Lifetime broadening in Compton scattering

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Recent high-resolution Compton scattering experiments revealed a smearing of the Fermi break in nearly free electron metals which contradicts Luttinger's theorem of a sharp Fermi surface even for the interacting electron gas. Sternemann *et al.* [Phys. Rev. B **62**, R7687 (2000)] explained this smearing by introducing a spectral function for the recoil electron with a finite lifetime due to final-state interactions. In contrast to the laborious *GW* approximation (*GWA*) used by these authors we calculate inelastic mean-free paths both for electron-hole and plasmon excitations which are of striking simplicity. The resulting lifetimes agree very well with those of GWA. In addition, we show that core excitation significantly shortens the lifetimes. The estimate of the influence of final-state interactions is extended to an analysis of $(e,2e)$ experiments.

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I. INTRODUCTION

In modern high-resolution Compton spectroscopy momentum resolution of about 0.02 a.u. can be achieved[1] which allows to obtain detailed information about the Fermi surface topology, especially if nearly free electron (NFE) metals are investigated. It thus gave rise to the ambition to study electron correlations in the interacting electron gas, a benchmark for many-particle theories during many years. It was one of the milestones of these theories that even for the interacting system the Fermi edge remains sharp (Luttinger theorem)[2] but the height Z_F of the break is reduced compared to the noninteracting free-electron gas $(Z_F=1)$.[3] It was thus very astonishing when Schülke *et al.*[4] found in a high-resolution Compton profile study of lithium a value of the renormalization constant Z_F close to zero, far away from any theoretical estimate.[5] Later Tanaka *et al.*[6] confirmed this result. Such a low Z_F value means an effective strong smearing of the Fermi break which was also observed for beryllium[1] and seemed to depend on the incident photon energy in the sense that at high energies (60 keV) this smearing became less pronounced,[7] but did not vanish even at such high photon energies. An early explanation of these disturbing experimental results was given by Kubo[8] based on a GW approximation for the band structure. But Schülke[9] showed that the plasmon-pole model used by Kubo had the deficiency that the inverse lifetime of the initial state did not vanish at the Fermi momentum as it has to do from quite general phase-space considerations. It is the vanishing of the lifetime at the Fermi momentum that is responsible for a sharp Fermi surface. Quantum Monte Carlo calculations of Compton profiles for Li by Fillipi and Ceperley[10] especially devoted to explain the experiments could only state a Z_F strongly different from zero.

II. THEORY

Another explanation for this effect which accounts at least partly for the observed smearing was proposed by Sternemann *et al.*[11] and relies on the assumption that the impulse approximation (IA),[12] which is the basis for the evaluation of Compton profiles, might be violated due to final-state interactions of the recoil electron. To proceed we start with the double-differential cross section for nonresonant inelastic x-ray scattering, differential with respect to a momentum and energy transfer k and ϵ,

$$\frac{d^2\sigma}{dk\,d\epsilon}=\frac{\omega_i'}{\omega_i}\left(\frac{d\sigma}{dk}\right)_{Th}S(k,\epsilon), \tag{1}$$

$(d\sigma/dk)_{Th}$ is the Thomson cross section, $S(k,\epsilon)$ the dynamical structure factor of the target material, and ω_i,ω_i' are the incident and scattered photon energy, respectively. Within the IA the dynamical structure factor $S^{\mathrm{IA}}(k,\epsilon)$ is connected with the conventional Compton profile $J(p_z)$ by

$$J(p_z)=k\int_{-\infty}^{+\infty}S^{\mathrm{IA}}(k,\epsilon)\,\delta(\epsilon-\epsilon_{\mathbf{p}+\mathbf{k}}+\epsilon_{\mathbf{p}})\,d\epsilon \tag{2}$$

with $\epsilon_{\mathbf{q}}=q^2/2$ and $p_z=\mathbf{p}\cdot\mathbf{k}/k$. The δ function in Eq. (2) results from the assumption of free electrons both in the initial and final states. Deviations from this assumption for the initial state took recourse to the concept of quasiparticles which keeps in essence the kinematics and thus the δ function correct but influences $S^{\mathrm{IA}}(k,\epsilon)$. Sternemann *et al.*[11] then proposed to consider in addition the final electron not as a free particle with an infinite lifetime, but to attribute a finite one due to its interaction with the rest of the solid. This interaction is quite different from the Coulomb scattering of the recoil electron with its own ion which leads to the well-studied Compton defect.[12,13] In the language of many-body theory the δ function of Eq. (2) has to be replaced by the spectral function $A(\mathbf{k},\epsilon)$ which accounts for finite lifetimes,[14] in order to get the corrected profile $J_c(p_z)$,

