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Electromagnetic characteristics of bilayer quantum Hall systems in the presence of interlayer coherence and tunneling

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The electromagnetic characteristics of bilayer quantum Hall systems in the presence of interlayer coherence and tunneling are studied by means of a pseudospin-texture effective theory and an algebraic framework of the single-mode approximation, with emphasis on clarifying the nature of the low-lying neutral collective mode responsible for interlayer tunneling phenomena. A long-wavelength effective theory, consisting of the collective mode as well as the cyclotron modes, is constructed. It is seen explicitly from the electromagnetic response that gauge invariance is kept exact, this implying, in particular, the absence of the Meissner effect in bilayer systems. Special emphasis is placed on exploring the advantage of looking into quantum Hall systems through their response; in particular, subtleties inherent to the standard Chern-Simons theories are critically examined.

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I. INTRODUCTION

The Chern-Simons (CS) theories, both bosonic[1–3] and fermionic,[4–6] realize the composite-boson and composite-fermion descriptions[7] of the fractional quantum Hall effect[8,9] (FQHE) and have been successful in describing various features of the FQHE. They, however, have some subtle limitations as well.[10] In particular, when applied to bilayer systems, they differ significantly in collective-excitation spectrum from the magnetoroton theory of Girvin, MacDonald, and Platzman,[11] based on the single-mode approximation (SMA).

The quantum Hall effect exhibits a variety of physics for bilayer (and multilayer) systems.[12–22] In a previous paper[23] we studied within the SMA theory the electromagnetic characteristics of bilayer systems in the absence of interlayer coherence and derived a long-wavelength effective theory that properly embodies the SMA spectrum of collective excitations. The effective theory was constructed from the electromagnetic response of the systems through functional bosonization,[24] without referring to the composite bosons or composite fermions. Thereby the relation between the SMA theory and the CS theories was examined.

The purpose of the present paper is to extend the program of looking into quantum Hall systems through their response to situations of particular interest, bilayer systems in the presence of interlayer coherence as well as tunneling, where phenomena such as a crossover between the tunneling and coherence regimes[14,22] and Josephson-like effects[15,16,25,26] attract attention. We study the electromagnetic characteristics of bilayer systems by means of (i) a pseudospin-texture effective theory and (ii) an algebraic framework of the single-mode approximation, with essentially the same results. Our analysis shows that proper account of the Landau-level projection is indispensable for deriving a low-energy effective theory of gauge-invariant form. The presence of interlayer coherence modifies even the leading long-wavelength features of the bilayer systems, and we critically examine the CS approach to clarify its validity and limitations.

In Sec. II we consider the projection of a bilayer system into the Landau levels. We study the electromagnetic characteristics of the bilayer system in Secs. III and IV. In Sec. V we comment on the CS approach. Section VI is devoted to a summary and discussion.

II. BILAYER SYSTEMS

Consider a bilayer system with average electron densities $\rho_{\rm av}^{(\alpha)}=(\rho_{\rm av}^{(1)},\rho_{\rm av}^{(2)})$ in the upper ($\alpha=1$) and lower ($\alpha=2$) layers. The two layers, each extending in the $\mathbf{x}=(x_1,x_2)$ plane, are taken to be situated at position $z^{(1)}=z_c+\frac{1}{2}d$ and $z^{(2)}=z_c-\frac{1}{2}d$ with separation d in the vertical (z) direction. The system is placed in a common strong perpendicular magnetic field $B_z=B>0$. We suppose that the electron fields $\psi^{(\alpha)}$ in each layer are fully spin polarized and assemble them into a pseudospin[20] doublet spinor $\Psi=(\psi^{(1)},\psi^{(2)})^{\rm tr}$. Our task in this paper is to study how the system responds to weak electromagnetic potentials $A_\mu(x;z)$ and $A_z(x;z)$ in three-dimensional space. [We suppose μ runs over (0,1,2) or (t,x_1,x_2) and denote $A_k=(A_1,A_2)=\mathbf{A}$ and $x=(t,\mathbf{x})$ for short.] We thus write the one-body Lagrangian in the form

$$L_1=\int d^2\mathbf{x}\Psi^\dagger(i\partial_t-\mathcal{H})\Psi, \tag{2.1}$$

$$\mathcal{H}=\frac{1}{2M}(\mathbf{p}+\mathbf{A}^B+\mathbf{A}^++\mathbf{A}^-\sigma_3)^2+A_0^++A_0^-\sigma_3, \tag{2.2}$$

where $A_\mu^\pm(x)=\frac{1}{2}\{A_\mu(x;z^{(1)})\pm A_\mu(x;z^{(2)})\}$ in terms of the potentials acting on each layer, or explicitly,

$$A_\mu^+(x)=A_\mu(x;z_c)+\cdots,$$

$$A_\mu^-(x)=(d/2)\partial_{z_c}A_\mu(x;z_c)+\cdots; \tag{2.3}$$

$\mathbf{A}^B=eB(-x_2,0)$ supplies a uniform magnetic field B; the electric charge $e>0$ has been suppressed by rescaling $eA_\mu\to A_\mu$. [For conciseness, we shall write $\psi^{(\alpha)}(x)=\psi^{(\alpha)}(\mathbf{x},z^{(\alpha)},t)$, etc., and suppress reference to the z coordinate or z_c unless necessary.] Let us denote the number density and pseudospin densities as

$$\{\rho(x), S^a(x)\} = \Psi^\dagger(x)\left\{1, \frac{1}{2}\sigma_a\right\}\Psi(x) \tag{2.4}$$

with the Pauli matrices σ_a ($a=1,2,3$). The A_0^+ coupled to $\rho=\rho^{(1)}+\rho^{(2)}$ probes the in-phase density fluctuations of the two layers while A_0^- coupled to $S^3=\frac{1}{2}(\rho^{(1)}-\rho^{(2)})$ probes the out-of-phase density fluctuations.

The electrons in the two layers are coupled through the intralayer and interlayer Coulomb potentials $V_{\mathbf{p}}^{11}=V_{\mathbf{p}}^{22}$ and $V_{\mathbf{p}}^{12}=V_{\mathbf{p}}^{21}$, respectively; $V_{\mathbf{p}}^{11}=e^2/(2\epsilon|\mathbf{p}|)$ and $V_{\mathbf{p}}^{12}=e^{-d|\mathbf{p}|}V_{\mathbf{p}}^{11}$ with ϵ being the dielectric constant of the substrate. The pseudospin structure of the Coulomb interaction is made manifest by rewriting it as

$$H^{\mathrm{C}}=\frac{1}{2}\sum_{\mathbf{p}}\,(V_{\mathbf{p}}^{+}\rho_{-\mathbf{p}}\rho_{\mathbf{p}}+4V_{\mathbf{p}}^{-}S_{-\mathbf{p}}^{3}S_{\mathbf{p}}^{3}), \tag{2.5}$$

$$V_{\mathbf{p}}^{\pm}=\frac{1}{2}(1\pm e^{-d|\mathbf{p}|})V_{\mathbf{p}}^{11}, \tag{2.6}$$

where $\rho_{\mathbf{p}}$ and $S_{\mathbf{p}}^a$ stand for the Fourier transforms of $\rho(x)$ and $S^a(x)$ with obvious time dependence suppressed.

Note here that the electromagnetic gauge transformations in space induce two sets of intralayer gauge transformations $A_\mu(x;z^{(\alpha)})\to A_\mu(x;z^{(\alpha)})+\partial_\mu\theta^{(\alpha)}(x)$ and $\psi^{(\alpha)}(x)\to e^{-i\theta^{(\alpha)}(x)}\psi^{(\alpha)}(x)$ with $\theta^{(\alpha)}(x)\equiv\theta(x;z^{(\alpha)})$, which can be regarded as totally independent (for $d\neq 0$) since $\theta(x;z)$ may have arbitrary dependence on z. The transformation laws read $\delta A_\mu^{\pm}(x)=\partial_\mu\theta^{\pm}(x)$ in terms of $\theta^{\pm}=\frac{1}{2}\{\theta^{(1)}\pm\theta^{(2)}\}$. Thus, for bilayer systems electromagnetic gauge invariance turns into two separate U(1) gauge symmetries, $\mathrm{U}(1)^{\mathrm{em}}\supset \mathrm{U}(1)^+\times\mathrm{U}(1)^-$. [We refer to this $\mathrm{U}(1)^-$ as "interlayer" gauge invariance below. Note that it disappears in the $d\to 0$ limit.]

