Penetration of vortices into the ferromagnet/type-II superconductor bilayer

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Vortex structures in the ferromagnet/type-II superconductor bilayer are investigated when the ferromagnet has domain structure and perpendicular magnetic anisotropy. It is found that two equilibrium vortex structures can be realized: straight vortices with alternating directions corresponding to the direction of the magnetization in the ferromagnetic domains and vortex semiloops connecting the ferromagnetic domains with opposite direction of the magnetization. These states are separated by an energy barrier. The values of the critical magnetization for the formation of these vortex structures are determined.

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I. INTRODUCTION

Interaction between a ferromagnet and vortices in type-II superconductors results in many interesting phenomena. The interplay between the regular pinning array of ferromagnetic dots and the periodic vortex lattice in type-II superconductors gives rise to pronounced commensurability effects in the critical current density as a function of the applied magnetic field.[1–4] Using scanning Hall probe microscopy it has been shown that single flux quanta with opposite polarity, which can be considered as an induced vortex-antivortex pair, are induced in the superconductor at the opposite poles of the dots.[5] The vortices created by the external field are preferentially pinned at the pole of the magnetic dot where a vortex of opposite polarity exists.[5] A local mixed state induced by a small ferromagnetic particle in a YBaCuO film has been studied obtaining evidence for entry of vortex semiloops and further breaking of these semiloops into vortex-antivortex pairs.[6] The giant vortex state around a magnetic dot which is embedded in a superconducting film was predicted in Ref. 7.

Much attention has recently been paid to superconductor/ferromagnet bilayers (SFB's). The onset of superconductivity in such structures[8] and its influence on the magnetic order[9–12] have been intensively studied. The ferromagnetic resonance measurements have shown that in Nb-Fe bilayers with thin Fe layers (10–15 Å) the average magnetic moment started to decay at T_c. This effect was explained by formation of a nonhomogeneous magnetic order (cryptoferromagnetic state).[10] It was found also that for a sufficiently thin ferromagnetic layer the domain structure can be fully suppressed (the period of the domain structure tends to infinity) and a single-domain state is realized.[11] An opposite effect was predicted for a thick ferromagnetic layer: below T_c the domains shrink due to expulsion of the magnetic field from the superconductor.[11,12]

The vortex properties in SFB's have been also investigated. It was found that the magnetostatic tangential field has a maximum near a domain wall and if the maximum field exceeds the lower critical field H_{c1} vortices can penetrate into a superconducting layer near the domain wall.[13] The domain wall can also pin vortices, and the interaction between the magnetic domain wall and a single vortex have been determined in Ref. 14. A two-dimensional vortex state has been considered in a SFB formed by a thin ferromagnetic film ($d_m \ll L$) having perpendicular magnetic anisotropy and a thin superconducting film ($d_s \ll \lambda_L$).[15] Here L is the period of the domain structure in the ferromagnet, λ_L is the London penetration depth and d_m and d_s are the thicknesses of the ferromagnet and the superconductor, respectively. In Ref. 15 it was shown that the SFB is split onto domains with alternating magnetization and vortex magnetic flux directions so that the direction of the magnetization in the ferromagnet and the magnetic flux direction in the superconductor coincide. Because of the attraction between negative and positive vortices in different domains the distribution of the vortices inside each domain is highly inhomogeneous.

In the present paper we consider a SFB consisting of a thick magnetic layer ($d_m \gg L$) with the magnetization perpendicular to the layer on top of a thick superconducting layer ($d_s \gg \lambda_L$). The magnetic field generated by the ferromagnet is strongly inhomogeneous in this case and the dependence of the vortex energy on its position is important for the vortex penetration process. In contrast to Ref. 15 dealing with the structures with many vortices per a domain, our goal was to find conditions for penetration of a first vortex into the domain. The first vortex can penetrate to the superconducting layer either from a domain center or around a domain wall. We have found the critical values of the magnetization for these two processes. The critical magnetization is determined by the condition that a state with a single vortex per one domain has an energy less than that of the Meissner state. In the case of vortex penetration from a domain center this is a straight vortex line crossing the superconducting layer. If vortices penetrate from around a domain wall, the first vortex is a semiloop, which connects two domains near the wall between them. The critical magnetizations for two vortex configurations can be larger or smaller with respect to each other. This means that both vortex configurations can be considered as those, which determine the transition to the mixed state of the superconducting layer. Our analysis refers to an *equilibrium* mixed state, which corresponds to an absolute energy minimum. However, even a vortex structure

