Decoupling and decommensuration in layered superconductors with columnar defects

A. Morozov and B. Horovitz

Department of Physics and Ilze Katz Center for Nanotechnology, Ben Gurion University, Beer Sheva 84105, Israel

P. Le Doussal

CNRS-Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France (Received 31 October 2002; published 28 April 2003)

We consider layered superconductors with a flux lattice perpendicular to the layers and random columnar defects parallel to the magnetic field B. We show that the decoupling transition temperature T_d , at which the Josephson coupling vanishes, is enhanced by columnar defects by an amount $\delta T_d/T_d \sim B^2$. Decoupling by increasing field can be followed by a reentrant recoupling transition for strong disorder. We also consider a commensurate component of the columnar density and show that its pinning potential is renormalized to zero above a critical long-wavelength disorder. This decommensuration transition may account for a recently observed kink in the melting line.

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The phase diagram of layered superconductors in a magnetic field B perpendicular to the layers is of considerable interest in view of recent experiments on high-temperature superconductors.¹⁻⁴ Columnar defects (CD) induced by heavy-ion bombardment provide an additional interesting probe. In particular, the irreversibility line at low temperatures is enhanced, 5,6 while within the liquid phase an onset of enhanced z-axis correlation⁷⁻⁹ was observed. Recent data¹⁰ indicate that CD produce a porous vortex matter in which ordered vortex crystallites are embedded in the "pores" of a rigid matrix of vortices pinned on the CD. A sharp kink in the melting curve signals an abrupt change from melting enhanced by the matrix at high fields to a more weakly enhanced melting at lower fields. Theoretical studies on CD have shown a "localization" transition within a bose glass phase. 11,12 Recent simulations 13 have been interpreted in terms of a Bragg-Bose glass with positional order which sets in as field increases. Also a "recoupling" crossover transition¹⁴ was studied in the vortex liquid phase.

In the absence of CD, theoretical studies have shown layer decoupling due to thermal fluctuations ^{15–17} or due to disorder. ^{17,18} At this phase transition, the Josephson coupling between layers vanishes at long scales, i.e., the critical current perpendicular to the layers vanishes and superconducting correlations in the *z* direction (perpendicular to the layers) become short range. Decoupling involves, in principle, also proliferation of point defects—vacancies and interstitials (VI). ¹⁹ The flux lattice is present even in the decoupled phase with the *z*-axis positional correlations maintained by magnetic couplings. In the case of point disorder, this phase would thus still exhibit Bragg-glass-type order without dislocations.

An increase in the critical current at a "second peak" transition has been interpreted as due to an apparent discontinuity in the tilt modulus at decoupling. Plasma resonance data^{21,22} have shown a significant jump at this transition, consistent with the decoupling scenario. Whether this transition is driven by decoupling alone, rather than by a sudden dislocation proliferation, ²³ remains to be investigated.

In the present work, we consider the effects of CD within the flux lattice phase, neglecting VI (whose role is discussed below). We find that the decoupling transition temperature $T_d(B)$ is enhanced by CD. In particular, for strong disorder, the low-field form $T_d(B) \sim 1/B$ becomes $T_d(B) \sim B$ at strong fields, hence decoupling followed by a reentrant transition into a coupled state, i.e., recoupling, is possible with increasing field. These predictions can test whether the second peak transition is of a decoupling type. We also allow for a finite component of the CD density, which is commensurate with the flux lattice, a component that usually needs to be specifically prepared.²⁴ We find that long-wavelength disorder renormalizes the commensurate coupling to zero, i.e., decommensuration, above a critical value of disorder. We propose that the matrix component of the porous vortex matter provides a commensurability potential for the embedded crystallites. At high fields, this enhances the crystallite melting temperature, while below the decommensuration transition, the crystallites decouple from the matrix, leading to a weaker enhancement of melting.

