Gilbert damping in magnetic multilayers

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We study the enhancement of the ferromagnetic relaxation rate in thin films due to the adjacent normal-metal layers. Using linear-response theory, we derive the dissipative torque produced by the s-d exchange interaction at the ferromagnet–normal-metal interface. For a slow precession, the enhancement of the Gilbert damping constant is proportional to the square of the s-d exchange constant times the zero-frequency limit of the frequency derivative of the local dynamic spin susceptibility of the normal metal at the interface. Electron-electron interactions increase the relaxation rate by the Stoner factor squared. We attribute the large anisotropic enhancements of the relaxation rate observed recently in multilayers containing palladium to this mechanism. For free electrons, the present theory compares favorably with recent spin-pumping results of Tserkovnyak *et al.* [Phys. Rev. Lett. **88**, 117601 (2002)].

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I. INTRODUCTION

Ferromagnetic multilayers have attracted much attention recently because of their applications in spintronics and high-density magnetic recording devices. The present paper is concerned with magnetic relaxation in a ferromagnetic film (F) imbedded between nonmagnetic metallic layers (N). In particular, we study the enhancement of the Gilbert damping in $N/F/N$ sandwiches as compared with that of a single ferromagnetic film. The Gilbert damping constant G is defined by the Landau-Lifshitz-Gilbert (LLG) equation of motion[1,2] as

$$\frac{1}{|\gamma|}\frac{\partial \boldsymbol{M}}{\partial t}=-[\boldsymbol{M}\times\boldsymbol{H}_{eff}]+\frac{G}{\gamma^2 M_s^2}\left(\boldsymbol{M}\times\frac{\partial \boldsymbol{M}}{\partial t}\right), \tag{1}$$

where $\boldsymbol{M}$ is the magnetization vector, M_s is the saturation magnetization, and $\boldsymbol{H}_{eff}$ is the effective field which is given by $\boldsymbol{H}_{eff}=-\partial E/\partial \boldsymbol{M}$ where E is the Gibbs free energy. The gyromagnetic ratio γ is a negative quantity $-g\mu_B/\hbar$ where g is the spectroscopic splitting factor, and μ_B is the Bohr magneton. The second term on the right-hand side of Eq. (1) represents the dissipative torque, the magnitude of which is proportional to G. The LLG equation describes well both the static and the dynamic properties of ultrathin ferromagnetic films and the $N/F/N$ sandwiches (for a review see Ref. 3). The experimental and theoretical aspects of spin relaxation in multilayers are covered in Ref. 4.

The present investigation was motivated by the seminal work of Berger[5] and by the experiments[6–8] inspired by his paper. He considers a bilayer $F_1/N/F_2$ in which the ferromagnet F_1 acts a spin polarizer of the conduction electron. The spin transfer between this electron and the precessing ferromagnet F_2 gives rise to the relaxation torque. Consequently, there is an enhanced electron-magnon scattering caused by the isotropic s-d exchange at the F/N interface. Being a surface effect, the enhancement of the Gilbert constant is presented as d^{-1} where d is the thickness of the ferromagnetic film. This unique feature is observed on the double ferromagnetic layer structures.[6,7] Also, the additional ferromagnetic relaxation (FMR) linewidth for permalloy–normal-metal sandwiches shows this d dependence.[8] It should be mentioned that already in 1987 a study of the FMR linewidth showed an appreciable increase in the Gilbert damping with decreasing thickness of the Fe ultrathin films grown on bulk Ag(001) substrates.[9]

A similar trend is seen in ferromagnetic films that are in contact with an antiferromagnetic layer. For instance, Stoecklein *et al.*[10] studied FMR of permalloy thin films exchange coupled to antiferromagnetic iron-manganese films. The measured linewidth is proportional to d^{-1}. The mechanism of the broadening is attributed to the dispersion of the exchange bias and anisotropy.[10] Breaking of the antiferromagnetic order into a random domain pattern due to surface roughness is conjectured to provide the source of the dispersion.

More recently, McMichael *et al.*[11] studied the FMR linewidth of thin films deposited on antiferromagnetic NiO. Compared to the uncoupled ferromagnetic films, the exchange-coupled films exhibit an additional linewidth that increases several times as the magnetization is rotated from the perpendicular to the in-plane direction. To explain this result, McMichael *et al.*[11] invoked the two-magnon model. Owing to the fluctuations of surface magnetic interactions, the uniform spin-wave mode excited by the FMR is allowed to decay into the continuum of other short-wave modes that are degenerate with the FMR mode. As the magnetization orientation goes from the in-plane to the perpendicular direction, the spin-wave manifold moves towards higher frequencies such that the number of short-wave modes degenerate as the FMR mode decreases. A decrease of the decay rate ensues, in qualitative agreement with the angular dependence of the linewidth. Arias and Mills [12] developed a detailed theory of the two-magnon contribution to the linewidth and resonance field shift of FMR in ultrathin films. Assuming surface defects in the form of bumps and pits, they derived the scattering matrix elements to be due to fluctuations of the Zeeman, dipolar, and surface anisotropy energies. It turns out that surface anisotropy yields a dominant contribution to the matrix element. Since it is generated at the surface, the matrix element exhibits a d^{-1} dependence on the film thickness. This implies a d^{-2} dependence for the FMR linewidth—an important signature of the two-magnon mechanism in ultrathin films. Azevedo *et al.*[13] measured the FMR linewidth and resonance field shift in thin films of NiFe deposited on a Si substrate as a function of the film thickness. Their data are consistent with the d^{-2} prediction of the two-magnon theory of Arias and Mills.[12] Subsequently, Rezende *et al.*[14] used FMR and Brillouin light scattering to study the spin-wave damping in NiFe/NiO sputtered on Si substrate. They found a dramatic (20-fold) increase of the damping compared to the Ni/Fe films on a Si substrate studied in Ref. 13. As a function of film thickness, the damping is again found in good agreement with the d^{-2} dependence. Rezende *et al.*[14] explained these results using an adaptation of the two-magnon model[12] in which the main source of scattering is fluctuations of the exchange coupling due to surface roughness.

Another signature of the two-magnon model of Ref. 12 is the dependence of the linewidth on the microwave frequency. Referring to Fig. 4 of Ref. 12, we see that this dependence is nonlinear. In the experimentally relevant frequency range (10–40 GHz), the linewidth can be approximated by a straight line. Subsequent extrapolation to zero frequency yields a "zero-field offset." Thus the zero-field linewidth is a measure of the strength of the mechanism of Arias and Mills.[12] This signature has been seen in a recent FMR study[15] of the crystalline Cr/Fe/GaAs ultrathin-film structure grown by molecular-beam epitaxy. Above 10 GHz, the in-plane FMR linewidth is linearly dependent on the microwave frequency with an appreciable zero-frequency offset. Also the dependence of the linewidth on the angle between the magnetization and the plane of the film is consistent with the two-magnon model. Specifically, the FMR linewidth decreases more than three times upon going from the in-plane to the perpendicular orientation of the magnetization.

Since our concern is experimental evidence of Gilbert damping due to s-d exchange at the F/N interface,[5] we need to examine the extent to which the two-magnon mechanism is present in the structures investigated in Refs. 6–8. Let us first consider the dependence of the FMR linewidth upon the angle between the magnetization and the plane of the film. Mizukami *et al.*[8] studied this dependence for a permalloy film sandwiched between two platinum cap layers and found that the linewidth does not change as the magnetization rotates from the in-plane to the perpendicular direction. For the permalloy film sandwiched between copper layers, there is a decrease of the linewidth seen upon such a rotation, however, there is a negligible change of the linewidth with the thickness of the film. For the double ferromagnetic layer, Heinrich *et al.*[7] found an additional linewidth that is 10% lower as the magnetization goes from the in-plane to the perpendicular direction. It should be noted that such a small decrease could be associated with crystalline anisotropy in the film rather than present evidence for the two-magnon mechanism. Also the dependence of the linewidth on the frequency is linear with a negligible zero-frequency offset. Thus, the angular dependence of the linewidth observed in Refs. 6 and 8 contrasts with the prediction of the two-magnon model.[10,11] What remains is to mention the dependence of the linewidth on the thickness of the ferromagnetic film. Except for the thicknesses of less than a characteristic coherence length,[5] the linewidth data of Refs. 6–8 can be fitted with a d^{-1} dependence in contrast with the d^{-2} prediction of the two-magnon model.[11] Presumably, another mechanism for the FMR linewidth, related to the theory of Berger,[5] is at work for ferromagnetic films in contact with a nonmagnetic metal.