The tunneling phenomena must respect electromagnetic gauge invariance. A naive choice of interlayer coupling $S^1+iS^2=\psi^{(1)\dagger}\psi^{(2)}$ should be promoted to a gauge-invariant form[27]

$$H^{\mathrm{tun}}=-\triangle_{SAS}\int d^2\mathbf{x}\;\frac{1}{2}\{\psi^{(1)\dagger}e^{-i\Gamma_z}\psi^{(2)}+\mathrm{H.c.}\} \tag{2.7}$$

with the line integral

$$\Gamma_z(x)=\int_{z^{(2)}}^{z^{(1)}}dz A_z(x;z)=dA_z(x;z_c)+\cdots, \tag{2.8}$$

connecting the two layers for each $\mathbf{x}$. Here Γ_z has the transformation law $\delta\Gamma_z=2\theta^-(x)$. The coupling strength $\triangle_{\mathrm{SAS}}$ characterizes the energy gap between the symmetric and antisymmetric states.

It is possible to gauge away Γ_z by setting $\Gamma_z'=\Gamma_z+2\theta^-=0$ so that the transformed fields

$$\psi'^{(1)}(x)=e^{i(1/2)\Gamma_z}\psi^{(1)}(x),$$

$$\psi'^{(2)}(x)=e^{-i(1/2)\Gamma_z}\psi^{(2)}(x),$$

$$A'^{-}_{\mu}(x)=A_\mu^-(x)-\tfrac{1}{2}\partial_\mu\Gamma_z=(d/2)(\partial_z A_\mu-\partial_\mu A_z)+\cdots, \tag{2.9}$$

and $A'^{+}_{\mu}(x)=A_\mu^+(x)$ are inert under $\mathrm{U}(1)^-$ gauge transformations. The $\psi'^{(\alpha)}(x)$ stand for the electron fields "projected" onto the common plane $z=z_c$ and undergo only the $\mathrm{U}(1)^+$ gauge transformations. Note that $A'^{-}_{\mu}(x)$ is gauge invariant and actually denotes a vertical electric field $A'^{-}_{0}\approx(d/2)E_z$ and in-plane magnetic fields $(A'^{-}_{1},A'^{-}_{2})\approx(d/2)(B_2,-B_1)$.

In view of this structure it is advantageous to restart with the Lagrangian written in terms of these $\psi'^{(\alpha)}$ and $A'^{\pm}_{\mu}$, and recover the effect of Γ_z at the very end. The interlayer gauge invariance is thereby kept exact. Accordingly, we shall from now on regard $\psi^{(\alpha)}$ and $A_\mu^{\pm}$ as denoting the transformed fields $\psi'^{(\alpha)}$ and $A'^{\pm}_{\mu}$.

In addition, it is rather natural and convenient to combine the one-body Lagrangian L_1 and H^{tun} into a formally $\mathrm{U}(1)\times\mathrm{SU}(2)$ gauge symmetric form by setting

$$A_\mu^-\sigma_3\to A_\mu^a\frac{1}{2}\sigma_a \tag{2.10}$$

in $\mathcal{H}$ of Eq. (2.2) and by identifying the SU(2) gauge field $A_\mu^a\equiv(A_\mu^1,A_\mu^2,A_\mu^3)$ with

$$A_\mu^1=-\triangle_{SAS}\delta_{\mu 0},\quad A_\mu^2=0,\quad A_\mu^3=2A'^{-}_{\mu}. \tag{2.11}$$

This SU(2) gauge symmetry, of course, is only superficial. The system has a global pseudospin SU(2) symmetry in the ideal limit $\triangle_{SAS}\to 0$ and $V_{\mathbf{p}}^-\to 0$ with $A'^{-}_{\mu}=0$; it gets broken to U(1) either for $\triangle_{SAS}=0$ or for $V_{\mathbf{p}}^-=0$.

The "interlayer" gauge invariance has to do with interlayer out-of-phase U(1) rotations induced by the z variation of $\theta(x;z)$, i.e., $\theta^-\propto\partial_z\theta(x;z)$. They are thus *distinct* from global U(1) rotations (with constant θ^-) about the S^3 axis, which have to do with charge conservation. As a result, the tunneling interaction $H^{\mathrm{tun}}\propto S'^{1}(x)$ defined in terms of $\psi'^{(a)}$ is gauge invariant but transforms covariantly (i.e., breaks invariance) under global $\mathrm{U}(1)^-$ rotations. (This in turn implies that there is no loss of generality in choosing $H^{\mathrm{tun}}\propto S'^{1}$.)

Let us now project our system onto the Landau levels. Let $|N\rangle=|n,y_0\rangle$ denote the Landau levels of a freely orbiting electron of energy $\omega_c(n+\frac{1}{2})$ with $n=0,1,2,\ldots$, and $y_0=\ell^2 p_x$, where $\omega_c\equiv eB/M$ and $\ell\equiv 1/\sqrt{eB}$; we frequently set $\ell\to 1$ below. We first pass into $N=(n,y_0)$ space via a unitary transformation $\Psi(\mathbf{x},t)=\Sigma_N\langle\mathbf{x}|N\rangle\Phi_n(y_0,t)$ and, by a subsequent unitary transformation $\Phi_n(y_0,t)\to\Psi_m(y_0,t)$, make the one-body Hamiltonian diagonal in level indices; the relevant transformation is constructed in powers of A_μ^+ and A_μ^a. The resulting projected Hamiltonian is an operator in $\mathbf{r}\equiv(r_1,r_2)=(i\ell^2\partial/\partial y_0,y_0)$ with uncertainty $[r_1,r_2]=i\ell^2$. Such a systematic procedure of projection, developed earlier,[23,28] is readily adapted to the present SU(2) case. As a matter of fact, for $\triangle_{SAS}\ll\omega_c$ the result is essentially the same as in the single-layer case.

Let us focus on the lowest Landau level $n=0$ in a strong magnetic field. The projected one-body Hamiltonian to $O(A^2)$ reads $\bar{H}^{\mathrm{cyc}}+\bar{H}^{\mathrm{em}}+\bar{H}^{\mathrm{tun}}$ with

$$\bar{H}^{\rm cyc}=\sum_{\alpha=1}^{2}\sum_{\mathbf{p}}\left\{\frac{\omega_c}{2}\delta_{\mathbf{p},0}+\mathcal{U}_{\mathbf{p}}^{(\alpha)}\right\}\bar{\rho}_{-\mathbf{p}}^{(\alpha)},$$

$$\bar{H}^{\rm em}=\sum_{\mathbf{p}}\{\chi_{\mathbf{p}}^{+}\bar{\rho}_{-\mathbf{p}}+2\chi_{\mathbf{p}}^{-}\bar{S}_{-\mathbf{p}}^{3}\},$$

$$\bar{H}^{\rm tun}=-\triangle_{SAS}\bar{S}_{\mathbf{p}=\mathbf{0}}^{1}, \tag{2.12}$$

with $\chi_{\mathbf{p}}^{\pm}=(A_0^{\pm})_{\mathbf{p}}+(1/2M)(A_{12}^{\pm})_{\mathbf{p}}$ and $A_{12}^{\pm}=\partial_1A_2^{\pm}-\partial_2A_1^{\pm}$; $(A_\mu^{\pm})_{\mathbf{p}}$ stands for the Fourier transform of $A_\mu^{\pm}(x)$. Here the projected charges $\bar{\rho}_{\mathbf{p}}=\bar{\rho}_{\mathbf{p}}^{(1)}+\bar{\rho}_{\mathbf{p}}^{(2)}$, $\bar{S}_{\mathbf{p}}^{3}=\frac{1}{2}(\bar{\rho}_{\mathbf{p}}^{(1)}-\bar{\rho}_{\mathbf{p}}^{(2)})$, etc., are defined by

$$\bar{\rho}_{\mathbf{p}}\equiv\int dy_0\ \Psi_0^{\dagger}(y_0,t)e^{-(1/4)\mathbf{p}^2}e^{-i\mathbf{p}\cdot\mathbf{r}}\Psi_0(y_0,t), \tag{2.13}$$