$$J_c(p_z)=k\int_{-\infty}^{+\infty}S^{\mathrm{IA}}(k,\epsilon)A(\mathbf{k},\epsilon-kp_z)\,d\epsilon, \tag{3}$$

where the spectral function is obtained from the Green's function G of the quasiparticle,

$$A(\mathbf{k},\epsilon)=\frac{1}{\pi}\left|\mathrm{Im}\,G(\mathbf{k},\epsilon)\right|=\frac{1}{\pi}\left|\mathrm{Im}\frac{1}{\epsilon-\epsilon_{\mathbf{k}}-\Sigma(\mathbf{k},\epsilon)}\right|. \tag{4}$$

Following Hedin's prescription,[15] Sternemann *et al.*[11] have calculated the spectral function $A(\mathbf{k},\epsilon)$ within the so-called G_0W_0 approximation where the self-energy $\Sigma(\mathbf{k},\epsilon)$ has been obtained with the help of Lindhard's random-phase approximation for the dielectric function.[16] Soininen *et al.*[14] confirmed in essence this result and were able to simplify the rather laborious G_0W_0 approximation by using for the dielectric function a simple plasmon-pole model. Both papers clearly revealed a much stronger lifetime broadening than an earlier estimate of Platzman and Tzoar.[17]

The final state spectral function $A(\mathbf{k},\epsilon)$ shows in essence a quasiparticle peak accompanied by at least one satellite (sidepeak) due to screened electron-electron interaction. In the following we will concentrate on the dominating quasiparticle peak whose shape is approximately Lorentzian, the width of which is determined by the inverse lifetime $\Gamma(k)$,

$$\Gamma(k)=\mathrm{Im}[\Sigma(k,\epsilon_k)], \tag{5}$$

where the quasiparticle energy ϵ has been approximated by the free particle energy ϵ_k, i.e., with the self-energy on-shell, see Ref. 11. Here and in the following we refer to the jellium, i.e., all quantities depend on the absolute value of $\mathbf{k}$ only. Instead of calculating $A(k,\epsilon)$ as in Refs. 11 and 14 by a rather complicated many-body theory we will give in the following very simple estimates of the inverse lifetime $\Gamma(k)$. We have several reasons for that: first, for the folding in Eq. (3) the exact shape of the quasiparticle part of $A(k,\epsilon)$ will not be as important as a good approximation to $\Gamma(k)$; second, the shape seems to depend on its usefulness since fully self-consistent GW self-energy calculation for the jellium by Holm and von Barth[18] assumes Gaussian-like spectral functions. But the most important reason will be the striking simplicity by which the lifetime τ_k can be obtained. Assuming that the phase memory of the quasiparticle is terminated by inelastic scattering processes of the recoil electron, one has[19]

$$\Gamma(k)=\frac{1}{2\tau_k}=\frac{v_k}{2\lambda_k}=kn\sigma/2, \tag{6}$$

v_k is the electron velocity, λ_k the inelastic mean-free path (IMFP), and σ the corresponding total cross section per jellium electron (we use atomic units, i.e., $e=m=\hbar=1$). $n=3/(4\pi r_s^3)$ is the electron density. This equation defines the mean electron-electron distance r_s, an important quantity since the material dependence of $\Gamma(k)$ will only be through r_s.

The total cross section is obtained from that one which is differential with respect to a momentum and energy transfer of the fast electron to the jellium electrons. Denoting these quantities by q and ω, respectively, one has

$$\frac{d^2\sigma}{dqd\omega}=\frac{k'}{k}\left(\frac{d\sigma}{dq}\right)_R S(q,\omega), \tag{7}$$

where k' is the final electron momentum. The Rutherford cross section reads $(d\sigma/dq)_R=8\pi/(kk'q^3)$. For the integration of Eq. (7) some care has to be taken for the use of appropriate expressions for the dynamical structure factor. From the theory of specific energy losses it is well known to make a distinction between close and distant collisions, i.e., those with large and small momentum transfer q.[20] A natural limit that separates these two regimes is the momentum q_c for which plasmon excitation decays into particle-hole pairs. We therefore assume that for $q<q_c$ plasmon excitation will dominate whereas for $q>q_c$ pair production. In the following we will also derive the specific energy loss since the corresponding expressions are well known and thus give confidence to the lifetime estimates. We assume that $k\gg k_F$, where $k_F=(9\pi/4)^{1/3}/r_s$ is the Fermi momentum. We start with single particle excitation. Using the sum rule for the zero-order moment of $S(q,\omega)$ at large momentum transfer[12]