A mechanism for additional Gilbert damping in $N/F/N$ structures has been recently proposed by Tserkovnyak *et al.*[16] These authors calculate the spin current pumped through the N-F contact by the precession of the magnetization vector $\boldsymbol{M}(t)$. The theory is based on extending the scattering approach of parametric charge pumping by Brouwer[17] to spin pumping.

Like the theory of Berger,[5] the additional damping of Ref. 16 scales inversely with the thickness of the ferromagnetic film, indicating that only the F/N interface is involved. However, the expression for excess G, which we call G', differs considerably from that of Ref. 5. In particular, G' vanishes with vanishing s-d exchange splitting. An attractive feature of this theory is that it links G' to the transport properties of the interface. Due to exchange polarization of the F/N contact, the reflection (r) and transmission (t) coefficients at the interface depend on the orientation of the conduction-electron spin with respect to the magnetization direction of the ferromagnet. The formula for G' involves differences such as $\Delta r = r^{\uparrow} - r^{\downarrow}$. Interestingly, similar quantities play a role in the theory of interlayer magnetic coupling proposed by Bruno.[18] For instance, the Ruderman-Kittel-Kasuya-Yosida (RKKY) coupling[19] between two-dimensional layers can be obtained by calculating the interlayer coupling energy in terms of the reflection coefficients $r^{\uparrow}$ and $r^{\downarrow}$ of the layers. These coefficients are obtained by solving a simple problem of scattering by δ-function potential. In the limit of weak exchange splitting (compared to the Fermi energy), the derived interlayer coupling agrees with the RKKY result of Yafet.[20]

These considerations prompt us to take another look at the theory of enhanced relaxation in multilayers. The fact that RKKY theory[18] involves transport properties at the interface similar to the spin pump theory of G' Ref. 16 suggests that a suitable generalization of RKKY theory to time-dependent magnetization $\boldsymbol{M}(t)$ may unravel the needed dissipative torque of Eq. (1). We note that the standard approach to RKKY coupling is to use a linear-response theory,[19,21] and calculate the conduction-electron spin density induced by the contact exchange potential. In applications to interlayer coupling between ferromagnetic layers with time-independent magnetization vectors, it is the static spin susceptibility of the electron gas which determines the coupling. In the present paper, we consider a response to a slowly varying time-dependent s-d exchange potential.

Owing to the dissipative part of the spin susceptibility, the spin density induced by the precession of $\boldsymbol{M}(t)$ will have a

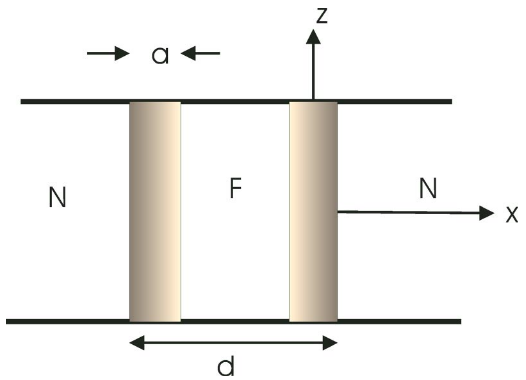

FIG. 1. A trilayer consisting of normal metals (N) adjacent to a ferromagnetic film (F) of thickness d. The s-d interaction generating the spin density $\boldsymbol{s}$ is assumed to take place in contact layers of thickness a of the order of the lattice constant.

component that is out of phase with $\boldsymbol{M}(t)$. Hence, such a component will have a time-dependence given by $d\boldsymbol{M}/dt$.

It is instructive to invoke the analogy to radiation damping of a charged particle in classical electrodynamics.[22] Thus, we put the dissipative torque of Eq. (1) in correspondence with the radiation reaction force acting on the particle. The first term of Eq. (1) corresponds to the external force. In view of this analogy, we rewrite the dissipative torque in terms of a reaction field $\boldsymbol{H}_r$,

$$\frac{G'}{\gamma^2 M_s^2}\left[\boldsymbol{M}\times\frac{d\boldsymbol{M}}{dt}\right]=-[\boldsymbol{M}\times\boldsymbol{H}_r^{(d)}], \tag{2}$$

where $\boldsymbol{H}_r^{(d)}$ represents the dissipative part of $\boldsymbol{H}_r$. For the $N/F/N$ system shown in Fig. 1, we find the reaction field (see Appendix A)

$$\boldsymbol{H}_r(t)=\frac{2Ja}{\gamma d}\langle \boldsymbol{s}(x=0,t)\rangle, \tag{3}$$

where J is the s-d exchange coupling constant, a is of order of the lattice constant, and d is the width of the ferromagnetic film. The quantity $\langle \boldsymbol{s}(x=0,t)\rangle$ is the spin density induced at the F/N interface by the s-d exchange interaction (x being the distance from the interface). The expression (3) is quite general in the sense that it can be used for both ballistic and diffusive cases. In the present paper we confine ourselves to the ballistic case and calculate the induced spin density using linear-response theory.[21]

Assuming a slow precession of $\boldsymbol{M}(t)$, we find two contributions. One is proportional to $\boldsymbol{M}(t)$ with a coefficient given by the real part of the local spin susceptibility at zero frequency. If the spin susceptibility is anisotropic, this term leads to an anisotropic shift of the FMR frequency. For isotropic susceptibility, the quantity on the right-hand side of Eq. (3) is a vector parallel to $\boldsymbol{M}(t)$ and the corresponding torque (2) vanishes.

The second contribution to $\langle \boldsymbol{s}(x=0,t)\rangle$ is proportional to $d\boldsymbol{M}/dt$. The coefficient of proportionality is the frequency derivative of the imaginary part of the susceptibility taken at $\omega=0$. Like the real part, this quantity is generally anisotropic. According to Eq. (2), it produces a dissipative torque leading to an anisotropic G'. Explicit evaluation of this term for an isotropic noninteracting electron gas yields a formula for G' which compares favorably with the spin pump theory of Tserkovnyak *et al.*[16]

The proposed formulation allows us to incorporate interactions between the electrons in the N region. The generalized Hartree-Fock approximation shows that the $\boldsymbol{q}$, ω-dependent spin susceptibility of the interacting electron gas is enhanced compared to the free-electron case.[21] Using these results, we find that the part of $\boldsymbol{H}_r$ that contributes to the FMR frequency shift is enhanced by the Stoner factor,

$$S_E=[1-UN(\epsilon_F)]^{-1}, \tag{4}$$

where U is the screened intra-atomic Coulomb interaction and $N(\epsilon_F)$ is the electron density of states, per atom, at the Fermi level.[21] On the other hand, we find that G' is enhanced by a factor of S_E^2. The Stoner enhancement is thought to be large in metals such as palladium and platinum. Recent results using multilayers with Pd layers as a spacer show a significant enhancement of interface damping exhibiting a fourfold anisotropy in keeping with the present theory.[23]

This paper is organized as follows. In Sec. II we derive the spin density induced by a two-dimensional layer of precessing spins, and the corresponding reaction field. Section III focuses on the dissipative part of the reaction field, and the excess damping G' for an isotropic electron gas. An expression for G' in a free-electron model is derived for the $N/F/N$ structure with N layers of both infinite and finite thicknesses. The enhancement of G' due to electron-electron interactions is considered within the generalized Hartree-Fock approximation. In Sec. IV we establish a relation between the spin-pumping theory of Ref. 16 and our free-electron result for G'. The role of spin sink in theories of Gilbert damping is discussed in Sec. V. The reaction field for the $N/F/N$ structure is derived in Appendix A. The role of spin relaxation in the theory of G' is considered in Appendix B. In Appendix C we derive the criterion for the validity of the slow precession approximation.

