

**Model of charge-density-wave current conversion and phase-slip dynamics in mesoscopic samples**

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A microscopic model of collective-to-normal current conversion in mesoscopic charge-density-wave (CDW) conductors is presented. The current conversion is described in terms of phase slips induced by CDW strain and thermal fluctuations. Cores of phase slips are described as dynamic solitons with suppressed order parameter centered at individual chains near current contacts. The size of the cores is of the order of the amplitude coherence length, and they are surrounded by long-range perturbations of the CDW phase and of electric field induced by the CDW distortion. If the contact spacing is shorter than decay length of the long-range perturbations, the dynamics of phase slips at opposite contacts is correlated. In this case the phase-slip voltage decreases with the contact spacing decreasing. The results are in qualitative agreement with experimental study of current conversion in submicron NbSe<sub>3</sub> wires. In the limit of a large applied electric field a decrease of the phase-slip voltage is predicted even at large distances between current contacts.

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**I. INTRODUCTION**

Formation of the charge-density wave (CDW) below the Peierls transition in quasi-one-dimensional conductors gives rise to a collective mechanism of electron transport related to the sliding motion of the CDW (for a review, see Refs. 1 and 2). At the interface between a CDW conductor and a normal metal contact, the current must be converted from the single-electron current in the normal metal to the collective CDW current inside the CDW conductor. This conversion occurs in a form of phase-slip processes induced by a CDW strain near current contacts that results in periodic suppression of the CDW amplitude while the CDW phase increases by  $2\pi$ .<sup>3,4</sup> Suppression of the CDW amplitude is induced by the CDW strain near current contacts. As a result of a phase slip this strain is removed and a wave front of the CDW is added or removed. The periodic suppression of the CDW amplitude in the phase-slip process may occur in the form of phase vortices (dislocations) moving across the sample,<sup>5</sup> in this case the CDW amplitude is suppressed in the center of the vortex. Phase-slip centers are also known in other systems with collective transport such as narrow superconducting channels<sup>6</sup> and superfluids.<sup>7</sup>

The current conversion near current contacts were extensively studied experimentally.<sup>8-15</sup> The strain of the CDW near contacts was found to induce an additional voltage drop called the phase-slip voltage,  $V_{PS}$ . Initially, the phase-slip voltage was experimentally studied in bulk crystals.<sup>8-11</sup> It was found that  $V_{PS}$  strongly decreases as the temperature increases but is independent of spacing between current contacts. First indications of modifications in the current conversion process on a submicron scale were obtained in experiments on arrays of NbSe<sub>3</sub> antidots.<sup>14</sup> Eventually, in a recent experimental study<sup>15</sup> of current conversion in submicron NbSe<sub>3</sub> wires a significant reduction of the phase-slip voltage was observed if the spacing between current contacts was smaller than 3  $\mu\text{m}$ .

The first theoretical studies of phase slips in the CDW materials were based on time-dependent Ginzburg-Landau equations<sup>3,16</sup> which are strictly valid for a gapless CDW con-

ductor in which the Peierls energy gap is suppressed by impurities ( $\Delta\tau \ll 1$ , where  $\Delta$  is the amplitude of the CDW order parameter, and  $\tau$  is the scattering time). This approach involved a solution a closed set of equations for the order parameter and the current density for a one-dimensional geometry. Later the structure of phase-slip centers was studied by means of microscopic approach for ordinary CDW conductors with nonzero energy gaps, and a solution describing the phase-slip center as an oscillating soliton was found.<sup>17</sup> This solution was also restricted to the one-dimensional geometry in which phase slip occurred in the whole cross section of a CDW conductor. The phase-slip voltage was found to be equal to  $2\Delta$ . This value does not agree with experimental data, because stimulation of phase slips by fluctuations was not taken into account.

Description of phase slips in the form of vortices<sup>5</sup> is based on equation of motion for the CDW phase, the CDW being considered as an elastic medium with topological defects. More detailed study of current conversion by means of vortices was given by Ong and Maki,<sup>18</sup> and later by Ramakrishna *et al.*<sup>19</sup> and Requardt *et al.*<sup>12</sup> In the model considered in these papers the CDW strain lowers the energy barrier for thermal nucleation of phase vortices. None of the models mentioned above can explain the reduction of the phase-slip voltage in short samples observed by Mantel *et al.*<sup>15</sup>