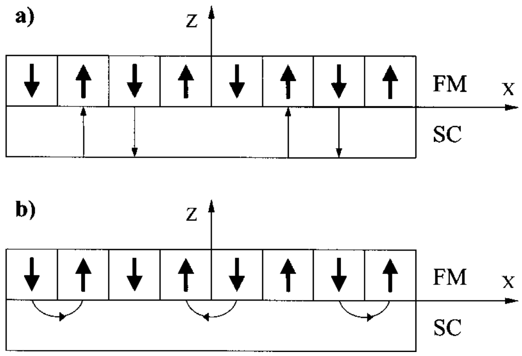

FIG. 1. Ferromagnet/type-II superconductor bilayer. Different vortex structures are shown by thin arrows: straight vortices (a) and semiloops (b). The magnetization vectors in the domains are shown by thick arrows.

with a larger energy can be metastable, and this should be taken into account considering the vortex penetration in reality. In particular, straight vortices in domain centers cannot appear without a semiloop around the domain wall as a transient state. The latter can be separated from the structure with straight vortices with a potential barrier, which can stabilize the semiloop configuration even if it does not correspond to an absolute minimum of the energy.

II. FIELD DISTRIBUTION IN THE MEISSNER STATE OF A SUPERCONDUCTOR

We consider a structure consisting of a ferromagnet having the thickness d_m and perpendicular magnetic anisotropy, on a surface of a superconductor with thickness d_s (Fig. 1). The easy magnetization axis is parallel to the z axis (the ferromagnet fills the space with $z>0$) and the boundary between the ferromagnet and the superconductor is along the xy plane. No exchange of electrons between the ferromagnet and the superconductor is assumed, i.e., they are coupled only by the magnetic field $\mathbf{H}$. We assume that the ferromagnet has the domain structure formed by stripes of width L parallel with the y axis. The lengths d_m, d_s, and L are presumed large compared with the London penetration depth λ_L and the domain wall thickness δ, which is the smallest length in our consideration. The influence of the superconductivity on L has been discussed in Refs. 11 and 12. In this paper we will not consider this effect, assuming that L is a fixed parameter. At $L \gg \delta$, a good approximation of $M(x)$ is a steplike function $M(x)=\pm M_0$ along the x axis inside the domains. The Fourier expansion of this function is

$$M(x)=\frac{-4M_0}{\pi}\sum_{k=0}^{\infty}\frac{\sin(2k+1)Qx}{(2k+1)}, \tag{1}$$

where $Q=\pi/L$. With account of the Maxwell equation $\nabla\cdot\mathbf{B}=\nabla\cdot(\mathbf{H}+4\pi\mathbf{M})=0$, the field $\mathbf{H}$ is induced by alternating magnetic charges $\nabla\cdot\mathbf{M}$ on the two surfaces of the ferromagnet. Thus the distribution of $\mathbf{H}$ inside the ferromagnet and in the empty space above satisfies the equations

$$\nabla\cdot\mathbf{H}=-4\pi[\delta(z)-\delta(z+d_m)]M(x), \tag{2}$$

$$\nabla\times\mathbf{H}=0. \tag{3}$$

Inside the superconductor the distribution of $\mathbf{H}$ is described by the London equation

$$\nabla^2\mathbf{H}-\lambda_L^{-2}\mathbf{H}=0. \tag{4}$$

The distributions of $\mathbf{H}$ inside the ferromagnet and inside the superconductor must satisfy the boundary conditions that the tangential component of $\mathbf{H}$ and the normal component of $\mathbf{B}$ are continuous across the interface between the ferromagnet and the superconductor.[16]

The solution for the magnetic field $\mathbf{H}_f$ inside the ferromagnet can be found as $\mathbf{H}_f=-\nabla\varphi$ where φ satisfies the Laplace equation $\nabla^2\varphi=0$. Taking into account that $d_m \gg L$ one can neglect the interaction between the top and the bottom boundary of the ferromagnet and find H_f for the bottom and the top interface of the ferromagnet separately. The solution for the border with vacuum (the top of the ferromagnet) was obtained in Ref. 16. The solution inside the ferromagnet near the bottom interface with the superconductor can be expressed with the Fourier series

$$\varphi=\sum \varphi_q \exp(-qz+iqx), \tag{5}$$

where $q=\pm(2k+1)Q$.