II. DYNAMIC RKKY

We consider a two-dimensional layer of aligned spins imbedded in a normal metal. Notice that a similar model was used by Yafet[20] to calculate the interlayer coupling for time independent magnetizations. Here we take into account the time-dependence of the precessing magnetization.

Our task is to derive the conduction-electron spin density induced by the s-d exchange interaction taking place at the magnetic layer. The Hamiltonian of the conduction electrons is

$$\hat{H}=\hat{H}_0-J\sum_i^{sheet}\int d^3r\,\boldsymbol{S}^{(i)}(t)\cdot\hat{\boldsymbol{s}}(r)\,\delta^3(\boldsymbol{r}-\boldsymbol{r}_i), \tag{5}$$

where $\hat{H}_0$ represents the Hamiltonian of the conduction s electrons in the absence of the interaction with spins of the d electrons of the ferromagnetic layer. As in the theory of time-independent RKKY interaction,[19,20] $\hat{H}_0$ corresponds to an infinite three-dimensional homogeneous Fermi gas. The second term in Eq. (5) is the s-d exchange interaction, with the spins of the ferromagnetic sheet consisting of a single atomic layer. J is the s-d exchange coupling constant, $\boldsymbol{S}^{(i)}(t)$ is the classical spin at the location $\boldsymbol{r}_i$, and $\hat{\boldsymbol{s}}(\boldsymbol{r})$ is the spin-density operator for the conduction electrons.

For a uniform precession of aligned spins, we have

$$\boldsymbol{S}^{(i)}(t)=\frac{\Omega}{\gamma}\boldsymbol{M}(t), \tag{6}$$

where Ω is the volume of the unit cell.

The second term of Eq. (5) acts as a time-dependent perturbation that will be treated using linear-response theory.[21] Thus, the expectation value of the μ component of the induced spin density is

$$\langle s_\mu(\boldsymbol{r},t)\rangle = J\sum_i^{sheet}\int d^3r'\int_{-\infty}^{\infty}dt'\,\chi_{\mu\nu}(\boldsymbol{r}t,\boldsymbol{r}'t')\times S_\nu^{(i)}(t')\,\delta^3(\boldsymbol{r}'-\boldsymbol{r}_i), \tag{7}$$

where

$$\chi_{\mu\nu}(\boldsymbol{r}t,\boldsymbol{r}'t')=\frac{i}{\hbar}\Theta(t-t')\langle[\hat{s}_\mu(\boldsymbol{r},t),\hat{s}_\nu(\boldsymbol{r}',t')]\rangle \tag{8}$$

is the retarded spin-correlation function (susceptibility),[21] with $\Theta(t-t')$ the unit step function.

Now, we evaluate the right-hand side of Eq. (7) assuming a slow precession. In Appendix C we derive a condition for the validity of this assumption. We note that the spin-[16] and charge[17] pumping theories also require a slow precession to ensure adiabatic evolution of the ground state.

Performing the $\boldsymbol{r}'$ integration, and applying the commutation law for the convolution, Eq. (7) reads

$$\langle s_\mu(\boldsymbol{r},t)\rangle = J\sum_i^{sheet}\int_{-\infty}^{\infty}dt'\,S_\nu^{(i)}(t-t')\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i,t'). \tag{9}$$

For slow precession, we write

$$S_\nu^{(i)}(t-t')\simeq S_\nu^{(i)}(t)-t'\frac{dS_\nu^{(i)}(t)}{dt}. \tag{10}$$

Introducing this expansion into Eq. (9), we obtain with use of Eq. (6)

$$\langle s_\mu(\boldsymbol{r},t)\rangle \simeq \frac{J\Omega}{\gamma}\lim_{\omega\to 0}\left[M_\nu(t)\sum_i^{sheet}\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i;\omega) - \frac{\partial M_\nu(t)}{\partial t}\sum_i^{sheet}\frac{\partial\,\mathrm{Im}\,\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i,\omega)}{\partial\omega}\right], \tag{11}$$

where

$$\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i,\omega)=\int_{-\infty}^{\infty}dt e^{i\omega t}\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i,t). \tag{12}$$

We note that the second term on the right-hand side of Eq. (11) follows by letting the derivative of the real part of the susceptibility, with respect to ω, equal zero at $\omega=0$. This is consistent with the reality condition, implying that the real part of the susceptibility is an even function of ω. The first term on the right-hand side of Eq. (11) corresponds to the static RKKY result of Yafet.[20] The second term is the dynamic generalization of RKKY for a slowly precessing magnetization.

For an infinite medium, the sheet sums in Eq. (11) can be evaluated by invoking translational invariance, and Fourier transforming $\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i,\omega)$ to $\boldsymbol{q}$ space.[18] Thus, we define a generic sum

$$X_{\mu\nu}(\boldsymbol{r},\omega)=\sum_i^{sheet}\chi_{\mu\nu}(\boldsymbol{r},\boldsymbol{r}_i,\omega) = \int\frac{d^3q}{(2\pi)^3}\sum_i^{sheet}\exp[i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{r}_i)]\chi_{\mu\nu}(\boldsymbol{q},\omega), \tag{13}$$

from which both terms of Eq. (11) can be deduced.

To perform the q integral on the right-hands side of this equation, we set

$$\boldsymbol{q}=\boldsymbol{q}_\parallel+\boldsymbol{q}_\perp, \tag{14}$$

where $\boldsymbol{q}_\parallel$ is confined to the sheet, and $\boldsymbol{q}_\perp$ is perpendicular to the sheet. For a square sheet of area L^2, we have

$$\int\frac{d^3q}{(2\pi)^3}=\int\frac{d^2q_\parallel}{(2\pi)^2}\int\frac{dq_\perp}{2\pi}=\frac{1}{L^2}\sum_{q_\parallel}\int\frac{dq_\perp}{2\pi}. \tag{15}$$

Assuming a continuous distribution of spins, the sheet sum in Eq. (13) is given by

$$\sum_i^{sheet}\exp[-i(\boldsymbol{q}_\parallel+\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_i]=N_s\delta_{\boldsymbol{q}_\parallel,0}, \tag{16}$$

where N_s is number of spins in the sheet. Using Eqs. (14)–(16), the right-hand side of Eq. (13) is evaluated, with the result

$$X_{\mu\nu}(\boldsymbol{r},\omega)=\frac{N_s}{L^2}\int\frac{dq_\perp}{2\pi}\exp(i\boldsymbol{q}_\perp\cdot\boldsymbol{r})\chi_{\mu\nu}(\boldsymbol{q}_\perp,\omega) = n_s\int\frac{dq_\perp}{2\pi}\exp(iq_\perp x)\chi_{\mu\nu}(q_\perp,\omega), \tag{17}$$

where $n_s=N_s/L^2$ is the sheet density. Owing to the symmetry of the model, the quantity $X_{\mu\nu}(\boldsymbol{r},\omega)$ depends only on the distance x from the sheet.

Also, the induced spin density is a function of x. Using Eqs. (11), (13), and (17), we obtain

$$\langle s_\mu(x,t)\rangle = \frac{J\Omega}{\gamma}\lim_{\omega\to 0}\left[X_{\mu\nu}(x,\omega)M_\nu(t) - \frac{\partial\,\mathrm{Im}\,X_{\mu\nu}(x,\omega)}{\partial\omega}\frac{dM_\nu(t)}{dt}\right]. \tag{18}$$

Using this result and Eq. (17) in Eq. (3), the μ component of the reaction field is

$$H_{r,\mu}(t)\simeq \frac{2J^2\Omega a n_s}{\gamma^2 d}\lim_{\omega\to 0}\left[\int_{-\infty}^{\infty}\frac{dq}{2\pi}\chi_{\mu\nu}(q,\omega)M_\nu(t) - \frac{\partial}{\partial\omega}\int_{-\infty}^{\infty}\frac{dq}{2\pi}\mathrm{Im}\,\chi_{\mu\nu}(q,\omega)\frac{dM_\nu(t)}{dt}\right]. \tag{19}$$

The first term on the right-hand side of this equation contributes to the torque $[\boldsymbol{M}\times\boldsymbol{H}_{eff}]$ only if $\chi_{\mu\nu}$ is anisotropic. In this case an anisotropic shift of the FMR frequency ensues. On the other hand, the second term contributes to the dissipative torque that is nonvanishing for both isotropic and anisotropic susceptibility.