In bulk samples a threshold field for the CDW sliding is determined by impurity pinning, and local CDW strains induced by pinning centers can compete with the CDW strain induced by the current conversion near contacts. Therefore, phase-slip centers may be distributed over large distances, for example, a conversion length about 40  $\mu\text{m}$  was reported<sup>13</sup> in NbSe<sub>3</sub>. In very short samples the threshold field is determined by contacts, and a large strain of the CDW which is compressed near one contact and stretched near the other dominates over local strains due to pinning centers. Therefore, one can expect that in short samples the impurity pinning can be neglected, and phase-slip centers are located near current contacts where the CDW strain is the largest. The theoretical model of current conversion from the normal

current to the CDW current in mesoscopic samples presented in this paper ignores the impurity pinning. The present model is based on the microscopic approach,<sup>17</sup> in which the cores of the phase-slip centers were described as dynamic solitons which produced phase slips at all conducting chains simultaneously. Here, in contrast to Ref. 17, we consider phase-slips occurring at individual chains, and thermal fluctuations stimulating the phase slips are taken into account. To describe temporal evolution of the CDW an equation for its phase averaged over the sample cross section is derived, while the evolution of individual phase slips is not studied. The phase-slip dynamics is found to depend on the sample length when the latter becomes comparable to or smaller than a decay length of long-range perturbations of the CDW phase induced by phase slips. The decay length is enhanced by a long-range electric field originating from charges induced by the CDW strain near current contacts.

The decay length depends on a screening which is determined by a distribution of the charge density between the CDW and single-particle excitations. Therefore, to compare the results with the measurements of current conversion in mesoscopic NbSe<sub>3</sub> samples<sup>15</sup> we focus on calculations within the model in which the CDW conductor consists of two types of conducting chains; chains in the CDW state and chains in the metallic state with a zero Peierls gap.

There is an important difference of the present model from the models based on the concept of dislocations moving across the sample.<sup>12,18,19</sup> That is, in our model we describe the phase slip as a local suppression of the CDW order parameter at conducting chains due to the combined action of the CDW strain and thermal fluctuations, the value of  $V_{PS}$  being related to an extra energy needed to suppress the CDW amplitude. On the other hand, in the models of current conversion due to moving dislocations, the periodic suppression of the CDW amplitude occurring when centers of dislocations cross the conducting chains is not considered, and  $V_{PS}$  is phenomenologically related to the energy barrier for the thermal nucleation of dislocations.

The paper is organized as follows. In Sec. II general equations for the CDW order parameter, current and charge densities are presented, and basic approximations are discussed. In Sec. III equations describing the dynamics of the CDW phase are derived and studied. First, equations describing the phase distribution in the quasiclassic region outside the cores of phase slips are derived, and a renormalization of the CDW elasticity and damping by electric field induced by the CDW deformation is discussed. It is very difficult to solve these equations and to match them at the cores of phase slips for a realistic geometry of samples with side contacts typically used in experimental studies of the phase-slip voltage. Therefore, for simplicity we neglect the current spreading near the contacts and consider an unrealistic but simple geometry with a one-dimensional distribution of the current. Then equations for the time dependence of variations of the CDW phase across the cores of the phase-slip centers are derived and studied. This variations consist of average and fluctuating parts. An equation for the average phase variations describing the phase-slip dynamics is solved in Sec. IV, and the

phase-slip voltage is calculated. Finally, Sec. V sums up the principal results.

## II. MODEL AND GENERAL EQUATIONS

Our approach to the problem is based on the kinetic equations<sup>20,21</sup> for quasiclassic Green's functions combined with self-consistency equations in which the CDW order parameter is expressed via the nonequilibrium Green's functions introduced by Keldysh.<sup>22</sup> These equations were derived within the mean-field approximation, and together with Maxwell's equations they provided a close set of equations describing the CDW dynamics. Each Green's function has the form of a matrix formed by the retarded,  $\hat{g}^R$ , and advanced,  $\hat{g}^A$ , Green's functions, and by the Green's function related to the distribution functions,  $\hat{g}^K$ , introduced by Keldysh:

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix}.$$

Furthermore, each of the components, in turn, is a matrix with respect to indices related to two parts of the Fermi surface of a quasi-one-dimensional metal at  $-p_F$  and  $+p_F$ . A discrete presentation of the equations with respect to numbers of conducting chains<sup>23</sup> is used, so each Green's function depends on two chain numbers.