We study the classical partition function of L/d Josephson coupled layers, where $L \rightarrow \infty$ is the total length in the z direction perpendicular to the layers and d is the interlayer spacing. The elastic energy of the transverse displacement fields u(q,k) in the absence of Josephson coupling can be written as 25,26

$$H_{el} = \frac{1}{2L} \sum_{k,a} \int_{a} (c_{66}q^2 + c_{44}^0(k)k_z^2) |u^a(q,k)|^2,$$
 (1)

where a replica index $a=1,2,\ldots,n$ is needed below for the disorder average. The elastic constants $\operatorname{are}^{25,26} c_{44}^0(k) = \tau/(8da_0^2\lambda_{ab}^2k_z^2)\ln(1+a_0^2k_z^2/4\pi)$ and $c_{66}=\tau/(16da_0^2)$, where k is the wave vector in the z direction, $k_z=(2/d)\sin(kd/2)$, $\tau=\Phi_0^2d/(4\pi^2\lambda_{ab}^2)$, λ_{ab} is the magnetic penetration length parallel to the layers, a_0^2 is the area of the flux lattice unit cell, and Φ_0 is the flux quantum, i.e., $\Phi_0=Ba_0^2$. Note that the Josephson coupling induces an additional term in c_{44} , c_{44} as also shown below. The decoupling transition of the pure system (for weak Josephson coupling) is given by $c_{44}^{16,17}$

$$T_d^0 = \frac{4a_0^4}{d^2} \left(\int_k \frac{dk}{c_{44}^0(k)} \right)^{-1} \tag{2}$$

and our principal aim is to obtain the corresponding T_d in the presence of correlated disorder.

Consider a distribution of CD whose positions within a layer are random and uncorrelated. Each of the CD has a radius b_0 and their average areal density n_{CD} is low, $n_{CD}b_0^2 \ll 1$. A flux line has a core of radius ξ_0 that usually satisfies $\xi_0 < b_0$. Once a flux line is partially inside a CD, it gains its core energy E_c per layer. The pinning potential per unit area is then $U_{pin}(\mathbf{r}) = (E_c/\xi_0^2) \Sigma_i p(\mathbf{r} - \mathbf{r}_i)$ with the sum on the CD positions and $p(\mathbf{r})$ is a shape function, e.g., $p(\mathbf{r}) = 1$ for $r < b_0$ and vanishes for $r > b_0$. The variance, neglecting CD overlaps, is, therefore,

$$\overline{U_{pin}(\mathbf{r})U_{pin}(\mathbf{r}')} \approx E_c^2 n_{CD} (b_0/\xi_0)^4 \delta^2(\mathbf{r} - \mathbf{r}'). \tag{3}$$

The average with respect to a flux density involves an additional factor $(\xi_0/a_0)^4$ due to the decomposition of a sharply peaked flux into harmonics with reciprocal vectors Q. The replica average at temperature T is then²⁷

$$\mathcal{H}_{dis} = -\sum_{ab} \left\{ \frac{1}{L} \sum_{k} \int \frac{d^{2}q}{(2\pi)^{2}} sq^{2}L \, \delta_{k,0} u^{a}(\mathbf{q}, k) u^{b}(-\mathbf{q}, -k) \right\}$$

$$+W\sum_{n,n'}\int d^2r\cos[\mathbf{Q}\cdot(\mathbf{u}_n^a(\mathbf{r})-\mathbf{u}_{n'}^b(\mathbf{r}))]\right\}/2T,\quad(4)$$

where $W = E_c^2 n_{CD} (b_0/a_0)^4$ and only the shortest most relevant²⁷ \mathbf{Q} is retained. The cos term above involves vectors \mathbf{Q} and \mathbf{u}_n^a which in the averages below yield $\langle \mathbf{Q} \cdot \mathbf{u} \rangle^2 = Q^2 \langle u^2 + u_l^2 \rangle / 2$; where u_l is the longitudinal displacement that is neglected for now as it has no effect on the decoupling transition. The parameter s measures a long-wavelength random torque coupled to a local bond angle²⁸ $\gamma = (\partial_x u_y - \partial_y u_x)/2$, since for transverse modes $(\nabla u)^2 = 4\gamma^2$.

The long-range Bragg glass properties depend on the non-linear cos term in Eq. (4). If this cos is expanded, it yields $\sim \Sigma_{ab} \int d^2r u^a(\mathbf{r},k=0) u^b(\mathbf{r},k=0)$, i.e., a k=0 quadratic term that has no effect on the decoupling transition. It is, therefore, essential to treat the Bragg glass nonlinearities properly.

