Stability of driven Josephson vortex lattice in layered superconductors revisited

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We analytically study stability of sliding lattice of Josephson vortices driven by a transport current in the stack direction in strong in-plane magnetic field. In contrast to recent findings we obtain that there are no diverse configurations of stable vortex lattices, and, hence, the stable sliding vortex lattice cannot be selected by boundary conditions. We find that in the bulk samples only the triangular (rhombic) lattice can be stable, its stability being limited by a critical velocity value. At higher velocities there are no simple stable lattices with single flux line per unit cell. Oblique sliding lattices are found to be never stable. Instability of such lattices is revealed beyond the linear approximation in perturbations of the lattice.

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I. INTRODUCTION

Strong anisotropy of layered superconductors, such as high-T_c superconductors, results in a number of specific features. In particular, magnetic field parallel to the conducting layers induces coreless vortices similar to fluxons in the conventional Josephson tunnel junctions.[1,2] In contrast to the fluxons in the conventional Josephson junctions, the current and magnetic field of a Josephson vortex in layered superconductors are spread over many conducting layers. The transport current flowing in the stack direction forces the vortices to slide along superconducting layers. Since in contrast to standard Abrikosov vortices the Josephson vortices are coreless, their motion does not involve perturbations of the amplitude of the superconducting order parameter and corresponds at low temperatures to an underdamped regime. So the velocity of such vortices, in principle, can be quite large. Many dynamical properties of Josephson vortices in layered superconducting crystals are similar to those of fluxons in artificially prepared stacked Josephson junctions (see, e.g., Ref. 3). The Josephson vortices can be arranged in a lattice; however, possibility of multiple metastable states corresponding to vortex configurations that do not form any lattice is predicted as well at small magnetic field.[4] At larger magnetic field the Josephson vortices form a lattice and, hence, in the flux-flow regime the coherent motion of the vortex lattice is expected. Such a regime must induce coherent electromagnetic radiation from the uniformly sliding lattice. The flux-flow regime of this lattice was observed in bismuth based cuprate superconductor mesa structures.[5–9] However, the regime of coherent sliding of the lattice was found to be limited by a maximum voltage and, hence, by a maximum lattice velocity.[6–9] Above this voltage either a different regime was observed[7,8] or no stable I-V curves were found in a close range of currents and voltages.[6] In addition, above the maximum velocity a broadband non-Josephson emission in the microwave region was observed.[7,8]

The upper limit for the velocity of the coherent flux-flow regime was obtained theoretically in our studies of stability of the vortex lattice,[10,11] and was related to the interaction of the vortices with the Josephson plasma modes excited by moving vortices. A regular motion of the Josephson vortices in a form of the rhombic lattice, usually called the triangular lattice, was found to be stable at velocities up to a critical value $v_{cr} \simeq cs/(2\lambda_{\parallel}\sqrt{\epsilon})$, where s is the period of the crystal lattice in the stack direction, $\lambda_{\parallel}$ is the London penetration depth, ϵ is the dielectric constant in the stack direction. This critical velocity was identified with the experimentally observed velocity limiting two regimes of the vortex motion.[6–9] We have found that at higher velocities sliding of a regular vortex lattice is frustrated due to a growth of fluctuations of the vortex lattice. The instability is induced by the interaction of the vortices with the Josephson plasma mode. The similar results were obtained both in the limit of large magnetic fields, when the distance between the vortices is smaller than the size of the nonlinear region near the vortex center, and for the case of lower magnetic fields[10,11] when the vortices are separated by distances much larger than the size of the nonlinear region.

However, qualitatively different conclusions on the stability of moving vortex lattice were made in recent papers by Koshelev and Aranson.[12,13] According to their calculations a stable vortex lattice motion can occur not only in a form of the triangular lattice, but also in a form of various oblique lattices (see Fig. 1), a particular experimentally realized lattice being uniquely selected by the boundary conditions. Regimes of stable motion of various oblique vortex lattices were found also for the velocities exceeding the critical value v_{cr}. The existence of a set of various stable vortex lattices

