Tunneling spectroscopy of spin-split states in quantum wells

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Tunneling between two-dimensional electron layers, where electron states are split by spin due to spin-orbit interaction, is studied theoretically. The expression for the tunneling current is derived and evaluated. The linear tunneling conductance shows two Lorentz-like peaks corresponding to the resonance contribution of two spin-split states. The current-voltage characteristics are essentially different from the case of the tunneling in the absence of spin splitting. They show peaks whose shape becomes almost rectangular in the limit of weak disorder. The position of these peaks is determined by the spin-splitting energy. The measurement of the tunneling current is suggested to be an efficient tool for direct investigation of the spin-split spectra in the quantum wells.

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I. INTRODUCTION

Spin splitting of the two-dimensional (2D) electron spectrum caused by the spin-orbit interaction in asymmetric quantum wells (Rashba effect[1]) has attracted substantial interest in past years. Apart from its significance from the point of view of fundamental physics, this effect is considered to be promising for application in spintronics,[2] since its magnitude can be modulated by the external gate modifying the confining potential. In this way, spin polarization of the electrons can be controlled. The spin splitting of 2D electron subbands is usually not large, of the order of 1 meV near the Fermi surface. Therefore, to study it, one should employ highly sensitive experimental techniques. Measurements of the beat patterns of Shubnikov–de Haas oscillations in the asymmetric quantum wells[3–8] have proven to be a reliable method for investigation of the Rashba effect.

In this paper we develop a theoretical background for the other method, which is supposed to give more direct and complementary information about the 2D subband spectrum, although it is more difficult to be realized experimentally. The idea of the method is to use the resonant tunneling between 2D electron states in the double quantum well systems with independent contacts to the 2D layers. First structures of this kind have been fabricated more than a decade ago.[9,10] Since then, the interlayer tunneling in such structures has been the subject of numerous experimental[9–15] and theoretical[13,16–22] studies. Measurements of the tunneling current in such structures[9–13] revealed peaks of the tunneling conductance in conditions in which the 2D levels in the two wells are aligned, reflecting the shape of the spectral functions within each layer, in accordance with theoretical predictions. A resonant tunneling transistor based on gate control of the tunneling current between 2D layers was demonstrated,[15] see Fig. 1. Investigation of the tunneling current in the magnetic field applied both parallel and perpendicular to the layers has shown a considerable modification of the electron energy spectrum by the magnetic field. Nevertheless, the experiments did not show any sign of spin splitting of the electron spectrum in zero magnetic field. In our opinion, the reason for this is that the existing double quantum well structures with independent contacts are based upon GaAs (AlGaAs) materials, in which spin-orbit interaction is not strong. As the technology advances to create independent contacts to InAs 2D layers, the observation of spin-splitting features should not be a problem. We consider this to be sufficient motivation for our study. The aim of our paper is to investigate the 2D-2D tunneling current in conditions in which the Rashba effect is not negligible. Below we show that the spin splitting leads to unusual dependence of the tunneling current on the electron densities in the layers and on the source-drain voltage.

The paper is organized as follows. In Sec. II we present a general formalism for a description of tunnel-coupled 2D electrons with a spin-split energy spectrum and derive an expression for the tunneling current. In Sec. III we calculate the linear tunneling conductance and study the tunneling current under a nonlinear regime, when the source-drain voltage is not small. Concluding remarks are given in Sec. IV.

II. FORMALISM

We consider a double-well system described by the Hamiltonian

$$\hat{H}=\begin{bmatrix} \hat{H}_0(\mathbf{x},z) & i\alpha(z)\hat{p}_- \\ -i\alpha(z)\hat{p}_+ & \hat{H}_0(\mathbf{x},z) \end{bmatrix}, \tag{1}$$

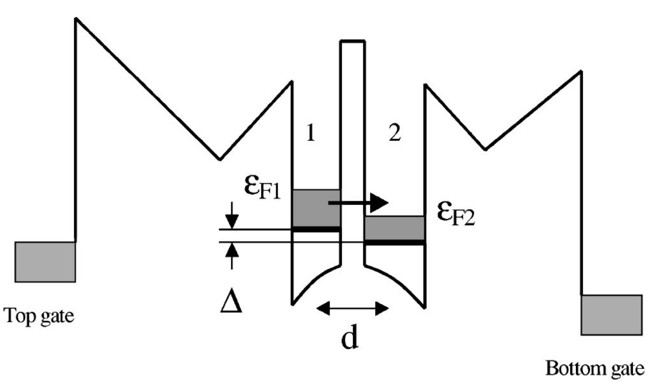

FIG. 1. Conduction-band profile of the structure containing a double quantum well with independent contacts to the 2D layers and two gates. An application of source-drain voltage leads to the tunneling current between the layers.

where $\hat{p}_{\pm}=\hat{p}_x \pm i\hat{p}_y$, and

$$\hat{H}_0(\mathbf{x},z)=\hat{p}_z \frac{1}{2m(z)}\hat{p}_z+\frac{\hat{p}^2}{2m(z)}+U(z)+V(\mathbf{x},z).$$

In these expressions, $\hat{p}_z$ and $\hat{\mathbf{p}}=(\hat{p}_x,\hat{p}_y)$ are the operators of momenta perpendicular and parallel to the quantum well planes, respectively, $\alpha(z)$ is a function which depends on spin-orbit interaction (see below), $U(z)$ is the double-well potential, and $V(\mathbf{x},z)$ is the random static potential created, for example, by impurities. The Hamiltonian (1) is a 2×2 matrix with respect to spin variables. It can be derived from the four-band Kane Hamiltonian[23] in a similar way as described in Ref. 24; such a consideration allows one to express $m(z)$ and $\alpha(z)$ through the material parameters of the structure. Using the basis of single-well orbitals $F_1(z)$ and $F_2(z)$, where 1 and 2 are the layer indices, we transform the Hamiltonian (1) to the following effective 2D Hamiltonian:

$$\hat{H}=\begin{pmatrix} \hat{h}_1 & \hat{1}t \\ \hat{1}t & \hat{h}_2 \end{pmatrix}. \tag{2}$$