We also allow for a commensurate term of the CD density of the form

$$\mathcal{H}_{com} = -y_c (2d/Q^2) \sum_{n,a} \int d^2 r \cos[\mathbf{Q} \cdot \mathbf{u}_n^a(\mathbf{r})].$$
 (5)

Consider next the Josephson phase, i.e., the relative superconducting phase of two neighboring layers. Each flux line can be viewed as a collection of point singularities, or pancake vortices, positioned one on top of the other in consecutive layers. Around each pancake vortex, the superconducting phase follows the angle $\alpha(\mathbf{r})$ that changes by 2π in a complete rotation. The Josephson phase involves then a nonsingular component $\theta_n(\mathbf{r})$ and a singular contribution from pancake vortices. The latter are positioned at $\mathbf{R}_l + \mathbf{u}_l^n$ in

the *n*th layer and at $\mathbf{R}_l + \mathbf{u}_l^{n+1}$ in the (n+1)th layer, where \mathbf{R}_l is the undistorted position of the *l*th flux line. The total Josephson phase is then

$$\theta_{n}(\mathbf{r}) + \sum_{l} \left[\alpha(\mathbf{r} - \mathbf{R}_{l} - \mathbf{u}_{l}^{n}) - \alpha(\mathbf{r} - \mathbf{R}_{l} - \mathbf{u}_{l}^{n+1}) \right]$$

$$\approx \theta_{n}(\mathbf{r}) + \sum_{l} \left(\mathbf{u}_{l}^{n} - \mathbf{u}_{l}^{n+1} \right) \nabla \alpha(\mathbf{r} - \mathbf{R}_{l}), \tag{6}$$

where the expansion is justified in the Bragg glass, since the correlation length in the z direction is $\gg d$. We define (including now the replica index) $b^a(\mathbf{q},k) = -2\pi de^{ikd/2}u^a(\mathbf{q},k)k_z/(qa_0^2)$ so that the Josephson phase is $\theta_n^a(\mathbf{r}) + \theta_n^a(\mathbf{r})$. Fluctuations of the $\theta_n(\mathbf{r})$ field involve the Josephson energy as well as magnetic-field terms,

$$\mathcal{H}_{J} = \frac{1}{2L} \sum_{k,a} \int \frac{d^{2}q}{(2\pi)^{2}} G_{f}^{-1}(q,k) |\theta^{a}(\mathbf{q},k)|^{2} -y_{J} \sum_{n,a} \int d^{2}r \cos[\theta_{n}^{a}(\mathbf{r}) + b_{n}^{a}(\mathbf{r})], \tag{7}$$

where 29 $G_f(q,k) = 4\pi d^3(\lambda_{ab}^{-2} + k_z^2)/(\tau q^2)$. The full Hamiltonian is then $\mathcal{H} = \mathcal{H}_{el} + \mathcal{H}_{dis} + \mathcal{H}_{com} + \mathcal{H}_J$.

We proceed to solve this system by the variational method allowing for replica symmetry breaking (RSB). The form of the variational Hamiltonian \mathcal{H}_0 is obtained by expanding the cos terms and replacing y_J , y_c , and W by variational parameters z_J , z_c , and $\sigma_{ab}(k)$, respectively. The Josephson term involves a θ ,b cross term that is eliminated by a shift $\tilde{\theta}^a(\mathbf{q},k) = \theta^a(\mathbf{q},k) - u^a(\mathbf{q},k)z_J(2\pi k_z/a_0^2q) \exp(ikd/2)/(G_f^{-1} + z_J/d)$. Hence (repeated indices are summed),

$$\mathcal{H}_{0} = \frac{1}{2L} \sum_{k} \int \frac{d^{2}q}{(2\pi)^{2}} \{ G_{ab}^{-1}(q,k) u^{a}(\mathbf{q},k) u^{b*}(\mathbf{q},k) + [G_{f}^{-1}(q,k) + z_{J}/d] | \widetilde{\theta}^{a}(\mathbf{q},k) |^{2} \},$$
(8)