The solution for the magnetic field $\mathbf{H}_s$ inside the superconductor can also be written as the Fourier series

$$\mathbf{H}_s=\sum \mathbf{H}_q \exp(-q_z z+iqx), \tag{6}$$

where $q_z^2=q^2+\lambda_L^{-2}$. Then, using the continuity condition for the tangential component of $\mathbf{H}$ and Eqs. (2) one gets

$$H_{sz}=-16M_0\sum_{k=0}^{\infty}\frac{\sin(2k+1)Qx}{(1+q_z/q)(2k+1)}\exp(q_z z), \tag{7}$$

$$H_{sx}=-16M_0\sum_{k=0}^{\infty}\frac{q_z\cos(2k+1)Qx}{q(1+q_z/q)(2k+1)}\exp(q_z z), \tag{8}$$

$$H_{fz}=16M_0\sum_{k=0}^{\infty}\frac{q_z\sin(2k+1)Qx}{q(1+q_z/q)(2k+1)}\exp(-qz), \tag{9}$$

$$H_{fx}=-16M_0\sum_{k=0}^{\infty}\frac{q_z\cos(2k+1)Qx}{q(1+q_z/q)(2k+1)}\exp(-qz). \tag{10}$$

These expression agree with those derived by Stankiewicz *et al.*[11] Figures 2 and 3 show the distribution of the magnetic field in the ferromagnet-superconductor bilayer calculated from Eqs. (7)–(10).

In the next section we consider the vortex structures when the parameter $\lambda_L/L \ll 1$. In this limit one can neglect penetration of the magnetic field into the superconductor. Then the $\mathbf{H}$ distribution can be found using the image charges with respect to the ferromagnet-superconductor interface, which plays a role of a mirror.[11–14] The $\mathbf{H}$ distribution can be found

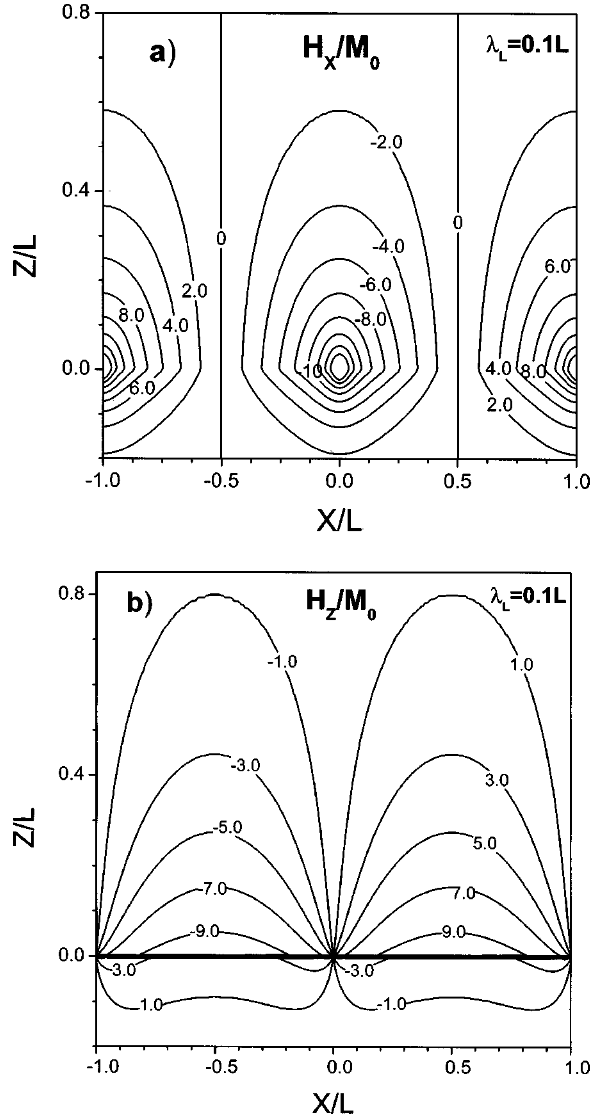

FIG. 2. Distribution of the H_x (a) and H_z (b) field components in the Meissner state of the superconductor for $\lambda_L/L=0.1$. Distances are given in units of the period of the domain structure L.

