Kohn-Luttinger pseudopairing in a two-dimensional Fermi liquid

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We consider possible superconducting instabilities in a two-dimensional Fermi system with short-ranged repulsive interactions between electrons. The possibility of an unusual superconducting paring due to the Kohn-Luttinger mechanism is examined. The quasiparticle scattering amplitude is shown to possess an attractive harmonic in second-order perturbation theory for finite values of the energy transfer. The corresponding singularity in the pairing vertex leads to a superconducting pairing of the electron excitations with finite energies. We identify the energy transfer in the Cooper channel as the binding energy of the excited pair. At low enough temperatures, the Fermi system is a mixture of normal electron excitations and fluctuating *d*-wave Cooper pairs possessing a finite gap.

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I. INTRODUCTION

Superconductivity induced by mechanisms other than electron-phonon interactions has been of long-standing interest. Throughout the last decade there has been a continuing theoretical search for unconventional superconductivity mechanisms, particularly in two-dimensional systems. This interest was, indeed, motivated by interesting superconducting materials such as high-T_c cuprates, organic superconductors as well as by the studies of ^{3}He films. Currently, there is no full understanding of the physical processes responsible for the pairing in those systems.

The Kohn-Luttinger effect is one of the oldest as well as among the most appealing and elegant physical effects, which might be considered within this quest. Back in 1965, Kohn and Luttinger[1] showed that any three-dimensional electron system with repulsive interactions between particles was unstable against a superconducting transition at extremely low temperatures. The origin of the effect is that the screening of the bare interaction leads to the well-known Friedel oscillations in the electron density and to similar oscillations in the scattering amplitude. The renormalized interaction acquires a long-ranged oscillatory component. Thus, there appear some regions where the effective interaction is attractive. This leads to the formation of Cooper pairs with nonzero orbital momenta $l \neq 0$. However, straightforward calculations showed that the transition temperature was extremely low (the estimate of Kohn and Luttinger[1] was $T_c \sim 10^{-40}$ K for some realistic parameters of the fermion system). This extreme low value of T_c was one of the reasons why the effect has not been much studied in recent years.

In the early nineties, Kagan and co-workers obtained a number of interesting results within the Kohn-Luttinger theory[2] (such as cascade transitions, the Kohn-Luttinger effect in a three dimensional system with long-ranged Coulomb interaction, Kohn-Luttinger superconductivity in the Hubbard model, etc.). One of the interesting results was that the temperature of the superconducting transition derived in the pioneering paper[1] was shown to be underestimated due to the unjustified extrapolation in the expression valid for large orbital momenta down to the value $l = 1$. The transition temperatures calculated in Ref. 2 were higher than the original estimate but still too low to attract much attention.

One of the most natural issues to be explored has been the status of the Kohn-Luttinger theory in two dimensions. First of all, Kohn-Luttinger physics is about the formation of bound states. It is very natural to expect that in the lower dimensionality it is easier to form bound states (i.e., Cooper pairs). However, a simple calculation of the polarization operator leads to the disappointing result: no singularity exists in second-order perturbation theory. Namely, the polarization operator reads (we use units $\hbar = c = 1$ throughout the paper)

$$\Pi(\mathbf{q}) = \begin{cases} \nu & \text{if} \quad q < 2k_{\mathrm{F}} \\ \nu[1 - \sqrt{1-(2k_F/q)^2}] & \text{if} \quad q > 2k_{\mathrm{F}}, \end{cases} \tag{1}$$

where $\nu = m/(2\pi)$ is the density of states at the Fermi line, k_{F} is the Fermi momentum, and $\mathbf{q}$ is the momentum transfer in the Cooper channel ($q = 2p_F \sin\phi/2$, where ϕ is the scattering angle). Let us remember that the attractive harmonics in the scattering amplitude in the three-dimensional case comes from the well-known logarithmic Kohn's singularity $\Pi_{\mathrm{sing}}(\phi) = (1+\cos\phi)\ln(1+\cos\phi)$ which exists in three dimensions on both sides of the Fermi surface. As can be seen from Eq. (1), the singularity in two dimensions is one sided, which suggests that no straightforward Kohn-Luttinger effect should exist in two dimensions.