$$\int_0^\infty S^{\mathrm{IA}}(q,\omega)d\omega=1 \tag{8}$$

we obtain for the inverse lifetime from Eqs. (6) and (7),

$$\Gamma_{sp}(k)=\frac{1}{2}kn\int_{q_{min}}^{q_{max}}\frac{d\sigma}{dq}dq=\frac{2\pi n}{k}\frac{1}{q_c^2} \tag{9}$$

with $q_{max}=k\gg q_{min}=q_c=(k_F^2+2\omega_p)^{1/2}-k_F$. Here ω_p is the plasmon frequency $\omega_p=(4\pi n)^{1/2}=(3/r_s^3)^{1/2}$. For the specific energy loss

$$\frac{dE}{dx}=-n\int\int\omega\frac{d^2\sigma}{dqd\omega}dqd\omega \tag{10}$$

one has in this case

$$\left(\frac{dE}{dx}\right)_{sp}=-\frac{4\pi n}{k^2}\ln\left(\frac{k}{q_c}\right), \tag{11}$$

where we have used the sum rule for the first-order moment,[12]

$$\int_0^\infty\omega S(q,\omega)d\omega=q^2/2. \tag{12}$$

For momentum transfers $q<q_c$, i.e., in the regime of the plasmon resonance, we approximate the dynamical structure factor by the plasmon-pole model[21]

$$S(q,\omega)=-\frac{1}{\phi_q}\mathrm{Im}\left(\frac{1}{\epsilon_M(q,\omega)}\right)=\frac{q^2\omega_p}{4\pi n}\delta(\omega^2-\omega_p^2), \tag{13}$$

where $\phi_q=4\pi/q^2$ is the Fourier transform of the Coulomb potential and $\epsilon_M(q,\omega)$ the macroscopic dielectric function. The model obeys the *f*-sum rule of Eq. (12). From Eqs. (6), (7), and (13) we get for the plasmon contribution

$$\Gamma_{pl}(k)=\frac{\omega_p}{2k}\int_{q_{min}}^{q_c}\frac{dq}{q}=\frac{\omega_p}{2k}\ln\left(\frac{kq_c}{\omega_p}\right), \tag{14}$$

and for the specific energy loss

$$\left(\frac{dE}{dx}\right)_{pl}=-\frac{\omega_p^2}{k^2}\ln\left(\frac{kq_c}{\omega_p}\right). \tag{15}$$

Interestingly, the total energy loss

$$\frac{dE}{dx}=\left(\frac{dE}{dx}\right)_{pl}+\left(\frac{dE}{dx}\right)_{sp}=-\frac{4\pi n}{k^2}\ln\left(\frac{k^2}{\omega_p}\right) \tag{16}$$

is independent of the plasmon cutoff momentum q_c which is well known from the stopping power theory of the electron gas.[16,20] Equation (16) is identical with Eq. (6.21) of Ref. 22. The total inverse lifetime for the jellium is $\Gamma_j(k)=\Gamma_{pl}(k)+\Gamma_{sp}(k)$.

To make contact with earlier work is a little difficult due to the huge amount of publications concerning IMFP's λ_k. Thus the citation of a few authors is a little arbitrary, but may guide the reader. The expression for the imaginary part of the self-energy as given by Eqs. (5)–(7) and (13) is the same as Eq. (4.7) of Ref. 22. The single-particle excitation of Eq. (9) corresponds in essence to Eq. (5.5) of the work of Ritchie.[22] Equation (14) is identical with Eq. (8) of Quinn[23] and the mean-free path for plasmon excitation of Ashley and Ritchie[24] [Eq. (8)]. It also coincides with an estimate of Lundqvist for the imaginary part of the self-energy[3,25] [Eq. (A10)]. Especially the mean-free path $\lambda_{pl}(k)$ is easily accessible to experimental determination due to the pronounced peaking of plasmon losses evaluated in electron-energy-loss spectroscopy. It has been confirmed that for NFE metals $\lambda_{pl}^{-1}=2\Gamma_{pl}/k$ with Γ_{pl} from Eq. (14) describes experiments very well.[26,27]