III. GILBERT DAMPING

In what follows, we consider the dissipative torque for an isotropic electron gas. Thus, $\chi_{\mu\nu}(\boldsymbol{q},\omega)=\chi(\boldsymbol{q},\omega)\delta_{\mu\nu}$, and the dissipative part of the reaction field $\boldsymbol{H}_r^{(d)}$ is according to Eq. (19) given by

$$\boldsymbol{H}_r^{(d)}(t)\simeq -\frac{2J^2\Omega a n_s}{\gamma^2 d}\lim_{\omega\to 0}\left[\frac{\partial}{\partial\omega}\int_{-\infty}^{\infty}\frac{dq}{2\pi}\mathrm{Im}\,\chi(q,\omega)\right]\frac{d\boldsymbol{M}(t)}{dt}. \tag{20}$$

Introducing this result in the right-hand side of Eq. (2), we obtain the damping enhancement constant G',

$$G'\simeq 2J^2\Omega n_s M_s^2\left(\frac{a}{d}\right)\lim_{\omega\to 0}\left[\frac{\partial}{\partial\omega}\int_{-\infty}^{\infty}\frac{dq}{2\pi}\mathrm{Im}\,\chi(q,\omega)\right]. \tag{21}$$

A. Independent electrons

First, we evaluate the expression (21) by disregarding the electron-electron interaction in the N regions. However, a finite splitting Δ of the $\uparrow$- and $\downarrow$-spin bands is assumed. The external magnetic field of the FMR experiment is one source of this splitting. For a system of infinite size, this splitting establishes a lower cutoff on the wave vector q. As shown below, this cutoff is essential for preventing the logarithmic divergence of Eq. (21). Due to the spin splitting, the susceptibility develops some anisotropy. Since transverse components of the reaction field (19) contribute to the dissipative torque, we need to use in Eq. (21) the transverse susceptibility[21]

$$\chi_T^{(0)}(\boldsymbol{q},\omega)=\frac{\hbar^2}{4}\int\frac{d^3k}{(2\pi)^3}\frac{f_{k+q\downarrow}-f_{k\uparrow}}{\hbar\omega-\epsilon_{k+q\downarrow}+\epsilon_{k\uparrow}+i\eta}, \tag{22}$$

where

$$\epsilon_{k+q\downarrow}-\epsilon_{k\uparrow}\simeq\frac{\hbar^2\boldsymbol{k}\cdot\boldsymbol{q}}{m}+\Delta. \tag{23}$$

Expanding the Fermi functions, we have

$$f_{k+q\downarrow}-f_{k\uparrow}\simeq\frac{\partial f}{\partial\epsilon_k}\left(\frac{\hbar^2\boldsymbol{k}.\boldsymbol{q}}{m}+\Delta\right). \tag{24}$$

Using Eqs. (23) and (24), the imaginary part of Eq. (22) becomes

$$\mathrm{Im}\,\chi_T^{(0)}(\boldsymbol{q},\omega)\simeq\frac{-\hbar^2}{16\pi}\int_0^\infty dk k^2\frac{\partial f}{\partial\epsilon_k}\times\int_0^\pi d\theta\sin\theta\left(\Delta+\frac{\hbar^2\boldsymbol{k}\cdot\boldsymbol{q}}{m}\right)\times\delta\left(\hbar\omega-\Delta-\frac{\hbar^2\boldsymbol{k}\cdot\boldsymbol{q}}{m}\right), \tag{25}$$

where we used polar coordinates to perform the $\boldsymbol{k}$ integration. Performing first the integration over the polar angle θ, we have

$$\int_0^\pi d\theta\sin\theta\left(\Delta+\frac{\hbar^2\boldsymbol{k}\cdot\boldsymbol{q}}{m}\right)\delta\left(\hbar\omega-\Delta-\frac{\hbar^2\boldsymbol{k}.\boldsymbol{q}}{m}\right)=\frac{m\omega}{\hbar kq}\Theta\left(k-\frac{|\hbar\omega-\Delta|m}{\hbar^2 q}\right). \tag{26}$$

Introducing this result into Eq. (25), and converting the k integral to ϵ_k integration, we obtain at $T=0$

$$\mathrm{Im}\,\chi_T^{(0)}(q,\omega)=\frac{m^2\omega}{16\pi\hbar q}\int_0^\infty d\epsilon\,\delta(\epsilon-\epsilon_F)\times\Theta\left(\sqrt{\frac{2m\epsilon}{\hbar^2}}-\frac{|\hbar\omega-\Delta|m}{\hbar^2 q}\right)=\frac{m^2\omega}{16\pi\hbar q}\Theta\left(k_F-\frac{|\hbar\omega-\Delta|m}{\hbar^2 q}\right). \tag{27}$$

The unit step function on the right-hand side of this equation equals 1 for $q>q_1$, and is zero for $q<q_1$, where

$$q_1=\frac{|\hbar\omega-\Delta|m}{\hbar^2 k_F}. \tag{28}$$

Thus, q_1 acts as a lower cutoff in the q integral of $\mathrm{Im}\,\chi(q,\omega)$. Since $|\hbar\omega-\Delta|\ll\epsilon_F$, the upper cutoff is given by $q_2\approx 2k_F$. We then get using Eq. (27)

$$\int_{-\infty}^{\infty}\frac{dq}{2\pi}\mathrm{Im}\,\chi_T^{(0)}(\boldsymbol{q},\omega)\approx\frac{m^2\omega}{16\pi^2\hbar}\int_{q_1}^{q_2}\frac{dq}{q}=\frac{m^2\omega}{16\pi^2\hbar}\ln\frac{4\epsilon_F}{|\hbar\omega-\Delta|}. \tag{29}$$

Introducing this result into Eq. (21), we obtain

$$G' \simeq \frac{(JM_s am)^2}{8\pi^2 \hbar d} \ln \frac{4\epsilon_F}{\Delta}, \tag{30}$$

where we assumed a cubic lattice to write $\Omega n_s a = a^2$. Strictly speaking, G' in this equation is not a simple Gilbert damping since its strength depends on the applied magnetic field, but it is a weak dependence.

Let us now examine the validity of the slow precession approximation (10) that was used to derive Eq. (30). In Appendix C we show that the term in Eq. (10) that is second order in t' can be neglected when the FMR frequency ω_r and the gap Δ satisfy the condition

$$\frac{\hbar \omega_r}{2\Delta \ln \frac{4\epsilon_F}{\Delta}} \ll 1. \tag{31}$$

For clean and infinitely thick N layers, the external magnetic field of the FMR experiment is the only source of the gap Δ in the present formulation. However, as shown in Sec. IV, if we go beyond linear-response theory, an effective gap of order J_{sd} is obtained as a result of static spin polarization of the electron gas. Restricting ourselves to linear response, we have $\Delta = g\mu_B H$. The magnetic field used by Mizukami *et al.*[8] ranges from 1 to 14 kOe as its orientation varies from the in-plane to the perpendicular direction. Consequently, $\Delta/\hbar$ ranges from 3.5×10^{10} s^{-1} to 48.5×10^{10} s^{-1}. The precessional frequency, $\omega_r \simeq 6\times 10^{10}$ s^{-1}, actually exceeds $\Delta/\hbar$ for the in-plane orientation of the external field. According to a simple intuitive argument that the precessional frequency must be much smaller than the lowest excitation energy of the fermion system, $\Delta/\hbar$, one would expect that the slow precession approximation fails for this orientation of the field. However, owing to the factor $2 \ln 4\epsilon_F/\Delta$, the inequality (31) is well satisfied over the entire range of the field. Taking $\epsilon_F \simeq 7$ eV, as appropriate for copper, the left-hand side of Eq. (31) ranges from 6×10^{-2} to 5×10^{-3} as the field varies from 1 to 14 kOe.

