Scattering of surface electrons over a superfluid helium film

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We find electron-interface scattering potentials for surface electrons floating over the surface of a helium film. The Hamiltonian for the electron scattering by the interface roughness of the solid substrate is obtained for substrates with an arbitrary dielectric constant. Our results for the ripplon-scattering Hamiltonian reproduce previous results obtained by using the electron-atom polarization model. However, this model gives correct results only for the electron-roughness scattering for substrates with a small atomic polarizability.

Surface electrons (SEs) floating over liquid helium constitute an important tool to investigate the properties of low-dimensional electronic systems in a broad area spanning from classical and quantum phase transitions up to analog quantum computers.[1,2] In particular, when SEs levitate over a helium film deposited on a solid substrate, intriguing phenomena are expected to occur like quantum melting, superfluidity in a single layer, and the formation of a ripplonic polaron, among others.[3] However, in contrast with SEs on bulk helium, now the electron interaction with substrate potentials plays a fundamental role in the transport properties of itinerant SEs and of the pinned Wigner crystal as well in the image field which determines the strength of the polaron state due to the coupling of SEs with oscillation modes of the interfaces.

Anomalies observed in the SE conductivity on thin films covering a hydrogen substrate[4] were explained successfully by the occurrence of a retrapping structural transition in the system of localized charges.[5] More recently a two-component SE model, in which one fraction corresponds to the free electron motion and the other one is associated with SE localization at trapped centers due to the roughness of the substrate, was proposed to explain the microwave absorption of SEs above a thin helium film.[6]

It is important to point out that all previous theories are based on nearly the same original Hamiltonian consisting of two parts: the Hamiltonian of free ripplons and a nonlocal polarization attraction depending on the interface displacement $\xi(\mathbf{r})$. It is assumed that the polarization interaction of the SE with the deformed surface is obtained by summing all one-particle electron-helium atom interactions, i.e., the electron interacts with the induced dipole moment μ of the helium atoms $V_{pol}(R) = -\alpha e^2/R^4$, where $R \gg a_s$, with R being the electron-atom distance, α the atomic polarizability, and a_s the scattering length.[5,7–12]

On the contrary, in the present work we obtain the potential energy for the surface electron located at $(\mathbf{r}, z)$, $z > 0$, over a helium film with a thickness d and a dielectric constant ε deposited on a solid substrate with a dielectric constant ε_s, by a different method, and compare our results with

those found in the above described electron-atom polarization model (EAPM). The approach is similar to that used to estimate the relativistic corrections to the image force acting over the surface of continuum media.[13–15] Our results depend essentially on the dielectric constant of the substrate, and we find the same results as in the EAPM when the dielectric constants are not so different from the helium dielectric constant. The calculation results for the matrix elements of the SE-interface defects show differences, for instance, in the cases of a helium film adsorved on a solid neon and metal substrate.

The electrostatic potential $\Phi^{(0)}(\mathbf{r}', z')$ in the case of ideal interfaces of the helium film can be obtained by solving Poisson's equation with appropriate boundary conditions. The result is obtained for all three regions: the vapor phase ($z' > 0$, approximated as the vacuum with $\varepsilon = 1$), the helium film ($-d < z' < 0$), and the substrate ($z' < -d$). In particular, for the vapor phase, we can write

$$\Phi_1^{(0)}(\mathbf{r}', z') = \Phi_{\text{vac}}(\mathbf{r}', z') + \Phi_1^{(\varepsilon)}(\mathbf{r}', z'), \qquad (1)$$

where $\Phi_{\text{vac}}(\mathbf{r}', z') = e/\sqrt{\rho^2 + (z' - z)^2}$ with $\rho = |\mathbf{r}' - \mathbf{r}|$ and

$$\Phi_1^{(\varepsilon)}(\mathbf{r}', z') = -\frac{Qe}{\sqrt{\rho^2 + (z' + z)^2}}$$
$$-\sum_{n=1}^{\infty} \frac{4Q_1 e(-a)^{n-1}}{\sqrt{\rho^2 + (z' + z + 2dn)^2}}. \qquad (2)$$