$$\bar{S}_{\mathbf{p}}^{a}\equiv\int dy_0\Psi_0^{\dagger}(y_0,t)e^{-(1/4)\mathbf{p}^2}e^{-i\mathbf{p}\cdot\mathbf{r}}\frac{\sigma_a}{2}\Psi_0(y_0,t), \tag{2.14}$$

where the two-spinor Ψ_0, defining the true lowest Landau level, obeys the canonical commutation relation $\{\Psi_0(y_0,t),\Psi_0^{\dagger}(y_0',t)\}=\delta(y_0-y_0')$. The $\mathcal{U}_{\mathbf{p}}^{(\alpha)}$ denote the contributions quadratic in $A_\mu^{(\alpha)}$, and are given (for $\mathbf{p}=0$) by $\mathcal{U}_{\mathbf{p}=\mathbf{0}}^{(\alpha)}=\int d^2\mathbf{x}\ \mathcal{U}^{(\alpha)}(x)$ with

$$\mathcal{U}^{(\alpha)}=\frac{1}{2}A_\mu^{(\alpha)}D\epsilon^{\mu\nu\rho}\partial_\nu A_\rho^{(\alpha)}-\frac{1}{2\omega_c}A_{k0}^{(\alpha)}DA_{k0}^{(\alpha)}+\cdots, \tag{2.15}$$

where $D=\omega_c^2/(\omega_c^2+\partial_t^2)$; $A_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, and $\epsilon^{\mu\nu\rho}$ is a totally antisymmetric tensor with $\epsilon^{012}=1$. Here we have retained terms to $O(\nabla^2/\omega_c)$; see Ref. 23 for an expression exact to all powers of ∂_k.

The charges $(\bar{\rho}_{\mathbf{p}},\bar{S}_{\mathbf{p}}^{a})$ obey an SU(2)$\times W_\infty$ algebra[20]

$$[\bar{\rho}_{\mathbf{p}},\bar{\rho}_{\mathbf{k}}]=-2is(p,k)\bar{\rho}_{\mathbf{p}+\mathbf{k}},\quad[\bar{\rho}_{\mathbf{p}},\bar{S}_{\mathbf{k}}^{a}]=-2is(p,k)\bar{S}_{\mathbf{p}+\mathbf{k}}^{a},$$

$$[\bar{S}_{\mathbf{p}}^{a},\bar{S}_{\mathbf{k}}^{b}]=c(p,k)i\epsilon^{abc}\bar{S}_{\mathbf{p}+\mathbf{k}}^{c}-\delta^{ab}\frac{i}{2}s(p,k)\bar{\rho}_{\mathbf{p}+\mathbf{k}}, \tag{2.16}$$

where

$$s(p,k)=\sin\left(\frac{\mathbf{p}\times\mathbf{k}}{2}\right)e^{(1/2)\mathbf{p}\cdot\mathbf{k}}, \tag{2.17}$$

$c(p,k)$ is given by $s(p,k)$ with sin→cos. It is important to note here that the projected charges themselves, $(\rho_{00}^{(\alpha)})_{\mathbf{p}}=\bar{\rho}_{\mathbf{p}}^{(\alpha)}+\triangle\bar{\rho}_{\mathbf{p}}^{(\alpha)}$, differ slightly[23] from $\bar{\rho}_{\mathbf{p}}^{(\alpha)}$ by $A_\mu^{(\alpha)}$-dependent corrections $\triangle\bar{\rho}_{\mathbf{p}}^{(\alpha)}$, which derive from the field-dependent projection employed. (See the Appendix.) As a result, the projected Coulomb interaction

$$\bar{H}^{\rm C}=\frac{1}{2}\sum_{\mathbf{p}}(V_{\mathbf{p}}^{+}\bar{\rho}_{-\mathbf{p}}\bar{\rho}_{\mathbf{p}}+4V_{\mathbf{p}}^{-}\bar{S}_{-\mathbf{p}}^{3}\bar{S}_{\mathbf{p}}^{3})+\triangle\bar{H}^{\rm C} \tag{2.18}$$

acquires a field-dependent piece $\triangle\bar{H}^{\rm C}$, which plays a crucial role, as we shall see.

The dynamics within the lowest Landau level is now governed by the Hamiltonian $\bar{H}=\bar{H}^{\rm C}+\bar{H}^{\rm cyc}+\bar{H}^{\rm em}+\bar{H}^{\rm tun}$. Suppose now that an incompressible many-body state $|G\rangle$ of uniform density $(\rho_{\rm av}^{(1)},\rho_{\rm av}^{(2)})$ is formed. Then, setting $\langle G|\bar{\rho}_{-\mathbf{p}}^{(\alpha)}|G\rangle=\rho_{\rm av}^{(\alpha)}(2\pi)^2\delta^2(\mathbf{p})$ in $\bar{H}^{\rm em}$ one obtains the effective action to $O(A^2)$:

$$S^{\rm cycl}=-\int dtd^2\mathbf{x}\sum_\alpha\rho_{\rm av}^{(\alpha)}\mathcal{U}^{(\alpha)}(x), \tag{2.19}$$

which summarizes the response due to the electromagnetic inter-Landau-level mixing, i.e., due to the cyclotron modes (one for each layer).

The electromagnetic interaction in $\bar{H}$ also gives rise to intra-Landau-level transitions. For single-layer systems the intra-Landau-level excitations are only dipole inactive[11] (i.e., the response vanishes faster than $\mathbf{k}^2$ for $\mathbf{k}\to0$) as a result of Kohn's theorem,[29] and the incompressible quantum Hall states show universal $O(\mathbf{k})$ and $O(\mathbf{k}^2)$ long-wavelength electromagnetic characteristics determined by the cyclotron mode alone.[23]

The situation changes drastically for bilayer systems, where both in-phase and out-of-phase collective excitations arise. In-phase excitations generally remain dipole inactive, as a consequence of invariance under translations of both layers. Out-of-phase collective excitations, in contrast, become dipole active[10,17] (in the absence of interlayer coherence) and modify the electromagnetic characteristics of the bilayer systems substantially.[23] Incompressible quantum Hall states well described by the Halperin (m,m,n) wave functions,[12] in particular, belong to this class of states. The presence of interlayer coherence is expected to cause further substantial changes in the systems, which we study in the following section.

III. INTERLAYER COHERENCE AND ELECTROMAGNETIC RESPONSE

In this section we study how the presence of interlayer coherence affects the electromagnetic properties of bilayer systems. The particular set of states of our concern are the ground states at filling $\nu=1/m$ for odd integers m, believed to have total pseudospin $S=N_e/2$, with their orbital wave functions well approximated by the Laughlin wave functions[9] or Halperin (m,m,m) wave functions.[12] For definiteness we shall concentrate on the $\nu=1$ ground state, but our analysis will apply to other cases as well.

Suppose first that the SU(2) breaking Coulomb interaction $V_{\mathbf{p}}^{-}=(e^2/4\epsilon)d+O(d^2)$ is negligibly weak (i.e., $d\to0$ and $\triangle_{\rm SAS}\neq0$). Then the $\nu=1$ ground state is given by the total pseudospin $S=N_e/2$ eigenstate $|G_0\rangle$, fully polarized in the S^1 direction via the tunneling interaction so that $\langle G_0|\bar{S}_{\mathbf{p}=\mathbf{0}}^{1}|G_0\rangle=\frac{1}{2}N_e$, or

$$\langle G_0|\bar{S}_{\mathbf{p}}^{a}|G_0\rangle=\delta^{a1}\frac{1}{2}\rho_0\delta_{\mathbf{p},\mathbf{0}},\quad\langle G_0|\bar{\rho}_{\mathbf{p}}|G_0\rangle=\rho_0\delta_{\mathbf{p},\mathbf{0}}, \tag{3.1}$$

where $\rho_0=\rho^{(1)}_{\rm av}+\rho^{(2)}_{\rm av}$ and $\delta_{\mathbf{p},0}\equiv(2\pi)^2\delta^2(\mathbf{p})$. We suppose that this $S^1_{\mathbf{p}=\mathbf{0}}=N_e/2$ eigenstate $|G_0\rangle$ continues to be a good approximation to the $\nu=1$ ground state as $V^-_{\mathbf{p}}$, kept weak, is turned on. It has been argued[20] and supported experimentally[22] that such $|G_0\rangle$ well approximates the ground state for $\triangle_{\rm SAS}\to 0$ with $V^-_{\mathbf{p}}\neq 0$, where interlayer coherence $\langle S^1\rangle\neq 0$ is realized spontaneously with $\langle S^3\rangle=0$ maintained so as to reduce the interlayer charging energy.