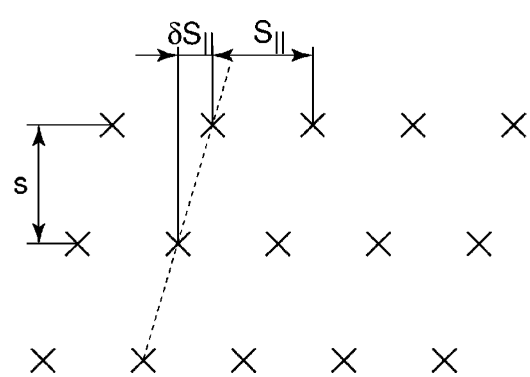

FIG. 1. The configuration of the vortex lattice is defined by $\chi = 2\pi\delta S_{\parallel}/S_{\parallel}$. The rectangular lattice corresponds to $\chi=0$, the oblique lattices are defined by $0<\chi<\pi$, and the triangular lattice corresponds to $\chi=\pi$.

for a given sliding velocity would lead to interesting physical consequences. If a particular vortex lattice is uniquely selected by the boundary conditions, then the radiation spectrum must also strongly depend on boundary conditions. Specifically, in contrast to other vortex lattices, the triangular one does not emit at the main radiation frequency related to the time needed to shift the lattice by its period. This happens since the emissions by adjacent vortex rows, which in the triangular lattice are shifted by the half-period, compensate each other. Therefore, in case of the triangular lattice the radiation is generated at even harmonics only. The oblique lattices, in which the shifts between the adjacent vortex rows are not equal to the half-period, can radiate both at even and odd harmonics as well. Furthermore, the intensity of the radiation maximum predicted at the Josephson plasma frequency[10,11] would be much larger if there exists a stable vortex lattice arrangement at $v > v_{cr}$.

To eliminate the discrepancy and to resolve the problem we reconsider stability of sliding vortex lattices and conclude that the oblique vortex lattices are not stable. Instability of the oblique lattices is found out from the analysis of equations of motion for perturbations of the lattice beyond the linear approximation in the perturbations (Sec. IV). We illustrate this statement by a particular case of the equilibrium state, i.e., of the lattice at rest (Sec. III). In the equilibrium state the results are especially transparent, because the stable vortex arrangement can be selected as that corresponding to a minimal energy. We conclude that oblique lattices correspond to a saddle point of the energy functional with respect to perturbations of a uniform lattice, and only the triangular lattice corresponds to a minimum. This is revealed when the terms beyond the quadratic ones are kept in the corresponding energy expansion. Therefore, to study stability of sliding vortex lattices by means of equations of motion one should proceed beyond the linear approximation with respect to perturbations.

II. MAIN EQUATIONS

Electrodynamic properties of layered superconductors can be described in terms of gauge invariant potentials which can be treated as superconducting momentum in layer n, $\mathbf{p}_n$, and gauge invariant phase difference between layers $n+1$ and n of the superconducting order parameter φ_n. In general case electric and magnetic fields in the superconductor depend also on gauge-invariant scalar potential $\mu_n = (1/2)\partial_t \chi_n + \Phi_n$, Φ_n is the electric potential in the nth layer. (We consider units with $e=\hbar=1$). We do not take into account here the components of the electric field related to μ_n since we consider low temperatures when the effects of branch imbalance related to μ_n can be ignored.

Equations for the moving vortex lattice are derived similar to our previous paper,[14] namely, substituting the expression for the current density $\mathbf{j}$ presented as a sum of superconducting and quasiparticle currents to the Maxwell equation

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\partial_t \mathbf{D}.$$

Then we find equations for $\mathbf{p}_n$ and φ_n, which have a form

$$\frac{c}{2s}\partial^2_{xx}\varphi_n - c\,\frac{\partial_x p_{n+1} - \partial_x p_n}{s} = \frac{4\pi}{c}\left[j_c \sin\varphi_n + \frac{\sigma_\perp}{2s}\partial_t \varphi_n \right] + \frac{\epsilon}{2cs}\partial^2_{tt}\varphi_n, \qquad (1)$$

$$-\frac{c}{2s^2}(\partial_x\varphi_n - \partial_x\varphi_{n-1}) + \frac{c}{s^2}(p_{n+1} + p_{n-1} - 2p_n) = \frac{4\pi}{c}\left[\frac{c^2}{4\pi\lambda_\parallel^2} p_n + \sigma_\parallel \partial_t p_n \right], \qquad (2)$$

Here $\lambda_\parallel$ is the magnetic penetration length for screening currents flowing in the plane direction, $\sigma_\parallel$ and $\sigma_\perp$ are quasiparticle conductivities for related directions.