Here $Q = (\varepsilon - 1)/(\varepsilon + 1)$, $Q_1 = \varepsilon(\varepsilon_s - \varepsilon)/[(\varepsilon + 1)^2(\varepsilon_s + \varepsilon)]$ and $a = (\varepsilon - 1)(\varepsilon_s - \varepsilon)/[(\varepsilon + 1)(\varepsilon_s + \varepsilon)]$. Evaluating the image force as $\mathbf{F}_{\text{im}}^{(0)} = -[e\boldsymbol{\nabla}\Phi_1^{(\varepsilon)}(\mathbf{r}', z')]|_{\mathbf{r}' = \mathbf{r}, z' = z}$ we conclude that only $(F_{\text{im}}^{(0)})_z = -\partial U(z)/\partial z$ does exist and the potential energy $U(z)$ is given by

$$U(z) = -\frac{Qe^2}{4z} - Q_1 e^2 \sum_{n=1}^{\infty} \frac{(-a)^{n-1}}{z + nd}. \qquad (3)$$

The above result can also be obtained by the method of images.[8] For liquid helium, $\varepsilon - 1 \simeq 0.0572$, which allows us

to discard the terms with $n>1$ in the summation of Eq. (3) and to write, within excellent accuracy, the simplified expression

$$U(z) = -\frac{Qe^2}{4z} - \frac{Q_1e^2}{z+d}. \tag{4}$$

However, if we consider the displacements $\xi_1(\mathbf{r}')$ of the vacuum-film interface and $\xi_2(\mathbf{r}')$ of the film-substrate interface, the situation changes drastically. Now, the electric potential should be written as $\Phi_j(\mathbf{r}',z') = \Phi_j^{(0)}(\mathbf{r}',z') + \Phi_j^{(1)}(\mathbf{r}',z')$ with $j=1$, 2, and 3 corresponding to the regions of the vacuum, film, and substrate respectively. $\Phi_j^{(1)}(\mathbf{r}',z')$ are the solutions of Poisson's equation satisfying the perturbed boundary conditions in the linearized form for $z'=0$ and $z'=-d$.[16] By Fourier transforming both Poisson's equation and the equations obtained from the boundary conditions we evaluate $\Phi_1^{(1)}(\mathbf{r}',z')$ and the additional image force $(F^{(1)})_z = -\partial\delta U(z)/\partial z$ [the radial components of the image force $(F^{(1)})_{\mathbf{r}}$ are still zero]. The final result is

$$\delta U(z) = -\frac{Qe^2}{4\pi}\int d^2\mathbf{r}'\,\xi_1(\mathbf{r}')\left\{\frac{1}{[\rho^2+z^2]^2} - \frac{Q[\rho^2-z^2]}{[\rho^2+z^2]^3}\right\}$$

$$-\frac{Q_1e^2}{\pi}\int d^2\mathbf{r}'\,\xi_2(\mathbf{r}')\left\{\frac{1}{[\rho^2+(z+d)^2]^2}\right.$$

$$\left.-\frac{Q_s[\rho^2-(z+d)^2]}{[\rho^2+(z+d)^2]^3}\right\}, \tag{5}$$

where $Q_s = (\varepsilon_s-\varepsilon)/(\varepsilon_s+\varepsilon)$. In order to calculate the Hamiltonian $\hat{H}_{er}$ of the interaction of the electron with surface vibrations (ripplons) we have to add the additional perturbation which comes from Eq. (4) i.e., $\delta U_{ad}(z) \simeq [Qe^2/4z^2 + Q_1e^2/(z+d)^2 + eE_\perp]\xi_1(\mathbf{r})$, where we have introduced the holding electric field $E_\perp$ which presses the electron against the helium surface. Furthermore the spatial structure of the interface excitations is quite complicated. However for ripplons with long wavelengths, $\hat{H}_{er}$ can be obtained by changing the electron coordinates $\mathbf{r}\to\mathbf{r}$, and $z\to z-\xi_1(\mathbf{r})$.[8] Because the electron does not adiabatically follow the profile of the interface film substrate and the electron-roughness interface scattering potential is independent of the electron wave function we use the same wave function in both scattering mechanisms.[17] To a good approximation $\hat{H}_{er(ei)}$ can be expanded in ξ_1 keeping the first-order term. This term is linear in $\xi_{(1,2)}(\mathbf{r}) = \sum_q\xi_{(1,2)q}e^{i\mathbf{q}\cdot\mathbf{r}}$,

$$\hat{H}_{er(ei)} = \sum_{\mathbf{q}}\xi_{(1,2)q}e^{i\mathbf{q}\cdot\mathbf{r}}V_{(r,i)q}, \tag{6}$$

with

$$V_{rq}(z) = \frac{Qe^2}{4}\left[\frac{1}{z^2} - \frac{qK_1(qz)}{\bar{\varepsilon}z} - Q\frac{q^2K_2(qz)}{2}\right]$$

