Bethe-Salpeter equation for quantum-well exciton states in an inhomogeneous magnetic field

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The trapping of excitons in a single quantum well due to the presence of a strong homogeneous magnetic field and a weak inhomogeneous cylindrical symmetric magnetic field, created by the deposition of a magnetized disk on top of the quantum well, both applied perpendicular to the x-y plane of confinement is studied theoretically. The numerical calculations are performed for GaAs/Al_xGa_{1-x}As quantum wells and the formation of bound exciton states with nonzero values for the center-of-mass exciton wave function only in a small area is predicted.

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I. INTRODUCTION

The purpose of the present paper is to apply the nonrelativistic Bethe-Salpeter (BS) approach to the problem of excitons in a single quantum well (SQW), trapped by a cylindrical symmetric inhomogeneous magnetic field. The first attempts to calculate the energy and the wave function of excitons in a quantum well, trapped due to an inhomogeneous magnetic field, were based on the Schrödinger equation for magnetoexcitons.[1] The main difference between those papers and our work is that we assume the high-field regime (when exciton cyclotron energy is much larger than the Coulomb energy). All calculations in Ref. 1 are done assuming the weak-field regime, and therefore, all magnetic-field-dependent terms in the exciton Hamiltonian can be treated within the perturbation theory. If we define the trapping energy as the difference between the exciton energy in the inhomogeneous applied magnetic field (consists of homogeneous and inhomogeneous fields) and the energy of the exciton in the homogeneous applied field for the same exciton state, then in the weak-field case the trapping energy is positive.[1] That means the exciton states are unbound exciton states and a large part of the center-of-mass exciton wave function is extended into a sufficiently large region of space. In the high-field regime case, which is under consideration in this paper, we can apply (i) the lowest Landau level (LLL) approximation and (ii) treat the Coulomb interaction by perturbation theory. Under those assumptions we have calculated a negative trapping energy. This corresponds to the bound exciton states. For the bound states the center-of-mass exciton wave function has nonzero values only in a sufficiently small area. The other important difference between the papers cited in Ref.1 and our work is that we apply BS approach instead of using the well-known Schrödinger equation for magnetoexcitons. In view of the fact that usually the excitons in the presence of a magnetic field are described by solving the corresponding Schrödinger equation, there may be a need to clarify the motivations for our approach. First, the great progress in possibilities of modern experimental technique in obtaining extremely high magnetic fields requires more accurate calculations and obviously, one possible way to improve the two-particle bound-state-energy calculations is to use the powerful methods, developing in the framework of quantum electrodynamics (QED)[2–4] Second, a phenomenon, known as magnetic catalysis, which cannot be described by using the Schrödinger-equation-based approach, has been predicted in field theories such as QED and quantum chromodynamics.[5] The magnetic catalysis manifests itself as a generation of an energy gap in the spectrum of fermions in an external constant magnetic field for any arbitrary weak attractive interaction between fermions. Quite recently, the nonrelativistic BS equation for two-particle electron-hole bound states in semiconductor quantum wells in the presence of a homogeneous magnetic field was solved, and it was shown that the phenomenon of a magnetic catalysis also takes place in exciton physics: the homogeneous magnetic field induces an energy gap in the exciton spectrum.[6] Third, calculated from the Schrödinger equation in-plane effective exciton mass in SQW or coupled quantum wells (CQW) does not depend on the electron and hole effective masses. For example, let us consider the case of CQW with finite QW widths $L_{c,v}$. According to the Ref. 7, in strong magnetic fields the effect of finite widths is wery weak. Thus, one can use the expression for the in-plane exciton mass, derived for pure two-dimensional magnetoexcitons:[8]

$$M_{eff}(B)=\frac{M_B}{\left(1+\frac{d^2}{l^2}\right)\exp\left(\frac{d^2}{2l^2}\right)\mathrm{erfc}\left(\frac{d}{\sqrt{2}l}\right)-\sqrt{\frac{2}{\pi}}\frac{d}{l}},$$

where d is a distance between the centers of the two layers, $M_B=2^{2/3}\epsilon_0\hbar^2/\sqrt{\pi}e^2l$, and $l=\sqrt{c\hbar/eB}$ is the magnetic length. In a magnetic field of 4 T, the calculated in-plane mass in the case of CQW with $\epsilon_0=12.5$ and $d=11.5$ nm is $M_{eff}(4\ \mathrm{T})=0.29m_0$, while the measured value in GaAs/$Al_{0.33}Ga_{0.67}$As CQW is $M_{eff}=0.58m_0$.[7] This discrepancy definitely indicates that the Schrödinger approach cannot provide the correct value of the in-plane exciton mass. In contrast, according to the BS approach the excitons in SQW acquire a finite in-plane mass not only due to the Coulomb interaction, but also due to an additional effective interaction V_{eff} as well (see Ref. 6).

The layout of the paper is as follows. In the following section, we derive the BS equation for magnetoexcitons in quantum wells in the presence of a strong external homogeneous magnetic field and a weak cylindrical symmetric mag-

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netic field, created by a magnetized disk on top of a quantum well. In Sec. III, some numerical results for the trapping energy and the corresponding center-of-mass wave function of magnetoexcitons in a GaAs/Al_xGa_{1-x}As single quantum well are presented.