Excluding superconducting momenta from Eqs. (1) and (2) one readily obtains the equation of motion for superconducting phases φ_n:

$$\left[\left(1 + \frac{\Omega_r}{\Omega_p^2}\partial_t \right) - \frac{\lambda_\parallel^2}{s^2}\partial^2_{nn} \right] (\partial^2_{tt}\varphi_n + \omega_r \partial_t \varphi_n + \omega_p^2 \sin\varphi_n) = \frac{c^2}{\epsilon}\left(1 + \frac{\Omega_r}{\Omega_p^2}\partial_t \right)\partial^2_{xx}\varphi_n. \qquad (3)$$

Here $\partial^2_{nn} f_n = f_{n+1} + f_{n-1} - 2f_n$ corresponds to the discrete version of the second derivative. $\Omega_r = 4\pi i \sigma_\parallel$, $\omega_r = 4\pi\sigma_\perp/\epsilon$ are relaxation frequencies. Ω_p and ω_p are plasma frequencies for the directions along and perpendicular to the layers, $\Omega_p = c/\lambda_\parallel$ is much larger than all typical frequencies of the considered problem, $\omega_p = c/\sqrt{\epsilon}\lambda_\perp \ll \Omega_p$, $\lambda_\perp$ is the magnetic penetration length for screening currents in the stack direction.

Studies of the vortex dynamics both in our papers and in papers by Koshelev and Aranson are based upon such equation.

We limit our study by the case of large magnetic field considered in Refs. 12 and 13 In high magnetic field the cores of Josephson vortices, i.e., nonlinear regions at the centers of the Josephson vortices, strongly overlap and solution of Eq. (3) can be found perturbatively as a sum of a linear term and a small oscillating correction:

$$\varphi_n^{(0)} = Y_n + \psi_n(Y_n). \qquad (4)$$

Here $Y_n = q_0 x - \omega_0 t + \chi n$, $\omega_0 = q_0 v$, $q_0 = 2\pi/S_\parallel$, $S_\parallel$ is the period of the chain of vortices along the superconducting layers, and χ specifies the type of the vortex lattice (see Fig. 1).

$$\psi_n(Y) = \Re \mathcal{G}(\omega_0, q_0, \chi) e^{iY_n}, \quad \mathcal{G}(\omega_0, q_0, \chi) = \frac{i\omega_p^2 K(\omega_0,\chi)}{\hat{c}^2(\omega_0) q_0^2 - K(\omega_0,\chi)(\omega_0^2 + i\omega_0\omega_r)}. \qquad (5)$$

Here $K(\omega, q_\perp) = 1 - i\Omega_r\omega/\Omega_p^2 + 4(\lambda_\parallel/s)^2 \sin^2 q_\perp/2$, $\hat{c}^2(\omega) = c^2(1 - i\Omega_r\omega/\Omega_p^2)/\sqrt{\epsilon}$. The denominator of $\mathcal{G}(\omega_0, q_0, \chi)$

does not become zero since $\omega_0 = q_0 v$ and vortex velocity is small enough, below the critical value v_{cr} mentioned above and discussed below in Sec. IV. Hence, for stable vortex lattice $|\psi_n(Y)| \ll 1$.

III. STATIONARY VORTEX LATTICE

At first we consider the vortex lattice at rest as the most simple and physically transparent case. Stability of a stationary vortex lattice is determined by its free energy. Minimum of the free energy as function of χ corresponds to the stable vortex lattice. The energy of the vortex lattice can be considered as a sum of the energy of magnetic field induced by superconducting electrons,

$$E^{(H)} = \frac{s}{8\pi} \sum_n \int dx H^2,$$

the kinetic energy of superconducting currents,

$$E^{(s)} = \sum_n \int dx \frac{ms}{2n_s} j_s^2,$$

and the Josephson energy of n Junctions,

$$E^{(J)} = \frac{\Phi_0 j_c}{2\pi c} \sum_n \int dx (1 - \cos\varphi_n).$$

Then the total energy can be readily presented as

$$E^{(tot)} = \sum_n \int dx \left[\frac{c^2}{8\pi s} \left(\frac{(\varphi_n')^2}{4} - \varphi_n'(p_{n+1} - p_n) + (p_{n+1} - p_n)^2 + \frac{p_n^2}{l^2} \right) + \frac{j_c}{2}(1 - \cos\varphi_n) \right]. \qquad (6)$$

Here $l = \lambda_\parallel / s$.

