (2002).
[9] Jian-Xin Zhu and A.V. Balatsky, Phys. Rev. Lett. **89**, 286802 (2002).
[10] A.V. Balatsky, Y. Manassen, and R. Salem, Philos. Mag. B **82**, 1291 (2002); cond-mat/0206325 (unpublished).
[11] J. Li, W.-D. Schneider, R. Berndt, and B. Delley, Phys. Rev. Lett. **80**, 2893 (1998).
[12] H. Manoharan, C.P. Lutz, and D. Eigler, Nature (London) **403**, 512 (2000).
[13] M. Sigrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991); V. Geshkenbein and A.I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 306 (1986) [JETP Lett. **43**, 395 (1986)]; D.A. Wollman, D.J. Van Harlingen, W.C. Lee, D.M. Ginsberg, and A.J. Leggett, Phys. Rev. Lett. **71**, 2134 (1993).
[14] The feedback effects of a tunneling current on a spin dynamics, a so-called back-action effect, have been considered in Refs. 7–10. It has been shown that the back-action effects are small and precession is not drastically affected by a tunneling current, provided the coupling is small enough. This allows us to treat the problem with a long decay time τ of precessing state $\omega_L \tau \gg 1$.
[15] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963); **11**, 104(E) (1963).
[16] Y. Maeno, T.M. Rice, and M. Sigrist, Phys. Today **54** (1), 42 (2001).
[17] D.F. Agterberg, T.M. Rice, and M. Sigrist, Phys. Rev. Lett. **78**, 3374 (1997).
[18] P.W. Anderson and W.F. Brinkman, Phys. Rev. Lett. **30**, 1108 (1973); P.W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).
[19] R. Joynt and L. Taillefer, Rev. Mod. Phys. **74**, 235 (2002).
[20] S.E. Barnes, J. Phys. C **10**, 2863 (1977); S.E. Barnes and F. Mehran, Phys. Rev. B **34**, 4537 (1986).
[21] A.M. Goldman, C.G. Kuper, and O.T. Valls, Phys. Rev. Lett. **52**, 1340 (1984).
[22] K. Baberschke, K.D. Bures, and S.E. Barnes, Phys. Rev. Lett. **53**, 98 (1984).
[23] F. Mehran, S.E. Barnes, C.C. Chi, R.L. Sandstrom, and C.P. Umbach, Phys. Rev. B **36**, 7281 (1987).
[24] S.H. Pan, E.W. Hudson, and J.C. Davis, Appl. Phys. Lett. **73**, 2992 (2001).
[25] O. Naaman, W. Teizer, and R.C. Dynes, Phys. Rev. Lett. **87**, 097004 (2001).
[26] J. Šmakov, I. Martin, and A.V. Balatsky, Phys. Rev. B **64**, 212506 (2001).