II. BETHE-SALPETER EQUATION FOR EXCITONS IN A QUANTUM WELL IN A NONHOMOGENEOUS MAGNETIC FIELD

We consider the electron-hole bound states in the presence of an inhomogeneous magnetic field in a quantum well of width L made with direct-gap semiconductor with nondegenerate and isotropic bands $E_c(\mathbf{k})=E_g+\mathbf{k}^2/2m_c$ and $E_v(\mathbf{k})=\mathbf{k}^2/2m_v$ [$E_c(\mathbf{k})$ and $E_v(\mathbf{k})$ are the dispersion laws for electrons and holes, respectively, E_g is the semiconductor band gap, m_c (m_v) is the electron (hole) effective mass and we set $\hbar=1$ throughout this paper]. According to the BS theory[2–4] the electron-hole bound states are described by the BS wave function $\Psi(\mathbf{r}_c,z_c,t_1;\mathbf{r}_v,z_v,t_2)$. This function determines the probability amplitude to find the electron at the point $(\mathbf{r}_c,z_c)$ at the moment t_1 and the hole at the point $(\mathbf{r}_v,z_v)$ at the moment t_2. The x-y plane has been taken to be the plane of confinement, $\mathbf{r}_c$ and $\mathbf{r}_v$ are the two-dimensional (2D) electron and hole radius, z_c and z_v are the corresponding z coordinates. In what follows, we will neglect the Elliott exchange interaction. In this approximation, the electron-hole bound states are known as mechanical excitons. The BS wave function of mechanical excitons $\Psi(\mathbf{r}_1,z_c,t_1;\mathbf{r}_2,z_v,t_2)$ satisfies the two-particle BS equation

$$\left\{\imath\frac{\partial}{\partial t_1}-E_g-\frac{1}{2m_c}\left[-\imath\boldsymbol{\nabla}_{\mathbf{r}_c}+\frac{e}{c}\mathbf{A}(\mathbf{r}_c)\right]^2-\frac{1}{2m_{cz}}\frac{\partial^2}{\partial z_c^2}-U_c(z_c)\right\}\left\{\imath\frac{\partial}{\partial t_2}-\frac{1}{2m_v}\left[-\imath\nabla_{\mathbf{r}_v}-\frac{e}{c}\mathbf{A}(\mathbf{r}_v)\right]^2-\frac{1}{2m_{vz}}\frac{\partial^2}{\partial z_v^2}-U_v(z_v)\right\}\Psi(\mathbf{r}_c,z_c,t_1;\mathbf{r}_v,z_v,t_2)$$
$$=-\imath I_C(\mathbf{r}_c,z_c,t_1;\mathbf{r}_v,z_v,t_2)\Psi(\mathbf{r}_c,z_c,t_1;\mathbf{r}_v,z_v,t_2). \tag{1}$$

Here, $U_{c,v}(z)$ denotes the corresponding confinement potential, $m_c(m_v)$ and $m_{cz}(m_{vz})$ are the electron (hole) in plane and z-axis effective masses, respectively. The irreducible kernel I_C represents the Coulomb attraction between electrons and holes that constitute the excitons. The vector potential of the magnetic field is denoted by $\mathbf{A}(\mathbf{r})$. The magnetic field is $\mathbf{B}(\mathbf{r})=\mathrm{rot}\mathbf{A}(\mathbf{r})$ and we have chosen the gauge such that $\mathrm{div}\mathbf{A}(\mathbf{r})=0$. In what follows, we will assume that the electron and hole motions along z direction are quantized into discrete levels due to the presence of confinement potentials. In our calculations, we take into account only the first electron E_{0c} and hole E_{0v} confinement levels.

In what follow, we will use 2D relative $\mathbf{r}=\mathbf{r}_c-\mathbf{r}_v$ and center of mass $\mathbf{R}=\alpha_c\mathbf{r}_c+\alpha_v\mathbf{r}_v$ coordinates. Here, $\alpha_{c,v}=m_{c,v}/M$ and $M=m_c+m_v$ is the exciton in-plane mass. To simplify the above BS equation, we assume the so-called adiabatic approximation, which is correct in the case when the exciton radius is smaller than the length scale over which the magnetic field varies. In the adiabatic approximation the vector potential can be expanded into $\mathbf{r}$ power series:

$$\mathbf{A}(\mathbf{r}_{c,v})=\mathbf{A}(\mathbf{R}\pm\alpha_{c,v}\mathbf{r})\approx\mathbf{A}(\mathbf{r})\pm\alpha_{c,v}\mathbf{r}\cdot[\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})]+\cdots.$$

In the adiabatic approximation the solution of the BS equation is found by separation of variables:

$$\Psi(\mathbf{r}_c,z_c,t_1;\mathbf{r}_v,z_v,t_2)=\exp\left\{-\imath\left[\frac{e}{c}\mathbf{r}\cdot\mathbf{A}(\mathbf{R})+E(\alpha_c t_1+\alpha_v t_2)\right]\right\}\times\Phi(\mathbf{R},\mathbf{r};t_1-t_2)\varphi_0(z_c)\phi_0(z_v). \tag{2}$$

Here E is the energy of the magnetoexcitons, and $\varphi_0(z_c)$ and $\phi_0(z_v)$ are the wave functions corresponding to the first confinement electron and hole levels. After some tedious but straightforward calculations, which are almost the same as in Ref. 6, we find the following effective interaction for the BS wave function:

$$\hat{H}\Phi(\mathbf{R},\mathbf{r})+\int d^2\mathbf{r}'\int d^2\mathbf{R}' I_{eff}(\mathbf{R},\mathbf{r};\mathbf{R}',\mathbf{r}';E)\Phi(\mathbf{R}',\mathbf{r}')=(E-E_{0c}-E_{ov}-E_g)\Phi(\mathbf{R},\mathbf{r}). \tag{3}$$