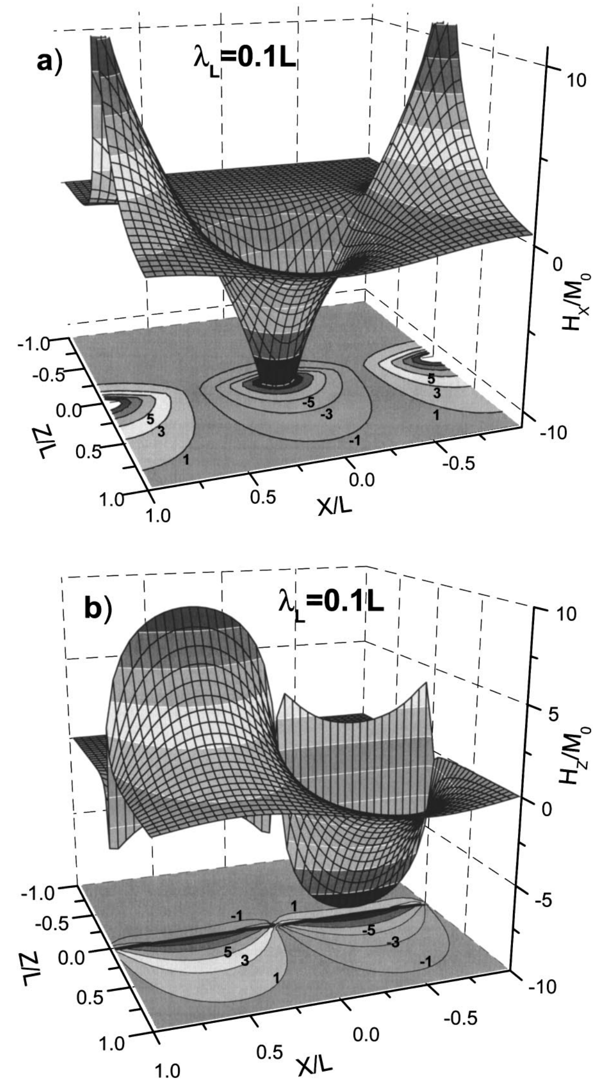

FIG. 3. Surface-contour plots of the H_x (a) and H_z (b) field components in the Meissner state of the superconductor for $\lambda_L/L=0.1$. Distances are given in units of the period of the domain structure L.

exactly in this limit using the method of complex variables.[12,13] According to this exact solution the normal and the tangential field at the interface $(z=0)$ for $0<x<L$ are $H_{fz}=4\pi M_0$ and

$$H_{fx}=8M_0\ln\tan(\pi x/2L), \tag{11}$$

correspondingly. These expressions follow from Eqs. (9) and (10) at $z=0$ in the limit $\lambda_L\to 0$, if one takes into account that $(0<x<\pi)$

$$\sum_{k=0}^{\infty}\frac{\cos(2k+1)x}{2k+1}=-\frac{1}{2}\ln\left(\tan\frac{x}{2}\right),$$

$$\sum_{k=0}^{\infty}\frac{\sin(2k+1)x}{2k+1}=\pi/4. \tag{12}$$

III. THE MIXED STATE IN SFB

In general the total energy of the bilayer can be written in the form

$$U=U_{\mathrm{SV}}+U_{\mathrm{VV}}+U_{\mathrm{VM}}+U_{\mathrm{MM}}+U_{\mathrm{DW}}, \tag{13}$$

where U_{SV} is the energy of single vortices, U_{VV} is the vortex-vortex energy of interaction, U_{VM} is the interaction energy between the vortices and the magnetic field generated by the domain structure, U_{MM} is the self-interaction energy of the ferromagnet, and U_{DW} is the surface tension energy of the domain walls. The latter is important only if we look for the equilibrium domain structure period, and further we shall ignore it. The rest terms all together are present in the energy

$$U=\frac{1}{8\pi}\int[\mathbf{H}^2+\lambda_L^2(\nabla\times\mathbf{H})^2]d^3r, \tag{14}$$

which includes the magnetic energy and the kinetic energy $\propto\lambda_L^2$ of supercurrents, the latter being present only inside the superconductor. In the mixed state the magnetic field $\mathbf{H}=\mathbf{H}_f+\mathbf{H}_v$ is generated not only by the magnetic charges $\nabla\cdot\mathbf{M}$ (the field $\mathbf{H}_f$) but also by vortex lines in the superconductor (the field $\mathbf{H}_v$). For simplicity we use here only one notation $\mathbf{H}_f$ for the whole magnetic field generated by the magnetic charges of the ferromagnet. For the Meissner state discussed in the previous section we assumed that the this field corresponded to $\mathbf{H}_f$ for $z>0$ and $\mathbf{H}_s$ for $z<0$ [see Eqs. (7)–(10)].