This Hamiltonian is a 4×4 matrix composed from 2×2 unit matrix blocks proportional to the tunneling matrix element t and 2×2 matrix blocks ($j=1,2$),

$$\hat{h}_j=\begin{bmatrix} \varepsilon_j+\hat{p}^2/2m+V_j(\mathbf{x}) & i\alpha_j\hat{p}_- \\ -i\alpha_j\hat{p}_+ & \varepsilon_j+\hat{p}^2/2m+V_j(\mathbf{x}) \end{bmatrix} \tag{3}$$

describing 2D states in the quantum wells 1 and 2. In Eq. (3) we introduced the 2D scattering potentials $V_j(\mathbf{x})=\int dz V(\mathbf{x},z)|F_j(z)|^2$ and Rashba velocities $\alpha_j=\int dz\,\alpha(z)|F_j(z)|^2$. The Hamiltonian (3) is the Rashba Hamiltonian describing spin splitting of 2D states in the layer j. For a double-well system, the spin splitting in the different layers is different, because α_j essentially depends on the layer index. If the directions of the potential gradients in the wells 1 and 2 are opposite to each other, as in Fig. 1, then the signs of α_1 and α_2 are different.

A calculation of the tunneling current for the problem described by the Hamiltonian (2) is very similar to the case of a spin-independent Hamiltonian, see details in Refs. 18 and 22. In the lowest order on t, the current is represented through a correlator of spectral functions:

$$I=\frac{2\pi e t^2}{\hbar S}\int d\varepsilon (f_{1\varepsilon}-f_{2\varepsilon})\sum_{\mathbf{pp}'} \mathrm{Tr}\langle \hat{A}_{\varepsilon 1}(\mathbf{p},\mathbf{p}')\hat{A}_{\varepsilon 2}(\mathbf{p}',\mathbf{p})\rangle, \tag{4}$$

where e is the absolute value of the elementary charge, S is the normalization square, $f_{j\varepsilon}$ are quasiequilibrium distribution functions in the layers, $\langle \dots \rangle$ denotes the average over the random potential, Tr denotes the matrix trace, and the spectral function $\hat{A}_{\varepsilon j}=(\hat{G}^A_{\varepsilon j}-\hat{G}^R_{\varepsilon j})/2\pi i$ is expressed through the advanced (A) and retarded (R) Green's functions. The latter, in the momentum representation, satisfies the following matrix equation:

$$[\varepsilon+i0-\hat{h}^{(0)}_{j\mathbf{p}}]\hat{G}^R_{\varepsilon j}(\mathbf{p},\mathbf{p}') + \frac{1}{S}\sum_{\mathbf{p}_1} V_j(|\mathbf{p}-\mathbf{p}_1|/\hbar)\hat{G}^R_{\varepsilon j}(\mathbf{p}_1,\mathbf{p}')=\hat{1}\delta_{\mathbf{p},\mathbf{p}'}, \tag{5}$$

$$\hat{h}^{(0)}_{j\mathbf{p}}=\pm\Delta/2+p^2/2m+\alpha_j(\hat{\sigma}_x p_y-\hat{\sigma}_y p_x),$$

where $V_j(\mathbf{q})=\int d\mathbf{x} V_j(\mathbf{x})e^{-i\mathbf{q}\cdot\mathbf{x}}$ is the Fourier transform of the potential and $\hat{\sigma}_i$ are the Pauli matrices. In the definition of the matrices $\hat{h}^{(0)}_{j\mathbf{p}}$, we introduced the interlayer level splitting energy $\Delta=\varepsilon_1-\varepsilon_2$ at $p=0$ and assumed that the upper (lower) sign corresponds to $j=1$ ($j=2$). Below we assume that there is no correlation between the scattering potentials in wells 1 and 2, which is always the case if the potentials are of short range. The correlator in Eq. (4) is expressed through the product of averaged Green's functions $G^{R,A}_{\varepsilon j}(\mathbf{p})$, and the current is given by

$$I=\frac{e t^2}{2\pi\hbar S}\int d\varepsilon (f_{1\varepsilon}-f_{2\varepsilon}) \times\sum_{\mathbf{p}}\sum_{s,s'=R,A}(-1)^l \mathrm{Tr}\,\hat{G}^s_{\varepsilon 1}(\mathbf{p})\hat{G}^{s'}_{\varepsilon 2}(\mathbf{p}), \tag{6}$$

where $l=1$ for $s=s'$ and $l=0$ for $s\neq s'$.