$$G_{ab}^{-1}(q,k) = [c_{66}q^2 + c_{44}(q,k)k_z^2 + z_c]\delta_{ab}$$

$$-sL\frac{q^2}{T}\delta_{k,0} - \sigma_{ab}(k). \tag{9}$$

The effect of the nonsingular θ^a is to shift $c_{44}^0(k)$ of Eq. (1) into $c_{44}(q,k)$

$$c_{44}(q,k) = c_{44}^{0}(k) + \frac{B^{2}}{4\pi(1+\lambda_{c}^{2}q^{2}+\lambda_{ab}^{2}k_{z}^{2})},$$
 (10)

where $\lambda_c^2 = \Phi_0^2/(16\pi^3 z_J d)$. Note that the limit $\lambda_c \to \infty$ at decoupling must be taken before $q \to 0$.

The variational method minimizes the free energy $F_0 + \langle \mathcal{H} - \mathcal{H}_0 \rangle_0$, where the free energy F_0 and the average $\langle \cdots \rangle_0$ correspond to \mathcal{H}_0 . This yields

$$\sigma_{ab}(k) = (WQ^2/2d^2T) \int_0^L dz \bigg[\cos kz \exp(-B_{ab}(z)/2) \\ -\delta_{ab} \sum_c \exp(-B_{ac}(z)/2) \bigg], \tag{11}$$

$$z_{J} = y_{J} \exp \left\{ - (T/2) \int_{q,k} \left[\frac{k_{z}^{2}}{q^{2}} \left(\frac{2\pi d}{a^{2}} \right)^{2} \left(\frac{G_{f}^{-1}(q,k)}{G_{f}^{-1}(q,k) + z_{J}/d} \right)^{2} \right.$$

$$\times G_{aa}(q,k) + (G_f^{-1}(q,k) + z_J/d)^{-1} \bigg] \bigg\},$$
 (12)

$$z_c = y_c \exp\left\{-(TQ^2/4) \int_{q,k} G_{aa}(q,k)\right\},$$
 (13)

where $\int_{q,k} = \int d^2q dk/(2\pi)^3$ and $B_{ab}(z)$ is given by

$$B_{ab}(z) = TQ^2 \int \frac{d^2q \, dk}{(2\pi)^3} [G_{aa}(q,k) - \cos(kz)G_{ab}(q,k)].$$

Since the disorder is z independent, the off-diagonal terms $B_{a\neq b}(z)$ are z independent so that $\sigma_{a\neq b}$ has only a k=0 component; hence RSB is present only at k=0. It is convenient to define $G_c^{-1}(q,k) = \sum_b G_{ab}^{-1}(q,k)$ so that for $k\neq 0$ $G_c(q,k) = G_{aa}(q,k)$. The RSB solution reduces here to a one-step form, 27 hence $G_c(q,k)$ can be written with self-energies in the form

$$G_c^{-1}(q,k) = c_{66}q^2 + c_{44}(q,k)k_z^2 + z_c + \sum_1 (1 - \delta_{k,0}) + I(k), \tag{14}$$

$$\Sigma_1 + z_c = (WQ^4/16\pi d^2c_{66})\exp(-B_+/2).$$
 (15)

Here, $B_+ = TQ^2 \int_{q,k} G_c(q,k)$ is a Debye Waller factor that is dominated by large q,k so that $c_{44}^0(k)$ can be used to obtain

$$|B_{+}| \lesssim \frac{16\pi T}{\tau} \ln \frac{c_{66}Q^{2}}{(\tau/8da_{0}^{2}\lambda_{ab}^{2}) + \Sigma_{1} + I(\pi/d) + z_{c}},$$
 (16)

while the function I(k) satisfies for $T \ll \tau$

$$I(k) = 4\pi c_{66}(\Sigma_1 + z_c) \int \frac{d^2q}{(2\pi)^2} [(c_{66}q^2 + z_c + \Sigma_1)^{-1} - G_c(q, k)].$$
(17)

Note that for $k \rightarrow 0$ this yields $I(k) \sim |k|$, while $I(k) \approx \Sigma_1 + z_c$ for large k, up to logarithmic terms. A condition for melting can be estimated by a Lindemann number $c_L \approx 0.15$ (Ref. 11) so that $\langle u_n^2(\mathbf{r})/a_0^2\rangle = B_+/4\pi^2 \approx c_L^2$. Hence, τ is a measure of the melting temperature and for $T \ll \tau$, B_+ is small. We note that it is essential to keep the nonsingular phase θ to obtain the correct structure factor $G_c(q,k)$ in Eq. (14).