The Hamilton $\hat{H}$ is defined as follows:

$$\hat{H}=-\frac{1}{2M}\nabla^2_{\mathbf{R}}-\frac{1}{2\mu}\nabla^2_{\mathbf{r}}-\frac{2\pi e^2}{\epsilon_0}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(|\mathbf{q}|)}{|\mathbf{q}|}\exp[\imath\mathbf{q}\cdot\mathbf{r}]+\frac{\imath e}{Mc}[\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}\cdot\mathbf{r}-\mathbf{r}\cdot\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}]\cdot\boldsymbol{\nabla}_{\mathbf{R}}-\frac{\imath e\gamma}{\mu c}[\mathbf{r}\cdot\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}]\cdot\boldsymbol{\nabla}_{\mathbf{r}}+\frac{\imath e}{2Mc}\triangle_{\mathbf{R}}[\mathbf{A}(\mathbf{R})\cdot\mathbf{r}]+\frac{e^2}{2Mc^2}[\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}\cdot\mathbf{r}-\mathbf{r}\cdot\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}]^2+\frac{e^2\gamma^2}{2\mu c^2}[\mathbf{r}\cdot\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}]^2, \tag{4}$$

where $\gamma=(m_v-m_c)/M$ and $f(|\mathbf{q}|)$ is the well-known structure factor. The effective potential I_{eff} has the form

$$I_{eff}(\mathbf{R},\mathbf{r};\mathbf{R}',\mathbf{r}';E)=\int\frac{d^2\mathbf{p}}{(2\pi)^2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\int\frac{d^2\mathbf{P}}{(2\pi)^2}\times\int\frac{d^2\mathbf{Q}}{(2\pi)^2}\exp\{\imath[\mathbf{P}\cdot\mathbf{R}-\mathbf{Q}\cdot\mathbf{R}'+\mathbf{p}\cdot\mathbf{r}-\mathbf{q}\cdot\mathbf{r}']\}I_{eff}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q};E), \tag{5}$$

where

$$I_{eff}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q};E)=\frac{[E_c(\mathbf{p}+\alpha_c\mathbf{P})-E_c(\mathbf{q}+\alpha_c\mathbf{Q})]Z_v^{\mathbf{A}}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q})-Z_{cv}^{\mathbf{A}}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q})}{E-E_{0c}-E_{0v}-E_c(\mathbf{p}+\alpha_c\mathbf{P})-E_v(\mathbf{q}-\alpha_v\mathbf{Q})+\imath 0^+}$$
$$+\frac{[E_v(\mathbf{p}-\alpha_v\mathbf{P})-E_v(\mathbf{q}-\alpha_v\mathbf{Q})]Z_c^{\mathbf{A}}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q})-Z_{cv}^{\mathbf{A}}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q})}{E-E_{0c}-E_{0v}-E_c(\mathbf{q}+\alpha_c\mathbf{Q})-E_v(\mathbf{p}-\alpha_v\mathbf{P})+\imath 0^+}. \quad (6)$$

where the following notations have been used:

$$Z_{c,v}^{\mathbf{A}}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q})=\int d^2\mathbf{r}\int d^2\mathbf{R}\exp\{\imath[(\mathbf{Q}-\mathbf{P})\cdot\mathbf{R}+(\mathbf{q}-\mathbf{p})\cdot\mathbf{r}]\}Z_{c,v}(\mathbf{Q},\mathbf{q},\mathbf{r},\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})),$$

$$Z_{cv}^{\mathbf{A}}(\mathbf{P},\mathbf{p};\mathbf{Q},\mathbf{q})=\int d^2\mathbf{r}\int d^2\mathbf{R}\exp\{\imath[(\mathbf{Q}-\mathbf{P})\cdot\mathbf{R}+(\mathbf{q}-\mathbf{p})\cdot\mathbf{r}]\}Z_{cv}(\mathbf{Q},\mathbf{q},\mathbf{r},\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})),$$

$$Z_c(\mathbf{Q},\mathbf{q},\mathbf{r},\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R}))=\frac{e}{Mc}\{\alpha_v\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]-\alpha_c[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\}\cdot\mathbf{Q}+\frac{e}{Mc}\left\{\frac{\alpha_v}{\alpha_c}\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]-[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\right\}\cdot\mathbf{q}$$
$$+\frac{e^2}{2Mc^2\alpha_c}\{\alpha_c[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}-\alpha_v\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\}^2,$$

$$Z_v(\mathbf{Q},\mathbf{q},\mathbf{r},\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R}))=\frac{e}{Mc}\{\alpha_c\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]-\alpha_v[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\}\cdot\mathbf{Q}+\frac{e}{Mc}\left\{\frac{\alpha_c}{\alpha_v}\mathbf{r}\cdot[\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})]-[\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\right\}\cdot\mathbf{q}$$
$$-\frac{e^2}{2Mc^2\alpha_v}\{\alpha_v[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}-\alpha_c\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\}^2.$$