The ferromagnet self-interaction energy $U_{\rm MM}$ is due to the contribution $\propto H_f^2$ in Eq. (14). This energy is relevant only if we want to determine the equilibrium domain structure. But we assume it to be known and shall neglect $U_{\rm MM}$ later on. Our goal is to estimate the critical values of the magnetization for the transition to the mixed state in the superconductor. As usual, this transition is determined by the condition that the energy of a single vortex inside the superconductor becomes negative and the *first* vortex appears in the superconductor. This means that the vortex density is negligible near the transition, and one may neglect the vortex-vortex interaction $U_{\rm VV}$ calculating the vortex self-energy contribution $\propto H_v^2$. The energy of the first vortex is $U_{\rm SV}=\phi_0 H_{c1} l/4\pi$, where l is the vortex length, ϕ_0 is the flux quantum, $H_{c1}=(\phi_0/4\pi\lambda_L^2)\ln(\lambda_L/\xi)$ is the lower critical field of the superconductor, and ξ is the coherence length. This energy takes into account only fields and supercurrents *inside* the superconductor. Meanwhile the vortex line generates also the magnetic field $\mathbf{H}_v=-\nabla\varphi_v$ outside the superconductor. The potential φ_v is similar to an electrostatic potential from a point charge located at the point $(r=0,z=0)$, where the vortex line exits from the superconductor (see Ref. 17, and references therein):

$$\varphi_v=\frac{\phi_0}{2\pi\sqrt{r^2+z^2}}. \tag{15}$$

Here $\mathbf{r}$ is the two-dimensional (2D) position vector in the interface plane. The integral $\int(\nabla\varphi_v)^2 d^3r$, which determines the energy of this field, is divergent at small distance. The divergence is cut off by the London penetration depth λ_L and eventually this yields the energy $\sim\phi_0^2/\lambda_L$, which is less by a factor λ_L/l than the vortex energy inside the superconductor proportional to its length l.

Another relevant energy is the interaction energy between the vortices and the magnetic field generated by the magnetic charges in the ferromagnet

$$U_{\rm VM}=\frac{1}{4\pi}\int(\mathbf{H}_v\cdot\mathbf{H}_f+\lambda_L^2\nabla\times\mathbf{H}_v\cdot\nabla\times\mathbf{H}_f)d^3r, \tag{16}$$

where $\mathbf{H}_v$ is the field induced by the vortices and $\mathbf{H}_f$ is the field induced by the magnetic charges [Eqs. (7)–(10)]. The energy $U_{\rm VM}$ can be divided into the integrals over the volume of the ferromagnet and the superconductor. The latter integral is proportional to $\phi_0 H_{sz}\lambda_L\propto\phi_0 M_0\lambda_L^2/L$. This contribution is small in the limit $\lambda_L\ll L$. Inside the ferromagnet the vortex field $\mathbf{H}_v=-\nabla\varphi_v$ is determined by Eq. (15). Transforming the integral over the ferromagnet by parts we arrive to the energy expression, which contains the magnetic charges $\epsilon_i\phi_0/2\pi$ of the vortices located at the vortex tips

$$U_{\rm VM}=-\sum_i\frac{\epsilon_i\phi_0\varphi(\mathbf{r}_i)}{4\pi}, \tag{17}$$

where $\epsilon_i=\pm1$ depends on the direction of the vortex flux and $\mathbf{r}_i$ is the position vector of the ith vortex tip in the interface plane. The magnetic potential $\varphi(\mathbf{r}_i)$ can be obtained by integration of Eq. (11):

$$\varphi(\mathbf{r}_i)=\int_0^{x_i}H_{fx}dx=\pm 8M_0\int_0^{x_i}\ln\tan(\pi x/2L)dx, \tag{18}$$

where x_i is the distance from the nearest left domain wall, the signs + and − are for the domains with the negative and the positive magnetization, respectively. With a proper choice of the sign ϵ_i of the vortex circulation every term in the energy $U_{\rm VM}$ is negative.

The energy $U_{\rm VM}(x)$ determines the force on the vortex, which is trying to shift the vortex along the axis x:

$$F_x(x)=-\frac{dU_{\rm VM}(x)}{dx}=\frac{\epsilon_i\phi_0}{4\pi}H_{fx}(x). \tag{19}$$

On the other hand, the Maxwell equations yield that $H_{fx}=-4\pi J_s(x)/c$, where the surface current $J_s(x)=\int j_s(x,z)dz$ is the integral over the density $j_s(x,z)$ of the Meissner screening currents flowing parallel to the axis y in the surface layer of thickness λ_L. Then one can see that the force $F_x(x)$ on the vortex is in fact the position-dependent Lorentz force exerted by the Meissner currents. This form of the force on the vortex has been also used in Ref. 18, where the problem of the vortex penetration in an inhomogeneous magnetic field has been considered. Further we shall consider two possibilities for the vortex penetration comparing the energy of the following two vortex configurations: a vortex-antivortex pair placed in neighboring domains and a vortex semiloop placed around a domain wall.