The decoupling transition is determined by the vanishing of z_J . Equation (12) can be written in the form

$$z_{J} = y_{J} \exp \left(-\frac{T}{2} \int_{q,k} \left(\frac{z_{J}}{d} + \left[G_{f}(q,k) + \left(\frac{2\pi dk_{z}}{a_{0}^{2}q}\right)^{2}\right] \right) \right) \times G_{c}(q,k;z_{J} = 0) \right)^{-1}.$$

The integral is dominated by $k \gg q$ so that for $k > 1/\lambda_{ab}$ the effect of the θ field via $G_f(q,k)$ is negligible for $a_0^2/8\pi\lambda_{ab}^2 \ll 1$. The q integration is then dominated by $c_{44}^0(k)$ (while c_{66} serves as a cutoff) leading to a $\ln z_J$,

$$z_{J} \sim y_{J} \exp \left\{ \frac{T}{8\pi} \int \frac{dk}{2\pi} \frac{(2\pi d/a_{0}^{2})^{2}}{c_{44}^{0}(k) + [\sum_{1} + z_{c} + I(k)]/k_{z}^{2}} \ln z_{J} \right\},\,$$

hence,

$$T_d = \frac{4a_0^4}{d^2} \left(\int_k \frac{dk}{c_{\Delta d}^{eff}(k)} \right)^{-1}, \tag{18}$$

where an effective elastic modulus is $c_{44}^{eff}(k) = c_{44}^0(k) + [\Sigma_1 + z_c + I(k)]/k_z^2$. The z_c term is obvious here by an expansion of \mathcal{H}_{com} , Eq. (5). However, the $\Sigma_1 + I(k)$ term cannot be derived by an expansion of \mathcal{H}_{dis} and a full RSB treatment is required. Σ_1 acts then as z_c and leads to a divergence of $c_{44}^{eff}(k)$ as $k \rightarrow 0$, as already noted in Ref. 27. Since large k dominates the integral in Eq. (18), we use $I(k) \approx \Sigma_1 + z_c$ so that

$$T_d \approx T_d^0 \left[1 + (\Sigma_1 + z_c) \frac{8d\lambda_{ab}^2 a_0^2}{\tau \ln(a_0/d)} \right], \tag{19}$$

where $T_d^0 = \tau a_0^2 \ln(a_0/d)/(4\pi\lambda_{ab}^2)$, from Eq. (2), is the transition temperature in the pure system. Since $Q \sim 1/a_0$ and $W \sim a_0^{-4}$, we have from Eq. (15) $\Sigma_1 + z_c \sim a_0^{-6}$. Thus, the change in T_d due to columnar defects is $\delta T_d/T_d^0 \sim a_0^{-4} \sim B^2$, up to $\ln B$ terms. From Eqs. (15) and (19), we obtain our first principal result

$$\frac{\delta T_d}{T_d^0} \approx \frac{2(4\pi)^3 E_c^2 n_{CD} b_0^4 \lambda_{ab}^2}{\tau^2 a_0^4 \ln(a_0/d)} \approx 10^2 \left(\frac{b_0}{a_0}\right)^4 n_{CD} \lambda_{ab}^2 \,, \quad (20)$$

where $E_c \approx 0.2\tau$ (Ref. 11). For strong disorder and strong fields, the CD can dominate and then $T_d \sim B$ increases with B. This allows a reentrant behavior, i.e., for a fixed temperature, as B is increased a decoupling occurs at $T_d \approx T_d^0 \sim 1/B$ and then a recoupling would occur at a higher field, assuming this field is still below melting.