The equations have a form

$$\begin{aligned} i\hbar v_F \frac{d\check{g}_{nm}}{dx} + t_{\perp} \sum_i (A_{nn+i}\check{g}_{n+im} - \check{g}_{nm+i}A_{m+i,m}) \\ + i\hbar \left( \sigma_z \frac{d\check{g}_{nm}}{dt_1} + \frac{d\check{g}_{nm}}{dt_2} \sigma_z \right) + (i\sigma_y \Delta_n - \Phi_n \sigma_z) \check{g}_{nm} \\ - \check{g}_{nm} (i\sigma_y \Delta_m - \sigma_z \Phi_m) + \frac{i\hbar}{2} v_f [\sigma_z \check{g}_{nn} \sigma_z \check{g}_{nm} \\ - \check{g}_{nm} \sigma_z \check{g}_{mm} \sigma_z] - \frac{i\hbar}{4} v_b [\sigma_x \check{g}_{nn} \sigma_x \check{g}_{nm} - \check{g}_{nm} \sigma_x \check{g}_{mm} \sigma_x \\ + \sigma_y \check{g}_{nn} \sigma_y \check{g}_{nm} - \check{g}_{nm} \sigma_y \check{g}_{mm} \sigma_y] = 0, \end{aligned} \quad (1)$$

where products assume a convolution with respect to time and matrix indices,  $v_F$  is the Fermi velocity in the chain direction,  $x$  is the coordinate along the chains,  $\Phi_n = e\phi_n - (v_F/2)(d\phi_n/dx)$ ,  $\phi_n$  is the matrix element of the electric potential calculated with Wannier functions related to chain number  $\mathbf{n}$ , and  $\Delta_n$  and  $\varphi_n$  are the amplitude and the phase of the order parameter at the  $n$ th chain, respectively:

$$A_{nm} = \sigma_z \cos \frac{\varphi_n - \varphi_m}{2} + i \sin \frac{\varphi_n - \varphi_m}{2}.$$

$\sigma_k$  are the Pauli matrices, and  $t_{\perp}$  is the transfer integral describing weak interchain interaction in the quasi-one-dimensional conductor; the summation in this term is limited by the neighbor chains.

The last terms in Eq. (1) describe the elastic scattering, and  $\nu_f$  and  $\nu_b$  are forward scattering and backscattering rates.

The order parameter satisfies the self-consistency equation

$$i \left( 1 + \frac{1}{\omega_Q^2} \frac{\partial^2}{\partial t^2} \right) (\sigma_y \cos \varphi_n + \sigma_x \sin \varphi_n) \Delta_n = \lambda/2 \int (\hat{g}_{nn}^K - \sigma_z \hat{g}_{nn}^K \sigma_z) d\epsilon, \quad (2)$$

where  $\omega_Q$  is the frequency of phonons with a wave vector equal to the wave vector of the Peierls instability, and  $\lambda$  is the electron-phonon coupling constant.

The charge and current densities at chain  $n$  can be calculated as

$$\rho_n = \frac{2}{\pi v_F} \left( \frac{1}{8} \int \text{Sp}(\sigma_z \hat{g}_{nn}^K) d\epsilon - \Phi \right). \quad (3)$$

$$j_n(x) = \frac{1}{4\pi} \int \text{Sp} \hat{g}_{nn}^K d\epsilon. \quad (4)$$

It is very difficult to solve these equations for the general case; therefore, we find an approximate solution taking into account that  $\Delta \gg \nu_b, \nu_f \gg \omega$  ( $\omega$  is a typical frequency of the problem), and having in mind the large anisotropy of the linear-chain compounds,  $\Delta \gg t_\perp$ . These inequalities permit us to divide the problem into two parts related to different length scales. Perturbations of the CDW order parameter at short distances of the order of  $\xi = v_F/\Delta$ , at individual chains, are considered neglecting time derivatives, the terms proportional to scattering terms, and the terms related to the inter-chain interaction. Then the short-range solution which describes a core of the phase slip must be matched with a long-wavelength quasiclassic solution varying at large scales like diffusion length or  $v/t_\perp$ . The short-range solution presents a soliton characterized by a rapid variation of the phase and a local suppression of the amplitude of the CDW. Solitons of such a type, called chord solitons, were introduced in linear-chain conductors by Brazovskii<sup>24</sup> as elementary excitations, and have already been used to describe the cores of phase slips.<sup>17</sup> In contrast to a previous study<sup>17</sup> the solution we use here describes large perturbations of the CDW amplitude at one conducting chain, perturbations of the CDW amplitude at the adjacent chains being negligibly small:  $\delta\Delta \propto t_\perp^2$ . In the leading approximation the solitonic solution for the order parameter and for the Green's functions have the forms (the chain numbers are dropped)