Our discussion up to now was restricted to conventional Compton scattering, where the inelastically scattered primary intensity is observed. In contrast, in so-called $(e,2e)$ or $(\gamma,e\gamma)$ experiments[28] both the scattered primary intensity and the intensity of the recoils are observed simultaneously. In this case all four quantities $\mathbf{k}$, ϵ and $\mathbf{p}',E'$, the final momentum and energy of the jellium electron, are measured. Then, the spectral electron momentum density (SEMD) ρ is proportional to[11]

$$\rho\propto\int_{-\epsilon+E_F}^{E_F}dE\int d^3\mathbf{p}A(\mathbf{p},E)A(\mathbf{p}',E')$$

$$\propto\int_{-\epsilon+E_F}^{E_F}A(\mathbf{p}'-\mathbf{k},E)A(\mathbf{p}',E')dE \tag{17}$$

with $E'=E+\epsilon$. For an infinitely long lifetime for the final state, one has

$$A(\mathbf{p}',E')=\delta(E'-E-\epsilon) \tag{18}$$

and therefore with $\epsilon\gg E_F$,

$$\rho\propto A(\mathbf{p}'-\mathbf{k},E'-\epsilon)\equiv A(\mathbf{p},E). \tag{19}$$

This is the result usually applied if experimental data are compared with results from many-body theories.[29] Assuming, for simplicity, a Lorentzian-like spectral function for the hole state and an on-shell approximation the SEMD will have a maximum at

$$E'-\epsilon\equiv E=\epsilon_{\mathbf{p}'-\mathbf{k}}+Z_{|\mathbf{p}'-\mathbf{k}|}\mathrm{Re}[\Sigma(\mathbf{p}'-\mathbf{k},\epsilon_{\mathbf{p}'-\mathbf{k}})], \tag{20}$$

where $Z_{|\mathbf{p}'-\mathbf{k}|}$ is the renormalization constant,[30] and the spectral function is broadened by an inverse lifetime $\Gamma_h(p)$ with $p=|\mathbf{p}'-\mathbf{k}|$. If in *addition* also the final particle state has a Lorentzian-like spectral function $A(\mathbf{p}',E')$ with an inverse lifetime $\Gamma_e(k)$ the convolution of Eq. (17) yields a Lorentzian SEMD with its maximum at the same E but now with an inverse lifetime $\Gamma=\Gamma_h(p)+\Gamma_e(k)$. $\Gamma_e(k)$ is identical with $\Gamma_j(k)$. These considerations demonstrate that also $(e,2e)$ experiments suffer from additional lifetime broadening due to final state interactions. This simple model shows that even at the Fermi momentum where $\Gamma_h(p=p_F)=0$ a smearing of the spectral momentum density will be observed unless the experiments are done at very high primary energies, i.e., at very large k. It might be argued that these results are rather trivial since it is well known from numerous fields of physics that the inverse lifetime of a transition is simply the sum of the inverse lifetimes for the initial and final states. We mention especially photoemission spectroscopy[31] which is closest to the results of $(e,2e)$ experiments. There it is known for more than twenty years that electron lifetimes can be measured if the hole state is close to the Fermi level.[32]

The question arises, if in addition to lifetime broadening polarization shifts $\Delta E=\mathrm{Re}[\Sigma(k,\epsilon_k)]$ are important. (We notice that both are connected by the Kramers-Kronig relations.) This holds also for the energy of the fast electrons in the $(e,2e)$ reaction where it is usually assumed that they behave as free particles. In reality they are dressed and the reduction of their energy compared to that of a bare particle $(\epsilon_{\mathbf{p}'})$ is just ΔE, i.e., in contrast to the assertation made above not E' but $\epsilon_{\mathbf{p}'}$ is measured. An estimate by Lindhard[16] yields for high energies $\Delta E\approx-\omega_p/k$, i.e., of the same order of magnitude as the inverse lifetime. These considerations are in accord with a discussion about the bandwidth in NFE metals by Yasuhara *et al.*[33] who emphasize that in photoemission experiments also the final-state energies are quasiparticle energies and not, as conventionally assumed, free-electron energies. In this case the neglection of ΔE can even lead to the wrong sign for the bandwidth change: narrowing instead of broadening. We notice that in photon Compton scattering polarization shifts both in the initial and final states *must* be neglected in order to describe experiments quantitatively.[11,14] Another point concerns lifetime contributions from the projectile. Equation (17) considers spectral functions of the jellium electron only and disregards from any similar effects of the projectile electron which is especially questionable if exchange scattering is remembered. It might be possible that additional lifetime broadening results from this effect.