In the gauge div$\mathbf{A}(\mathbf{R})=0$ the following relation takes place $[(\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R}))\cdot\mathbf{r}-\mathbf{r}\cdot(\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R}))]=\mathbf{r}\times\mathbf{B}(\mathbf{R})$. We can simplify Eq. (3) by applying the following transformation:

$$\Phi(\mathbf{R},\mathbf{r})\rightarrow\exp\left\{-\imath\frac{\gamma e}{2c}\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\right\}\Phi(\mathbf{R},\mathbf{r}).$$

This transformation leads to the appearance of r^{-3} terms. They should be neglected in the adiabatic approximation. After this transformation the BS equation (3) assumes the form

$$\hat{\mathrm{H}}\Phi(\mathbf{R},\mathbf{r})+\int d^2\mathbf{r}'\int d^2\mathbf{R}'\exp\left\{\imath\frac{\gamma e}{2c}\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\right\}$$
$$\times I_{eff}(\mathbf{R},\mathbf{r};\mathbf{R}',\mathbf{r}';E)$$
$$\times\exp\left\{-\imath\frac{\gamma e}{2c}\mathbf{r}'\cdot[\boldsymbol{\nabla}_{\mathbf{R}'}\mathbf{A}(\mathbf{R}')]\cdot\mathbf{r}'\right\}\Phi(\mathbf{R}',\mathbf{r}')$$
$$=(E-E_{0c}-E_{ov}-E_g)\Phi(\mathbf{R},\mathbf{r}), \quad (7)$$

where

$$\hat{H}=-\frac{1}{2M}\nabla_{\mathbf{R}}^2-\frac{1}{2\mu}\nabla_{\mathbf{r}}^2-\frac{2\pi e^2}{\epsilon_0}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(|\mathbf{q}|)}{|\mathbf{q}|}\exp[\imath\mathbf{q}\cdot\mathbf{r}]$$
$$+\frac{\imath e}{Mc}\mathbf{r}\cdot[\mathbf{B}(\mathbf{R})\times\boldsymbol{\nabla}_{\mathbf{R}}]-\frac{\imath e}{2Mc}\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\times\mathbf{B}(\mathbf{R})]$$
$$-\frac{\imath e\gamma}{2\mu c}\mathbf{B}(\mathbf{R})\cdot(\mathbf{r}\times\boldsymbol{\nabla}_{\mathbf{r}})-\frac{\imath e\gamma}{4\mu c}[\{\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}\cdot\mathbf{r}$$
$$+\mathbf{r}\cdot\{(\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})\}]+\frac{e^2}{8\mu c^2}[\mathbf{r}\times\mathbf{B}(\mathbf{R})]^2. \quad (8)$$

The Eq. (7) is our BS equation for the exciton wave function. In comparison to the well-known Schrodinger equation used in Ref. 1, the above BS equation contains additionally: (i) an effective nonlocal interaction I_{eff}, which depends on the exciton energy; (ii) an additional term $-\imath e\gamma/(4\mu c)[(\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R}))\cdot\mathbf{r}+\mathbf{r}\cdot(\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R}))]$. In the case of a constant magnetic field the last term is equal to zero, but it should be taken into account, when the inhomogeneous magnetic field is applied.

Let us first consider the case of a homogeneous magnetic field $\mathbf{B}_0=(0,0,B_0)$. The vector potential in Cartesian coordinates $\mathbf{R}=(X,Y,0)$ in the gauge div$\mathbf{A}(\mathbf{R})=0$ is given by $\mathbf{A}(\mathbf{R})=\{-1/2B_0Y,1/2B_0X,0\}$ and the relation $\mathbf{r}\cdot[\boldsymbol{\nabla}_{\mathbf{R}}\mathbf{A}(\mathbf{R})]=(1/2)\mathbf{B}_0\times\mathbf{r}$ takes place. In the case of a homogeneous magnetic field the exciton center of mass moves through the crystal with a constant momentum $\mathbf{Q}$ and the wave function $\Phi(\mathbf{R},\mathbf{r};t_1-t_2)$ can be written as $\Phi(\mathbf{R},\mathbf{r};t_1-t_2) = \exp(\imath\mathbf{Q}\cdot\mathbf{R})\Phi_{\mathbf{Q}}(\mathbf{r};t_1-t_2)$. Thus, the BS equation for the exciton wave function $\Phi_{\mathbf{Q}}(\mathbf{r};t_1-t_2=0)=\Phi_{\mathbf{Q}}(\mathbf{r})$ assumes the form

$$\left\{\frac{\mathbf{Q}^2}{2M}-\frac{1}{2\mu}\nabla_{\mathbf{r}}^2-\frac{2\pi e^2}{\epsilon_0}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{f(|\mathbf{q}|)}{|\mathbf{q}|}\exp[\imath\mathbf{q}\cdot\mathbf{r}]+\frac{e}{Mc}(\mathbf{B}_0\times\mathbf{r})\cdot\mathbf{Q}-\frac{\imath e\gamma}{2\mu c}(\mathbf{B}_0\times\mathbf{r})\cdot\nabla_{\mathbf{r}}+\frac{e^2}{8\mu c^2}[\mathbf{r}\times\mathbf{B}_0]^2\right\}\Phi_{\mathbf{Q}}(\mathbf{r})+\int d^2\mathbf{r}' V_{eff}(\mathbf{r};\mathbf{r}';\mathbf{Q};E(\mathbf{Q}))\Phi_{\mathbf{Q}}(\mathbf{r}') = [E(\mathbf{Q})-E_g-E_{0c}-E_{0v}]\Phi_{\mathbf{Q}}(\mathbf{r}), \tag{9}$$

where the exact form of the effective potential $V_{eff}(\mathbf{r};\mathbf{r}';\mathbf{Q};E)$ is derived in Ref. 6.