Next we address the commensurability term, which, unlike the Josephson coupling, depends also on the longitudinal $u_l(\mathbf{q},k)$ component. We, therefore, add longitudinal energy terms: first, an elastic energy of the form (1) with c_{66} and c_{44}^0 replaced by c_{11} and c_{44}^l , respectively;²⁶ and second, the usual long-wavelength disorder coupled to $\nabla \cdot \mathbf{u}$ (Ref. 27) which yields the form of the first term of Eq. (4) with s replaced by s^l . Since σ_{ab} originates from the W term in Eq. (4), Σ_1 and I(k) are common to both longitudinal and transverse parts, while the location of the one-step solution changes by a factor $c_{11}/(c_{11}+c_{66})$. Since c_{66}/c_{11}

 $=a_0^2/16\pi\lambda_{ab}^2 \ll 1$, the effect on Σ_1 is small, yet the structure factor for the longitudinal modes [analog of Eq. (14)] is significantly modified by the same Σ_1 and I(k).

The equation for z_c depends also on the k=0 component of $G_{aa}(q,k)$ which involves the long-wavelength disorder parameters s, s^l . Using inversion methods for G_{ab} (Refs. 27,30), we obtain our second principal result

$$z_c \sim y_c \left(\frac{z_c}{\Sigma_1 + z_c}\right)^{1/2} (z_c)^{sQ^2/(16\pi c_{66}^2) + s^l Q^2/(16\pi c_{11}^2)}.$$
 (21)

Hence, at some critical s, s^l [where the powers of z_c on both sides of Eq. (21) equal], the commensurability potential is renormalized to zero. We note that this renormalization is driven by k=0 terms, i.e., the same derivation is valid for a 2D system with point disorder.³¹

Long-wavelength disorder can generate dislocations²⁸ at $s > c_{66}^2 a_0^2 / 16\pi$, i.e., below decommensuration. Furthermore, on very long scales, dislocations will be induced by shortwavelength CD disorder as the system is effectively two-dimensional.^{28,32} We limit our discussion to a Bragg glass domain that ignores these very long scale effects.

The decoupling description neglects point defects, i.e., the nucleation of VI. The latter were studied in the absence of Josephson coupling and were shown to be generated by point disorder, leading to logarithmically correlated disorder for VI. Disordered CD, however, induce only a k=0 component of disorder which has exponentially decreasing correlations $\sim (q^2 + \lambda_{ab}^{-2})^{-2} (q^2 + \Sigma_1)^{-1}$, hence, the defect transition is not affected by the CD. The true decoupling, which allows for both Josephson phase fluctuations and for point defects, lies near the above decoupling for not too small Josephson coupling. 19,29

Finally, we address the data¹⁰ on the melting curve showing a kink at fields $B_k \gg B_{\phi}$. Within the proposed porous

vortex model, 10 we suggest that the "vortex matrix," pinned by the random CD, forms a commensurate potential. The lowest harmonic of this potential which couples to the flux periodicity has wave vector \mathbf{Q} [harmonics with $\mathbf{Q}' > \mathbf{Q}$ have a $(z_c/(z_c+\Sigma_1))^{Q'^2/2Q^2}$ factor in Eq. (21), forcing a $z_c=0$ solution]. Since s is a second-order effect, we consider s^l $=W/d^2 \sim B_{ab}B^2$, hence decommensuration occurs at B $\sim B_{\phi}$, the bare proportionality constant being, however, too small to account for the data. The parameters s, s^l are relevant parameters within RG (Ref. 28) so that their renormalized values can be large. The main result is then that elasticity dominates at large B, while disorder dominates at low B, driving $z_c \rightarrow 0$, in qualitative agreement with the data. We propose then to search for an additional phase transition line within the solid phase, corresponding to decommensuration, which meets the melting curve at B_k .

In conclusion, we have shown that columnar defects enhance the decoupling transition so that $\delta T_d/T_d^0 \sim B^2$. However, in contrast, the melting temperature involves the same ratio within a logarithm [see Eq. (16)]; hence at weak disorder the enhancement is also $\sim B^2$, while at strong disorder it is only a weak $\ln B$ effect. The B^2 enhancement at strong disorder can, therefore, be useful in identifying a decoupling transition. Furthermore, for strong CD disorder a possibility of a reentrant transition has been found, i.e., with increasing field, decoupling is followed by recoupling. We have also studied effects of a commensurate CD density and shown that its potential vanishes above a critical value of long-wavelength disorder. This decommensuration transition may account for the unusual kink in the melting curve data.

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