In 1993, Chubukov[3] showed that this simple scenario was not the complete story in two dimensions. A two-sided singularity exists, but to find it one should go beyond second-order perturbation theory. The corresponding transition temperature derived by Chubukov reads $T_c(l) \propto \exp[-l^2/2f_0^3]$, where f_0 is the dimensionless *s*-wave scattering amplitude. Having applied this result to a realistic experiment on ^{3}He-^{4}He mixture films, the numerical value was found as $T_c(l=1) = 10^{-4}$K.

Let us also mention a recent paper of Guinea *et al.*[4] in which the Kohn-Luttinger physics was phenomenologically incorporated in a model of high-T_c cuprates. Within this model the shape of the gap anisotropy has been explored as a function of doping.

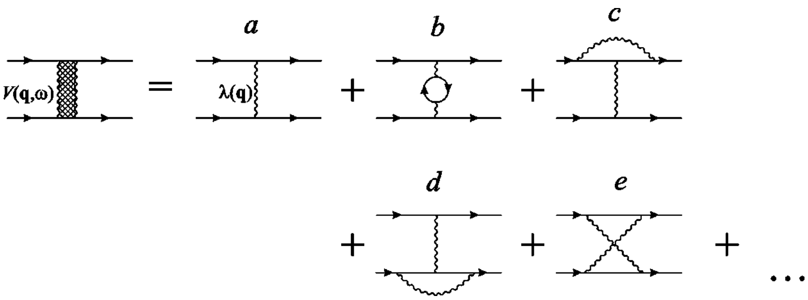

FIG. 1. Renormalization of the scattering amplitude by Friedel oscillations. Kohn-Luttinger theory. If the bare coupling is **q**-independent diagrams *b*, *c*, and *d* cancel each other out.

The main idea of the present paper is to search for an effective attractive interaction by taking into account the frequency dependence of the polarization operator, instead of going into higher order perturbation theory. The account for dynamical screening, as we shall see below, yields a two-sided singularity. Thus, we are looking for a dynamical Kohn-Luttinger effect rather than the original static pairing problem as in Refs. 1 and 3. Due to the energy dependence of the effective electron-electron coupling, the Cooper problem turns into an integral equation, similar to the Eliashberg equation in the strong coupling theory of superconductivity.[5]

Our paper is structured as follows. In Sec. II, we rederive the expression for the polarization operator as a function of momentum $\mathbf{q}$ and Matsubara frequency ω. Using this result, we formulate the Cooper problem and derive the corresponding Bethe-Salpeter equation for the pairing vertex $\mathcal{T}(\mathbf{q};\varepsilon,\varepsilon')$.

In Sec. III, we consider spherical harmonics of the effective interaction $\mathcal{V}_l(\omega)$ and show that *d* harmonic, which corresponds to the orbital momentum $l=2$, yields the strongest effective attraction.

In Sec. IV, we use the explicit expression for the interaction in the *d* channel and derive an integral equation for the pairing vertex. Studying this equation, we show that the pairing vertex may diverge if the incoming particles have high enough energies. We estimate the temperature at which the pairing with the typical binding energy of ω commences. We conclude that at low enough temperature the system is a mixture of low-lying electron excitations and fluctuating Cooper pairs. We estimate the temperature T_* at which the effect of this fluctuaing pairs becomes essential and may strongly change transport and thermodynamic properties of the system.

In Sec. V, we briefly discuss the case of long-ranged Coulomb interactions. We argue that Kohn-Luttinger physics strongly depends on the screening properties. In a purely two-dimensional system we do not expect any superconducting instability to survive. If transport is two-dimensional but screening is three-dimensional, the system is qualitatively described by our theory and Ref. 3.

II. COOPER PROBLEM

Let us start by calculating the effective electron-electron interaction $\mathcal{V}(\mathbf{q},\omega)$ (where $\omega=2\pi nT$ is the bosonic Matsubara frequency). In second-order perturbation theory there are four diagrams to be considered, which are shown in Fig. 1. If the bare potential $\lambda(\mathbf{q})$ is short ranged the diagrams *b*, *c*, and *d* cancel each other out, and only *e* contributes to the renormalized interaction. The latter diagram is functionally identical to *b*, but depends on $p+p'$ rather then on $p-p'$, where $p=(\mathbf{p},\varepsilon)$. Thus, knowing the two-dimensional polarization operator, we readily obtain the total effective electron-electron coupling.