$$\Delta(x) = \Delta [\cos \theta + i \sin \theta \tanh(x/\zeta)], \quad (5)$$

$$\hat{g}^K = (\hat{g}^R - \hat{g}^A) \tanh\left(\frac{\epsilon - \mu}{2T}\right) + \hat{g}^{(a)}, \quad (6)$$

$$\hat{g}^{R(A)} = \pm \left[ i\Delta(x)\sigma_y + \epsilon\sigma_z + (i\sigma_y - \sigma_z) \frac{\Delta^2 \sin^2 \theta}{2[\epsilon - \Delta \cos \theta] \cosh^2(x/\zeta)} \right] / \sqrt{\epsilon^2 - \Delta^2}, \quad (7)$$

where  $\zeta = \xi/\sin \theta$ , and  $\mu$  is the shift of the chemical potential from the mid-gap position, it appears due to nonuniformity induced by the CDW strain. The singularities in the energy,  $\epsilon$ , must be circumvented assuming that the retarded (advanced) Green's function is analytical in the upper (lower) half-plane. The anomalous Green's function  $\hat{g}^{(a)}$  describes nonequilibrium perturbations of electronic distribution smoothly varying at quasiclassic distances.

Brazovskii<sup>24</sup> found that only the soliton with  $\theta = \pi/2$ , corresponding to a zero CDW amplitude at the soliton center, can exist in the equilibrium state, since for  $\theta \neq \pi/2$  the self-consistency condition for the phase of the order parameter is not satisfied. Here we consider the case when the chemical potential is shifted from the midgap position due to strain of the CDW. Then the self-consistency condition [Eq. (2)] relates variation of the phase across the soliton,  $2\theta$ , to the position of the chemical potential. In the limit of low temperatures,  $T \ll \Delta$ , the position of the chemical potential at the soliton approximately coincides with the subgap electron level localized at the soliton (7),  $\mu = \Delta \cos \theta$  (for more details, see Ref. 17). Then the variation of the phase at the core of the phase slip,  $2\theta$ , and the related shift of the chemical potential,  $\mu$ , in Eqs. (5)–(7) can be considered as parameters slowly varying in time. These parameters will be determined by matching the solution at the core,  $x \lesssim \xi$ , with quasiclassic solutions at large distances from the phase-slip center,  $x \gg \xi$ .

A solution of Eq. (1) far from the soliton is described by Eqs. (5)–(7), considered in the limit of large distances  $x \gg \zeta$ . It gives a solutions for the retarded and advanced Green's functions in the quasiclassic region as well. The non-trivial problem here is to find the nonequilibrium part  $\hat{g}^{(a)}$ . Solutions for  $\hat{g}^{(a)}$  are available in different limits.<sup>21,23</sup> However, we do not discuss the form of the anomalous Green's function  $\hat{g}^{(a)}$  here because  $\mu$  and  $\theta$  are readily obtained from the equation of motion for the CDW phase, and expressions for current and charge densities derived from solutions for  $\hat{g}^{(a)}$  in the quasiclassic region.<sup>21,23</sup> These expressions are physically transparent and their different limiting forms are well-known.<sup>1,2</sup>

To be more determined and closer to the experimentally studied case of NbSe<sub>3</sub> we must consider a CDW conductor which, in addition to quasiparticle excitations via the Peierls gap, contains also normal electrons at the Fermi level. Therefore, we focus on a model in which in addition to the conducting chains with nonzero gap,  $\Delta$ , there are chains of another type in which the CDW is absent in the lowest approximation in  $t_\perp$ .

The equation for linear charge density in the CDW chains reads (see Ref. 25)

$$q_n = -N_{CDW} \frac{e}{\pi} \frac{\partial \varphi}{\partial x} - N_Q \frac{2e^2}{\pi \hbar v_F} T \sinh \frac{\mu_n}{T}, \quad (8)$$

where the first term describes the CDW charge,  $N_{CDW}$  is the fraction of electrons condensed in the CDW, and  $N_Q = 1 - N_{CDW}$  is the fraction of the charge related to quasiparticles excited above the gap. The second term describes the contribution to the charge density due to a variation of local value of the chemical potential; this term has a form analogous to the case of ordinary semiconductors. The charge density at the normal chains can be found from Eq. (8), with  $N_{CDW} = 0$ .