The variation of the energy functional (6) over p_n and φ_n yields Eqs. (1) and (2) with dissipative terms equal to zero.

To determine the stable vortex configuration we substitute expressions for $p_n(Y_n)$ and $\varphi_n^{(0)}$ of Eqs. (4) and (5) to the equation for the energy density (6). After some algebra the expression for the energy density of the vortex lattice reads

$$E^{(tot)} = \frac{c}{8\pi s} \sum_n \int dx \left(\frac{2l^4 \sin^2(\chi/2)^2 [\sin(\chi/2) - 1]^2}{\lambda_\perp^4 q_0^2} - \frac{l^2 \sin^2(\chi/2)}{\lambda_\perp^4 q_0^2} + \frac{1}{4} q_0^2 - \frac{1}{8\lambda_\perp^4 q_0^2} + \frac{1}{2\lambda_\perp^2} \right). \qquad (7)$$

From Eq. (7) one can see that the energy minimum corresponds to $\chi = \pi$. It means that in accordance with previous results (see, e.g., Ref. 2), only the triangular lattice is stable (Fig. 2). The difference between the energies of the triangular and the rectangular lattices is small and proportional to q_0^{-2}. Note that the triangular lattice corresponds to the minimum of the bulk energy, which means that the existence of the triangular lattice in the equilibrium state is imposed by the forces acting in the bulk, but not by the boundary conditions at the surface, as it was obtained in Ref. 13.

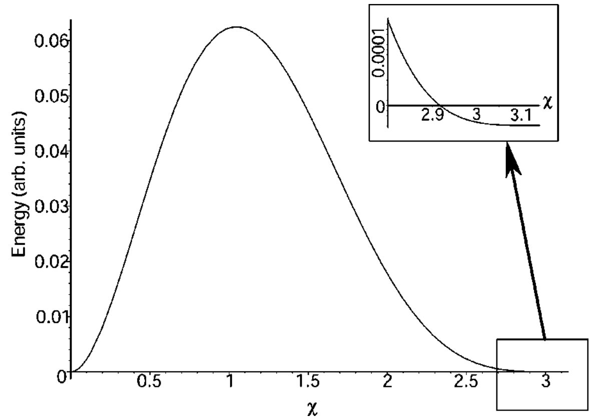

FIG. 2. Dependence of the vortex energy on χ. The curve is calculated for $l = \lambda_\parallel / s = 160$.

IV. STABILITY OF SLIDING JOSEPHSON VORTEX LATTICE

Now we consider a sliding vortex lattice. We adopt here the method used in Ref. 15 to study a system of two coupled Josephson junctions. The solution of Eq. (3) for φ_n can be presented in form

$$\varphi_n(x,t) = \varphi_n^{(0)}(Y) + \delta\varphi_n(x,t), \qquad (8)$$

where $\varphi_n^{(0)}(Y)$ given by Eqs. (4) and (5) describes the uniform motion of the vortex lattice with the velocity v, and $\delta\varphi_n(x,t)$ corresponds to small perturbations of the uniformly sliding lattice, $|\delta\varphi_n(x,t)| \ll 1$.

The vortex lattice is stable if all possible perturbations $\delta\varphi_n(x,t)$ decrease with time and there are no increasing solutions for $\delta\varphi_n(x,t)$. If in the linear approximation in $\delta\varphi_n(x,t)$ there are solutions, which neither decay nor increase, then the linear approximation in perturbation is not sufficient and one should take into account higher-order perturbations in $\delta\varphi_n(x,t)$.

The equation for $\delta\varphi$ in the linear approximation we derive substituting Eq. (5) into Eq. (3) and performing the Fourier transformation over x, t, and n. It reads

$$F(\omega, q_\parallel, q_\perp)\,\delta\varphi(\omega, q_\parallel, q_\perp) + f(\omega + \omega_0, q_\parallel + q_0, q_\perp + \chi) \times \delta\varphi(\omega + \omega_0, q_\parallel + q_0, q_\perp + \chi) + f(\omega - \omega_0, q_\parallel - q_0, q_\perp - \chi) \times \delta\varphi(\omega - \omega_0, q_\parallel - q_0, q_\perp - \chi) = 0. \qquad (9)$$