A. Penetration of a vortex from a domain center

In this case straight vortices with alternating directions corresponding to the direction of the magnetization in the ferromagnetic domains are expected [Fig. 1(a)]. In the considered vortex structure the vortex length is $l=d_s$, and the self-energy of the vortices does not depend on their position, $U_{\rm SV}=\phi_0 H_{c1} d_s/4\pi$, giving a constant contribution to the total energy. We consider the vortex-antivortex pair with the vortices placed symmetrically around the domain wall with separation $2x$. Figure 4 shows the dependence of normalized energy of the vortex-antivortex pair $U_v(x)/U_{v0}$ obtained from Eq. (17) for different values of M_{c1}/M_0, where $U_v(x)=[U_{\rm VM}(x)+2U_{\rm SV}]$ and $U_{v0}=4M_0\phi_0 L/\pi$. As can be seen from this picture the vortices have the minimum energy at the center of the domain. Increasing of M_0 results in lowering of the energy without shifting the minimum position. The minimum vortex energy U_v becomes negative if $M_0>M_{c1}$, where

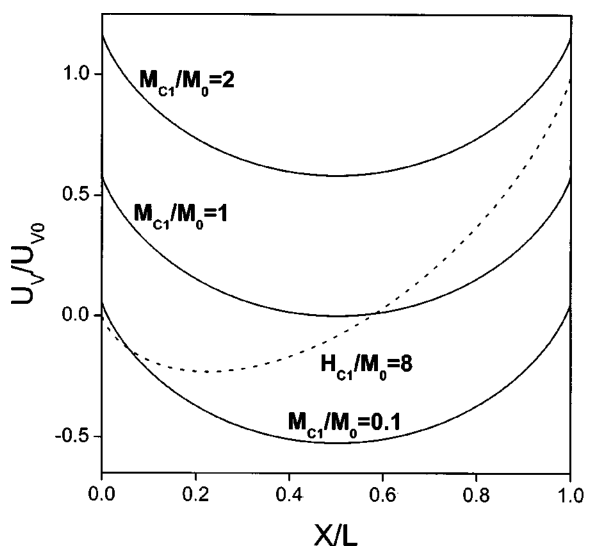

FIG. 4. Dependences of the energy $U_v(x)/U_{v0}$ of the vortex-antivortex pair at different M_{c1}/M_0 (solid lines) and the semiloop (dashed line) at $H_{c1}/M_0=8$.

$$M_{c1}=\frac{H_{c1}d_s}{8\alpha L}, \tag{20}$$

with $\alpha=-\int_0^{0.5}\ln\tan(\pi x/2)\,dx=0.583$. The critical magnetization M_{c1} determines the transition to the equilibrium mixed state. For $d_s\ll L$, M_{c1} can be considerably less than H_{c1}.

Let us show that the interaction $U_{\rm VV}$ between vortices in neighboring domains is not essential in the limit of small λ_L. The energy $U_{\rm VV}$ can be considered as the interaction between the magnetic charges located at the points where the vortices exit from the superconductor.[17] It can be written as

$$U_{\rm VV}=\sum_{i,j}\frac{\phi_0^2}{8\pi^2|\mathbf{r}_i-\mathbf{r}_j|}, \tag{21}$$

where $\mathbf{r}_i$ is a two-dimensional vector in the plane $z=0$ determining the positions of the vortices. This energy $\sim\phi_0^2/L$ is by a factor $\lambda_L^2/d_sL\ll 1$ smaller than the self energy of the vortices $U_{\rm SV}=\phi_0H_{c1}d_s/4\pi$ and therefore can be neglected.

For $M_0>M_{c1}$ many vortices are formed inside the superconductor and their interaction becomes important. Since there is a potential well for vortices located in the domain center, one can expect that the equilibrium vortex distribution has a shape of a dome placed in the center of the domain. At $M_0\gg M_{c1}$ the dome expands reaching the domain border. Then the attraction of the vortices and antivortices in the neighboring domain becomes important. This energy can be calculated in the way similar to the thin SFB.[15]