Further characterization of this $S^1_{\mathbf{p}=0}=N_e/2$ state is given by the static structure factors

$$\langle G_0|\bar{S}^{a'}_{\mathbf{p}}\bar{S}^{b'}_{\mathbf{k}}|G_0\rangle=\delta_{\mathbf{p}+\mathbf{k},\mathbf{0}}(\delta^{a'b'}+i\epsilon^{a'b'1})\frac{1}{4}\rho_0 e^{-(1/2)\mathbf{p}^2},$$

$$\langle G_0|\bar{\rho}_{\mathbf{p}}\bar{\rho}_{\mathbf{k}}|G_0\rangle=\rho_0^2\delta_{\mathbf{p},\mathbf{0}}\delta_{\mathbf{k},\mathbf{0}}\equiv R_{\mathbf{p},\mathbf{k}},$$

$$\langle G_0|\bar{\rho}_{\mathbf{p}}\bar{S}^a_{\mathbf{k}}|G_0\rangle=2\langle G_0|\bar{S}^1_{\mathbf{p}}\bar{S}^a_{\mathbf{k}}|G_0\rangle=\delta^{a1}\frac{1}{2}R_{\mathbf{p},\mathbf{k}},\qquad(3.2)$$

where b' runs over (2,3). These relations are readily derived by rewriting $\bar{\rho}_{\mathbf{p}}$ and $\bar{S}^a_{\mathbf{p}}$ in terms of the eigenspinors (ψ_S,ψ_A) of S^1, and by noting that $|G_0\rangle$ involves no ψ_A component (of $S^1=-\frac{1}{2}$). For the partially filled ψ_S Landau level of $\nu=1/3,1/5,\cdots$ one has to retain in $R_{\mathbf{p},\mathbf{k}}$ a term[11] $\rho_0\delta_{\mathbf{p}+\mathbf{k},\mathbf{0}}\bar{s}^+(\mathbf{p})$ with $\bar{s}^+(\mathbf{p})\sim O(\mathbf{p}^4)$, which vanishes in the present $\nu=1$ case.

The correlations characteristic of interlayer order $\bar{S}^1_{\mathbf{p}=0}=N_e/2$ are involved in the structure factor

$$\bar{s}^-(\mathbf{p})=\frac{2}{N_e}\langle G_0|\bar{S}^3_{-\mathbf{p}}\bar{S}^3_{\mathbf{p}}|G_0\rangle=\frac{1}{2}e^{(-1/2)\mathbf{p}^2},\qquad(3.3)$$

which is nonvanishing for $\mathbf{p}\to 0$, in contrast to the case of the Halperin (m,m,n) states where[17,10] $\bar{s}^-(\mathbf{p})\sim\mathbf{p}^2$.

Let us now study low-energy excitations over this ground state. With polarization $\langle G_0|\bar{S}^1_{\mathbf{p}=\mathbf{0}}|G_0\rangle=N_e/2$, the Coulomb interaction $\bar{H}^{\rm Coul}$ has an (approximate) U(1) symmetry about the $\bar{S}^1$ axis, yielding two Nambu-Goldstone (NG) modes $\{\Omega^2_{\mathbf{p}}(t),\Omega^3_{\mathbf{p}}(t)\}$. These NG modes constitute the low-energy collective excitations in the system, and one can employ the technique[20] of nonlinear realizations of the pseudospin symmetry for their description. To this end let $\Psi^{\rm cl}_0(y_0,t)$ denote a classical configuration or the ground-state configuration, characterized by the expectation values in Eqs. (3.1) and (3.2). Let us set $\Omega[\mathbf{r},t]\equiv\Sigma_a(\sigma^a/2)\Sigma_{\mathbf{p}}\Omega^a_{\mathbf{p}}(t)e^{i\mathbf{p}\cdot\mathbf{r}}$ (with $a=2,3$) and write the electron field Ψ_0 in the form of a small rotation in pseudospin from $\Psi^{\rm cl}_0$,

$$\Psi_0(y_0,t)=e^{-i\Omega[\mathbf{r},t]}\Psi^{\rm cl}_0(y_0,t).\qquad(3.4)$$

Here the NG modes serve as pseudospin textures in which the local pseudospin alignment varies slowly with position. Rewriting the Lagrangian in favor of $\Psi^{\rm cl}_0$ and $\Omega^a_{\mathbf{p}}$, and replacing the products of $(\Psi^{\rm cl}_0)^\dagger$ and $\Psi^{\rm cl}_0$ by the expectation values (3.1) and (3.2) then yields a low-energy effective Lagrangian for the NG modes (Ω^2,Ω^3).

To facilitate such transcription it is convenient to express Eq. (3.4) in operator form

$$\Psi_0(y_0,t)=\mathcal{P}\Psi^{\rm cl}_0(y_0,t)\mathcal{P}^{-1},$$

$$\mathcal{P}=e^{i\Omega\cdot\bar{S}^{\rm cl}},\quad \Omega\cdot\bar{S}^{\rm cl}\equiv\sum_{\mathbf{p}}e^{(1/4)\mathbf{p}^2}\Omega^a_{\mathbf{p}}(\bar{S}^{\rm cl}_{-\mathbf{p}})^a.\qquad(3.5)$$

Here $(\bar{S}^{\rm cl}_{\mathbf{p}})^a$ stand for $\bar{S}^a_{\mathbf{p}}$ with Ψ_0 replaced by $\Psi^{\rm cl}_0$, and obey the same algebra (2.16) as $\bar{S}^a_{\mathbf{p}}$. Repeated use of the algebra then enables one to express $\bar{S}^a=\mathcal{P}(\bar{S}^{\rm cl})^a\mathcal{P}^{-1}$ and $\bar{\rho}^a=\mathcal{P}(\bar{\rho}^{\rm cl})\mathcal{P}^{-1}$ in powers of $\bar{\rho}^{\rm cl}$ and $(\bar{S}^{\rm cl})^a$. [Remember that the characterization (3.1) and (3.2) from now on applies to $(\bar{S}^{\rm cl}_{\mathbf{p}})^a$ and $\bar{\rho}^{\rm cl}_{\mathbf{p}}$.] In particular, for their expectation values one obtains to $O(\Omega^2)$,

$$\langle\bar{\rho}_{\mathbf{p}}\rangle=\rho_0\left[\delta_{\mathbf{p},\mathbf{0}}+\frac{1}{2}\gamma_{\mathbf{p}}\sum_{\mathbf{k}}\sin\left(\frac{\mathbf{p}\times\mathbf{k}}{2}\right)\epsilon^{1a'b'}\Omega^{a'}_{\mathbf{k}}\Omega^{b'}_{\mathbf{p}-\mathbf{k}}\right],$$

$$\langle\bar{S}^1_{\mathbf{p}}\rangle=\frac{\rho_0}{2}\left[\delta_{\mathbf{p},\mathbf{0}}-\frac{1}{2}\gamma_{\mathbf{p}}\sum_{\mathbf{k}}\cos\left(\frac{\mathbf{p}\times\mathbf{k}}{2}\right)\Omega^{a'}_{\mathbf{k}}\Omega^{a'}_{\mathbf{p}-\mathbf{k}}\right],$$

$$\langle\bar{S}^2_{\mathbf{p}}\rangle=\frac{\rho_0}{2}\gamma_{\mathbf{p}}\Omega^3_{\mathbf{p}},\quad\langle\bar{S}^3_{\mathbf{p}}\rangle=-\frac{\rho_0}{2}\gamma_{\mathbf{p}}\Omega^2_{\mathbf{p}},\qquad(3.6)$$

where $\langle\cdots\rangle=\langle G_0|\cdots|G_0\rangle$ for short and $\gamma_{\mathbf{p}}=e^{-\frac{1}{4}\mathbf{p}^2}$; a' and b' run over (2,3). These expressions suggest us to rename, following Moon *et al.*,[20] $(m_2)_{\mathbf{p}}=\Omega^3_{\mathbf{p}}$ and $(m_3)_{\mathbf{p}}=-\Omega^2_{\mathbf{p}}$ so that their x space representatives $m_a(x)=[m_1(x),m_2(x),m_3(x)]$ stand for the pseudospin density [with normalization $\Sigma^3_{a=1}(m_a)^2\approx 1$ classically]. Actually it is possible to generalize Eq. (3.6) to all powers of m_a, if one ignores their derivatives $\partial_k m_a$:

$$\langle\bar{S}^1(x)\rangle\approx\frac{\rho_0}{2}\cos|\mathbf{m}|,\ \langle\bar{S}^2(x)\rangle\approx\frac{\rho_0}{2}\frac{m_2}{|\mathbf{m}|}\sin|\mathbf{m}|,\qquad(3.7)$$

where $\mathbf{m}=[m_2(x),m_3(x)]$; $\langle\bar{\rho}(x)\rangle\approx\rho_0$, etc.