Here $q_\parallel$ and $q_\perp$ ($|q_\perp| < \pi$) are the wave vectors in the directions parallel and perpendicular to the superconducting layers, respectively,

$$F(\omega,q_{\|},q_{\perp})=\hat{c}(\omega)^2q_{\|}^2-K(\omega,q_{\perp})[\omega^2+i\omega\omega_r-(\omega_p^2/2)\Im\mathcal{G}(\omega_0,q_0,\chi)],$$

$$f(\omega,q_{\|},q_{\perp})=K(\omega,q_{\perp})\omega_p^2/2.$$

The values of the perturbation with shifted arguments, $\delta\varphi(\omega\pm\omega_0,q_{\|}\pm q_0,q_{\perp}\pm\chi)$, are small compared to $\delta\varphi(\omega,q_{\|},q_{\perp})$ with small values of the arguments. So using perturbation approach with respect to $S_{\|}/\lambda_J$ and neglecting $\delta\varphi(\omega\pm 2\omega_0,q_{\|}\pm 2q_0,q_{\perp}\pm 2\chi)$, we express the shifted values in terms of $\delta\varphi(\omega,q_{\|},q_{\perp})$ and find a simple equation of motion for $\delta\varphi$, which determines the eigenmodes of the vortex lattice:

$$\left(F(\omega,q_{\|},q_{\perp})-\frac{f(\omega,q_{\|},q_{\perp})f(\omega+\omega_0,q_{\|}+q_0,q_{\perp}+\chi)}{F(\omega+\omega_0,q_{\|}+q_0,q_{\perp}+\chi)}-\frac{f(\omega,q_{\|},q_{\perp})f(\omega-\omega_0,q_{\|}-q_0,q_{\perp}-\chi)}{F(\omega-\omega_0,q_{\|}-q_0,q_{\perp}-\chi)}\right)\times\delta\varphi(\omega,q_{\|},q_{\perp})=0. \tag{10}$$

In this expression last two terms originate from interaction of the first harmonic of oscillating field induced by the sliding lattice with the Josephson plasma mode.

As it follows from Eq. (5) $\mathcal{G}(\omega_0,q_0,\chi)\propto(S_{\|}/\lambda_J)^2$ is small, and one may neglect terms with $\mathcal{G}(\omega_0,q_0,\chi)$ in the expressions for F in the denominators of the last terms of Eq. (10). Such simplifications allow us to calculate from Eq. (10) the dispersion relation defining the spectrum of collective oscillations:

$$\omega^2+i\omega\omega_r=\frac{\hat{c}^2(\omega)q_{\|}^2}{K(\omega,q_{\perp})}+\frac{\omega_p^2}{2}\left(\Im\mathcal{G}(\omega_0,q_0,\chi)+\frac{i}{2}\mathcal{G}(\omega+\omega_0,q_{\|}+q_0,q_{\perp}+\chi)+\frac{i}{2}\mathcal{G}(\omega-\omega_0,q_{\|}-q_0,q_{\perp}-\chi)\right). \tag{11}$$

Consider first the triangular vortex lattice, $\chi=\pi$. In the long-wavelength limit, for the lattice in rest we obtain a soundlike spectrum with the velocities $c_{\|}=c/\sqrt{\epsilon}$ and $c_{\perp}=(\lambda S_{\|}/\sqrt{8\epsilon\pi\lambda_{\perp}^2})c$ in the in-plane and in-stack directions, respectively. These velocities coincide with ones obtained by Volkov.[16]

For the sliding triangular lattice the imaginary part of the frequency of the eigenmodes, which determines the decrement of damping, is non-negative for small velocities $v<v_{cr}\simeq(s/2\sqrt{\epsilon}\lambda_{\|})c$. Thus, at such velocities the triangular lattice is stable. At larger velocities we find that any lattice with a single vortex per unit cell is unstable. We demonstrate this below analytically for the case when the damping is not too large, $\Omega_r\ll\Omega_p/(sq_0)$, and at lattice velocities $v\lesssim v_{cr}$ all the terms with relaxation frequencies in the right-hand side of Eq. (11) can be neglected. Then after some simple algebra the dispersion relation (11) for $q_{\|}=0$ and $\omega\ll\omega_0$ can be presented in the form