The averaged Green's function is written through the self-energy function, according to $\hat{G}^s_{\varepsilon j}(\mathbf{p})=[\varepsilon-\hat{h}^{(0)}_{j\mathbf{p}}-\hat{\Sigma}^s_{\varepsilon j}(\mathbf{p})]^{-1}$. The self-energy, in the Born approximation, satisfies the following equation:

$$\hat{\Sigma}^s_{\varepsilon j}(\mathbf{p})=\frac{1}{S}\sum_{\mathbf{p}_1} w_j(|\mathbf{p}-\mathbf{p}_1|/\hbar)\hat{G}^s_{\varepsilon j}(\mathbf{p}_1), \tag{7}$$

where the binary correlation function of the scattering potential is defined by the relation $\langle V_j(\mathbf{q})V_j(\mathbf{q}')\rangle=\delta(\mathbf{q}+\mathbf{q}')w_j(q)$. Applying the approximation of short-range scattering potential, when $w_j(q)\simeq w_j$ is independent of q, we find that $\hat{\Sigma}^{A,R}_{\varepsilon j}(\mathbf{p})$ are diagonal and momentum independent: $\hat{\Sigma}^{A,R}_{\varepsilon j}(\mathbf{p})=[\mathrm{Re}\,\Sigma_{\varepsilon j}\pm i\hbar/2\tau_j]\hat{1}$. Below we omit the real part in this expression, since it always can be accounted for by proper shifts of ε and Δ. The imaginary part is expressed through the scattering time of 2D electrons defined as $\tau_j=\hbar^3/mw_j$. As a result, we obtain the matrix $\hat{G}^s_{\varepsilon j}(\mathbf{p})$ in the form

$$\hat{G}^s_{\varepsilon j}(\mathbf{p})=\begin{bmatrix} (L^s_{j+}+L^s_{j-})/2 & \frac{ip_-}{2p}(L^s_{j+}-L^s_{j-}) \\ -\frac{ip_+}{2p}(L^s_{j+}-L^s_{j-}) & (L^s_{j+}+L^s_{j-})/2 \end{bmatrix}, \tag{8}$$

where $L^R_{1\pm}=L^{A*}_{1\pm}=[\varepsilon-\Delta/2-p^2/2m\mp\alpha_1 p+i\hbar/2\tau_1]^{-1}$ and $L^R_{2\pm}=L^{A*}_{2\pm}=[\varepsilon+\Delta/2-p^2/2m\mp\alpha_2 p+i\hbar/2\tau_2]^{-1}$. Substituting Eq. (8) into Eq. (6), we finally rewrite the latter as

$$I=\frac{et^2}{(2\pi)^2\hbar\tau_1\tau_2}\int d\varepsilon(f_{1\varepsilon}-f_{2\varepsilon})\int_0^\infty p\,dp$$
$$\times\sum_{\sigma=\pm}[(\varepsilon-\Delta/2-p^2/2m-\sigma\alpha_1 p)^2+(\hbar/2\tau_1)^2]^{-1}$$
$$\times[(\varepsilon+\Delta/2-p^2/2m-\sigma\alpha_2 p)^2+(\hbar/2\tau_2)^2]^{-1}. \tag{9}$$

Under the integral over p in Eq. (9), we have a sum of the products of spectral densities belonging to spin-split 2D subbands of the layers 1 and 2. Indeed, if the scattering is negligibly small, the poles of the expression under the integral are $\varepsilon=\Delta/2+p^2/2m+\sigma\alpha_1 p$ and $\varepsilon=-\Delta/2+p^2/2m+\sigma\alpha_2 p$. Each of these equations describe spin-split subbands characterized by the spin number $\sigma=\pm$. This spin number is conserved in the tunneling. Since the tunneling current depends on the Rashba velocities α_1 and α_2, the spin-splitting effects can be probed by tunneling. Below we analyze the tunneling current described by Eq. (9) for different transport regimes.

III. RESULTS

If a source-drain voltage V is applied across the structure by means of independent contacting to the layers, the distribution functions can be approximated as $f_{j\varepsilon}=\{\exp[(\varepsilon-\varepsilon_{Fj})/T]+1\}^{-1}$, with quasi-Fermi levels $\varepsilon_{F1}=\varepsilon_F+eV/2$ and $\varepsilon_{F2}=\varepsilon_F-eV/2$. We start our consideration with the linear transport case, when $V\to 0$ and $f_{1\varepsilon}-f_{2\varepsilon}\simeq -eV(df_\varepsilon/d\varepsilon)$, where $f_\varepsilon=\{\exp[(\varepsilon-\varepsilon_F)/T]+1\}^{-1}$ is the equilibrium distribution function. It is convenient to introduce the tunneling conductance $G_T=I/V$. For strongly degenerate electrons, it is given by

$$G_T=\frac{e^2t^2}{(2\pi)^2\hbar\tau_1\tau_2}\sum_{\sigma=\pm}\int_0^\infty p\,dp\left\{\left[\frac{n_1}{\rho_{2D}}-\frac{(p+\sigma m\alpha_1)^2}{2m}\right]^2+\left(\frac{\hbar}{2\tau_1}\right)^2\right\}^{-1}\left\{\left[\frac{n_2}{\rho_{2D}}-\frac{(p+\sigma m\alpha_2)^2}{2m}\right]^2+\left(\frac{\hbar}{2\tau_2}\right)^2\right\}^{-1}, \tag{10}$$

where we made use of the relations $n_1=\rho_{2D}(\varepsilon_{F1}-\Delta/2+m\alpha_1^2/2)$ and $n_2=\rho_{2D}(\varepsilon_{F2}+\Delta/2+m\alpha_2^2/2)$ expressing the 2D electron densities in the layers n_1 and n_2 through the characteristic energies and constant 2D density of states $\rho_{2D}=m/\pi\hbar^2$. Below, for the sake of simplicity, we assume that the scattering is symmetric, $\tau_1=\tau_2=\tau$. A calculation of the integral over p under conditions $n_{1,2}/\rho_{2D}\gg\hbar/2\tau$ gives us the following result:

$$G_T=\frac{16e^2t^2m\tau n_1n_2}{\pi\hbar^4(\sqrt{n_1}+\sqrt{n_2})^4}\times\sum_{\sigma=\pm}\left\{1+\left[\frac{4\tau}{\hbar}\frac{\sqrt{n_1n_2}}{\rho_{2D}}\frac{\sqrt{n_1}-\sqrt{n_2}-\sigma\gamma(\alpha_1-\alpha_2)}{\sqrt{n_1}+\sqrt{n_2}}\right]^2\right\}^{-1}, \tag{11}$$

where $\gamma=m/\sqrt{2\pi\hbar^2}$. In the calculation we also assumed that the Rashba velocities α_j are small in comparison to the Fermi velocity p_F/m, which allowed us to neglect $\gamma|\alpha_j|$ in comparison to $\sqrt{n_j}$. However, it is important to keep the terms $\gamma\alpha_j$ in the denominator, where they can be comparable to $\sqrt{n_1}-\sqrt{n_2}$.