The polarization operator is defined as

$$\pi(\mathbf{q},\omega_m)=T\sum_{\varepsilon_n}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\mathcal{G}_{\varepsilon_n+(\omega_m/2)}\left(\mathbf{k}+\frac{\mathbf{q}}{2}\right)\mathcal{G}_{\varepsilon_n-(\omega_m/2)}\left(\mathbf{k}-\frac{\mathbf{q}}{2}\right), \tag{2}$$

where $\mathcal{G}_\varepsilon(\mathbf{k})=(i\varepsilon-\xi_\mathbf{k})^{-1}$ is the Matsubara Green function, $\xi_\mathbf{k}=(\mathbf{k}^2-k_\mathrm{F}^2)/2m$, and $\varepsilon_n=(2n+1)\pi T$ is the Fermionic Matsubara frequency. After the sum over ε_n is evaluated, Eq. (2) takes on the form

$$\pi(\mathbf{q},\omega_m)=2\,\mathrm{Re}\left[\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{f(\mathbf{k})}{\varepsilon(\mathbf{k})-\varepsilon(\mathbf{k}-\mathbf{q})-i\omega_m}\right], \tag{3}$$

where $f(\mathbf{k})$ is the Fermi distribution. At not very high temperatures $T\ll\varepsilon_\mathrm{F}$, it can be written as $f(\mathbf{k})=\theta(k_\mathrm{F}-|\mathbf{k}|)$ and after a straightforward calculation we obtain

$$\pi(z)=\nu\,\mathrm{Re}\left[1-\frac{1}{\mathrm{Re}\,z}\sqrt{z^2-1}\right], \tag{4}$$

where we have introduced the complex variable z for compactness:

$$z=\frac{q}{2k_\mathrm{F}}+\frac{i|\omega_m|}{v_\mathrm{F}q},$$

and $v_\mathrm{F}=k_\mathrm{F}/m$ is the Fermi-velocity. One can easily check that Eq. (3) reproduces Eq. (1) if $\omega_m=0$. To get the expression for the polarization operator $\Pi(\mathbf{q},\omega)$ as a function of the real frequency,[6] one has to do the analytical continuation in Eq. (4). Let us note here that only the real part of the polarization operator renormalizes the scattering amplitude. The imaginary part is not relevant to this renormalization. The latter quantity is proportional to the density of the electron-hole pairs.

Let us now formulate the Cooper problem for the case under consideration. We are looking for a singularity in the Cooper channel (see the diagrammatic equation in Fig. 2). After averaging over the spin indices, the Bethe-Salpeter equation can be written as

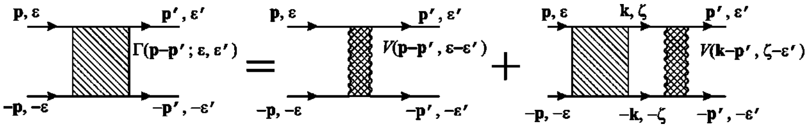

FIG. 2. The Bethe-Salpeter equation for the irreducible vertex Γ (Cooper problem).

$$\mathcal{T}(\mathbf{q};\varepsilon,\varepsilon')=\mathcal{V}(\mathbf{q},\omega)-T\sum_{\zeta}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\mathcal{T}(\mathbf{k}-\mathbf{p};\varepsilon,\zeta)\times\mathcal{G}_{\zeta}(\mathbf{k})\mathcal{G}_{-\zeta}(-\mathbf{k})\times\mathcal{V}(\mathbf{k}-\mathbf{p}',\zeta-\varepsilon'),\tag{5}$$

where ζ, ε, and ε' are fermionic Matsubara frequencies, $\mathbf{q}=\mathbf{p}-\mathbf{p}'$, and $\omega=\varepsilon-\varepsilon'$. Let us emphasize that $\mathcal{V}(\mathbf{q},\omega)$ is the renormalized interaction which depends on the momentum and energy transfer.