The described process of the vortex penetration in SFB with $d_s\ll L$ is similar to that in a flat superconductor in perpendicular magnetic field, where due to demagnetization effects the field distribution is also strongly inhomogeneous.[18] The vortex has a minimum of energy in the center of the superconductor, or in the domain center in our case (see Fig. 4 in our paper and Fig. 1 in Ref. 18). In a flat superconductor the vortex energy in the center is zero at $H_{\rm eq}=H_{c1}d/2w$, where d is the thickness and w the semiwidth of the sample. Equation (20) for M_{c1} looks similar to the formula for $H_{\rm eq}$ if we substitute d_s and L by d and w, respectively, the difference is only a numerical factor. Also the dome vortex profile is realized in a flat superconductor.[18] However, the current distribution $j_s(x)$ is different in these cases: $j_s(x)\propto\ln(L/x)$ near the domain border of SFB and $j_s(x)\propto 1/x^{1/2}$ near the boundary of a flat superconductor[18] (in the latter case x is the distance from the boundary). This results in different conditions for the nonequilibrium vortex penetration to the center of the superconductor (see Sec. IV).

B. Penetration of a vortex semiloop from a domain wall

The vortex configuration for this case is shown on Fig. 1(b). As in the previous case, most important are the energy of the interaction between the vortex and the magnetic field of the ferromagnet $U_{\rm VM}$ and the self-energy of the vortex $U_{\rm SV}$. The energy $U_{\rm SV}$ depends on the length and the shape of the vortices. We shall consider the "macroscopic" vortex loops with length significantly exceeding λ_L. The shape of the loop can be approximated by a straight line with length $2x$ connected with the surface by two tips at opposite ends with length of larger but of the same order as λ_L. The elastic energy of the tips with curvature radius of order λ_L cannot exceed the total line tension energy $\sim\phi_0H_{c1}\lambda_L$, and if $x\gg\lambda_L$, the total energy of the loop is approximately given by the energy of the straight segment of the loop $U_{\rm SV}\approx 2\phi_0H_{c1}x/4\pi$. The energy $U_{\rm VM}$ is given by Eq. (17) as before, and for a single semiloop we have two terms in this expression with opposite vortex signs (two tips of the semiloop). Finally the single-vortex energy is

$$U_v=U_{\rm SV}+U_{\rm VM}=\frac{\phi_0}{2\pi}\left[H_{c1}x-\int_0^{x_i}H_{fx}(x)\,dx\right]. \tag{22}$$

If the ferromagnetic layer is absent and the superconductor has a border with vacuum, where a constant magnetic field $\mathbf{H}$ is applied, the field H_{fx} in Eq. (22) must be replaced by H. Then the total energy is $U_v=(\phi_0/4\pi)(H_{c1}-H)2x$. This yields a trivial result that the energy of the vortex state with respect to the Meissner state is positive at $H<H_{c1}$ and negative at $H>H_{c1}$. Turning back to the case of the ferromagnet with domains, the tangential field is not uniform and is given by Eq. (11). The energy U_v has a minimum at $H_{c1}=H_{fx}(x_0)$, where according to Eq. (11) $x_0=(2L/\pi)\arctan[\exp(-H_{c1}/8M_0)]$. The energy in the minimum is negative if H_{fx} is a decreasing function of x. This means that the semiloop configuration is stable if the field $H_{fx}(x)>H_{c1}$ at $x\to 0$. However, one should remember that our calculation was based on the assumption that $x\gg\lambda_L$. We can use the equality $x_0\approx\lambda_L$ as a condition that a vortex semiloop becomes a stable configuration. This yields the critical magnetization for formation of the semiloop:

$$M_{cl}=\frac{H_{c1}}{8\ln(2L/\pi\lambda_L)}. \tag{23}$$

The dashed line in Fig. 4 is the dependence $U_v(x)/U_{v0}$ obtained from Eq. (22) at $H_{c1}/M_0=8$. As can be seen from this figure and from comparison between Eqs. (20) and (23) the vortex semiloop has much higher energy than the vortex-antivortex pair if $d_s \ll L$.

For $M_0 > M_{cl}$ many loops with different size can be formed inside the superconductor and their interaction may be important. Vortices fill the superconducting layer from domain walls to domain centers. The difference between the structures, which were considered in the present and the previous subsection, is that they have different points of higher vortex concentration. This can be detected experimentally by low energy muon measurements.[19]