$$+\frac{Q_1e^2}{(z+d)^2} + eE_\perp \tag{7}$$

and

$$V_{iq}(z) = -Q_1e^2\left\{\frac{qK_1[q(z+d)]}{\bar{\varepsilon}_s(z+d)} + \frac{Q_s}{2}q^2K_2[q(z+d)]\right\}, \tag{8}$$

where $\bar{\varepsilon} = (\varepsilon+1)/2$, $\bar{\varepsilon}_s = (\varepsilon_s+\varepsilon)/2\varepsilon$, and $K_j(x)$ is the modified j-order Bessel function. Obtaining Eqs. (5)–(8), we have omitted the terms which depend on $z+nd$ and are proportional to ξ_1. These contributions of the coefficients $Q[(\varepsilon_s-\varepsilon)/(\varepsilon_s+\varepsilon)]^n \ll Q_1$ are negligible in comparison with $Q_1/(z+d)^2$ which appears in $V_{rq}^{(im)}(z)$ coming from $\delta U_{ad}(z)$. For the very interesting and special case of a metal substrate, we can take $\bar{\varepsilon}_s\to\infty$ and $Q_s=1$ to obtain $V_{iq}(z) = -Q_1e^2q^2K_2[q(z+d)]/2$.

Now we compare our results to those obtained in the framework of the EAPM. The results for the electron-ripplon interaction[8] and the electron-interface roughness interaction[17] in this model are

$$V_{rq}^\mu(z) = \frac{Qe^2}{4}\left[\frac{1}{z^2} - \frac{qK_1(qz)}{z}\right] + \frac{Q_1e^2}{(z+d)^2} + eE_\perp \tag{9}$$

and

$$V_{iq}^\mu(z) = -Q_1e^2\frac{qK_1[q(z+d)]}{z+d}. \tag{10}$$

Equations (7) and (8) are formally different from the corresponding Eqs. (9) and (10) in the EAPM. Because $\varepsilon\simeq 1$ and $Q\ll 1$, $V_{rq}(z)$ and $V_{rq}^\mu(z)$ are almost coincident for arbitrary values of the argument, which justifies the applicability of the EAPM to obtain the Hamiltonian for the electron-ripplon scattering. However, for the electron interaction with the interface roughness of the helium film-solid substrate, the expressions for $V_{iq}(z)$ and $V_{iq}^\mu(z)$ agree only for solid substrates with ε_s close to 1, such as solid neon ($\varepsilon_s\simeq 1.19$), and, as a consequence $Q_s\ll 1$. For $\varepsilon_s>1$, the expressions are quite different and the explanation is that Eq. (10) is valid only for small ε_s when, in the first approximation, the screening of the electron field in the solid substrate can be disregarded and the induced field above can be represented as a sum of contributions from each polarized atom. At large ε_s, the field acting on a chosen atom in the bulk is screened by the other atoms. In the extreme limit of a metal substrate, there is no electric field inside the metal and the image field is created by inducing interface charges. In the long wavelength limit ($qz\ll 1$), where $K_1(qz)\simeq 1/qz$ and $K_2(qz)\simeq 2/(qz)^2$, $V_{iq}(z)$ and $V_{iq}^{(\mu)}(z)$, are the same. However, for the calculation of the surface electron mobility one cannot restrict oneself to this limit because wave numbers with $qz\sim 1$ make an essential contribution, especially in the nonlinear regime.[8]

The matrix elements of the scattering potential which contribute to the collision frequency are written as $\langle 1|V_{iq}(z)|1\rangle = -Q_1e^2q^2\varphi_{ei}(q/2\gamma_1)/2$, where $\gamma_1(d,E_\perp)$ is the localization length of the electron wave function in the z
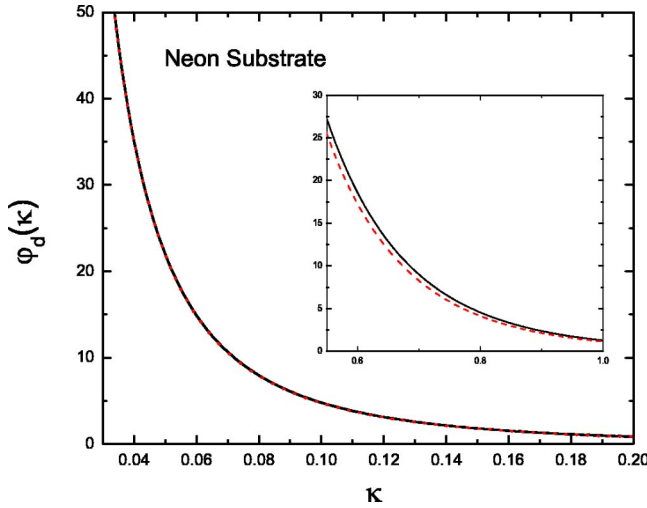
FIG. 1. Matrix elements for the electron-interface defects interaction calculated in this work (solid line) and in the EAPM for a solid neon substrate. The values of the ordinate axis, shown in the inset, are 10^3 higher than the corresponding values in the main figure.