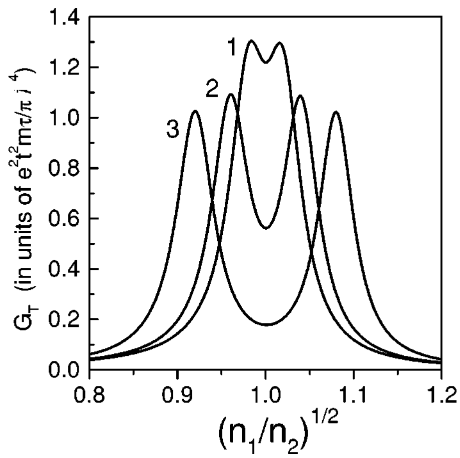

FIG. 2. Linear tunneling conductance as a function of $\sqrt{n_1/n_2}$ at $\tau n_2/\hbar\rho_{2D}=20$, $(\alpha_1-\alpha_2)m/\sqrt{2\pi\hbar^2n_2}=0.02$ (1), 0.04 (2), and 0.08 (3).

The most important difference between the expression (11) and corresponding result[16] in the limit of $\alpha_{1,2}\to 0$ is the shift of the tunneling resonance. At zero-spin splitting, the resonance in the tunneling conductance occurs for matched electron densities, $n_1=n_2$, or, equivalently, at $\Delta=0$. At nonzero-spin splitting, there are two resonances, determined by the condition $\sqrt{n_1}-\sqrt{n_2}=\pm\gamma(\alpha_1-\alpha_2)$. As a result, the dependence of G_T on $\sqrt{n_1}-\sqrt{n_2}$ has two peaks, each one corresponding to the contribution of different spin states into the tunneling. The double-peak structure is resolved under conditions

$$|\alpha_1-\alpha_2|p_F>\hbar/\tau, \tag{12}$$

i.e., when the spin splitting at the Fermi level exceeds the collision-broadening energy. The dependence of G_T on $\sqrt{n_1/n_2}$ at fixed n_2 and $\alpha_1-\alpha_2$ is shown in Fig. 2. It is well described by a superposition of two Lorentzian lines. The line shape is independent of the sign of $\alpha_1-\alpha_2$. In experiment, the densities n_1 and n_2 can be varied by biasing the top and bottom gates, see Fig. 1. We stress that due to large electron densities and the small effective mass of electrons in InAs quantum wells, the dimensionless parameter $\tau n_2/\hbar\rho_{2D}=20$ chosen in the calculations of Fig. 2 corresponds to a considerably large broadening: using $n_2=10^{12}$ cm^{-2} and $m=0.03m_0$, we have an estimate $\hbar/\tau\simeq 4$ meV. On the other hand, using the same n_2 and m, we find that the parameter $(\alpha_1-\alpha_2)\gamma/\sqrt{n_2}=0.04$ corresponds to $\hbar(\alpha_1-\alpha_2)\simeq 2.5\times10^{-11}$ eV·m, which is a reasonable estimate (see the end of this section). The typical measured linewidths $\hbar/\tau$ of the resonant tunneling peaks in GaAs/AlGaAs double-well structures are about 0.5 meV at low temperatures, see Ref. 13. Our calculations, therefore, demonstrate that the splitting of the resonant tunneling peak in the InAs-based double-well structures can withstand a disorder which is much stronger than the disorder in the existing GaAs/AlGaAs structures.

The usage of the double quantum well structures with independent contacts to the 2D layers and two gates, as shown in Fig. 1, gives one the freedom to change parameters ε_{F1}, ε_{F2}, and Δ. If ε_{F1} and ε_{F2} are made considerably different, the tunneling current flows through the structure in the nonlinear regime. In contrast to the linear tunneling conductance, which always depends on the scattering, the nonlinear tunneling current, as shown below, becomes scattering independent if the broadening energies $\hbar/\tau_{1,2}$ are small enough. Recalling Eq. (9), we write the tunneling current as

$$I=\frac{et^2}{(2\pi)^2\hbar\tau_1\tau_2}\sum_{\sigma=\pm}\int d\varepsilon\left(\frac{1}{e^{\varepsilon/T-eV/2T}+1}-\frac{1}{e^{\varepsilon/T+eV/2T}+1}\right)$$
$$\times\int_0^\infty p\,dp\left\{\left[\varepsilon-\frac{eV}{2}+\frac{n_1}{\rho_{2D}}-\frac{(p+\sigma m\alpha_1)^2}{2m}\right]^2+\frac{\hbar^2}{4\tau_1^2}\right\}^{-1}$$
$$\times\left\{\left[\varepsilon+\frac{eV}{2}+\frac{n_2}{\rho_{2D}}-\frac{(p+\sigma m\alpha_2)^2}{2m}\right]^2+\frac{\hbar^2}{4\tau_2^2}\right\}^{-1}. \quad (13)$$