Let us now consider only electrons in the very vicinity of the Fermi surface so that $q=2k_{\mathrm{F}}\sin(\phi/2)$, where ϕ is the scattering angle. Following the standard route in the Kohn-Luttinger theory, we expand $\mathcal{V}$ and $\mathcal{T}$ in series of the normalized eigenfunctions of the angular momentum,

$$\mathcal{V}(\mathbf{q},\omega)=\sum_{l}\mathcal{V}_l(\omega)\Phi_l(\phi)\tag{6}$$

and

$$\mathcal{T}(\mathbf{q},;\varepsilon,\varepsilon')=\sum_{l}\mathcal{T}_l(\varepsilon,\varepsilon')\Phi_l(\phi),\tag{7}$$

where

$$\Phi_l(\phi)=\frac{1}{\sqrt{2\pi}}e^{il\phi}.$$

Then, Eq. (5) takes on the form

$$\mathcal{T}_l(\varepsilon,\varepsilon')=\mathcal{V}_l(\omega)-T\sum_{\zeta}\mathcal{T}_l(\varepsilon,\zeta)\mathcal{C}(\zeta)\mathcal{V}_l(\zeta-\varepsilon'),\tag{8}$$

where, as usual, $\mathcal{C}(\zeta)$ is the Cooperon, which is the source of the BCS logarithm:

$$\mathcal{C}(\zeta)=\int|\mathcal{G}_{\zeta}(\mathbf{k})|^2\frac{d^2\mathbf{k}}{(2\pi)^2}=\frac{\pi\nu}{|\zeta|}.\tag{9}$$

Let us note that Eq. (8) is exact at any temperature. However, we shall consider only the case of low temperatures to avoid technical difficulties connected with the analytical continuation in Eq. (8). In the limit $T\to 0$, the procedure of the analytical continuation reduces to the simple Feynman rotation and all the Matsubara sums involved may be replaced by the corresponding integrals with the temperature serving as a "low-energy cutoff." The main result we are deriving in the present paper can be noticed in this limit as well.

III. EFFECTIVE ATTRACTION IN THE *d* CHANNEL

The next step is to evaluate the spherical harmonics of the renormalized interaction. At this point, let us assume that the initial electron-electron interaction is defined only for the energies smaller than some threshold value $\tilde{\omega}\ll\varepsilon_{\mathrm{F}}$, which will be serving as the high-energy cutoff (just as the Debye frequency in the classical weak-coupling BCS theory). In this case, when performing actual calculations we can expand on $\omega/\varepsilon_{\mathrm{F}}$.

The l harmonics of polarization operator (4) can be written as

$$\pi_l(\omega_m)=\sqrt{\frac{2}{\pi}}\int_0^{2\pi}\pi(\phi,\omega_m)\cos l\phi\, d\phi.\tag{10}$$

Keeping in mind that $\omega\ll\varepsilon_{\mathrm{F}}$ and evaluating the integral with the logarithmic accuracy, we obtain, for even orbital momenta $l=2n$,

$$\pi_{2n}(\omega)=-\sqrt{\frac{2}{\pi}}\nu\frac{|\omega|}{2\varepsilon_{\mathrm{F}}}\left\{\frac{3}{2}\ln\frac{2\varepsilon_{\mathrm{F}}}{|\omega|}-2\left[\psi\left(n+\frac{1}{2}\right)-\psi\left(\frac{1}{2}\right)\right]\right\},\tag{11}$$

and, for $l=2n+1$,

$$\pi_{2n+1}(\omega)=-\sqrt{\frac{2}{\pi}}\nu\frac{|\omega|}{2\varepsilon_{\mathrm{F}}}\left\{\frac{1}{2}\ln\frac{2\varepsilon_{\mathrm{F}}}{|\omega|}-2n\frac{|\omega|}{2\varepsilon_{\mathrm{F}}}[\mathbf{C}+\psi(n+1)]\right\},\tag{12}$$

where ψ is the logarithmic derivative of the gamma function and $\mathbf{C}\approx 0.577$ is the Euler's constant.