Moon *et al.*[20] earlier made such a pseudospin-texture calculation and showed that the Coulomb interaction leads to the following low-energy effective Hamiltonian to $O(\Omega^2)$ and $O(\mathbf{p}^2)$,

$$\langle\bar{H}^{\rm C}\rangle=\sum_{\mathbf{p}}\left\{\beta[\mathbf{p}]|(m_3)_{\mathbf{p}}|^2+\frac{1}{2}\rho^E_s\mathbf{p}^2|(m_2)_{\mathbf{p}}|^2\right\},\qquad(3.8)$$

with

$$\rho_s=\frac{1}{8}\rho_0\sum_{\mathbf{p}}V^{11}_{\mathbf{p}}\mathbf{p}^2e^{(-1/2)\mathbf{p}^2}=\frac{e^2}{4\pi\epsilon\ell}\frac{\nu}{16\sqrt{2\pi}},$$

$$\rho^E_s=\rho_s\{1-\sqrt{8/\pi}\hat{d}+(3/2)\hat{d}^2+\cdots\},$$

$$\beta[\mathbf{p}]=\rho_s\{(c_0/\ell^2)+(c_1/\ell)|\mathbf{p}|+\tfrac{1}{2}(1+c_2)\mathbf{p}^2\},$$

$$c_0\approx\hat{d}^2,\quad c_1\approx-\sqrt{2/\pi}\hat{d}^2,\quad c_2\approx-\sqrt{8/\pi}\hat{d}(1-\hat{d}^2/3),\qquad(3.9)$$

where we have used the pseudospin stiffness ρ_s as a common factor and recorded some corrections in powers of $\hat{d}\equiv d/\ell$; ρ_s^E is given by the same expression as ρ_s with $V_{\mathbf{p}}^{11}\to V_{\mathbf{p}}^{12}$.

Substituting $\Psi_0=e^{-i\Omega[\mathbf{r},t]}\Psi_0^{\rm cl}$ into the electronic kinetic term $\langle\Psi_0^\dagger i\partial_t\Psi_0\rangle$ yields Berry's phase,[30] which turns into the kinetic term of the NG modes

$$L^{\rm kin}=-\frac{1}{4}\rho_0\sum_{\mathbf{k}}\epsilon^{ab1}\Omega^a_{-\mathbf{k}}\partial_t\Omega^b_{\mathbf{k}} \tag{3.10}$$

to $O(\Omega^2)$. This shows that $\Omega^2=-m_3$ is canonically conjugate to $\Omega^3=m_2$.

Substitution of Eqs. (3.6) and (3.7) into $\overline{H}^{\rm em}+\overline{H}^{\rm tun}$ yields the coupling of the NG modes to external fields, $\langle G_0|\overline{H}^{\rm em}+\overline{H}^{\rm tun}|G_0\rangle=\int d^2\mathbf{x}\mathcal{H}_A$ with

$$\mathcal{H}_A=\rho_0\bigg\{A_0^+ +\frac{1}{2}\chi^+\epsilon^{ij}\partial_i m_2\partial_j m_3+\chi^-\gamma m_3 -\frac{1}{2}\triangle_{SAS}\cos|\mathbf{m}|\bigg\} \tag{3.11}$$

to $O(m^2)$ and $O(\partial^2)$, where $\mathbf{m}=(m_2,m_3)$, $\gamma=e^{(1/4)\nabla^2}$, and $\chi^\pm=A_0^\pm+(1/2M)A_{12}^\pm$.

Similarly, the field-dependent Coulomb interaction $\triangle\overline{H}^{\rm C}$ leads to the effective interaction

$$\langle\triangle\overline{H}^{\rm C}\rangle=2\rho_s^E\int d^2\mathbf{x}\{m_2\partial_j A_j^- +(A_j^-)^2+\cdots\}; \tag{3.12}$$

see the Appendix for details.

Collecting terms so far obtained yields the effective action $S_{\rm eff}^{\rm coll}=\int dt d^2\mathbf{x}\mathcal{L}^{\rm coll}$ with

$$\begin{aligned}\mathcal{L}^{\rm coll}=&\frac{\rho_0}{2}m_3(\dot{m}_2-2A_0^-)-m_3(\beta[\mathbf{p}]+\tfrac{1}{4}\rho_0\triangle_{SAS})m_3\\&-\frac{1}{2}\rho_s^E(\partial_j m_2-2A_j^-)^2+\frac{1}{2}\rho_0\triangle_{SAS}\cos m_2\\&-\rho_0A_0^+(1+\tfrac{1}{2}\epsilon^{ij}\partial_i m_2\partial_j m_3),\end{aligned} \tag{3.13}$$

where $\mathbf{p}\to-i\nabla$ in $\beta[\mathbf{p}]$. Here we have simplified the result slightly by retaining only terms that contribute to the $O(\nabla^2)$ electromagnetic response eventually. The $\mathcal{L}^{\rm coll}$ is essentially the Lagrangian of a nonlinear sigma model that supports classical topological excitations,[31,32] Skyrmions, which constitute the low-lying charged excitations of the system; see Eq. (5.3) in Sec. V. Note that Eq. (3.13) correctly involves the topological charge density[32] $(\rho_0/2)\epsilon^{ij}\partial_i m_2\partial_j m_3$, which implies that the Skyrmions carry electric charge of a multiple of νe.

Let us here focus on the neutral collective excitations described by the field m_2 or m_3. Eliminating m_3 from $\mathcal{L}^{\rm coll}$ yields the Lagrangian of the neutral field m_2

$$\begin{aligned}\mathcal{L}^{\rm coll}_{m_2}=&\frac{1}{2}\rho_s^E\bigg[\frac{1}{v^2}(\partial_t m_2-2A'^-_0)^2-(\partial_j m_2-2A'^-_j)^2\bigg]\\&+\frac{1}{2}\rho_0\triangle_{SAS}\cos m_2,\end{aligned} \tag{3.14}$$

with

$$v^2=2(\rho_s^E/\rho_0^2)(4\beta[0]+\rho_0\triangle_{SAS}),$$

$$2A'^-_\mu=2A^-_\mu-\partial_\mu\Gamma\approx d(\partial_z A_\mu-\partial_\mu A_z). \tag{3.15}$$

Here we have indicated explicitly that A_μ^- so far used actually stands for A'^-_μ; we have also isolated the $\rho_0A_0^+$ term that detects the charge of the ground state $|G_0\rangle$.