From now on we limit ourselves to the case of an inhomogeneous cylindrical symmetric magnetic field. In this case, it is more convenient to use cylindrical coordinates $\mathbf{B}=(X,Y,0)=(R\cos\varphi,R\sin\varphi,0)$ and $\mathbf{r}=(x,y,0)=(r\cos\theta,r\sin\theta,0)$. We assume that the cylindrical symmetric magnetic field is along the z direction and can be written as a sum of a homogeneous magnetic field $\mathbf{B}_0=(0,0,B_0)$ and a cylinder symmetric part $\mathbf{B}_1(R)=(0,0,B_1(R))$. The total magnetic field is $\mathbf{B}(R)=\mathbf{B}_0+\mathbf{B}_1(R)=(0,0,B(R))$, where $B(R)=B_0+B_1(R)$ and $R^2=\sqrt{X^2+Y^2}$. In the case under consideration the vector potential is given by

$$\mathbf{A}(\mathbf{R})=\{-B_0Y/2-A(R)Y/R,B_0X/2-A(R)X/R,0)\},$$

where $A(R)$ is a function of the cylindrical coordinate R. From the relation $\mathbf{B}=\mathrm{rot}\mathbf{A}(\mathbf{R})$, we find $B_1(R)=dA/dR+A(R)/R$.

It can be proved that the following relation, $\mathbf{r}\cdot[\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})]=(1/2)\mathbf{B}_0\times\mathbf{r}+\mathbf{F}$, takes place, where the components of the vector $\mathbf{F}=(F_1,F_2,0)$ are

$$F_1=-r\sin\varphi\cos(\theta-\varphi)\frac{dA(R)}{dR}-r\cos\varphi\sin(\theta-\varphi)\frac{A(R)}{R},$$

$$F_2=r\cos\varphi\cos(\theta-\varphi)\frac{dA(R)}{dR}-r\sin\varphi\sin(\theta-\varphi)\frac{A(R)}{R}.$$

In cylindrical coordinates the Hamiltonian (8) assumes the form

$$\hat{H}=\hat{H}_1+\hat{H}_2+\hat{H}_3,$$

$$\hat{H}_1=-\frac{1}{2M}\left\{\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right\},$$

$$\hat{H}_2=-\frac{1}{2\mu}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right\}-\frac{2\pi e^2}{\epsilon_0}\int dq f(|\mathbf{q}|)J_0(qr)-\frac{\imath e\gamma B(R)}{2\mu c}\frac{\partial}{\partial\theta}+\frac{e^2B^2(R)}{8\mu c^2}r^2,$$

$$\hat{H}_3=\frac{\imath eB(R)r}{Mc}\left\{\sin(\theta-\varphi)\frac{\partial}{\partial R}-\frac{1}{R}\cos(\theta-\varphi)\frac{\partial}{\partial\varphi}\right\}+\frac{\imath er}{2Mc}\sin(\theta-\varphi)\frac{dB(R)}{dR}+\frac{\imath e\gamma}{2\mu c}\left\{\frac{A(R)}{R}-\frac{dA(R)}{dR}\right\}\times\left\{\sin[2(\theta-\varphi)]r\frac{\partial}{\partial r}+\cos[2(\theta-\varphi)]\frac{\partial}{\partial\theta}\right\}, \tag{10}$$

where $J_0(qr)$ is the Bessel function.

To simplify the calculations we will assume: (i) the homogeneous magnetic field $\mathbf{B}_0$ is a strong magnetic field and (ii) $|\mathbf{B}_0|\gg|\mathbf{B}_1(R)|$. Those assumptions allow us to apply the LLL approximation and to treat the Coulomb interaction by perturbation theory. By replacing $B(R)\approx B_0$ in the term

$$\int d^2\mathbf{r}'\int d^2\mathbf{R}'\exp\left\{\imath\frac{\gamma e}{2c}\mathbf{r}\cdot[\nabla_{\mathbf{R}}\mathbf{A}(\mathbf{R})]\cdot\mathbf{r}\right\}\times I_{eff}(\mathbf{R},\mathbf{r};\mathbf{R}',\mathbf{r}';E)\times\exp\left\{-\imath\frac{\gamma e}{2c}\mathbf{r}'\cdot[\nabla_{\mathbf{R}'}\mathbf{A}(\mathbf{R}')]\cdot\mathbf{r}'\right\}\Phi(\mathbf{R}',\mathbf{r}'),$$

we obtain the following approximation for it:

$$\int d^2\mathbf{r}'\int d^2\mathbf{R}'\int\frac{d^2\mathbf{K}}{(2\pi)^2}\exp\{\imath\mathbf{K}\cdot(\mathbf{R}-\mathbf{R}')\}\times V_{eff}(\mathbf{r},\mathbf{r}';\mathbf{K};E)\Phi(\mathbf{R}',\mathbf{r}'),$$