$$\omega^2+i\omega\omega_r+\frac{2\omega_p^2\sin^2(q_{\perp}/2)[2\cos^2(q_{\perp}/2)+\cos\chi(\gamma-\cos\chi)]}{\lambda_J^2q_0^2(v/v_{cr})^2(\gamma+\cos\chi)[\gamma+\cos(\chi+q_{\perp})][\gamma+\cos(\chi-q_{\perp})]}=0, \tag{12}$$

where $\gamma=(v_{cr}/v)^2-1$. The lattice described by a given value χ is unstable if the last term in the right-hand side of Eq. (12) is positive. For the triangular lattice, $\chi=\pi$, this term is not positive at $v<v_{cr}$. It is not positive also for oblique lattices with values of χ around π at small velocities when $|\gamma|>1$, stability of such lattices will be analyzed below beyond the linear approximation in perturbations. For the rectangular lattice and the oblique lattices with positive values of $\cos\chi$, this term is positive and such lattices are unstable. At large velocities, $v>v_{cr}$, when $|\gamma|<1$, the last term several times changes its sign as a function of $q_{\perp}$ for any value of χ. Therefore, in this case there are values of $q_{\perp}$ for which the imaginary part of the frequency of the oscillations becomes positive. This means that fluctuations increase with time resulting in an instability of the lattice at such velocities. It can be shown easily that the instability remains at larger velocities, $v\gg v_{cr}$, as well. The value of v_{cr} is about $3\times10^5 m/c$, for typical parameters $\lambda_{\|}/s=100$ and $\epsilon\approx20$. This estimate of the critical velocity coincides with the velocity limiting flux-flow regime in Refs. 6 and 7. The instability of the lattice at large velocities was also confirmed by our numerical study of Eq. (11) with parameters corresponding to BSCCO crystals.

Now we consider in more details the case of arbitrary χ and $v<v_{cr}$. The sign of the imaginary part of ω, which determines the decrement of damping, sets conditions for stability of the vortex lattice. A positive damping factor for a given χ and for all values of $q_{\|}$ and $q_{\perp}$ means the stability of the vortex lattice. While a negative damping factor, in other words, a positive increment of growth, manifests the instability. Analytical and numerical calculations show that the damping factor is not negative for the whole region of values of χ for a given value of the vortex velocity including zero velocity. This result was interpreted in Refs. 12 and 13 as an evidence of the stability of these lattices. However, the existence of a set of different stable lattices at zero velocity ap-

parently contradicts to the studies of the equilibrium lattice[2] and to the results of the preceding section according to which only the triangular lattice must be stable. The point is that though for the oblique lattices the damping factor is positive for finite values of **q**, it is equal to zero for $\mathbf{q}=\mathbf{0}$. This indicates that the linear approximation on $\delta\varphi$ does not give an ultimate answer on the stability problem, and further study is needed. The situation can be clarified if we consider the particular example of the equilibrium state, then the stable solution can be selected, using the energy considerations. If the series expansion of the energy with respect to perturbations near the extremum does not contain the quadratic term, then contribution of the potential energy to the linearized equation of motion becomes zero suggesting a zero damping coefficient in a corresponding equation of motion. Therefore, in order to study stability in such a case one must take into account the next order perturbations in the energy expansion in perturbations. If the third-order term in perturbations in the expansion of the potential energy is nonzero, then the extremum of the energy corresponds to a saddle point and, hence, to an instability.

These considerations cannot be directly applied to the nonequilibrium case the stability of which should be studied by means of equations of motion, but they point out that to find whether solutions with zero damping factor in the linear approximation are really stable, one must include in the equation of motion for the phase the next order terms in perturbations $\delta\varphi$. Therefore, we present the perturbed phase in the form

$$\varphi_n=\varphi_n^{(0)}(\chi)+\delta\varphi_n+\phi_n . \tag{13}$$

where, again, $\varphi_n^{(0)}(Y)$ given by Eqs. (4) and (5) describes the uniform motion of the vortex lattice with the velocity v, and $\delta\varphi_n(x,t)+\phi_n(x,t)$ corresponds to perturbations of the uniformly sliding lattice. Further, we choose $\delta\varphi_n=\Re\{[\mathcal{G}_n(\chi+\delta\chi_n)-\mathcal{G}_n(\chi)]\exp[iY_n(\chi)]\}$, where $\delta\chi_n$ is a smoothly varying function of the layer number n. The initial perturbation $\delta\varphi_n$ describes distortion of the oscillating part of the phase corresponding to the shift of the neighboring vortex rows by phase $\delta\chi_n$. The small term ϕ_n in the perturbation to be found from the equations of motion (2). Thus, we calculate temporal evolution of an additional perturbation ϕ_n in the vortex lattice with the initial perturbations related to $\delta\chi_n$.