From Eqs. (11) and (12) we see that the dependence on the orbital momentum is very weak. The effective interaction can be written as

$$\mathcal{V}_l(\omega)=\pi_l(\omega)\{\lambda(0)(-1)^{-l}+2[\lambda(0)\lambda(2k_{\mathrm{F}})-\lambda^2(2k_{\mathrm{F}})]\},\tag{13}$$

where $\lambda(\mathbf{q})$ is the Fourier component of the bare interaction potential.

If the initial interaction is $\mathbf{q}$ independent, we see that the effective interaction is attractive only for the even values of the orbital momentum $l=2n\neq 0$. The effective attraction is the strongest for $l=2$. The corresponding d harmonics reads[8]

$$\mathcal{V}_d(\omega)=-\frac{3}{\sqrt{2\pi}}\nu\lambda^2\frac{|\omega|}{2\varepsilon_{\mathrm{F}}}\ln\frac{2\varepsilon_{\mathrm{F}}}{|\omega|}.\tag{14}$$

IV. PAIRING AT FINITE ENERGIES

We can substitute result (14) into the Bethe-Salpeter equation (8) which turns into an integral equation (at $T\to 0$) with a well defined kernel $K(\varepsilon,\varepsilon')=\mathcal{V}_d(\varepsilon-\varepsilon')\mathcal{C}(\varepsilon')$. One can easily see that if the incoming particles have zero energies, the Cooper singularity gets canceled. However, at finite energies the Cooper logarithm survives being cut off by the energy transfer.

For further treatment, let us define the following auxiliary dimensionless variables and functions:

$$x=\varepsilon/2\varepsilon_{\rm F},$$

$$\tilde{x}=\tilde{\omega}/\varepsilon_{\rm F},$$

$$g_0(x)=-|x|\ln\frac{1}{|x|},$$

$$g(x,x')=\left[\frac{3}{\sqrt{2\pi}}\nu\lambda^2\right]^{-1}\mathcal{T}(\varepsilon,\varepsilon'),$$

and

$$\kappa=\frac{3\pi}{(2\pi)^{3/2}}(\lambda\nu)^2.$$

In these notations, Eq. (8) takes on the form

$$g(x,x')=-g_0(x-x')+\kappa\int dy\frac{g_0(x-y)}{|y|}g(y,x'). \tag{15}$$

The integral in Eq. (15) is defined in such a way that the large-y singularities are cut off by $\tilde{\omega}/\varepsilon_{\rm F}$ and low-y singularities at $\tau=T/2\varepsilon_{\rm F}$.

It is hard to solve Eq. (15) exactly. However, we are mostly interested not in the detailed solution but in the possibility of a singularity in the pairing vertex $g(x,x')$ which would be a signal of a superconducting pairing (but not necessarily a global superconducting instability). Let us emphasize here that $g(0,0)=0$ by construction, and it cannot diverge simply because there is no attraction in this case, unless we take into account the higher order diagrams. At finite energy transfers, a large Cooper logarithm appears which yields a divergence of $g(x,x')$ which we interpret as the appearance of fluctuating Cooper pairs built up of the electronic excitations with finite energies. One of the ways to search for the singularity is to consider the eigenvalue problem for the kernel of integral equation (15):

$$\Delta(x)=\kappa\int\frac{|x-y|}{|y|}\ln\frac{1}{|x-y|}\Delta(y)dy. \tag{16}$$

The singularity exists if there is a nontrivial solution of this equation. To get some qualitative estimates let us approximate the corresponding eigenvector by the following trial function:

$$\Delta(x)=\Delta_0+\Delta_1|x|, \tag{17}$$

where Δ_0 and Δ_1 are some weak (logarithmic) functions of x. From Eqs. (16) and (17) we can derive the self-consistency equation which yields the estimate for the threshold temperature at which the pairing with the typical energy transfer of ω commences[7]:

$$T_p(\omega)\sim\omega\exp\left\{-\frac{1}{k^2\tilde{x}^2\ln(2\varepsilon_{\rm F}/\omega)}\right\}. \tag{18}$$