This collective mode m_2 gives rise to an electromagnetic response of the form

$$\begin{aligned}\mathcal{L}^{\rm coll}_{\rm em}=&2\rho_s^E(A^-_{j0}\mathcal{D}A^-_{j0}-v^2A^-_{12}\mathcal{D}A^-_{12})\\&+\rho_0\triangle_{SAS}(A'^-_0\mathcal{D}A'^-_0-v^2A'^-_j\mathcal{D}A'^-_j)\end{aligned} \tag{3.16}$$

in compact notation, where $\mathcal{D}=1/\{\omega_{\mathbf{p}}^2-(i\partial_t)^2\}$ and

$$\omega_{\mathbf{p}}^2=\bigg\{\triangle_{SAS}+\frac{4}{\rho_0}\beta[\mathbf{p}]\bigg\}\bigg\{\triangle_{SAS}+\frac{2\rho_s^E}{\rho_0}\mathbf{p}^2\bigg\} \tag{3.17}$$

with $\mathbf{p}\to-i\boldsymbol{\nabla}$. Here we have recovered $\beta[\mathbf{p}]$ to obtain the dispersion more accurately. In terms of the field strengths in three-dimensional space one can write $\mathcal{L}^{\rm coll}_{\rm em}$ as

$$\begin{aligned}\mathcal{L}^{\rm coll}_{\rm em}\approx&\frac{1}{2}\rho_s^E d^2(\partial_z\mathbf{E}_\parallel\mathcal{D}\partial_z\mathbf{E}_\parallel-v^2\partial_z B_\perp\mathcal{D}\partial_z B_\perp)\\&+\frac{\rho_0}{4}\triangle_{SAS}d^2(E_\perp\mathcal{D}E_\perp-v^2\mathbf{B}_\parallel\mathcal{D}\mathbf{B}_\parallel)\end{aligned} \tag{3.18}$$

in obvious notation.

The response due to the cyclotron modes in Eq. (2.19) is generally suppressed by powers of $1/\omega_c$ compared with the collective-mode contribution, except for the Hall-drift or Chern-Simons term

$$\mathcal{L}^{\rm cyc}_{A^-}=-\frac{\rho_0}{2}A^-_\mu\frac{\omega_c^2}{\omega_c^2-\omega^2}\epsilon^{\mu\nu\rho}\partial_\nu A^-_\rho+\cdots, \tag{3.19}$$

which thus combines with $\mathcal{L}^{\rm coll}_{\rm em}$ to form the principal out-of-phase response of the system at long wavelengths. Note here that the collective mode gives rise to no such Hall-drift term, unlike for the (m,m,n) states.[23] This implies that no appreciable interlayer Hall drag is expected for the present $\nu=1$ state, in contrast to the case[33] of the (gapful) (m,m,n) states. (Note that the cyclotron modes alone yield no interlayer Hall drag.)

Some comments are in order here. First, the effective Lagrangian (3.14) essentially agrees with that derived earlier[20,27] if one sets $m_2=\hat{m}_2-\Gamma_z$, where $\hat{m}_2$ is taken to undergo the gauge transformation $\delta\hat{m}_2=2\theta^-$. The earlier derivations focused on the spectrum of the low-lying mode and its coupling to weak external electromagnetism was only guessed on the ground of gauge invariance. A direct derivation of such electromagnetic coupling, as shown in our approach, is quite nontrivial, since it requires proper account of the Landau-level projection, especially the field-dependent Coulomb interaction.

Second, in our approach electromagnetic gauge invariance is kept exact at each step of discussion by use of the gauge-covariant fields $\psi'^{(a)}(x)$ and $\overline{A'^{-}_{\mu}}(x)$ in Eq. (2.9). Recall that the pseudospin densities $\overline{S'^{a}}$ are gauge invariant while S^1 and S^2, defined in terms of $\psi^{(a)}$, are gauge variant so that $\overline{S^1}+i\overline{S^2}=e^{i\Gamma_z}(\overline{S'^1}+i\overline{S'^2})$. Our characterization of interlayer coherence $\langle G_0|\overline{S'^1_{\mathbf{p}=0}}|G_0\rangle=\frac{1}{2}N_e$ therefore is a sensible gauge-invariant statement and, as a result, the related order parameters

$$\langle\overline{S^1_{\mathbf{p}=0}}\rangle=\frac{1}{2}N_e\cos\Gamma_z,\quad \langle\overline{S^2_{\mathbf{p}=0}}\rangle=\frac{1}{2}N_e\sin\Gamma_z, \tag{3.20}$$

rotate in the pseudospin 1-2 plane under electromagnetic gauge transformations $\Gamma_z\to\Gamma_z+2\theta^-$, or under the action of in-plane magnetic fields $\partial_j\Gamma_z$. In other words, a naive choice $\langle\overline{S^a_{\mathbf{p}=0}}\rangle\propto\delta^{a1}$ is not physically acceptable unless layer spacing $d\to 0$. This is the real reason why we have restarted with $\psi'^{(a)}(x)$ and $A'^{-}_{\mu}(x)$ after Eq. (2.9).

We have handled two NG modes (m_2,m_3) associated with SU(2)→U(1) breaking. They, being gauge invariant, are neutral physical fields. They, however, happen to form a pair of canonical conjugates and thus actually describe only one physical mode m_2. Note here that, since $m_2\sim\Omega^3$, a shift $m_2\to m_2+\text{const}$ induces a rotation about the S^3 axis so that

$$i[\overline{S^3_{\mathbf{p}=0}},m_2]=1\neq 0. \tag{3.21}$$

This shows that m_2 can also be interpreted as an NG mode associated with the spontaneous breaking of the global U(1) symmetry about the S^3 axis.[15,16] Because this global $\mathrm{U}(1)^-$ is only approximate, m_2 is a pseudo-NG mode and acquires a finite energy gap $\propto\triangle_{SAS}$. In the absence of tunneling ($\triangle_{SAS}=0$ but $V^-_{\mathbf{p}}\neq 0$), the $\mathrm{U}(1)^-$ becomes exact but spontaneously broken; the energy gap closes and m_2 disperses linearly.

Unlike the global $\mathrm{U}(1)^-$, the gauged $\mathrm{U}(1)^-$ or $\mathrm{U}(1)^{\mathrm{em}}$ is kept exact, as seen clearly from the gauge-invariant response (3.16). This implies, in particular, that there is no Anderson-Higgs mechanism or no Meissner effect working in the present bilayer system. Here we see a peculiar instance of spontaneous breaking of a global symmetry with the related gauge symmetry kept exact; this derives from the special character of the "interlayer" gauge invariance remarked in Sec. II.

Finally, one can use the effective theory to discuss the tunneling phenomena. The equation of motion of m_2 implies the conservation law for the three-current $-\partial\mathcal{L}^{\mathrm{coll}}_{m_2}/\partial A^-_{\mu}=j^{(1)}_{\mu}-j^{(2)}_{\mu}$, from which one can read off the tunneling current $j^{\mathrm{tun}}_z\sim-\partial_t\rho^{(1)}$ as

$$j^{\mathrm{tun}}_z=\frac{1}{2}e\rho_0\triangle_{SAS}\sin m_2. \tag{3.22}$$

Adding a source term $a_z j^{\mathrm{tun}}_z$ to $\mathcal{L}^{\mathrm{coll}}_{m_2}$ and calculating the response yields the tunneling current

$$j^{\mathrm{tun}}_z=\frac{1}{2}e\rho_0\triangle_{SAS}\frac{1}{\omega^2-\omega^2_{\mathbf{p}}}\partial_t V_z \tag{3.23}$$

in response to an alternating interlayer voltage $V_z=-2A'_0\approx-dE_{\perp}$.

IV. RELATION TO THE SINGLE-MODE APPROXIMATION

In this section we present a derivation of the electromagnetic response (3.16) by an alternative means, the single-mode approximation (SMA). Let us first suppose that $V^-_{\mathbf{p}}=0$, in which case the ground state is exactly given by the $\overline{S'^1_{\mathbf{p}=0}}=N_e/2$ eigenstate $|G_0\rangle$ in Eq. (3.1).