Let us first calculate the current in the limit $\hbar/\tau_{1,2}\to 0$. The spectral densities under the integrals are reduced to the δ functions. In the case of strongly degenerate electron gas, when $T\to 0$, we obtain

$$I=\frac{et^2}{\hbar^3}\sum_{\sigma=\pm}\int_{-eV/2}^{eV/2}d\varepsilon'\int_0^\infty p\,dp$$
$$\times\delta[\varepsilon'-eV/2+n_1/\rho_{2D}-(p+\sigma m\alpha_1)^2/2m]$$
$$\times\delta[\varepsilon'+eV/2+n_2/\rho_{2D}-(p+\sigma m\alpha_2)^2/2m]. \quad (14)$$

An elementary integration in this expression gives a simple result: the contribution comes either from the branch $\sigma=+$ or from $\sigma=-$, and the current is

$$I=\frac{et^2|\Delta|}{\hbar^3(\alpha_1-\alpha_2)^2}\frac{V}{|V|}, \quad (15)$$

if V falls in the interval described by the following nonequalities:

$$(\sqrt{n_1}+\sqrt{n_2}-\gamma|\alpha_1-\alpha_2|)<\frac{\rho_{2D}|eV|}{|\sqrt{n_1}-\sqrt{n_2}\pm\gamma|\alpha_1-\alpha_2||}<(\sqrt{n_1}+\sqrt{n_2}+\gamma|\alpha_1-\alpha_2|). \quad (16)$$

If V is beyond this interval, the current is equal to zero.

The tunneling current (15) is independent of the parameters characterizing the scattering. This remarkable property appears because of the intersection of the 2D electron spectra of different layers (with the same spin σ) in the energy-momentum space, see Fig. 3. In these conditions, the electron tunnel near the intersection points with conservation of the momentum. Such scattering-independent tunneling be-

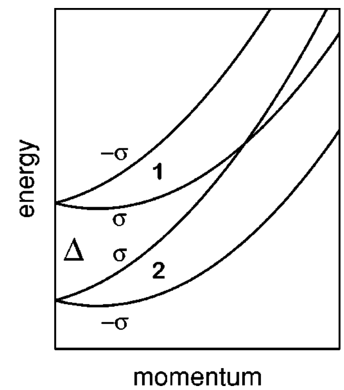

FIG. 3. Intersection of the branches σ of the spin-split electron spectrum belonging to the 2D layers (1) and (2). If the sign of Δ is changed, the intersection occurs for the other spin branch $-\sigma$.

tween 2D layers also takes place[11,18] in the double quantum wells if there is a magnetic field parallel to the layers. This magnetic field shifts the 2D electron spectra of different layers in the momentum space with respect to each other. The consequences of the Rashba effect are similar. The intersection of the spectra takes place on a circle $|\mathbf{p}|=p_0=|\Delta|/|\alpha_1-\alpha_2|$. If $T\to 0$, the tunneling current is nonzero if the energy corresponding to this intersection stays in the interval between the quasi-Fermi energies of 2D electrons in the layers. This condition is formally given by Eq. (16). Only one spin state $\sigma=\sigma_0$ contributes to the current. If the sign of the factor $\Delta/(\alpha_1-\alpha_2)$ is positive (negative), $\sigma_0=-$ $(\sigma_0=+)$. A sharp change of the current from zero to the value given by Eq. (15) is smoothed at finite temperatures. In this case, using Eq. (13) at $\hbar/\tau_{1,2}\to 0$, we obtain

$$I=\frac{et^2|\Delta|}{\hbar^3(\alpha_1-\alpha_2)^2}\frac{\sinh(eV/2T)}{\cosh(\varepsilon_0/T)+\cosh(eV/2T)},$$

$$\varepsilon_0=\frac{[(n_1-n_2)/\rho_{2D}-eV]^2}{2m(\alpha_1-\alpha_2)^2}-\frac{n_1+n_2}{2\rho_{2D}}+\frac{m(\alpha_1-\alpha_2)^2}{8}, \quad (17)$$

where ε_0 is the energy corresponding to the intersection of the spectra. The last term in ε_0 is small in comparison to the other terms and can be neglected. The influence of the disorder (finite $\hbar/\tau_{1,2}$) on the tunneling current in the nonlinear regime is also important. To ensure that only one spin state $\sigma=\sigma_0$ contributes to the current in the vicinity of $|\mathbf{p}|=p_0$, we assume that $|\alpha_1-\alpha_2|p_0\gg\hbar/\tau_{1,2}$. Then, Eq. (13) at $T=0$ is rewritten as

$$I=\frac{et^2}{(2\pi)^2\hbar\tau_1\tau_2}\int_{-eV/2}^{eV/2}d\varepsilon\int_{-p_0}^{\infty}d\,\delta p(p_0+\delta p)$$
$$\times[(\varepsilon-\varepsilon_0-v_1\delta p-\delta p^2/2m)^2+(\hbar/2\tau_1)^2]^{-1}$$
$$\times[(\varepsilon-\varepsilon_0-v_2\delta p-\delta p^2/2m)^2+(\hbar/2\tau_2)^2]^{-1}, \quad (18)$$

where $\delta p=p-p_0$ and $v_j=p_0/m+\sigma_0\alpha_j$ are the group velocities of electrons at $p=p_0$. Under the assumed condition $|\alpha_1-\alpha_2|p_0\gg\hbar/\tau_{1,2}$, we can neglect the terms quadratic on

δp in the denominators, neglect δp in comparison to p_0, and shift the lower limit of the integral over δp to $-\infty$. In these approximations, the integrals over δp and ε are calculated analytically, and we find

$$I=\frac{et^2|\Delta|}{\pi\hbar^3(\alpha_1-\alpha_2)^2}\left[\arctan\frac{\varepsilon_0+eV/2}{\Gamma}-\arctan\frac{\varepsilon_0-eV/2}{\Gamma}\right],$$

$$\Gamma=\frac{\hbar|\Delta|}{\tau m(\alpha_1-\alpha_2)^2}, \tag{19}$$

where $\tau^{-1}=(\tau_1^{-1}+\tau_2^{-1})/2$.