It is interesting that spin relaxation of the conduction electrons can provide an effective gap and a finite cutoff length even in the limit of infinitely thick N layers. As shown in Appendix B, the resulting cutoff length is $q_1^{-1} \approx v_F \tau_s$ where τ_s is the spin-relaxation time. The corresponding value of the effective gap is $\Delta_{eff} \approx \hbar/\tau_s$. The same quantity determines the linewidth of the electron paramagnetic resonance of conduction electrons. Its magnitude is presumably not negligible in comparison with the magnetic splitting $g\mu_B H$.

In real samples, the gap is dominated by the effect of the finite thickness D of the normal layers. As shown in the next subsection, the boundary conditions at the outer surfaces of the sandwich yield a cutoff $q_1 \approx D^{-1}$. The corresponding energy gap is

$$\Delta \approx \hbar q_1 v_F \approx \frac{\hbar v_F}{D}. \tag{32}$$

We see that this gap becomes comparable with $\hbar \omega_r$ when $D \approx v_F/\omega_r$. For $v_F \approx 10^8$ cm s^{-1}, and $\omega_r \approx 6\times 10^{10}$ s^{-1}, the corresponding value of D is 1.6×10^{-3} cm. The thickness of the N layers in real samples is usually 10^{-3} smaller than this value, ensuring that the condition for slow precession is well satisfied.

B. Finite normal layers

We consider a ferromagnetic sheet imbedded between two N layers each of thickness D. Strictly speaking, in a finite system, the translational invariance invoked in the evaluation of the sheet sum (13) is broken. This complicates the calculation of the induced spin density. Nevertheless, a simplified approximate evaluation of $X_{\mu\nu}(x,\omega)$ can be carried out if $x \simeq 0$. In this case, the translational invariance is restored locally since the boundary at $x = D$ plays a small role. The boundary condition (BC) at $x = D$ is taken in the form

$$\langle s_\mu(x,t)\rangle|_{x=D} = 0. \tag{33}$$

This condition follows from assuming that there is an infinite potential step at the normal-metal–vacuum boundary. Thus, the N-electron wave functions are forced to equal zero at the boundary and so does the magnetization density. Applying this BC to Eq. (18), the finite-D version of Eq. (17) reads

$$X_{\mu\nu}(x,\omega) \simeq \frac{n_s}{D} \sum_{n=1}^{n_{max}} \cos(q_n x)\chi_{\mu\nu}(q_n,\omega), \tag{34}$$

where $q_n = \pi(n-1/2)/D$, and $n_{max} \simeq D/a$ since it is the lattice spacing which determines the highest value of q_n. In view of this, Eq. (21) is changed to

$$G' \simeq \frac{2a}{dD} J^2 \Omega n_s M_s^2 \lim_{\omega\to 0}\left[\frac{\partial}{\partial\omega}\sum_{n=1}^{n_{max}} \operatorname{Im}\chi(q_n,\omega)\right]. \tag{35}$$

For noninteracting electrons without spin splitting, we have

$$\sum_{n=1}^{n_{max}} \operatorname{Im}\chi(q_n,\omega) \simeq \frac{m^2\omega}{16\pi\hbar}\sum_{n=1}^{n_{max}} q_n^{-1}. \tag{36}$$

For $D/a \gg 1$, we use the definition of the Euler constant $\gamma_E \simeq 1.78$ to obtain

$$\sum_{n=1}^{n_{max}} q_n^{-1} \approx \frac{D}{\pi}\ln(4\gamma_E n_{max}). \tag{37}$$

Using Eqs. (35)–(37), we obtain

$$G' \approx \frac{(JM_s am)^2}{8\pi^2\hbar d}\ln(D/a). \tag{38}$$

As expected, the boundary conditions in a finite slab imply a cutoff $q_1 \approx D^{-1}$.

We now make an order-of-magnitude estimate of Eq. (38) for an iron film of thickness $d = D = 10a$ where $a = 4\times 10^{-8}$ cm. The constant J can be estimated by relating it to the atomic exchange integral J_{sd},

$$J \approx \frac{2J_{sd}\Omega}{\hbar^2}. \tag{39}$$

Taking $J_{sd}=0.1$ eV and $M_s=1.7\times10^3$ G, Eq. (38) yields $G'\approx10^8$ s^{-1}. This agrees with the interface damping observed recently in the double-layer structure by Urban *et al.*[6] It should be pointed out that the second ferromagnetic layer in this experiment plays a crucial role in establishing the spin sink needed to prevent spin accumulation in the N layers (see Sec. V.)

C. Electron-electron interactions

We now calculate G' by taking into account interactions between electrons in the normal metal. The generalized Hartree-Fock approximation for the Hubbard model yields the following expression for the transverse susceptibility:[24]

$$\chi_T(q,\omega)=\frac{\chi_T^{(0)}(q,\omega)}{1-\tilde{U}\chi_T^{(0)}(q,\omega)}, \tag{40}$$

where $\tilde{U}=4\Omega U/\hbar^2$, and U is the screened intra-atomic Coulomb energy. Using this formula, we have

$$\lim_{\omega\to0}\frac{\partial}{\partial\omega}[\mathrm{Im}\,\chi_T(q,\omega)] =[1-\tilde{U}\chi_T^{(0)}(q,0)]^{-2}\lim_{\omega\to0}\frac{\partial}{\partial\omega}[\mathrm{Im}\,\chi_T^{(0)}(q,\omega)]. \tag{41}$$

To simplify the evaluation of the q integral of this quantity, we take advantage of the weak dependence of the static susceptibility on q for $q<q_F$, and make the approximation

$$\chi_T^{(0)}(q,0)\approx\chi_T^{(0)}(0,0)=\frac{\hbar^2}{4\Omega}N(\epsilon_F), \tag{42}$$

where $N(\epsilon_F)$ is the density of states, per atom, at the Fermi energy. Using Eqs. (41) and (42) in Eq. (35), we obtain the enhancement, G', for interacting electrons in finite-N layers,

$$G'\approx\frac{(JM_s amS_E)^2}{8\pi^2\hbar d}\ln(D/a), \tag{43}$$

where S_E is the Stoner factor defined in Eq. (4). An estimate of this factor for palladium and platinum can be made using the giant magnetic moments of dissolved $3d$ atoms.[25] We use the fact that the giant moment is proportional to the Stoner factor of the host. In this way, we find S_E for palladium and platinum equal to 10 and 6, respectively. Thus, large values of G' are expected for sandwiches containing Pd and Pt as normal layers. Mizukami *et al.*[8] measured the Gilbert damping constant in $N/F/N$ sandwiches, with F as a thin film of a permalloy (Py). These measurements show that G' for the Pd/Py/Pd system is well above that for the Cu/Py/Cu system. However, it is about twice as small as G' for the Pt/Py/Pt. On the other hand, assuming all other parameters unchanged, Eq. (43) implies G' that is $(10/6)^2$ times larger for Pd. Thus, we have to deal with a net factor of 5 discrepancy between the experiment[8] and our theory. One possible explanation is that other parameters in Eq. (43), such as J, have larger values in Pt. Another possibility is the suppression of G' by spin accumulation in the normal layers. Since the spin-orbit coupling constant of Pt is larger (by a factor of 3) than that for Pd, the spin-lattice relaxation rate in Pt is, according to the theory of Elliot,[26] an order-of-magnitude stronger than that in Pd. As pointed out by Tserkovnyak *et al.*,[16] spin-accumulation takes place when the spin relaxation rate is small. This is presumably responsible for the absence of a measurable G' in Cu/Py/Cu system.[8]

More convincing evidence for Stoner enhancement of interface damping comes from the recent FMR studies on 20Au/40Fe/40Au/3Pd/$[\mathrm{Fe/Pd}]_5$/14Fe/GaAs(001) samples.[23] Compared to a single-layer structure, these samples show a G' that is enhanced by a factor of 4 and exhibits a strong fourfold in-plane anisotropy. Apparently, the presence of a second Fe layer provides an efficient spin sink. Thus G' is determined by the exchange-enhanced susceptibility rather than the bottleneck due to a weak spin-lattice relaxation in the N layers.