This estimate can be alternatively derived by considering the resolvent of the integral equation straightforwardly. Namely, one can formally rewrite Eq. (15) as follows:

$$\hat{g}=g_0+\kappa\hat{K}\hat{g},$$

where $\hat{K}$ is the operator with the kernel in the x representation being equal to $K(x,y)=(|x-y|/|y|)\ln(1/|x-y|)$. The solution of this equation has can be formally written as

$$\hat{g}=\hat{R}(\kappa)g_0=[1-\kappa\hat{K}]^{-1}g_0,$$

where $\hat{R}(\kappa)$ is the resolvent, which can be also written as:

$$\hat{R}(\kappa)=\sum_{n=0}^{\infty}\kappa^n\hat{K}^n, \tag{19}$$

where $\hat{K}^n$ can be found by evaluating the convolution of the corresponding kernels in the x representation:

$$K^{(n)}(x,y)=\int K(x,z)K^{(n-1)}(z,y)dz.$$

Studying the geometric series [Eq. (19)], one can see that its $2n$ term contains the logarithm $\ln^n(\omega/T)$, with ω being the typical energy of the electrons in the Cooper channel. Summing up the series, we reproduce Eq. (18).

The integral equation (15) and the corresponding eigenproblem (16) are mathematically well defined for any $x\lesssim\tau$ (i.e., $\omega\lesssim T$). However, it does not make too much sense to study the structure of the solutions at such energies in the framework of our formalism based on the Matsubara technique. Thus, result (18) has the following domain of applicability:

$$T\ll\omega\lesssim\tilde{\omega}\ll\varepsilon_{\rm F}.$$

Working in this domain, the replacement of the Matsubara sums by the integrals is legitimate and our interpretation of $\omega\gg T$ as a real energy of a pair is valid as well. Let us now briefly discuss how the appearance of the fluctuating pairs affects the physical properties of the system. The correction to the conductivity is described by the diagrams similar to the ones in the conventional fluctuation theory[9] (see, e.g., Fig. 3, where the Aslamazov-Larkin-like diagram is shown). It is a rather difficult problem to calculate the corresponding contributions in the case under consideration. However, we can get some qualitative insight by noting that the analytical continuation to the real frequencies in the expression for the conductivity contains the factor $\coth(\omega/2T)$, which is basically the Bose distribution for the fluctuating Cooper pairs (the density of the Cooper pairs). This factor and the corresponding correction are exponentially small unless there exist Cooper pairs with $\omega\sim T$. Using Eq. (18), we can estimate

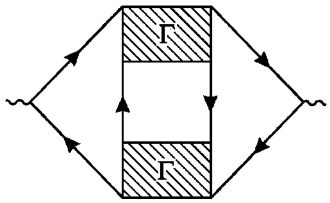

FIG. 3. Aslamazov-Larkin contribution to the conductivity. Small wavy lines correspond to the factor ev. Shaded boxes are the pairing vertexes which in the case under consideration are functions of the four variables $\Gamma(k_1,k_2;k_1',k_2')$.

the temperature T_* at which such pairs appear. It is defined by the condition $T_p(T_*)\sim T_*$. Thus, we readily obtain[7]

$$T_*\sim\varepsilon_F\exp\left\{-\left[\frac{(2\pi)^{3/2}}{3\pi}\frac{\varepsilon_F}{\tilde{\omega}}\right]\frac{1}{(\lambda\nu)^4}\right\}. \tag{20}$$

At this temperature, contribution to the conductivity due to the preformed Cooper pairs may become comparable to the Drude conductivity of a normal metal.

V. LONG-RANGE COULOMB INTERACTION

Until now, we have been studying a Fermi system with the bare electron-electron coupling being short-ranged. It is worth considering the case when the initial interaction is the long-range Coulomb repulsion. In this case our treatment is not applicable since the momentum dependence of the Coulomb interaction becomes crucial. However, we can get some qualitative insight into the problem without cumbersome calculations. There are several possibilities one can consider.