To derive equation for ϕ we insert φ_n expressed according to Eq. (13) into Eq. (2) keeping perturbations $\delta\varphi$. Then we act similar to the derivation of Eqs. (11) and (10), i.e., we use perturbational approach with respect to $S_\parallel/\lambda_J$, linearize the equation with respect to ϕ, and express the values of $\phi(\omega\pm\omega_0,q_\parallel\pm q_0,q_\perp\pm\chi)$ with shifted arguments in terms of $\phi(\omega,q_\parallel,q_\perp)$. Furthermore, for brevity we again consider the case when the magnetic field or damping are not too large, $(sq_0)\Omega_r\ll\Omega_p$, and one can neglect the terms with the relaxation frequencies. Then in the long-wavelength and low-frequency limits ($q_\parallel=0$, small $q_\perp$ and ω) we find simple equation for ϕ,

$$\omega_r\partial_t\phi_n+\frac{\omega_p^2}{4}\left[2\,\delta\chi_n\frac{\partial\Im\mathcal{G}(\omega_0,q_0,\chi)}{\partial\chi}\phi_n-\frac{\partial^2\Im\mathcal{G}(\omega_0,q_0,\chi)}{\partial\chi^2}\partial_{nn}^2\phi_n\right]=0. \tag{14}$$

We seek solution of this equation in the form $\phi_n=\Phi(n)\exp(-\alpha t)$. Then Eq. (14) acquires a form of the Schrödinger equation

$$-\frac{1}{2m}\partial_{nn}^2\Phi+V(n)\Phi=E\Phi, \tag{15}$$

where we assume the continuous limit with respect to layer numbers and use the notations

$$\frac{1}{m}=-\frac{\cos\chi+u^2\sin^2(\chi/2)(2-\cos\chi)}{[1-u^2\sin^2(\chi/2)]^3},V(n)=\frac{\sin\chi}{[1-u^2\sin^2(\chi/2)]^2}\delta\chi_n,u=\frac{v}{v_{cr}},E=\alpha\omega_r\frac{\lambda_J^2q_\parallel^2}{\omega_p^2}.$$

Note that the lattices with positive values of $\cos\chi$ are unstable [see Eq. (12)], therefore, for the states we consider $m>0$ if the velocity of the lattice is small enough.

If we find a form of $\delta\chi_n$ corresponding to a potential $V(n)$ for which the Schrödinger equation (15) has solutions describing bound states, $E<0$, then Eq. (14) has increasing solutions, which means the instability of the vortex lattice. As long as the potential depends both on $\sin\chi$ and on $\delta\chi_n$ one can always choose perturbation $\delta\chi_n$ in the form of the potential well. As long as there are bound states in any one-dimensional quantum well, then the bound solutions of Eq. (15) can be found unless χ is a multiple of π. As it was shown above in the linear approximation in lattice perturbations the rectangular lattice, $\chi=0$, is not stable due to perturbations with finite $q_\perp$, so there is only solution $\chi=\pi$ for which the bound states cannot be found. This means that there is only one simple vortex lattice with one flux quantum per cell, which could be stable, namely, the triangular one, $\chi=\pi$. Note that one can find both decaying and increasing perturbations around the lattice state defined by the parameter $\chi\neq\pi$, this is typical for a behavior near the saddle point.

V. CONCLUSIONS

In the frame of a simple theoretical model we studied stability of sliding lattices of the Josephson vortices induced by strong magnetic field applied parallel to conducting layers of layered superconductor and driven by a transport current in the stack direction. We found that in contrast to recent theoretical predictions[12,13] the type of the stable sliding lattice is not selected by the boundary conditions, but there is only one possible stable vortex lattice arrangement of sliding the Josephson vortices containing single flux line per unit cell, namely, the triangular lattice. In other words, an experimentalist cannot have an influence on the structure of the sliding vortex lattice manipulating with the experimental conditions. Sliding of vortices in the form of the regular lattice is found to be stable up to the critical velocity only. At higher velocities there is no sliding regime with the vortices coherently arranged in an oblique, triangular, or rectangular vortex lattices.

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