We now digress, for a moment, to consider the enhancement of the FMR frequency shift due to interactions. Applying Eq. (40) to the static anisotropic susceptibility, and making the approximation (42), we have $\chi_{\mu\nu}(q,0)\approx S_E\chi_{\mu\nu}^{(0)}(q,0)$. Using this result in Eq. (19), we see that the frequency shift for the interacting electrons is S_E times that for the independent electrons. This prediction could be used, in conjunction with the data for the anisotropic G', to clarify experimentally the role of interactions in the FMR of multilayers.

IV. RELATION TO SPIN-PUMPING THEORY

We now show that for free electrons there is a similarity between our formula (30) for G' and the spin-pumping theory of Tserkovnyak *et al.*[16] According to these authors, the excess damping produced by pumping of spins into adjacent N layers is $G'=\gamma M_s\alpha'$ where

$$\alpha'=\frac{g_L\mu_B[A_r^{(L)}+A_r^{(R)}]}{4\pi M_s L^2 d}, \tag{44}$$

where g_L is the Landé factor, μ_B is the Bohr magneton, and $A_r^{(L)}$ and $A_r^{(R)}$ are the interface parameters for the left and right N layers, respectively. In terms of the elements of the 2×2 scattering matrix, for a symmetric $N/F/N$ sandwich, these parameters are

$$A_r^{(L)}=A_r^{(R)}=A_r=\frac{1}{2}\sum_{mn}\{|r_{mn}^{\uparrow}-r_{mn}^{\downarrow}|^2+|t_{mn}'^{\uparrow}-t_{mn}'^{\downarrow}|^2\}, \tag{45}$$

where $(r_{mn}^{\uparrow},r_{mn}^{\downarrow})$ and $(t_{mn}^{\uparrow},t_{mn}^{\downarrow})$ are the reflection and transmission coefficients for electrons with up and down spins. The expression (45) is to be evaluated with the transverse modes (m,n) taken at the Fermi energy.

Following Bruno,[18] we consider the scattering of N electrons by a ferromagnetic monolayer. Due to the conservation of transverse momentum, $r_{mn}=r_m\delta_{mn}$, where $r_m=r_0(k_\perp)$, and $k_\perp$ is the component of the electron wave vector perpendicular to the monolayer.

The reflection coefficients $r_0(k_\perp)$ are found by solving the one-dimensional scattering problem for the potential

$$v(x)=v_0\delta(x), \tag{46}$$

where v_0 is given by the interface coupling constant J, and by the magnitude of the atomic spin S of the ferromagnet

$$v_0=\pm\frac{\hbar}{2}JSn_s, \tag{47}$$

where the $(+,-)$ signs correspond to the $(\downarrow,\uparrow)$ electron spins, respectively.

The reflection coefficients for this problem are[18]

$$r_0^{\uparrow}=\frac{-i\beta}{k_\perp+i\beta}, \tag{48}$$

$$r_0^{\downarrow}=\frac{i\beta}{k_\perp-i\beta}$$

where

$$\beta=\frac{mv_0}{\hbar^2}. \tag{49}$$

The transmission coefficients are[27]

$$t_0^{\uparrow}=\frac{ik_\perp}{ik_\perp-\beta}, \tag{50}$$

$$t_0^{\downarrow}=\frac{ik_\perp}{ik_\perp+\beta}.$$

Equations (48) and (50) imply

$$|r_0^{\uparrow}-r_0^{\downarrow}|^2_{\epsilon_F}=|t_0^{\uparrow}-t_0^{\downarrow}|^2_{\epsilon_F}=\frac{4\beta^2(k_F^2-k_\parallel^2)}{(k_F^2-k_\parallel^2+\beta^2)^2}, \tag{51}$$

where we applied the identity $(k_\parallel^2+k_\perp^2)_{\epsilon_F}=k_F^2$. Using this result, the transverse-mode sum (45) becomes a sum over the in-plane wave vectors $\boldsymbol{k}_\parallel$. Converting the $\boldsymbol{k}_\parallel$ sum to a two-dimensional integral, we have

$$A_r\simeq\frac{L^2}{2\pi}\int_0^{k_F}dk_\parallel k_\parallel\frac{4\beta^2(k_F^2-k_\parallel^2)}{(k_F^2-k_\parallel^2+\beta^2)^2}. \tag{52}$$

Evaluating this integral, we get $A_r\simeq(L^2\beta^2/\pi)F(\beta)$ where

$$F(\beta)\simeq\ln\frac{k_F^2+\beta^2}{\beta^2}-\frac{k_F^2}{k_F^2+\beta^2}. \tag{53}$$

Inserting Eqs. (52) and (53) into Eq. (44) and expressing β with use of Eqs. (47) and (49), we obtain

$$\alpha'\simeq\frac{\gamma(mSJn_s)^2}{8\pi^2\hbar M_s d}F(\beta). \tag{54}$$

We can bring this result closer to the form of our Eq. (30) by letting $n_s=a^{-2}$ and $M_s=\gamma Sa^{-3}$, yielding

$$\gamma S^2 n_s^2=\frac{M_s^2a^2}{\gamma}. \tag{55}$$

Furthermore, assuming $\beta\ll k_F$, the function $F(\beta)$ can be approximated by

$$F(\beta)\approx 2\ln\frac{k_F}{1.65\beta}\approx 2\ln\frac{\epsilon_F}{J_{sd}}. \tag{56}$$

From Eqs. (54)–(56) we have

$$G'=\gamma M_s\alpha'\approx\frac{(JM_s am)^2}{4\pi^2\hbar d}\ln\frac{\epsilon_F}{J_{sd}}. \tag{57}$$

This equation shows a remarkable similarity with the expression (30). Note, however, that the logarithmic terms do not match. If we consider an infinite system, and ignore the cutoff due to the magnetic length (32), the gap Δ vanishes, and Eq. (30) becomes logarithmically divergent. On the other hand, Eq. (57) shows that there is an effective gap $\Delta\approx J_{sd}$ corresponding to a finite cutoff $q_1\approx k_F(J_{sd}/\epsilon_F)$. Thus, the spin-pumping theory is divergence-free even for an infinite system.

The presence of an effective gap, $\Delta\approx J_{sd}$, in the spin-pumping theory is presumably linked to the fact that, in contrast to linear-response theory, it is of infinite order in the coupling constant J. This is seen in Eq. (54) where $F(\beta)$ is a nonlinear function of β given in Eq. (53). This kind of nonlinearity is generic in the scattering approach to transport [see Eq. (51)]. In fact, Bruno[18] derives an exact expression for the static RKKY coupling that goes beyond the linear-response result of Yafet.[20]

We now show that an effective gap of order J_{sd} can be obtained in the framework of the present theory if the strong static RKKY spin polarization of the N layers induced by the longitudinal ferromagnetic magnetization is included at the outset. To analyze the x dependence of this polarization, we use Eq. (11). For static M_z, we obtain with use of Eq. (6)

$$\langle s_z(x)\rangle\approx JS_z\sum_i^{sheet}\chi(\boldsymbol{x},\boldsymbol{r}_i,\omega=0), \tag{58}$$

where $\boldsymbol{x}$ is a fixed vector of length x perpendicular to the ferromagnetic sheet. For $D=\infty$, and a continuous distribution of the ferromagnetic moments, the sheet sum on the right-hand side of Eq. (58) becomes

$$\sum_i^{sheet}\chi(\boldsymbol{x},\boldsymbol{r}_i)\approx 2\pi n_s\int_0^\infty dr_i r_i\chi(\boldsymbol{x}-\boldsymbol{r}_i). \tag{59}$$