First, we can study a system in which both transport and screening are two dimensional. In this case, we can readily conclude that there is no possibility for Kohn-Luttinger pairing because the long-wavelength Thomas-Fermi screening is weak,

$$V(\mathbf{r})=\int\frac{d^2\mathbf{q}}{(2\pi)^2}\frac{2\pi e^2}{q}\frac{1}{\epsilon(q)}e^{i\mathbf{qr}}\propto\frac{1}{r^3}, \tag{21}$$

where $a_0=1/me^2$ is the effective two-dimensional screening length and the Thomas-Fermi dielectric function has the standard long-wavelength form

$$\epsilon(q)=1+\frac{2}{a_0q}. \tag{22}$$

We can now calculate the spherical harmonics of the screened Coulomb interaction,

$$V_l=\sqrt{\frac{2}{\pi}}\int\frac{2\pi e^2}{q+2/a_0}\cos l\phi\, d\phi,\quad q=2k_F\sin\frac{\phi}{2}, \tag{23}$$

which are certainly all repulsive and remain repulsive even after Friedel oscillations are taken into account (see Fig. 1). Thus, even going beyond this long-wavelength Thomas-Fermi analysis, we do not expect any pairing instability for the long-ranged Coulomb interaction to appear, as long as both the transport and screening are two dimensional. However, the account for the dynamically screened Coulomb interaction may lead to other important effects such as the renormalization of the Fermi-liquid parameters (effective mass, g factor, etc.). This issue is currently being investigated by the authors, and the results will be reported elsewhere.

Second, one can consider a system in which the transport is two dimensional but the screening is three dimensional. In this case the Coulomb interaction is well screened and decays exponentially at large distances $V(r)\propto\exp(-r/d)$ (d is the screening length). In the limit $k_Fd\ll1$, the potential becomes effectively short ranged and, thus, the theory developed in the present paper is qualitatively valid. Let us note that in this model the high-energy cutoff is basically the Fermi energy which violates the assumption $\tilde{\omega}\ll\varepsilon$ we used in our calculations. This, however, should not change main qualitative result of the paper.

There is also an intermediate situation which may exist when the two-dimensional Fermi liquid lives in the very close vicinity of a metallic substrate. In this situation, each two-dimensional electron produces an image in the metallic substrate so that the bare electron-electron interaction decays only as r^{-3} at large distances. In this case, there is no simple answer as to whether the Kohn-Luttinger pairing exists or not. Presumably, the Kohn-Luttinger pairing in such a setup is possible if the Fermi liquid is dilute enough, so that Friedel oscillations may compete with the initial dipole-dipole coupling.

VI. CONCLUSION

Before concluding, we point out that earlier theoretical work in the literature has considered[10] the possibility of bound states and Cooper pairing in a dilute two-dimensional (2D) system of fermions interacting via a short-ranged repulsive interaction. Engelbrecht and Randeria have considered a regular expansion in the T matrix in two dimensions analogous to the expansion on the dilute gas parameter $k_Fa\ll1$ in three dimensions.[11] Apart from the three-dimensional result, they have found an unusual pole in the particle-particle channel. Although we do not find any obvious connection between our microscopic analysis and this earlier work,[10] the claim of a new 2D collective mode interpreted as a bound excitation of two holes is somewhat reminiscent of our finding in this paper that a Kohn-Luttinger-type superconducting pairing is possible at finite excitation energies. Whether there is a deep connection between our work and the earlier results[10] remains unclear at this stage.

Summarizing, we have shown that a clean two-dimensional Fermi system with a short-range repulsive interaction between electrons becomes unstable against the formation of d-wave Cooper pairs with a finite binding energy at a low-enough temperature. Thus, the low-temperature state of the system is a mixture of low-lying electron excitations and preformed fluctuating Cooper pairs. The carriers may noticeably change the physical properties of the system such as conductivity, susceptibility, etc. at a temperature T_* [see Eq. (20)]. Let us note that from our theory it follows that the fluctuating pairs appear within the normal state having a finite gap which is connected with the binding energy. Note that there is no global superconductivity specifically predicted in our theory, only a pseudo-pairing at finite excitation energies. The results obtained in the present paper may be relevant to the pseudogap experiments in high-T_c superconductors.[12]

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[7] The approximation we use allows us to determine the exponents in Eqs. (18) and (20) up to a numerical factor only.

[8] Strictly speaking in Eq. (14), we have to replace the bare interaction λ with the full *s*-wave scattering amplitude f_0. In the leading approximation the renormalized amplitude is $f_0=[(\lambda\nu)^{-1}+\ln(ak_{\rm F})^{-2}]^{-1}$, where a is the range of the potential.

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