To find the current-voltage characteristics of the double-layer system, one should consider the electrostatic problem for a structure of the geometry shown in Fig. 1. Such a consideration relates the electron densities and level splitting Δ to the voltages applied to the gates and contacts. We have carried it out for a structure with mirrorlike symmetry, when the widths of the wells are the same and the donor densities, the distances from the wells to the doping planes, and the distances to the gates are equal to each other on both sides. The donor impurities in the doping layers are assumed to be completely ionized, so that an application of the voltages does not change the density of the charged donors. For the sake of simplicity, we consider the situation in which the gates are grounded and the voltage $V/2$ $(-V/2)$ is applied to the first (second) layer. Neglecting a small correction of the order of d/L, where d is the distance between the centers of 2D layers and L is the thickness of the structure from one gate to the other, we obtain the following equation:

$$\left(1+\frac{2d}{a_B}\right)(n_1-n_2)-\left(\frac{1}{3}-\frac{5}{4\pi^2}\right)\frac{a}{a_B}(n_1-n_2)$$
$$-\frac{2^{3/2}}{\pi^{3/2}a_B}(\sqrt{n_1}-\sqrt{n_2})=\rho_{2D}eV, \tag{20}$$

where $a_B=\hbar^2\epsilon/e^2m$ is the Bohr radius expressed through the static dielectric permittivity ϵ, which is assumed to be constant across the layers, and a is the quantum well width. Two last terms in the left-hand side of this equation appear because of the influence of the electron-electron interaction on the Fermi energies in the wells, taken into account within the Hartree-Fock approximation using the ground-state wave function for rectangular hard-wall confinement potential and neglecting the terms of the order of $(p_Fa/\hbar\pi)^2$ and higher in the expansion of the exchange interaction energy term.[25] Equation (20) should be accompanied by the relation $n_1+n_2=2n_0$, where n_0 is the equilibrium electron density in each layer (at $V=0$), determined by the doping. Because of the small value of the spin-splitting energy in comparison to the Fermi energy, the conditions (16) of nonzero tunneling current correspond to $|\sqrt{n_1}-\sqrt{n_2}|\ll 2n_0$, when one can expand the factor $\sqrt{n_1}-\sqrt{n_2}$ in the series of n_1-n_2 as $\sqrt{n_1}-\sqrt{n_2}\simeq(n_1-n_2)/2\sqrt{n_0}$. Taking this into account, we derive from Eq. (20) a simple linear dependence,

$$n_1=n_0+\frac{a_B\rho_{2D}}{4d^*}eV,\quad n_2=n_0-\frac{a_B\rho_{2D}}{4d^*}eV, \tag{21}$$

where $d^*=d+a_B/2-a(1/6-5/8\pi^2)-1/\sqrt{2\pi^3n_0}$. For typical electron densities $n_0>10^{12}$ cm^{-2} in the InAs quantum wells, the principal contribution of the exchange term into d^* is estimated as $1/\sqrt{2\pi^3n_0}<1.3$ nm. It is more than an order-of-magnitude smaller than either the typical interlayer distance $d\sim20$ nm or the Bohr radius a_B. Therefore, this exchange term can be neglected, in agreement with the statement that the Hartree approximation is expected to work quite well in the two-dimensional systems based on InAs semiconductors, see Ref. 26, p. 468. The term $a(1/6-5/8\pi^2)\simeq0.1a$ is also small in comparison to $d+a_B/2$, so that one can write $d^*=d+a_B/2$ with high accuracy. As for the energy Δ, it is connected to n_1, n_2, and V by a simple relation $n_1-n_2=\rho_{2D}(eV-\Delta)$ and, together with n_1-n_2, is directly proportional to V. According to the definition, d^* is always larger than $a_B/2$, which gives us $|n_1-n_2|<\rho_{2D}|eV|$ and the sign of Δ always coincides with the sign of the applied voltage V. The consideration presented above does not take into account the minor corrections due to the spin splitting of the spectrum.

To estimate the velocities α_1 and α_2, we use the following expression for $\alpha(z)$:

$$\alpha(z)\simeq\frac{\hbar P^2}{3}\frac{d}{dz}\left[\frac{1}{\varepsilon_g-v(z)-V_p(z)}\right.$$
$$\left.-\frac{1}{\varepsilon_g+\Delta_s-v(z)-V_s(z)}\right], \tag{22}$$

where P is the Kane's velocity, ε_g is the energy gap, and Δ_s is the energy distance from the valence band to the spin-split band in the quantum well regions. The valence-band discontinuity energy $V_p(z)$ and spin-split-band discontinuity energy $V_s(z)$ are defined as step functions equal to zero in the quantum well regions and to constants V_p and V_s in the barrier regions. The presence of $V_p(z)$ and $V_s(z)$ leads to singular terms in $\alpha(z)$ proportional to δ functions at the interfaces. Nevertheless, such terms can be neglected if the velocities $\alpha_j=\int dz\,\alpha(z)|F_j(z)|^2$ are calculated on the basis of the wave functions of hard-wall confinement, i.e., the underbarrier penetration of the wave functions is not taken into account. In this case, only the terms proportional to $dv(z)/dz$ remain significant, and

$$\alpha(z)\simeq\frac{2\hbar P^2}{3}\frac{\Delta_s(\varepsilon_g+\Delta_s/2)}{\varepsilon_g^2(\varepsilon_g+\Delta_s)^2}\frac{dv(z)}{dz}. \tag{23}$$