We consider the phonon-roton mode coupled to A^-_0 and represent it as $|\phi^-_{\mathbf{k}}\rangle\sim\overline{S^3_{\mathbf{k}}}|G_0\rangle$. The basic quantity in the SMA is the static structure factor $\overline{s^-}(\mathbf{k})\sim\langle\phi^-_{\mathbf{k}}|\phi^-_{\mathbf{k}}\rangle$, which, in view of Eq. (3.3), is given by

$$\overline{s^-}(\mathbf{k})=(1/2)e^{(-1/2)\mathbf{k}^2}. \tag{4.1}$$

To determine the collective-excitation spectrum in the SMA one considers the (projected) oscillator strength

$$\overline{f^-}(\mathbf{k})=(2/N_e)\langle G_0|\overline{S^3_{-\mathbf{k}}}[\overline{H},\overline{S^3_{\mathbf{k}}}]|G_0\rangle, \tag{4.2}$$

which is calculable[10,17] by the use of algebra (2.16). With $\overline{H}^{\mathrm{tun}}=-\triangle_{SAS}\overline{S'^1_{\mathbf{p}=0}}$ included, it is given to $O(\mathbf{k}^2)$ by

$$\overline{f^-}(\mathbf{k})=\frac{1}{2}e^{(-1/2)\mathbf{k}^2}[\triangle_{SAS}+2(\rho^E_s/\rho_0)\mathbf{k}^2+\cdots]. \tag{4.3}$$

Here the coefficient of the $\mathbf{k}^2$ term derives from the general expression

$$\frac{1}{2}\sum_{\mathbf{p}}\mathbf{p}^2V^{12}_{\mathbf{p}}\{\overline{s^-}(\mathbf{p})-\overline{s^+}(\mathbf{p})\} \tag{4.4}$$

upon substitution of $\overline{s^-}(\mathbf{p})$ above; $\overline{s^+}(\mathbf{p})=0$ for $\nu=1$. Saturating $\overline{f^-}(\mathbf{k})$ with the single mode $|\phi^-_{\mathbf{k}}\rangle$ then yields the SMA excitation spectrum $\epsilon^-_{\mathbf{k}}=\overline{f^-}(\mathbf{k})/\overline{s^-}(\mathbf{k})$ or

$$\epsilon^-_{\mathbf{k}}=\triangle_{SAS}+2(\rho^E_s/\rho_0)\mathbf{k}^2+\cdots. \tag{4.5}$$

This agrees with the spectrum derived by the pseudospin-texture calculation in Eq. (3.17) with $V^-_{\mathbf{p}}\to 0$.

To calculate the electromagnetic response one may resort to the previous SMA analysis,[23] which, though developed originally for the case of a dipole-active response $\overline{s^-}(\mathbf{k})=(c^-/2)\mathbf{k}^2+\cdots$, is adapted to the present case as well: One may simply replace $2\overline{s^-}(\mathbf{k})\epsilon^-_{\mathbf{k}}$ in Eq. (3.20) of Ref. 23 by $2\overline{f^-}(\mathbf{k})=\triangle_{SAS}e^{-(1/2)\mathbf{k}^2}+2(\rho^E_s/\rho_0)\mathbf{k}^2$ and $c^-\epsilon^-_0$ in Eq. (3.28) of Ref. 23 by Eq. (4.4) or $2\rho^E_s/\rho_0$. Then our result

(3.16) is correctly reproduced, apart from the $v^2 A_{12}^- \cdots A_{12}^-$ and $v^2 A'^-_j \cdots A'^-_j$ terms, which, being $O(v^2) \sim O([\bar{H}^C]^2)$ higher in the Coulomb interaction, were not covered in the previous SMA treatment.

It is possible to include the effect of $V_{\mathbf{p}}^-$ and make the agreement complete if one appeals to the low-energy effective theory in Eq. (3.13). With the identification $\overline{S_{\mathbf{p}}^3} = (\rho_0/2)\gamma_{\mathbf{p}}(m_3)_{\mathbf{p}}$, as implied by Eq. (3.6), one can calculate $\overline{s^-}(\mathbf{k})$ from the vacuum expectation value $\propto (\gamma_{\mathbf{k}})^2 \langle 0|(m_3)_{-\mathbf{k}}(m_3)_{\mathbf{k}}|0\rangle$. The task is thus reduced to determining the uncertainty $\langle 0|(m_3)^2|0\rangle$ for a collection of harmonic oscillators described by the Hamiltonian

$$H^{\mathrm{coll}} \approx \sum_{\mathbf{k}} \frac{1}{4}[g^{(12)}|(m_2)_{\mathbf{k}}|^2 + g^{(11)}|(m_3)_{\mathbf{k}}|^2], \tag{4.6}$$

where $g^{(12)} \equiv \rho_0 \triangle_{SAS} + 2\rho_s^E \mathbf{k}^2$ and $g^{(11)} \equiv \rho_0 \triangle_{SAS} + 4\beta[\mathbf{k}]$. Via rescaling $(m_3)^2(\rho_0/2)\sqrt{g^{(11)}/g^{(12)}}$ is seen to attain the minimum uncertainty $(\hbar/2)\int d^2\mathbf{x}$, yielding

$$\overline{s^-}(\mathbf{k}) = \frac{1}{2} e^{(-1/2)\mathbf{k}^2} \sqrt{g^{(12)}/g^{(11)}}. \tag{4.7}$$

The $\overline{f^-}(\mathbf{k})$ in Eq. (4.3), being already exact to $O(V_{\mathbf{p}}^-)$, remains unmodified. The excitation spectrum and the response thereby agree with those in Eq. (3.16). The $\overline{s^-}(\mathbf{k})$ above neatly summarizes the effect of squeezing[34] in pseudospin of the ground state due to two competing sources of SU(2) breaking, $V_{\mathbf{p}}^-$ and $\triangle_{SAS}$. It is seen from $\langle (m_3)^2\rangle / \langle (m_2)^2\rangle = g^{(12)}/g^{(11)} \propto [\overline{s^-}(\mathbf{k})]^2$ that $\langle (m_3)^2\rangle$ gets rapidly squeezed with decreasing $\triangle_{SAS}$, i.e., in passing from the tunneling regime to the correlation regime (where $\overline{s^-}(\mathbf{k}) \propto |\mathbf{k}|$ for $\triangle_{SAS}=0$). It is an advantage of the pseudospin-texture theory that it accommodates different types of correlations in a single framework.

V. COMPARISON WITH THE CHERN-SIMONS APPROACH

In this section we examine the bilayer system within the Chern-Simons theory. For the $\nu=1$ quantum Hall state, as naively described by the (1,1,1) state, one introduces a single CS field[16] to convert the electron fields $\psi'^{(\alpha)}$ [of Eq. (2.9)] into the composite-boson fields $\psi_{\mathrm{cb}}^{(\alpha)}$.

Let us set $\psi_{\mathrm{cb}}^{(\alpha)}(x) = \sqrt{\rho^{(\alpha)}(x)}\, e^{i\eta^{(\alpha)}(x)}$, rewrite the Lagrangian in favor of $\rho^{\pm} = \rho^{(1)} \pm \rho^{(2)}$ and $\eta^{\pm} = \eta^{(1)} \pm \eta^{(2)}$, and expand it around the mean field $\rho^+(x) \sim \rho_0$. Then the (ρ^+, η^+) sector, coupled to A_μ^+, is seen to be essentially the same as in the single-layer case. The (ρ^-, η^-) sector, on the other hand, is sensitive to the SU(2) breaking interactions $\propto V_{\mathbf{p}}^-$ or $\triangle_{SAS}$. Integration over ρ^- leads to a low-energy Lagrangian, that takes essentially the same form as $\mathcal{L}_{m_2}^{\mathrm{coll}}$ in Eq. (3.14) with $m_2 \to \eta^-$, apart from some differences in scale.

The difference is subtle for the $(\partial_0 m_2 - 2A'_0)^2$ term

$$v^2/\rho_s^E \leftrightarrow 4V_{\mathbf{p}=0}^- + 2\triangle_{SAS}/\rho_0. \tag{5.1}$$

These coincide if $V_{\mathbf{p}=0}^-$ reads $V_{\mathbf{p}=0}^- - (1/\rho_0)\Sigma_{\mathbf{p}} V_{\mathbf{p}}^- e^{-(1/2)\mathbf{p}^2}$; this shows the importance of Landau-level projection, of which no explicit account is taken in the CS approach. For the $(\partial_j m_2 - 2A'_j)^2$ term the discrepancy is

$$\rho_s^E \leftrightarrow (\rho_0/4M) = \omega_c/(8\pi). \tag{5.2}$$

Here we see that the CS approach attributes the pseudospin stiffness improperly to inter-Landau-level processes. Another difficulty is that an important Hall-drift response (3.19) is missing from the CS theory.

All these subtleties derive from the fact that the CS approach, because of the lack of the Landau-level projection, fails to distinguish between the cyclotron modes and the collective modes. The flux attachment in the CS approach properly introduces some crucial correlations among electrons, but unfortunately not all of them.