FIG. 2. Same as Fig. 1 for a metal substrate.

direction.[12] For the sake of comparison we evaluate the exact $\varphi_{ei}(\kappa)$ and $\varphi_{ei}^{\mu}(\kappa)$, where $\kappa = q/2\gamma_1$, in the EAPM. The results are

$$\varphi_{id}^{\mu}(\kappa) = \frac{e^{2\gamma_1 d}}{\kappa} \int_{2\gamma_1 d}^{\infty} (t - 2\gamma_1 d)^2 e^{-x} K_1(\kappa t) \frac{dt}{t} \quad (11)$$

and

$$\varphi_{id}(\kappa) = \frac{\varphi_{id}^{\mu}(\kappa)}{\bar{\varepsilon}_s} + \frac{Q_s}{2} e^{2\gamma_1 d} \int_{2\gamma_1 d}^{\infty} (t - 2\gamma_1 d)^2 e^{-x} K_2(\kappa t) dt. \quad (12)$$

The functions $\varphi_{id}(\kappa)$ and $\varphi_{id}^{\mu}(\kappa)$ are depicted in Fig. 1 for the solid neon, and in Fig. 2 for the metal substrate. One can see no sensible differences for the solid neon whereas only a qualitative agreement is achieved for a metal substrate. Then one can apply either function for calculating and adjusting the experimental data to the surface electron mobility over a helium film adsorbed on substrates with weak polarizability.[12] However, $\varphi_{id}(\kappa)$ should be used in the case of substrates where screening effects are appreciable.

One should emphasize that the results obtained in this work are valid for substrates where the roughness amplitude ξ_s is much smaller than d, such that the defect scattering can
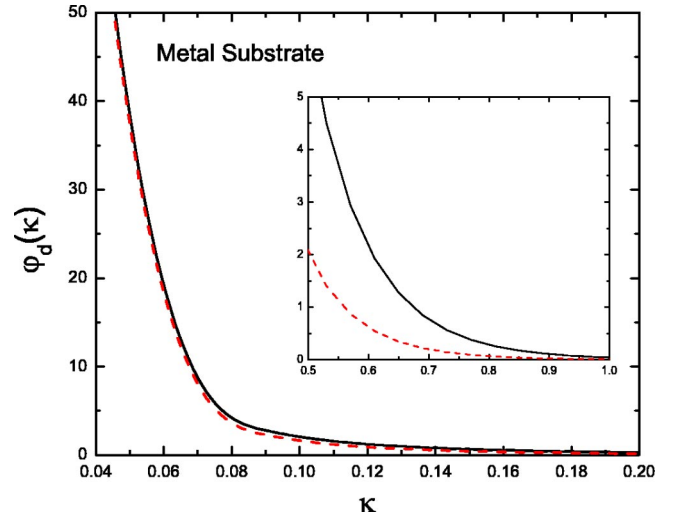
be treated perturbatively in the framework of the Born approximation. In this case, the approach developed here can explain reasonably the experimental data of the SE mobility on helium films covering solid neon and glass,[18,19] because the quality of the substrate-film interface is rather good. However, the characteristic roughness parameters obtained by adjusting the experimental data of Ref. 19 to EAPM calculations[12] should be revised. Unfortunately, as far as we know, there are no mobility data available for substrates with larger dielectric constants. On the other hand, for larger ξ_s, trapped charges can strongly contribute to the electron scattering,[20] and other possible mechanisms are necessary to explain the dependence of the mobility on the temperature and film thickness, as for instance, discussed in Ref. 6.

In conclusion, we have determined scattering potentials for surface electrons localized over a helium film. We calculated the perturbed electron potential energy due to the profile of the interphase boundaries. Due to the small polarizability of the liquid helium the Hamiltonian for electron-ripplon scattering coincides with that obtained in the largely used EAPM. For solid substrates with a large dielectric constant the result for the electron–interface roughness scattering potential differs from that obtained in the EAPM.

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