Using $v(z)$ calculated from the Poisson equation, and expressing P^2 through the effective mass m according to $m^{-1}=4P^2/3\varepsilon_g+2P^2/3(\varepsilon_g+\Delta)$, we find

$$\alpha_1=\beta(n_1-\rho_{2D}eVa_B/2d^*),$$

$$\alpha_2=\beta(-n_2-\rho_{2D}eVa_B/2d^*), \tag{24}$$

$$\beta = \frac{2\hbar}{m\rho_{2D}a_B}\frac{\Delta_s(\varepsilon_g+\Delta_s/2)}{\varepsilon_g(\varepsilon_g+\Delta_s)(3\varepsilon_g+2\Delta_s)}.$$

The parameter β depends only on material properties. We stress that for $m=0.03m_0$, $\epsilon=12$, $\varepsilon_g=0.41$ eV, and $\Delta_s=0.38$ eV (parameters ε_g and Δ_s for InAs are taken from Ref. 27), Eq. (24) gives us $\hbar|\alpha_{1,2}|\sim 10^{-11}$ eV·m for 2D electron densities $n_{1,2}\sim 10^{12}$ cm^{-2}, typical for InAs-based heterostructures. This estimate of $\alpha_{1,2}$ is in good agreement with the values obtained from experimental data, see Ref. 6, and references therein. In the symmetric structure at $V=0$, one has $n_1=n_2$ and $\alpha_1=-\alpha_2$. A comparison of Eqs. (16), (21), and (24) shows that the tunneling current flows under conditions when the relative changes of electron densities n_j and Rashba velocities α_j induced by the applied voltage V are small. This property is a consequence of the smallness of the spin-splitting energy $|\alpha_1-\alpha_2|p_0$ in comparison to the Fermi energy n_0/ρ_{2D}.

Let us apply the above results to the calculation of the current-voltage characteristics. In the symmetric structure we consider, the term $|\alpha_1-\alpha_2|$ entering the expressions for the tunneling current is equal to $\beta(n_1+n_2)=2\beta n_0$. Therefore, this term appears to be independent of the applied voltage, and the variation of the maximal value of the tunneling current at low temperatures is determined by the variation of Δ. Since Δ is proportional to V, it is convenient to introduce the nonlinear (voltage-dependent) conductance $G_T(V)$. Using Eqs. (17) and (21), we obtain, in the case of zero disorder

$$G_T(V)=\frac{e^2t^2r}{\hbar^3(\alpha_1-\alpha_2)^2}\sinh\frac{|eV|}{2T}\left[\cosh\left(\frac{e^2V^2}{2E_0T}-\frac{n_0}{\rho_{2D}T}\right)+\cosh\frac{eV}{2T}\right]^{-1}, \tag{25}$$

where $r=(1-a_B/2d^*)$ is a constant of the order of unity, and the characteristic energy E_0 is defined as $E_0=m(\alpha_1-\alpha_2)^2/r^2$. The conductance (25) has two identical symmetric peaks centered at $V=\pm V_0$, where $V_0=e^{-1}\sqrt{2n_0E_0/\rho_{2D}}$. The width of the peaks at low temperature, when $V_0\gg 2T$, is equal to E_0/e. The peaks in this case have an almost rectangular shape. In Fig. 4 we plot $G_T(V)$ in the region of positive V for 4.2 K and 40 K, substituting $(\alpha_1-\alpha_2)^2=4\beta^2n_0^2$, using the material parameters given above, and assuming $a_B/2d^*=0.5$ and $n_0=2\times 10^{12}$ cm^{-2}. The temperature-induced broadening of the peak becomes significant when $T\sim V_0/2$, and it is negligible at liquid-helium temperature. The effect of disorder on the broadening is far more considerable. At $T=0$, using Eqs. (19) and (21), we obtain

$$G_T(V)=\frac{e^2t^2r}{\pi\hbar^3(\alpha_1-\alpha_2)^2}\left[\arctan\frac{\tau r(eV-eV_0^2/V+E_0)}{2\hbar}-\arctan\frac{\tau r(eV-eV_0^2/V-E_0)}{2\hbar}\right]. \tag{26}$$

The broadening described by this equation can be neglected only if

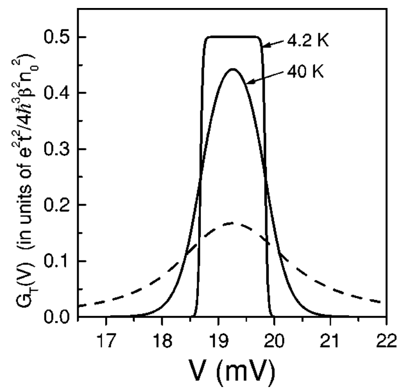

FIG. 4. Nonlinear tunneling conductance for a symmetric double quantum well structure with grounded gates as a function of the source-drain voltage for $T=4.2$ K and 40 K at $\hbar/\tau=0$ (solid lines) and for $T=0$ at $\hbar/\tau=0.5$ meV (dashed line). The parameters used in the calculation are given in the text.

$$\hbar/\tau \ll m(\alpha_1-\alpha_2)^2. \tag{27}$$

This condition is difficult to satisfy for reasonable parameters. The large effect of the disorder is explained qualitatively if we take into account that the group velocities of electrons v_1 and v_2 near the intersection point are close to each other, see Fig. 3, and the scattering permits the electrons to tunnel through in a rather broad region of energies around this point. In Fig. 4 we show the shape of the peak at $\hbar/\tau=0.5$ meV, which is the typical broadening energy of the resonant tunneling peaks in GaAs/AlGaAs double-well structures.[13] The rectangular shape, which would exist at $\hbar/\tau\to 0$, is lost. Nevertheless, the peak itself is well defined, as far as the condition (12) is satisfied, and the position of its maximum is the same as in the absence of the disorder.