Total current transported along a chain containing the CDW is equal to

$$j_n = -S \sigma_{\parallel} \frac{\partial V_n}{\partial x} + (1-b) \frac{2e^3}{\pi \hbar v_F} \frac{\partial \varphi_n}{\partial t}. \quad (9)$$

Here the first term describes the single-particle current driven by the gradient of the electrochemical potential

$$e V_n = \Phi_n - \mu_n, \quad (10)$$

the conductivity  $\sigma_{\parallel}$  is proportional to the density of quasiparticles thermally activated over the Peierls gap  $N_Q$ , and  $S$  is the area per single conducting chain. The second term describes the CDW current. Factor  $1-b$  reflects a decrease of the CDW current at finite temperatures. Typically  $T \ll \Delta$ , and this factor can be approximated by unity. The expression for the current flowing along the chains which do not contain the CDW gap is described by Eq. (8) with  $b=1$ , i.e., without the second term describing the CDW current, and the corresponding single-particle conductivity is equal to a normal metallic conductivity.

The current between neighbor chains  $n$  and  $n+i$  can be modeled as

$$j_{n,n+i} = \sigma_{n,n+i} (V_{n+i} - V_n) / s, \quad (11)$$

where the conductivity  $\sigma_{n,n+i}$  has different values for currents between different types of chains (normal and normal, CDW and normal, or CDW and CDW). The equation for the CDW phase reads

$$\begin{aligned} & \frac{\hbar}{2v_F} \frac{m^*}{m} \frac{\partial^2 \varphi_n}{\partial t^2} + \gamma_0 \frac{\partial \varphi_n}{\partial t} - N_{CDW} \frac{\hbar v_F}{2} \frac{\partial^2 \varphi_n}{\partial x^2} \\ & + J \sum_i \sin(\varphi_n - \varphi_{n+i}) \\ & = N_{CDW} e E_n + (b - N_Q) \frac{\partial V_n}{\partial x}, \end{aligned} \quad (12)$$

where  $m$  is the electron mass, and  $m^* = [1 + 4\Delta^2 / (\hbar^2 \lambda \omega_Q^2)] m$  is “the effective mass of the CDW.” Below, for simplicity, we consider the case of relatively low frequencies when this term can be omitted. Further,  $J = N_{CDW} t_1^2 / (\hbar v_F)$ , and  $\gamma_0$  is the bare friction coefficient. Summation in the term with  $J$  describing the interchain interaction is limited by the nearest neighbors. Pinning in Eq.

(12) is neglected because in the case of very short distances between current contacts the onset of the sliding of the CDW is controlled by the phase-slip voltage rather than by the impurity pinning.

Note that, in principle, kinetic coefficients in equations for the current, charge, and phase depend on relaxation times in a complicated way and may be frequency dependent. Simple equations like Eqs. (8)–(12), with constant kinetic coefficients, can be strictly derived from Eqs. (1) provided typical frequencies of the problem are smaller than both the energy and momentum relaxation rates of the electrons, and one can use the concept of the chemical potential  $\mu$  (see Ref. 26 for details).

In the right-hand side of Eq. (12) derived from the microscopic approach there are two contributions to the force driving the CDW. The first contribution is due to the electric field in the chain direction,  $E_n$ , which includes both an externally applied field and an electric field due to charges induced by deformations of the CDW. The second force is due to the gradient of the electrochemical potential. The latter contribution is not included, usually, in phenomenological equations of motion for the phase.<sup>1,2</sup> Below we assume the condition  $T \ll \Delta$ , which usually holds true in CDW conductors in a wide temperature region. In this case the second term in Eq. (12) is proportional to the exponentially small quasiparticle density, and can be neglected.

Thus the single-particle current and the CDW are driven by different forces. The CDW is driven mainly by the electric field, i.e., by the gradient of the electric potential, while the single-particle current is driven by the gradient of the electrochemical potential described by Eq. (10). Equivalently, the single-particle current can be presented as a sum of two components, one of which is the current driven by the electric field and the other the diffusion current presented by the term proportional to the gradient of the chemical potential. It is important to note that the voltage is equivalent to the difference of electrochemical potentials which drives the single-particle current, while the difference in the electric potentials is not a voltage and cannot be measured by a voltmeter. For example, a difference of electric potentials arises across the contacts of two different conductors even in the equilibrium case, but it produces neither voltage nor current. Furthermore, the constancy of the electrochemical potential, in other words the constancy of the sum of the potential energy and of the chemical potential, is the necessary condition of the equilibrium state<sup>27</sup> in which current and voltage are absent while the electric field can be nonzero.