Using the three-dimensional RKKY range function for a point source,[19] and making a substitution $\boldsymbol{r}=\boldsymbol{x}-\boldsymbol{r}_i$, we obtain from Eqs. (58) and (59)

$$\langle s_z(x)\rangle \approx \frac{JS_z n_s m}{4\pi^2}\int_x^\infty \frac{dr}{r^3}[\sin(2k_F r)-2k_F r\cos(2k_F r)]. \tag{60}$$

Using integration by parts, the x dependence of the induced spin density takes the form of the range function derived by Yafet:[20]

$$\langle s_z(x)\rangle \approx \frac{JS_z n_s m k_F^2}{2\pi^2}\left[\frac{\pi}{2}-Si(X)-\frac{\cos X}{X}+\frac{\sin X}{X^2}\right], \tag{61}$$

where $X=2k_F r$, and $Si(X)$ is the sine integral. In contrast with the diverging three-dimensional range function for a point source,[19] the pseudo one-dimensional function in Eq. (61) converges for $x\to 0$ to a finite value

$$\langle s_z(x=0)\rangle \approx \frac{JS_z n_s m k_F^2}{4\pi}. \tag{62}$$

We are interested in the effective spin splitting which is responsible for this spin density. In a homogeneous electron gas of density n, with the spin splitting Δ, there is a uniform spin density

$$\langle s_z\rangle = \frac{3\hbar n\Delta}{16\epsilon_F}. \tag{63}$$

If $s_z(x)$ decayed slowly on the scale of the Fermi wavelength, it would be a good approximation to calculate G' from the susceptibility of a homogeneous electron gas with a spin splitting obtained by equating the spin densities (62) and (63). In this way we get a maximum spin splitting Δ_{max} given by

$$\Delta_{max} \approx \frac{4J\epsilon_F S_z n_s m k_F^2}{3\pi\hbar n} \sim 4\pi J_{sd}. \tag{64}$$

Actually, the decay of $s_z(x)$ is neither slow nor fast on the scale of the Fermi wavelength. Expanding the right-hand side of Eq. (61) for small X, we see that $\langle s_z(x)\rangle \sim \langle s_z(0)\rangle(1-0.8k_F x)$. Hence, the spin density drops to a zero value at a distance of about one Fermi wavelength from the ferromagnetic sheet. Exact calculation of the local dynamic susceptibility in an electron gas with such an inhomogeneity is a formidable task. Instead we argue, similar to Sec. III B, that the local susceptibility at $x\simeq 0$ is little influenced by the regions of $x \gg k_F^{-1}$. Hence, it can be estimated using an effective gap that is less than Δ_{max}, and to an order-of-magnitude accuracy given by J_{sd}. Using this gap in Eq. (30), we obtain a formula for G' which compares favorably with the spin-pumping result (57).

V. DISCUSSION

Our numerical estimate of G' based on Eq. (38) suggests that a substantial enhancement of the FMR linewidth, that is independent of the atomic number Z, should be observed in $N/F/N$ systems. In contrast, the data of Mizukami *et al.*[8] on trilayers containing permalloy films show a strong dependence of G' on Z. In fact, for Cu, which has the smallest Z of the N metals studied, there is a complete absence of a $1/d$-dependent G'. Tserkovnyak *et al.*[16] proposed that it is the spin accumulation in the N layer that is responsible for such a suppression of the ferromagnetic relaxation in the copper layer. Note that the theory of spin pumping assumes at the outset that the spin system in the N layer is kept in thermal equilibrium during the precession. For that one needs an efficient spin-sink mechanism. The data of Ref. 8 indicate that spin-lattice relaxation via spin-orbit coupling[26] provides the required spin sink. In fact, metals with larger Z exhibit generally larger measured values of G'. This trend is in agreement with the fact that the spin-lattice relaxation rate scales as Z^4.[28] In a recent paper, Tserkovnyak *et al.*[29] studied the role of spin accumulation in magnetization dynamics in F/N and $N/F/N$ multilayers. By taking into account the backflow of the spin current, the original idea[16] of relaxation suppression by spin accumulation in the N layers is given a firm theoretical foundation.

Also the theory of G', presented in Sec. III, assumes that the electron spins in the N layers are in thermal equilibrium. This can be established either by the spin-lattice relaxation in the bulk or by surface relaxation. One way to include these effects into the reaction field of Eq. (3) is to calculate the quantity $\langle \boldsymbol{s}(0,t)\rangle$ using the Bloch equation with diffusion.[30] This equation is to be solved with the BC that allows the electron spin to be flipped upon collision with the surface. Such a BC has been proposed by Dyson.[30] In terms of the spin density $\langle \boldsymbol{s}\rangle$ this so-called "evaporation" BC reads

$$\frac{\partial\langle \boldsymbol{s}\rangle}{\partial x} = \frac{3p}{4\Lambda}\langle \boldsymbol{s}\rangle, \tag{65}$$

where p is the probability that the spin flip will take place upon reflection from the boundary, and Λ is the mean free path in the bulk. Due to surface irregularities and paramagnetic surface impurities, the probability p can be large enough to provide the necessary spin sink even for layers with small bulk disorder.

Alternatively, the spin density and the reaction field can be obtained from the time-dependent 2×2 matrix kinetic equations driven by precessing magnetization of the ferromagnet.[31] Such an approach is inspired by the work of Kamberský[32] on intrinsic damping due to spin-orbit coupling in bulk ferromagnets. This work invokes the idea of a "breathing Fermi surface:" The chemical potential varies in response to the time-dependent perturbation. However, the distribution of the electrons does not respond instantaneously to the perturbation. There is a time lag characterized by a relaxation time τ. In Ref. 23 we apply this idea to the case of dynamic interlayer exchange coupling. In this case it is the spacer electrons which are affected by the spin-dependent potential at the interfaces, and the time variation of this potential is due to the precession of the ferromagnetic moment. Thus, the relevant relaxation time is the transverse spin-relaxation time τ_{spin}. The resulting effective damping field is, like Eq. (20), proportional to $d\boldsymbol{M}(t)/dt$, implying Gilbert damping. However, distinct from Eq. (20), it is also proportional to τ_{spin}.

This brings us to question. Whether there is a system to which the ballistic theory of the present paper is applicable. We believe that the double-layer structure studied in Ref. 6 is a good example of such a system. Here, the precessing layer F_1 deposits spin current into the N spacer and the second layer F_2 acts as an absorber of the transverse component of the spin current—thus providing an effective spin sink. Detailed analysis of this mechanism has been presented in beautiful papers by Stiles and Zangwill.[33,34] These authors showed that there is an oscillatory, power-law, decay of the transmitted transverse-spin current that is caused both by cancellations due to a distribution of precessional frequencies and the rotation of the spin of the incoming spin upon reflection. Consequently, almost complete cancellation of the transverse spin takes place after propagation into the ferromagnet by a few lattice constants. This finding also supports our assumption that the excitation of transverse components of $\langle \boldsymbol{s}(x,t)\rangle$, via s-d exchange, is confined to the N/F interface layer (see Appendix A).

Since our results for the Gilbert damping constant G' are based on a $T=0$ theory, whereas the FMR studies are done at finite temperatures, it is important to have an estimate of the temperature region over which Eqs. (30) and (38) are valid. By considering thermal excitation across the energy gap Δ, the criterion of validity is $k_B T \ll \Delta$. For N layers of infinite thickness, G' is given by Eq. (30) where $\Delta = g\mu_B H$. For the external magnetic field ranging from 1 to 10 kOe, the temperature $T=\Delta/k_B$ ranges from 0.2 K to 2 K. On the other hand, for N layers of finite thickness D, the energy gap is given by Eq. (32). Taking $D=100k_F^{-1}$, $v_F = 10^8$ cm s^{-1}, and $k_F=10^8$ cm^{-1}, the corresponding temperature $T=\Delta/k_B$ is 800 K. It should be pointed out that the expression (57), derived from spin-pumping theory,[16] involves an effective gap $\Delta \approx J_{sd}$. For $J_{sd}\approx 1$ eV, the corresponding temperature Δ/k_B is of order 10^4 K.