III. RESULTS AND DISCUSSION

Since for the experimenter the smearing effect is most interesting on a p_z scale, we will show in the following $(\Delta p_z)_{FWHM}=2.35\Gamma_j/k$, assuming a Gaussian-like broadening. Figure 1 shows $(\Delta p_z)_{sp}$ (dashed curve), $(\Delta p_z)_{pl}$ (dash-dotted curve), and the sum (solid curve). The smearing (Δp_z) is plotted as a function of the incident photon energy ω_i assuming a maximum of inelasticity, i.e., a scattering angle of 180°, for which

$$k=2c\omega_i\frac{1+\omega_i}{1+2\omega_i} \tag{21}$$

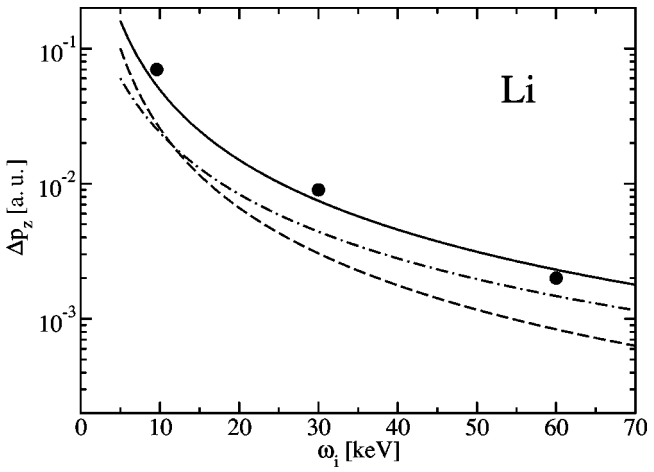

FIG. 1. The momentum smearing Δp_z in case of Li ($r_s=3.27$) as a function of the incident photon energy ω_i. Dashed curve, $(\Delta p_z)_{sp}$; dash-dotted curve, $(\Delta p_z)_{pl}$; solid curve, the sum $(\Delta p_z)_{pl}+(\Delta p_z)_{sp}$. Dots: the G_0W_0A of Ref. 11.

holds, where ω_i is measured in units of the electron rest mass. The curves hold for Li ($r_s=3.27$). If the sum (solid curve) is compared with the G_0W_0 result of Sternemann *et al.*[11] (dots) nearly perfect agreement is recognized. As an example, the inverse lifetime Γ_j is 3.0 eV at $\omega_i=10$ keV and decreases to 0.78 eV at 60 keV. In Fig. 2 a similar comparison is made with the results of Soininen *et al.*[14] in case of beryllium ($r_s=1.87$). Now the agreement is slightly worse and the coincidence is best for $(\Delta p_z)_{pl}$. This might be understood if it is realized that for the calculation of $A(k,\epsilon)$ no contribution from the particle-hole pair continuum (single-particle excitation in our terminology) can be expected within the plasmon-pole model. The authors' finding that an artificial increase of the plasmon cutoff q_c by a factor 2.6 in order to achieve agreement between the plasmon-pole model and the G_0W_0 approximation shows in the same direction: a comparison of Eqs. (9) and (14) demonstrates that an increase of q_c favors Γ_{pl} on account of Γ_{sp}. The importance

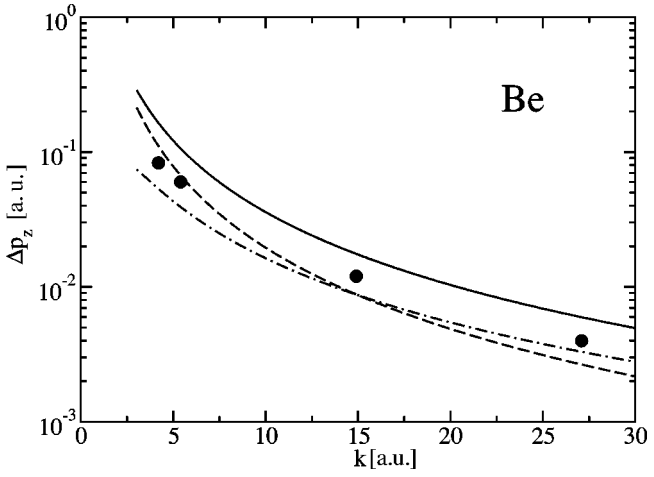

FIG. 2. Δp_z for Be ($r_s=1.87$) as a function of the transferred momentum k. Dashed curve, $(\Delta p_z)_{sp}$; dash-dotted curve, $(\Delta p_z)_{pl}$; solid curve, the sum. Dots: the plasmon-pole calculation of Ref. 14.