So, according to Eq. (10), shifts of the chemical potential  $\mu$  induced by the CDW strain near current contacts contribute to the voltage. The phase-slip voltage  $V_{PS}$  is usually determined (see, e.g., Refs. 8 and 15) as a difference of voltage measurements in transposed and normal configurations. In the transposed configuration presented in Fig. 1 current is injected through the middle contacts 2 and 3 at  $|x|=a$ , while the voltage is measured between contacts 1 and 4. In this configuration the current conversion occurs between the voltage probes and contributes to the measured voltage. In the normal configuration, current is injected at contacts 1 and 4 and voltage is measured between contacts 2 and 3 so that the

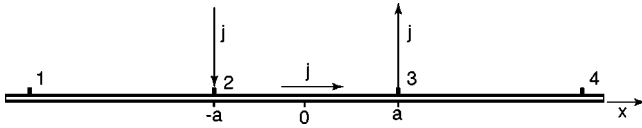


FIG. 1. The transposed configuration for study of the phase-slip voltage.

current conversion occurs beyond the voltage probes and the phase-slip process does not contribute to the measured voltage.

Now we can express  $V_{PS}$  in terms of shifts  $\mu$  near current contacts at  $|x|=a$ . We integrate Eq. (12) over  $x$  and average it over sample cross-section. Then we use the obtained equation together with Eq. (10) to express the CDW current [the second term in Eq. (9)] in terms of voltage and difference of the chemical potentials. This enables us to calculate the phase-slip voltage as the difference of voltages measured in two configurations at the same values of the CDW current:

$$V_{PS} = \frac{1 - N_Q}{1 - b} [\mu(a) - \mu(-a)] \approx [\mu(a) - \mu(-a)]. \quad (13)$$

In the last equality we neglected factors  $b$  and  $N_Q$ , which describe the quasiparticle contributions to the current and charge densities and are exponentially small at  $T \ll \Delta$ . Thus the phase-slip voltage is determined by the difference of chemical potentials at current contacts.<sup>17,28</sup>

### III. EQUATIONS DESCRIBING PHASE-SLIP DYNAMICS

#### A. Renormalized equation for the CDW phase in the quasiclassic region

In order to find solutions for phase distribution in the quasiclassic region we insert expressions for the current into the Poisson and continuity equations. Then we exclude potentials  $\mu$ ,  $V$ , and  $\Phi$  from Eq. (12) and find a quasiclassic equation of motion for the CDW phase.

Below, assuming that  $T \ll \Delta$ , we neglect, where possible, factors like  $N_Q$  and  $b$  proportional to the small density of quasiparticles thermally excited over the Peierls gap. Then expressions for current and charge densities [Eqs. (8)–(11)] become linear, and one can use the Fourier transformation with respect to time,  $t \leftrightarrow \omega$ , coordinate along the chains,  $x \leftrightarrow q_{\parallel}$ , and the chain numbers,  $\mathbf{n} \leftrightarrow \mathbf{q}_{\perp}$ . At this stage we must consider definite arrangements of the conducting chains with and without the CDW distortion. The reason for this is the necessity to take into account screening by normal electrons located at the chains in the normal state. The screening may depend quantitatively on the chain arrangement, but the qualitative character of the screening is expected to be independent of the details of the crystal structure. Therefore, for simplicity, we consider a square lattice with an alternating sequence of normal and CDW chains. Furthermore, also for simplicity, we use a discrete form of space derivatives in the perpendicular direction, e.g.,  $\partial_n \phi \rightarrow (\phi_{n+1} - \phi_n)/s$ , where  $s = \sqrt{S}$  is the interchain spacing. Then inserting these expressions into the Poisson equation and continuity equations, and

using the Fourier transformation, after some algebra we express potentials  $\mu$ ,  $V$ , and  $\Phi$  in terms of the CDW phase. In particular, we find the relation between the shift of the chemical potential and of the phase gradient:

$$\mu_n = -g \frac{\hbar v_F}{2} \frac{\partial \varphi_n}{\partial x}. \quad (14)$$

Then we exclude potentials from Eq. (12) and find an equation of motion for the CDW phase that does not contain potentials which, in turn, depend on the phase distribution. General expressions for coefficients in such equation of motion are rather complicated, but they are quite simple in the limit which we need,  $\omega \ll Dq_{\parallel}^2$  (where  $D$  is the diffusion coefficient for the chain direction). In this limit an equation which is similar to Eq. (12) is obtained:

$$\tilde{\gamma} \frac{\partial \varphi_n}{\partial t} + \frac{\hbar v_F}{2} (1 + g) \frac{\partial^2 \varphi_n}{\partial x^2} + J \sum_i \sin(\varphi_n - \varphi_{n+i}) = e \bar{E}. \quad (15)$$