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APPENDIX A: DERIVATION OF EQ. (3)

We consider a trilayer shown in Fig. 1, and derive the reaction field from the torque equation

$$[\boldsymbol{M}_f(t)\times \boldsymbol{H}_r(t)] = \boldsymbol{T}(t), \tag{A1}$$

where $\boldsymbol{M}_f(t)$ is the net magnetic moment of the ferromagnetic film, and $\boldsymbol{T}(t)$ is the torque due to the s-d interaction. In what follows, we assume that this torque is contributed only by the s-d interaction with the outmost magnetic planes. As pointed out by Bruno and Chappert,[36] this assumption is substantiated experimentally by the fact that the observed interlayer exchange coupling in $F/N/F$ systems is roughly independent of the thickness of the ferromagnetic layers (see also Ref. 4). Theoretical understanding of this fact stems from the work of Stiles and Zangwill[34] discussed in Sec. V. Since only interface regions contribute to the torque, we pick a magnetic atom in the plane $x=0$, and consider the local magnetic field $\boldsymbol{H}^{(i)}(t)$ acting on its magnetic moment $\boldsymbol{M}^{(i)}(t)$. Using Eq. (5), the expectation value of the s-d exchange energy of this atom is $-J\boldsymbol{S}^{(i)}(t)\cdot\langle \boldsymbol{s}(0,t)\rangle$. If we write this quantity as $-\boldsymbol{M}^{(i)}(t)\cdot \boldsymbol{H}^{(i)}(t)$, where $\boldsymbol{M}^{(i)}(t)=\gamma \boldsymbol{S}^{(i)}(t)$, the local magnetic field is

$$\boldsymbol{H}^{(i)}(t) = \frac{J}{\gamma}\langle \boldsymbol{s}(0,t)\rangle. \tag{A2}$$

For a square film of area L^2, the number of interface atoms is L^2/a^2. Thus, using Eq. (A2), the net torque contributed by both interfaces is

$$\boldsymbol{T} = \frac{2L^2 J}{a^2\gamma}[\boldsymbol{M}^{(i)}(t)\times\langle \boldsymbol{s}(0,t)\rangle]. \tag{A3}$$

Noting that the net magnetic moment of the film is $\boldsymbol{M}_f = \boldsymbol{M}^{(i)} L^2 d/a^3$ and using Eq. (A3) in Eq. (A1) yields Eq. (3). A similar approach has been used to deduce the effective field in ultrathin layers in the presence of interfaces [see Eq. (1.6) in Ref. 3).

APPENDIX B: SPIN RELAXATION AND INFRARED CUTOFF

To include spin relaxation into the theory of G', we start from Eq. (22) and replace the infinitesimal quantity η by $\Gamma = \hbar/\tau_s$, where τ_s is the spin-relaxation time. Thus, the electron-hole pairs with flipped spin are assumed to relax with a frequency $\Gamma/\hbar$. A similar assumption has been made in the theory of magnon relaxation via s-d interaction.[35] Moreover, we neglect the splitting Δ. Making these changes in Eqs. (22)–(24), we obtain

$$\mathrm{Im}\,\chi_T(q,\omega) \approx -\frac{\hbar^2\Gamma}{16\pi^2}\int_0^\infty dk k^2 \frac{\partial f}{\partial \epsilon_k} I(k,q), \tag{B1}$$

where

$$I(k,q) = \int_0^\pi d\theta \sin\theta \frac{\hbar^2 kq\cos\theta/m}{(\hbar\omega - \hbar^2 kq\cos\theta/m)^2 + \Gamma^2}. \tag{B2}$$

Expanding the integrand to order ω, the θ integration yields

$$I(k,q) = \frac{2\hbar\omega}{\Gamma^2 x}\left[\tan^{-1}x - \frac{x}{1+x^2}\right], \tag{B3}$$

where $x=\hbar^2 kq/(\Gamma m)$. Using Eq. (B3), the k integration in Eq. (B1) yields at $T=0$

$$\mathrm{Im}\,\chi_T(q,\omega)\approx\frac{m^2\omega}{8\pi^2\hbar q}\left[\tan^{-1}(v_F\tau_s q)-\frac{v_F\tau_s q}{1+(v_F\tau_s q)^2}\right]. \tag{B4}$$

Consistent with Eq. (27), this expression is equal to $m^2\omega/(16\pi\hbar q)$ in the limit $\tau_s\to\infty$. The evaluation of the q integral of this expression is done by approximating $\tan^{-1}x$ by $\pi x/2$ for $x<1$, and by $\pi/2$ for $x>1$. For $\epsilon_F\gg\Gamma$, we obtain

$$\int_{-q_2}^{q_2}\frac{dq}{2\pi}\mathrm{Im}\,\chi_T(q,\omega)\approx\frac{m^2\omega}{16\pi^2\hbar}\ln(v_F\tau_s q_2). \tag{B5}$$

Introducing this result into Eq. (21), the damping enhancement in the presence of spin relaxation is given by

$$G'\approx\frac{(JM_s am)^2}{8\pi^2\hbar d}\ln\frac{q_2}{q_1}, \tag{B6}$$

where $q_1\approx(v_F\tau_s)^{-1}$ is the infrared cutoff mentioned at the end of Sec. III A.

APPENDIX C: VALIDITY OF SLOW PRECESSION APPROXIMATION

To find the criterion for validity of the approximation used in Eq. (10) we expand $S_\nu^{(i)}(t-t')$ up to second order in t',

$$S_\nu^{(i)}(t-t')\simeq S_\nu^{(i)}(t)-t'\frac{dS_\nu^{(i)}(t)}{dt}+\frac{t'^2}{2}\frac{d^2S_\nu^{(i)}(t)}{dt^2}. \tag{C1}$$

Introducing this expansion into Eq. (9), we obtain an induced spin density that is given by Eq. (11), plus a term $\langle s_\mu^{(2)}(t)\rangle$ generated by the t'^2 term in Eq. (C1). For the component, say, $\mu=x$, we have

$$\langle s_x^{(2)}(\boldsymbol{r},t)\rangle=-\frac{J\Omega}{2\gamma}\lim_{\omega\to 0}\mathrm{Re}\left[\frac{d^2M^+(t)}{dt^2}\sum_i^{sheet}\partial_\omega^2\chi(\boldsymbol{r},\boldsymbol{r}_i)\right], \tag{C2}$$

where $M^+=M_x+iM_y$. Assuming a circular precession, we have

$$M_x(t)=m_1\cos\omega_r t, M_y(t)=-m_1\sin\omega_r t. \tag{C3}$$

On the right-hand side of Eq. (C2), we have two terms. One is proportional to d^2M_x/dt^2, the other one to $d^2M_y(t)/dt^2$. Using Eq. (C3), we see that only the latter term oscillates in phase with the $dM_x(t)/dt$ term which corresponds to the second, Gilbert damping, term of Eq. (11). Denoting this damping term as $\langle s_x^{(1)}(\boldsymbol{r},t)\rangle$, we calculate using Eqs. (C2), (C3), and (19) the ratio

$$R_{21}=\frac{\langle s_x^{(2)}(0,t)\rangle}{\langle s_x^{(1)}(0,t)\rangle}\simeq\frac{\omega_r}{2}\lim_{\omega\to 0}\left[\frac{\partial_\omega^2 A(\omega)}{\partial_\omega A(\omega}\right), \tag{C4}$$

where

$$A(\omega)=\int_{-\infty}^{\infty}\frac{dq}{2\pi}\mathrm{Im}\,\chi(q,\omega). \tag{C5}$$

For a free-electron gas, the expression on the right-hand side of this equation is given by Eq. (29). Using this form of $A(\omega)$ on the right-hand side of Eq. (C4), we obtain

$$R_{21}\simeq\frac{\hbar\omega_r}{2\Delta\ln\dfrac{4\epsilon_F}{\Delta}}. \tag{C6}$$

If $R_{21}\ll 1$, the t'^2 term in the expansion (C1) plays a negligible role in comparison with the t' term. In this way, we obtain the condition of slow precession approximation quoted in Eq. (31).

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