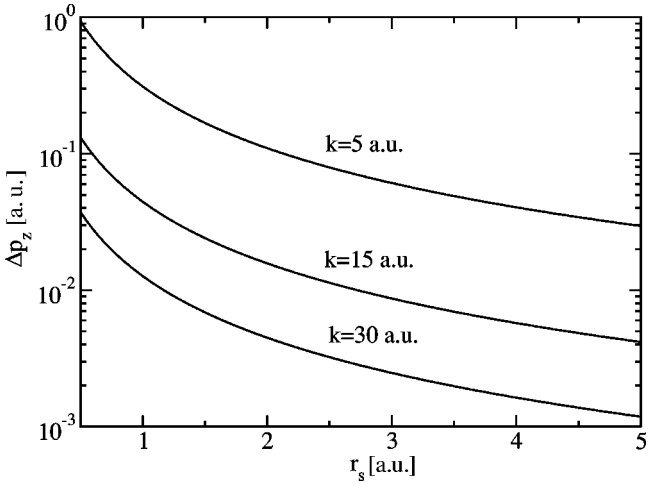

FIG. 3. Δp_z as a function of r_s for different momentum transfers k.

of the contribution from single-particle excitation has also been demonstrated by Quinn.[23] Our lifetime estimate agrees within about 30% with that of Lindhard[16] in the momentum regime of Figs. 1 and 2. Figure 3 demonstrates the material dependence of the smearing effect, i.e., Δp_z as a function of r_s for different momentum transfers k. It confirms the tendency found in Ref. 14 that for constant momentum transfer k the smearing decreases with increasing r_s.

Finally, we demonstrate that the lifetime can be shortened considerably by core-state excitation. This can be accounted for by either calculating the dynamic structure factor, which is proportional to the generalized oscillator strength,[34] for each core state separately, or one establishes a local plasma frequency $\omega_p(r)$ according to the locally varying electron density $n(r)$ in an atom, a procedure early developed by Lindhard and Scharff[35] in course of stopping power theories and later applied to the calculation of IMFP's by Ritchie and co-workers.[24,36] Then, by a kind of phase-space averaging,[35] one writes

$$\Gamma(k)=\int_{\Omega_{WS}}\frac{d^3\mathbf{r}}{\Omega_{WS}}\Gamma_j[k;r_s(r)], \tag{22}$$

where Ω_{WS} is the volume of the Wigner-Seitz cell. Evaluating $\Gamma(k)$ in case of core states we have assumed an atomic heliumlike core for Li and a neonlike one for Na and used for the calculation of $n(r)$ the Roothaan-Hartree-Fock wave functions of Bunge *et al.*[37] Figure 4 shows $k^2\Delta p_z$ due to inverse lifetime contributions from the jellium and the core states separately. Figure 4(a) holds for lithium, Figure 4(b) for sodium. The small step in case of core excitation for Na at $k=8.87$ a.u. is due to the neglection of the K-shell contribution, which cannot be excited for smaller k. Both parts indicate a significant lifetime reduction due to core-state excitation. The increase of Δp_z is approximately 40% in case of Li and 70% for Na. We mention that core contributions are not accounted for by the G_0W_0 calculations of Refs. 11 and 14. With respect to $(e,2e)$ experiments we estimate for an electron energy of 20 keV still a final state inverse lifetime Γ_e of about 0.9 eV for both Li and Na.