In contrast to Eq. (12), the right-hand side of Eq. (15) contains an externally applied electric field  $\bar{E}$  only, while electric fields induced by deformations of the CDW are represented by a renormalization of the damping and elasticity coefficients described by the operators which in the Fourier representation have a form

$$g = \frac{2e^2}{\hbar v_F}, \quad \tilde{\gamma} = \gamma_0 + \frac{v_F \kappa^2 q_{\parallel}^2}{8\pi(\sigma_{N\perp} q_{\perp}^2 + \sigma_{N\parallel} q_{\parallel}^2)}, \quad (16)$$

where  $\kappa$  is the inverse Thomas-Fermi screening length,  $\sigma_{N\perp}$  is the conductivity describing the current between the normal chains, and  $\sigma_{N\parallel}$  is the conductivity along the normal chains. The dimensionless parameter  $g$  looks like the fine structure constant with the velocity of light substituted for the Fermi velocity. For the typical value of the Fermi velocity<sup>1,2</sup>  $v_F \approx 2 \times 10^7$  cm/s, the parameter  $g$  can be estimated as  $g \approx 24$ , so in contrast to the fine structure constant it is not small. Therefore, we consider parameter  $g$  to be large, and the expression for  $g$  [Eq. (16)] is already presented under the assumption  $g \gg 1$ . Besides, Eq. (16) is presented in the long wavelength limit, while a more accurate expression for  $g$  contains the additional factor  $4/(4 + \epsilon_{\Delta} q_{\parallel}^2 S)$ , where  $\epsilon_{\Delta} \approx \kappa^2 \xi^2 / 6$  is the CDW gap dielectric function.

The renormalization increases the damping coefficient drastically as well. The bare damping coefficient  $\gamma_0$  is small, being proportional to the density of quasiparticles thermally excited via the Peierls gap.<sup>20,23</sup> The renormalized damping coefficient for the phase averaged over all chains is much larger than  $\gamma_0$ , and equals  $\gamma = \tilde{\gamma}(\mathbf{q}_{\perp} = 0) \approx \hbar/l$ , where  $l$  is the mean free path along the chains in the normal state.

Now we briefly discuss the case of a linear-chain conductor in which all conducting chains are in the CDW state like in most CDW conductors, and there are no electrons from the chains in the normal state that can effectively screen the charge density induced by CDW distortions. Then one must take into account the screening by quasiparticles thermally excited above the Peierls gap. In general, the problem of

screening by such quasiparticles is more difficult than that of screening by electrons from the chains in the normal state because of the nonlinearity in the second term in Eq. (8) for the charge density. However, the solution can be simplified at high temperatures, when the quasiparticle density is large enough to provide effective screening, so that the maximal shift of the chemical potential is small enough,  $\mu \ll T$ . Therefore, the second term in Eq. (8) can be linearized, and the problem can be solved by means of the Fourier transformation similar to the previous case of screening by electrons in the normal chains. Note that in this limit the Poisson equation is reduced to the quasineutrality condition. One obtains an equation for the CDW phase with renormalized stiffness and damping, and relation between phase and chemical potential resembling Eqs. (15) and (16):

$$\left( \gamma_0 + \frac{2e^4(1-b)^2}{\hbar v_F S \sigma_{\parallel}} \right) \frac{\partial \varphi_n}{\partial t} + \frac{(1-N_Q)}{N_Q} \frac{\hbar v_F}{2} \frac{\partial^2 \varphi_n}{\partial x^2} + J \sum_i \sin(\varphi_n - \varphi_{n+i}) = e \frac{j}{\sigma_{\parallel}}, \quad (17)$$

$$\mu_n = - \frac{\hbar v_F}{2N_Q} \frac{\partial \varphi_n}{\partial x}. \quad (18)$$

The total current density  $j$  in the right-hand side of Eq. (17) does not depend on coordinates in the case of the one-dimensional current distribution considered below. Thus the structure of equations for the phase and of relations between the phase gradient and chemical potential  $\mu$  is similar to Eqs. (15) and (16), but the role of the renormalization factor  $g$  of the CDW stiffness is played by  $1/N_Q$ . A rapid exponential decrease of the fraction  $N_Q$  of single-electron excitations with decreasing temperature leads to an increase of  $\mu$ . Therefore, the screening by quasiparticles becomes nonlinear at lower temperatures, and Eqs. (17) and (18) are no longer valid and a more thorough analysis is needed.