Finally, it will be instructive to refer to the full effective Lagrangian to make it clear what is missing. Let us employ the decomposition[31] $\psi_{\mathrm{cb}}^{(\alpha)}(x) = \sqrt{\rho(x)}Z^\alpha(x)$ in terms of a CP^1 field $Z=(Z^1,Z^2)^{\mathrm{tr}}$ with $Z^\dagger Z=1$, particularly suited for studying the dynamics of Skyrmions and vortices. Making use of the dual transformation of Lee and Zhang[2] then enables one to rewrite the Lagrangian in terms of Z^α and a vector field b_μ (representing the cyclotron mode coupled to A_μ^+):

$$\mathcal{L}^{\mathrm{CS}} = -(A_\mu^B + A_\mu^+ - iZ^\dagger D_\mu Z)(\rho_0\delta_{0\mu} + \epsilon^{\mu\nu\rho}\partial_\nu b_\rho) + \frac{\pi}{\nu}\left\{ b_\mu \epsilon^{\mu\nu\rho}\partial_\nu b_\rho + \frac{1}{\omega_c}(b_{k0})^2 \right\} - \frac{1}{2}K\{|D_k Z^\alpha|^2 + (Z^\dagger D_k Z)^2\}$$
$$+ \frac{1}{2}\rho_0 \triangle_{SAS} Z^\dagger \sigma_1 Z + \cdots, \tag{5.3}$$

where only the principal terms are shown; $D_\mu = \partial_\mu + iA_\mu^-\sigma_3$ and $b_{k0} = \partial_k b_0 - \partial_0 b_k$. The last two terms, constituting a CP^1 nonlinear sigma model with a breaking interaction, essentially coincide with our $\mathcal{L}^{\mathrm{coll}}$ in Eq. (3.13) if one replaces the stiffness $K=\rho_0/M$ in this CS theory by $K = 4\rho_s$ [in accordance with Eq. (5.2)] and includes some SU(2) breaking terms coming from $V_{\mathbf{p}}^-$. The full effective Lagrangian is obtained by supplying to this modified $\mathcal{L}^{\mathrm{CS}}$ the missing cyclotron-mode contribution with another vector field b_μ^-:

$$\mathcal{L}^- = -A_\mu^- \epsilon^{\mu\nu\rho}\partial_\nu b_\rho^- + \frac{\pi}{\nu}\left[b_\mu^- \epsilon^{\mu\nu\rho}\partial_\nu b_\rho^- + \frac{1}{\omega_c}(b_{k0}^-)^2\right] + \cdots. \tag{5.4}$$

VI. SUMMARY AND DISCUSSION

In this paper we have studied the electromagnetic characteristics of bilayer quantum Hall systems in the presence of interlayer coherence and tunneling by means of a pseudospin-texture effective theory and the single-mode approximation (SMA). It will be clear from the analysis that a proper choice of the fields to start with, as well as proper account of the Landau-level mixing, is crucial for deriving a long-wavelength effective theory in gauge-invariant form. We have seen from the response that electromagnetic gauge invariance is kept exact, this, in particular, implying the absence of the Anderson-Higgs mechanism or the Meissner effect in bilayer systems. The response also shows that no appreciable Hall drag is expected for the $\nu=1$ state, in contrast to the case of the gapful (m,m,n) states. We have further seen that the identification of the low-lying neutral collective mode with a (pseudo) Nambu-Goldstone mode offers a peculiar instance of a spontaneously broken (approximate) global symmetry with the related gauge symmetry kept intact. Our approach offers a critical look into the Chern-Simons theories, and we have observed that the lack of the Landau-level projection is the principal source of subtleties inherent to them.

The idea underlying our approach is to explore the quantum Hall systems via their electromagnetic response, which in some cases is calculable without the details of the microscopic dynamics. An immediate example is the case of single-layer systems where it is generally known that the tintra-Landau-level collective excitations are dipole inactive; the leading long-wavelength response of the single-layer systems to $O(\mathbf{k}^2)$, therefore, is governed by the cyclotron mode alone. The second example is offered by bilayer systems (without interlayer coherence), for which one can construct from the response an effective gauge theory properly realizing the SMA spectrum of collective excitations. The third example is the analysis of the effects of interlayer coherence and tunneling given in the present paper. These would combine to enforce again the fact that incompressibility is the key character of the quantum Hall states and prove that studying the response offers not only a fresh look at the quantum Hall systems but also a practical means for constructing effective theories without referring to composite bosons and fermions.

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APPENDIX: FIELD-DEPENDENT COULOMB INTERACTION

In this appendix we display some expressions related to the field-dependent Coulomb interaction $\triangle\bar{H}^{\mathrm{C}}$. The charge densities $\rho_{\mathbf{p}}^{(\alpha)}$ projected unto the lowest Landau level read $\bar{\rho}_{\mathbf{p}}^{(\alpha)} + \triangle\bar{\rho}_{\mathbf{p}}^{(\alpha)}$ with

$$\triangle\bar{\rho}_{\mathbf{p}}^{(\alpha)} = \sum_{\mathbf{k}} u_{\mathbf{p},\mathbf{k}}^{(\alpha)}\bar{\rho}_{\mathbf{p}-\mathbf{k}}^{(\alpha)} + \sum_{\mathbf{q},\mathbf{k}} w_{\mathbf{p},\mathbf{q},\mathbf{k}}^{(\alpha)}\bar{\rho}_{\mathbf{p}-\mathbf{q}-\mathbf{k}}^{(\alpha)} + \cdots,$$

$$u_{\mathbf{p},\mathbf{k}}^{(\alpha)} = i\epsilon^{0jk}p_j(A_k^{(\alpha)})_{\mathbf{k}} + \cdots,$$

$$w_{\mathbf{p},\mathbf{q},\mathbf{k}}^{(\alpha)} = -\frac{1}{4}\mathbf{p}^2\sum_{\mathbf{q},\mathbf{k}} (A_i^{(\alpha)})_{\mathbf{q}}(A_i^{(\alpha)})_{\mathbf{k}} + \cdots, \tag{A1}$$

where we have retained only terms with no derivatives acting on $A_\mu^{(\alpha)}$, the portion relevant to our discussion. They give rise to the field-dependent piece $\triangle\bar{H}^{\mathrm{C}}$ in the Coulomb interaction. See Ref. 23 for the explicit form of the $O(A)$ contribution, which involves operator products of the form

$$\bar{I}_{\mathbf{p},\mathbf{k}}^{+} = \{\bar{\rho}_{-\mathbf{p}}, \bar{\rho}_{\mathbf{p}-\mathbf{k}}\}, \quad \bar{I}_{\mathbf{p},\mathbf{k}}^{-} = 2\{\bar{\rho}_{-\mathbf{p}}, \bar{S}_{\mathbf{p}-\mathbf{k}}^{3}\}, \tag{A2}$$

and those with $\bar{\rho}\leftrightarrow 2\bar{S}^3$ in the above.

In Sec. III we evaluate the expectation value $\langle\triangle\bar{H}^{\mathrm{C}}\rangle = \langle G_0|\triangle\bar{H}^{\mathrm{C}}|G_0\rangle$ to derive an effective electromagnetic coupling following from $\triangle\bar{H}^{\mathrm{C}}$. A direct calculation to $O(\Omega)$ shows that $\langle\bar{I}_{\mathbf{p},\mathbf{k}}^{-}\rangle = -\langle\bar{I}_{\mathbf{k}-\mathbf{p},\mathbf{k}}^{-}\rangle \propto \Omega_{-\mathbf{k}}^{3}$, while $\langle\bar{I}_{\mathbf{p},\mathbf{k}}^{+}\rangle \propto \delta_{\mathbf{k},\mathbf{0}}$ and $\langle\bar{S}_{-\mathbf{p}}^{3}\bar{S}_{\mathbf{p}-\mathbf{k}}^{3}\rangle \propto \delta_{\mathbf{k},\mathbf{0}}$ fail to contribute. As a result, the $O(A)$ coupling is written as

$$\rho_0\sum_{\mathbf{p},\mathbf{k}} u_{\mathbf{p},\mathbf{k}}^{-} V_{\mathbf{p}}^{12}\gamma_{\mathbf{p}}\gamma_{\mathbf{k}-\mathbf{p}}\sin\left(\frac{\mathbf{p}\times\mathbf{k}}{2}\right)\Omega_{-\mathbf{k}}^{3}. \tag{A3}$$

The calculation of the $O(A^2)$ term is somewhat tedious, though straightforward, eventually leading to Eq. (3.12).

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