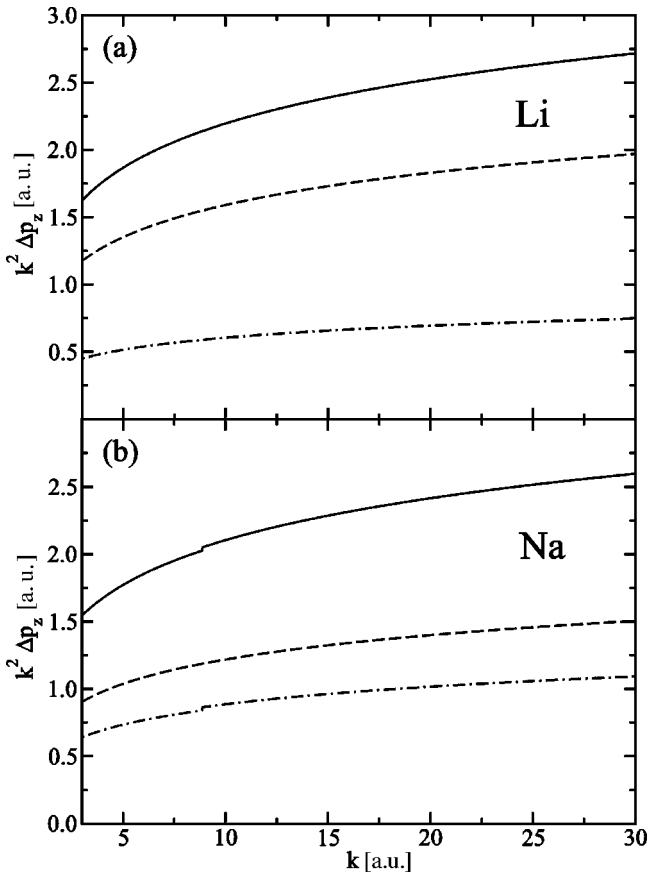

FIG. 4. $k^2\Delta p_z$ due to inverse lifetime contributions from the jellium (dashed curve), the core states (dash-dotted curve), and the sum (solid curve). (a) holds for Li, (b) for Na.

IV. CONCLUSION

Even if Luttinger's theorem of a sharp Fermi surface holds, neither Compton scattering nor $(e,2e)$ experiments or photoemission will be able to measure a sharp edge due to lifetime broadening caused by final state interactions of the ejected electron. The comparison of our extremely simple calculation with the by far more elaborate work of Refs. 11 and 14 has shown that it is able to reproduce quantitatively the influence of lifetime broadening. The background for this simplification is the subdivision of the problem in hard and soft collisions, or in other words electron-hole pair production and the excitation of the plasmon resonance. This allows the use of different approximations for the dielectric function. On the other hand there are certainly deficiencies of this simple treatment since it does consider neither exchange nor dynamically induced correlations in the electron gas, where the most prominent one is particle-hole pair interaction with plasmons,[25] which is accounted for in a full self-energy calculation. The neglection of this contribution would, in principle, underestimate lifetime broadening, but as it is seen from the work of Soininen *et al.*[14] the spectral weight of the corresponding satellite diminishes strongly with increasing momentum transfer k and becomes almost negligible for $k > 5$ a.u. The estimated inverse lifetimes Γ are considerably smaller than the plasmon energy ω_p which is approximately the energy difference between the quasiparticle peak and the first satellite. Thus, no strong quasiparticle (plasmon-pair) mixing is expected, which would violate the quasiparticle picture.

We emphasize that this final state lifetime broadening accounts only partly for the smearing of the Fermi break. There remains an additional broadening that is apparently independent of the recoil energy and seems to be an intrinsic problem of considerably stronger electron-electron correlations than predicted up to now by many-body theory.[7,11] To account for these correlations Barbiellini and Bansil[38] have recently calculated electron momentum densities by using BCS-like many-body wave functions in which individual elements involve singlet electron pairs (''geminals'') rather than one-particle orbitals. On the other hand, Schülke *et al.*[39] have shown that in case of Li a careful treatment of the Lam-Platzman correction[40] including recent but, nevertheless, conventional electron correlation schemes shortens the gap between experiment and theory.

Our extremely simple approach treats lifetime broadening as an incoherent process. Thus the question arises what happens for targets with thicknesses less than the IMFP λ_k as they are used in $(\gamma,e\gamma)$ or $(e,2e)$ experiments.[28] Can lifetime broadening be suppressed in this case? Or is the reverse true, i.e., does the quasiparticle lose its phase memory if it leaves the solid and becomes a free particle? Then, the role of λ_k would be overtaken by the foil thickness d and the lifetime is given by d/v_k. For $d=10$ nm and 25 keV electrons we estimate a quasiparticle lifetime of 4.6 a.u. or a broadening of 5.9 eV, by far larger than the experimental energy resolution.[28]

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