We are looking for a solution of the BS equation (7) in the form:

$$\Phi_{snm}(\mathbf{R},\mathbf{r})=\Psi(R)\exp(-\imath s\varphi)\phi_{nm}(r,\theta;R),$$

$$\phi_{nm}(r,\theta;R)=\exp(\imath m\theta)\phi_{nm}(r;R),$$

$$\phi_{nm}(r;R)=\sqrt{\frac{n!}{2^{|m|+1}\pi(n+|m|)!}}\left[\frac{r}{l(R)}\right]^{|m|}L_n^{|m|}\left(\frac{r^2}{2l^2(R)}\right)\times\exp\left\{-\frac{r^2}{4l^2(R)}\right\},$$

where $s=0,\pm1,\pm2,\ldots$, $n=0,1,\ldots$, $m=0,\pm1,\pm2,\ldots$, $l(R)=\sqrt{c/eB(R)}$ is the magnetic length, and $L_n^{|m|}(x)$ are the Laguerre polynomials. The functions $\phi_{nm}(r,\theta;R)$ are the wave functions of the Hamiltonian of the 2D electron-hole pair in a perpendicular magnetic field $\mathbf{B}(R)$:

$$H_{2D}=-\frac{1}{2\mu}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right\}-\frac{\imath e\gamma B(R)}{2\mu c}\frac{\partial}{\partial\theta}+\frac{e^2B^2(R)}{8\mu c^2}r^2.$$

The corresponding energy is $W_{nm}(R)=\Omega(R)\{n+\frac{1}{2}[|m|+\gamma m+1]\}$, where $\Omega(R)=(eB(R)/\mu c)$ denotes the distance between Landau levels. In the LLL approximation $s=n=m=0$ and the wave function of the ground state is given by

$$\Phi_{s=0n=0m=0}(\mathbf{R},\mathbf{r})=\Psi(R)\frac{1}{\sqrt{2\pi}}\frac{1}{l(R)}\exp\left\{-\frac{r^2}{4l^2(R)}\right\}.$$

In this approximation the center-of-mass motion wave function $\Psi(R)$ satisfies the Schrödinger equation

$$-\frac{1}{2M}\left\{\frac{d^2}{dR^2}+\frac{1}{R}\frac{d}{dR}\right\}\Psi(R)+V(R)\Psi(R)=(E-E^0_{exc})\Psi(R), \tag{11}$$

where E^0_{exc} is the exciton energy in a homogeneous magnetic field $\mathbf{B}_0=(0,0,B_0)$. According to the results, obtained in Ref. 6, the exciton energy can be written in the form

$$E^0_{exc}=E_g+E_{0c}+E_{0v}+\Omega_0\left[\frac{1}{2}+\Delta\right]-\delta E_C,$$

where $\Omega_0=(eB_0/\mu c)$ is the exciton cyclotron energy in a homogeneous magnetic field $\mathbf{B}_0$. Δ is the energy gap generated by the homogeneous magnetic field and δE_C is the first-order correction in Coulomb interaction. The effective potential of the center-of-mass motion of the exciton $V(R)$ to the first-order in the inhomogeneous magnetic field $B_1(R)/B_0$ is given by

$$\frac{V(R)}{\Omega_0}=\frac{B_1(R)}{B_0}\left\{\frac{1}{2}-\frac{l_0}{2a_0}\int_0^{+\infty}dy f\left(|\mathbf{q}|\to y\frac{L}{l_0}\right)\times y^2\exp\left(-\frac{y^2}{2}\right)\right\}, \tag{12}$$

where $l_0=\sqrt{c/eB_0}$ is the magnetic length in a homogeneous magnetic field $\mathbf{B}_0$ and $a_0=\epsilon_0/\mu e^2$ is the effective Bohr radius. We will use the magnetic length l_0 for the unit length and the exciton cyclotron energy Ω_0 for the energy unit. The exciton trapping energy $\varepsilon=(E-E^{(0)}_{exc})/\Omega_0$ is defined as the difference between the exciton energy E/Ω_0 in the inhomogeneous magnetic field $\mathbf{B}(R)=\mathbf{B}_0+\mathbf{B}_1$ and the exciton energy $E^{(0)}_{exc}$ in the homogeneous applied field $\mathbf{B}_0$ for the same exciton state.

In what follows, we will consider the case when a magnetized disk of radius a on the top of the quantum well creates an inhomogeneous magnetic field $\mathbf{B}_1$. The distance of the magnetized disk to the x-y plane is denoted by d. In this case, the magnetic field $B_1(R)$ is given by the following equation:[1]

$$B_1(R)=4B_1\frac{(a+R)a}{(a+R)^2+d^2}\sqrt{\frac{a}{R}}\frac{1}{p}\left[-E_1(p^2)+\left(1-\frac{p^2}{2}\right)K(p^2)\right]+\frac{(a^2-R^2+d^2)a}{R^2\sqrt{Ra}}\times p^3\left[-\frac{dE_1(p^2)}{dp^2}-\frac{1}{2}K(p^2)+\left(1-\frac{p^2}{2}\right)\frac{dK(p^2)}{dp^2}\right],$$

where $p=2\sqrt{aR/[(a+R)^2+d^2]}$, $B_1=hMa$, h is the disk thickness and M is the magnetization of the disk. $K(x)$ and $E(x)$ are the elliptic integrals of first and second type, respectively.