IV. NUCLEATION OF THE VORTICES IN SFB

Thus, at $M_0 > M_{c1}$ or M_{cl} there are two possible vortex configurations: vortex semiloop, located at the walls, and straight vortices at the centers of the domains. If $M_{c1} < M_{cl}$ the configuration with vortices in domain centers is energetically more preferable. However, to determine which vortex structure would be formed we should consider the process of the vortex nucleation. One can compare our case with the usual situation when the uniform external field is applied to a superconductor where two experimental situations are possible: magnetizing the sample after cooling in zero field (ZFC) or cooling the sample in a magnetic field (FC). If the external field is applied after ZFC there are two potential barriers preventing the vortices from entering the sample: the Bean-Livingston barrier (BL) (Ref. 20) and the geometrical barrier.[18] If the energy of the BL barrier or the geometrical barrier exceeds the thermal energy the situation can be strongly nonequilibrium, i.e., there are no vortices for fields higher than H_{c1}. In contrast, in the FC experiment the vortices are always formed if $H > H_{c1}$ and the density of the vortices $n \approx H/\phi_0$.[21]

For SFB's, a situation analogous to the ZFC procedure is realized when the Curie temperature is lower than the superconducting transition temperature T_c. Then vortex loops (with sizes $\sim\lambda_L$) should be formed first near the domain walls where the magnetic field of the ferromagnet ($\sim 4\pi M_0$) has the maximum (see Figs. 2 and 3). At the initial stage the process of the vortex nucleation is determined by the BL barrier[20,21] and the usual formulas[22] are applicable with $H = 8M_0 \ln(2L/\pi\delta)$. The height of the energy barrier is $U_{\rm BL} = (\pi+2)\phi_0 H_{c1}\lambda_L/4\pi$ (Ref. 22) in fields of the order of H_{c1}. The energy $U_{\rm BL}$ decreases when the magnetic field increases because the position of the saddle point moves to the surface.[22] But because the magnetic field of the ferromagnet is inhomogeneous the vortex nuclei cannot transform to straight vortices parallel to the surface. Instead, the vortex semiloops with the length $\approx x_0 + 2\lambda_L$ are formed. In contrast to the case of superconductor in uniform external field, where the configuration of the vortex semiloops has the saddle point in the energy profile, in SFB the vortex semiloop has the minimum of the energy. If $M_{c1} < M_{cl}$ ($d_s \ll L$) this is a local minimum. To reach the equilibrium configuration with straight vortices inside domains the vortex semiloop should be separated in two vortices, i.e., to expand to the lower boundary of the superconductor. This process requires an energy of the order of the self-energy of the two vortices $U_{\rm SV} = H_{c1}\phi_0 d_s/2\pi$, i.e., a second energy barrier appears. This barrier does not depend on the magnetization of the ferromagnet. It is determined by the geometry (the dependence on the size) and the equilibrium properties (the dependence on H_{c1}) of the superconductor. This is similar to the geometrical barrier for the penetration of the vortices in flat superconductors in perpendicular magnetic field with the value being proportional to the thickness of the superconductor.[18] Because $U_{\rm SV} \gg k_B T$ there should be no appreciable thermal activation over such an extended barrier for the considered values of the parameters and the vortex structure with semiloops near domain walls is formed. The geometrical barrier is suppressed at penetration magnetization M_{c1}^*. At $M_0 \approx M_{c1}^*$ the vortex semiloop near the domain wall transforms in two straight vortices of opposite polarity which move to the neighboring domain centers. Estimating from the equality between the Lorentz force at $x \sim d_s$ and the line tension force, which is $\sim\epsilon_0/d_s$,[18] we get $M_{c1}^* \sim H_{c1}/\ln(L/d_s)$. This is different from the value of the penetration field $H_p \sim H_{c1}(d/w)^{1/2}$ for a flat superconductor in perpendicular magnetic field in the ZFC procedure.[18]

If the Curie temperature is higher than the superconducting transition temperature the situation is similar to the FC procedure. Because H_{c1} is small near T_c the vortices should fill the whole space when the temperature decreases below T_c. After further decrease of the temperature the situation of $M_0 < M_{c1}$, M_{cl} can occur and vortices should exit from the superconducting layer. The BL barrier cannot effectively stop the exit of the semiloops when the magnetic field is lower than H_{c1}.[21,23] In contrast, the straight vortices inside the domains can be trapped because $U_{\rm SV} \gg U_{\rm BL}, k_B T$.

V. CONCLUSIONS

The Meissner and the vortex states of the ferromagnet/type-II superconductor bilayer were investigated when the ferromagnet has domain structure and perpendicular magnetic anisotropy. We have calculated the values of the critical magnetization for the formation of two vortex structures: (i) straight vortices with alternating directions corresponding to the direction of the magnetization in the ferromagnetic domains and (ii) vortex semiloops connecting the ferromagnetic domains with opposite direction of the magnetization. Different processes of the vortex nucleation are discussed.

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