IV. CONCLUSIONS

We calculated the tunneling current between the 2D layers with a spin-split electron energy spectrum and demonstrated that the measurements of the tunneling conductance in both linear and nonlinear regimes can reveal specific features of the Rashba effect. The linear tunneling conductance shows a superposition of two Lorentz-like peaks corresponding to a resonance contribution of two spin-split subbands. The nonlinear conductance, as a function of the applied voltage, shows a peak which becomes rectangular in the limit of weak disorder. In this limit, the width and height of the peak are independent of the disorder. The position of the peak depends on the Rashba velocities describing the magnitude of the spin splitting.

In contrast, for 2D-2D tunneling in the absence of spin splitting, there is only one Lorentz peak[13] of the tunneling conductance as a function of the relative density difference or interlayer level splitting, though its width is given by the same relation: it is proportional to the product of the Fermi energy by the scattering time. In the nonlinear regime, the tunneling current between the layers without spin splitting decreases[13] with the applied source-drain voltage V for symmetric structures. Indeed, the application of this voltage drives the system out of the tunneling resonance because the interlayer level splitting Δ increases with V. The decrease of the current follows the Lorentzian dependence, because the tunneling current is proportional to $[\Delta^2+(\hbar/\tau)^2]^{-2}$. If the double quantum well structure is not symmetric (for example, if the wells are of different widths), the tunneling current-voltage dependence has a similar Lorentz-like resonant peak around the voltage corresponding to $\Delta=0$. Therefore, there are essential qualitative differences between the behavior of 2D-2D tunneling current in the absence of spin splitting and that of the case we studied.

Let us discuss which information can be obtained from the measurements of the tunneling current between 2D layers with a spin-split spectrum. If the gate-voltage dependence of the electron densities n_1 and n_2 is known, the measurements of the width and position of the peaks of linear tunneling conductance give us both the broadening energy $\hbar/\tau$ and spin-splitting energy, and the latter is described in terms of the quantity $|\alpha_1-\alpha_2|$. Knowing $\hbar/\tau$ and $|\alpha_1-\alpha_2|$ and measuring the height of the tunneling conductance peaks, one can find the tunneling matrix element t, which is usually not known precisely from theoretical estimates. The measurements of the current-voltage characteristics in the nonlinear regime provide similar information. In particular, the quantity $|\alpha_1-\alpha_2|$ is determined by the position of the peak V_0 if the total density of electrons in the structure $2n_0$ is known. The measurements of the width and height of the peak allow one to extract $\hbar/\tau$ and t. If the spin splitting is so strong that the condition (27) is satisfied, the measurement of the width of the peak provides complementary information about the quantity $|\alpha_1-\alpha_2|$.

From the tunneling experiment, one can determine directly only the absolute value of the difference of Rashba velocities, $|\alpha_1-\alpha_2|$. From a first look, this quantity seems less important than the quantities α_1 and α_2 themselves. Nevertheless, knowing $|\alpha_1-\alpha_2|$, it is possible to determine α_1 and α_2, because an additional relation between α_1 and α_2 can often be deduced from the consideration of the symmetry of the confining potentials in the wells. In the simplest case, one can choose for the measurements a mirrorlike symmetric structure, where $\alpha_1=-\alpha_2$ and $|\alpha_1|=|\alpha_2|=|\alpha_1-\alpha_2|/2$. In more complex cases, one can compare the shapes of the confining potentials in the wells by solving an electrostatic problem for the structure under consideration.

The important feature of the tunnel contact between 2D layers with Rashba spin splitting is its selectivity with respect to the spin number σ. Indeed, a single peak of the tunneling current corresponds to the resonant tunneling of either $\sigma=+$ or $\sigma=-$ states, see Fig. 3. By adjusting the gate voltages, one can switch from $\sigma=-$ to $\sigma=+$ tunneling. Therefore, in principle, the tunnel contact between 2D layers can work as a spin filter. A spin-filter resonant tunneling diode based on a similar idea has been recently proposed.[28] Nevertheless, it seems difficult to realize such a kind of spin filter in spin electronics. First of all, the electrons in $\sigma=+$ or $\sigma=-$ states coming after the passage of the tunnel contact are not polarized in a definite direction. The direction of the spin polarization depends on the direction of their momenta. Therefore, an additional filtering by direction of momentum is necessary. Most important, one has to find some way to extract the electrons that just tunneled before $\sigma=-$ and $\sigma=+$ states become intermixed by the scattering. In any case, the spin filtering can become possible only near the edge of the tunnel contact area, where the description of the tunneling requires a special consideration beyond the scope of this paper.

In conclusion, we have demonstrated that Rashba spin splitting leads to unusual behavior of the tunneling current between 2D layers and suggested that the measurements of this current can be used for direct investigation of the spin-split electron spectra in quantum wells. A fabrication and experimental investigation of double quantum well structures with independently contacted InAs 2D layers, which could verify our theoretical predictions, remains an issue for future advances in microstructure science.

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