III. NUMERICAL RESULTS FOR EXCITONS IN A GaAs/AlGaAs QUANTUM WELL IN A NONHOMOGENEOUS MAGNETIC FIELD

Let us first discuss the exciton energy $E^0_{exc}=E_g+E_{0c}+E_{0v}+\Omega_0[1/2+\Delta]-\delta E_C$ in a homogeneous magnetic field $\mathbf{B}_0=(0,0,B_0)$. In Table I of Ref. 6 the calculated values of E_{0c}, E_{0v}, δE_C, and $\Omega_0[1/2+\Delta]$ in the case of a single GaAs quantum-well sandwiched between two $Al_xGa_{1-x}As$ layers for various well widths L and magnetic fields $\mathbf{B}_0=(0,0,B_0)$ are presented. The corresponding calculations were done assuming the following parameters: $\epsilon_0=12.5$, $m_c=m_{cz}=0.067m_0$, $E_g=1.52$ eV at $T=0$ K and E_g

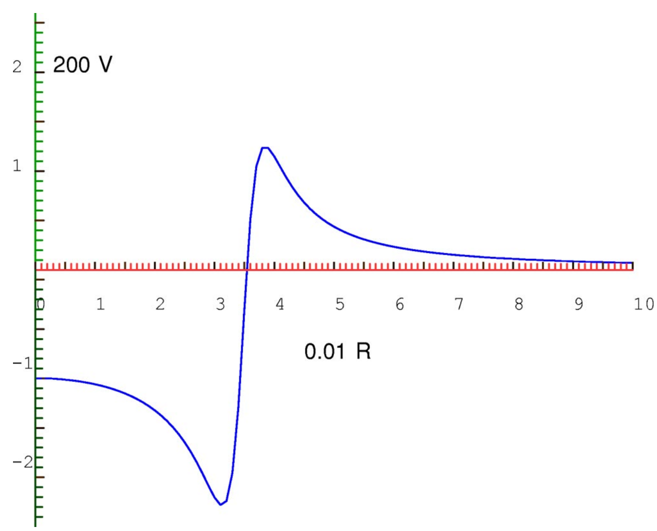

FIG. 1. The effective potential of the center-of-mass motion of the heavy-hole exciton $V(R)$ for the $GaAs/Al_{0.3}Ga_{0.7}As$ single quantum well ($m_c=0.067m_0$, $\gamma_1=7.36$, and $\gamma_2=2.57$) of width $L=4.03$ nm as a function of R. The homogeneous magnetic field is $B_0=20$ T and the magnetized disk parameters are as follows: $a=2$ μm, $d=0.2$ μm, $B_1=-0.05$ T. We use the magnetic length $l_0=0.005\,736$ μm for the unit length and the exciton cyclotron energy $\Omega_0=0.05756$ eV for the energy unit.

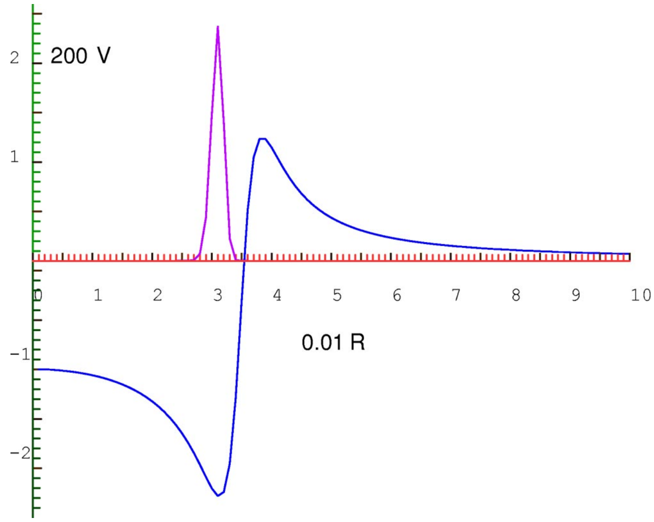

FIG. 2. The potential $V(R)$ and the numerical calculated function $\Psi(R)$ for heavy-hole excitons in GaAs/$Al_{0.3}Ga_{0.7}$As single quantum well ($m_c = 0.067 m_0$, $\gamma_1 = 7.36$, and $\gamma_2 = 2.57$) of width $L = 4.03$ nm as a function of R. The homogeneous magnetic field is $B_0 = 20$ T and the magnetized disk parameters are as follows: $a = 2$ μm, $d = 0.2$ μm, $B_1 = -0.05$ T. We use the magnetic length $l_0 = 0.005\,736$ μm for the unit length and the exciton cyclotron energy $\Omega_0 = 0.057\,56$ eV for the energy unit.

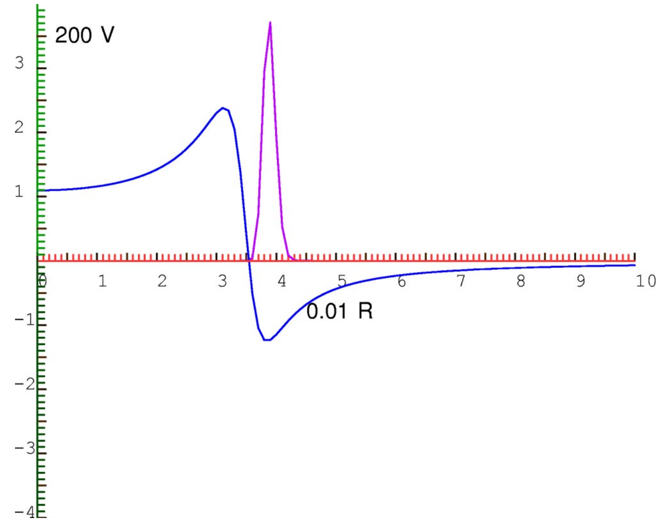

FIG. 3. The potential $V(R)$ and the numerical calculated function $\Psi(R)$ for heavy-hole excitons in GaAs/$Al_{0.3}Ga_{0.7}$As single quantum well ($m_c = 0.067 m_0$, $\gamma_1 = 7.36$ and $\gamma_2 = 2.57$) of width $L = 4.03$ nm as a function of R. The homogeneous magnetic field is $B_0 = 20$ T and the magnetized disk parameters are as follows: $a = 2$ μm, $d = 0.2$ μm, $B_1 = 0.05$ T. We use the magnetic length $l_0 = 0.005\,736$ μm for the unit length and the exciton cyclotron energy $\Omega_0 = 0.057\,56$ eV for the energy unit.

$= 1.512$ eV at $T = 55$ K. Two different groups of Luttinger parameters were used: (1) $\gamma_1 = 6.9$ and $\gamma_2 = 2.7$ (Ref. 9) and (2) $\gamma_1 = 6.9$ and $\gamma_2 = 2.7$.[10] The parameters characterizing the magnetized disk are chosen to be $B_1 = -0.05$ T, $a = 2$ μm and $d = 0.2$ μm. The homogeneous magnetic field is assumed to be $B_0 = 20$ T. For the GaAs/$Al_{0.3}Ga_{0.7}$As single quantum well of width $L = 4.03$ nm the calculated exciton energy is $E^0_{exc} = 1.646$ eV ($\gamma_1 = 6.9$ and $\gamma_2 = 2.7$). The effective potential of the center-of-mass motion of the exciton $V(R)$ defined by Eq. (12) is plotted in Fig. 1.

Equation (11) was solved numerically using a shooting method described by Pruess and Fulton.[11] The approach begins by explicitly writing the eigenvalue problem in Sturm-Liouville form $-(d/dx)\{p(x)d\psi/dx\} + q(x)\psi(x) = Ew(x)\psi(x)$. Then one defines an initial mesh on $[0,\infty)$ comprised of N subintervals $0 = x_1 < x_2 < \cdots < x_N < \infty$, and the coeffcient functions $p(x), q(x)$, and $w(x)$ are approximated by piecewise constant functions on each subinterval. This ‘‘piecewise-constant’’ approximation to the Sturm-Liouville equation can be solved exactly, which leads to an algorithm for propagating ψ_n outward from the origin. For a fixed mesh, we iterate on the approximate eigenvalue until the boundary condition at ∞ is satisfied.[12] This algorithm gives a ground state energy eigenvalue of $E_{\text{ground}} = -4.00\,001$ for the two-dimensional Coulomb problem, with potential $V(x) = -2/x$ and other parameters set to unity. The analytical value for the ground-state energy eigenvalue is -4. In these calculations, parameters were adjusted until this algorithm yielded the same energy eigenvalue to four significant places for both the radial form of the Schrödinger equation, and its pseudo-Cartesian form obtained from the substitution $\psi(r) \to \phi(r)/\sqrt{(r)}$.

We have applied the above-described numerical method to the case when the homogeneous and the inhomogeneous magnetic fields point in different directions, i.e., $B_1 = -0.05$ T, and we have found that the exciton trapping energy is $\varepsilon = -0.0113$. The potential $V(R)$ and the numerically calculated function $\Psi(R)$ are shown in Fig. 2. In the case when the homogeneous and the inhomogeneous magnetic fields point in the same direction, i.e., $B_1 = 0.05$ T, the potential $V(R)$ and the numerically calculated function $\Psi(R)$ are shown in Fig. 3. In this case the exciton trapping energy is calculated to be $\varepsilon = -0.0057$. Notice that for both cases, the trapping energies are negative, so we have exciton bound states. In the case of a weak constant magnetic field $\mathbf{B}_0$, discussed in Ref. 1, the exciton trapping energy was found to be positive (i.e., unbound states). As a result in the weak-field regime case, a large part of the center-of-mass exciton wave function is extended into a large area. In the high-field regime case, we have nonzero values for the center-of-mass exciton wave function only in a small area (1.377 $\mu\text{m} \leqslant R \leqslant 1.721$ μm for $B_1 = -0.05$ T, and $2.008\mu\text{m} \leqslant R \leqslant 2.581$ μm for $B_1 = 0.05$ T).

IV. SUMMARY

We have applied the Bethe-Salpeter formalism for the problem of excitons in a single quantum well in the presence of an external strong constant magnetic field and a weak cylindrical symmetric magnetic field, created by a magnetized disk on top of the quantum well. We have predicted the formation of bound exciton states with nonzero values for the center-of-mass exciton wave function only in a sufficiently small area. This effect of exciton trapping can be used to design new functional nanoelectronic devices. A new device can be designed on the base of existence of a ground bound exciton state and a number of excited bound states. For example, we have calculated that in the case when $B_1 = -0.05$ T, the excited exciton energies are $\varepsilon_1 = -0.0102$, $\varepsilon_2 = -0.0093$, $\varepsilon_3 = -0.0085$. In the case when the homogeneous and inhomogeneous magnetic fields point in the same direction, i.e. $B_1 = 0.05$ T, the excited exciton energies are $\varepsilon_1 = -0.0046$, $\varepsilon_2 = -0.0038$, $\varepsilon_3 = -0.0031$.

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