### B. Equations for variation of the CDW phase at current contacts

In this section we need to consider a definite geometry of the current distribution explicitly. As already mentioned above, we consider the simplest one-dimensional current distribution, the current density being uniform between contacts 2 and 3, at  $|x| < a$ , and equal to zero in the region outside the contacts, at  $|x| > a$  (Fig. 1). It can be shown that the results obtained for this geometry are very similar to the case of current flowing between current contacts at butt ends of a sample, with the boundary condition at the contacts consisting of a constancy of the CDW phase. Such boundary conditions imply that the CDW current at the contacts equals zero.

We denote the variation of the CDW phase across the chord solitons near the current contacts at  $x = a$  and  $x = -a$  as  $2\theta_{n+}$  and  $2\theta_{n-}$ , respectively. Each of these phase variations consist of an average part  $\langle \theta \rangle$  and a fluctuating part  $\delta\theta_n$  (where it is possible we omit subscripts  $\pm$  for brevity). To find the CDW current and  $I$ - $V$  curves we need to calculate

average values  $\langle \theta_+ \rangle$  and  $\langle \theta_- \rangle$ . We assume that the sample consists of many chains, and that the statistical averaging is equivalent to ensemble averaging, so that  $\langle \theta \rangle$  does not depend on the chain index  $n$ :

$$\theta_n = \langle \theta \rangle + \delta\theta_n, \quad \langle \cos \theta_n \rangle = \cos \langle \theta \rangle \langle \cos \delta\theta_n \rangle, \\ \langle \cos \delta\theta_n \rangle = \exp(-\langle \delta\theta_n^2 \rangle / 2). \quad (19)$$

The last equality is valid if fluctuations are Gaussian.

In order to derive equations describing dynamics of phase variations  $\theta_{n\pm}$  we have to solve Eq. (15) outside the region of the chord solitons with an additional condition that perturbations of the phase decay at  $|x| \rightarrow \infty$ , and to match the solutions at  $x = \pm a$ . The matching conditions are the following: first, the phase variations at the solitons are equal to  $2\theta_{n\pm}$ , and, second, the spatial derivative of the phase is continuous at each chord soliton and is related to the chemical potential according to Eq. (14), the chemical potential being equal to

$$\mu_{n\pm} = \Delta \cos \theta_{n\pm}. \quad (20)$$

The main difficulty in solving Eq. (15) originates from the nonlinear terms describing the interchain interaction. Note that this difficulty does not appear in the derivation of the equations for the averaged values of  $\langle \theta_{\pm} \rangle$  because the spatial averaging of the equations in the perpendicular direction involves summation of the term describing the interchain interaction in Eq. (15) over all conducting chains; this gives zero. To overcome this difficulty in the general equations for the nonaveraged  $\theta_{n\pm}$ , we model the sum in the term proportional to  $J$  in Eq. (15) by the discrete form of second spatial derivatives in the directions perpendicular to the chains. As noted above, this substitution does not affect equations for the average phase  $\langle \theta_{\pm} \rangle$ . In fact, such a model corresponds to a standard form of the equation for the phase in which the interchain interaction is presented by the elastic term.<sup>1,2</sup> In this case Eq. (15) becomes linear and one can use the Fourier representation with respect to frequency and chain numbers and find equations for the Fourier transforms  $\theta_{\pm}$ ,

$$2\theta_- - \frac{C_- + \delta\mu_- / \Delta}{\xi k} (2 + \tanh ak + \coth ak) \\ - \frac{C_+ + \delta\mu_+ / \Delta}{\xi k} (\tanh ak - \coth ak) = F, \\ 2\theta_+ - \frac{C_+ + \delta\mu_+ / \Delta}{\xi k} (2 + \tanh ak + \coth ak) \\ - \frac{C_- + \delta\mu_- / \Delta}{\xi k} (\tanh ak - \coth ak) = -F, \quad (21)$$

where  $C_{\pm}$  are the Fourier transforms of  $\cos \theta_{n\pm}$ , respectively, and  $\delta\mu$  is an extraneous random force describing fluctuations of the chemical potential in the quasiclassic region outside the phase-slip center. Correlation functions for  $\delta\mu$  are discussed below. An increment  $k$  describes spatial variations of the phase along the chains, which may be presented in the form of a linear combination of the exponents

$\exp(\pm kx)$  with  $k^2 = q_{\perp}^2 S / l_{\perp}^2 - 2i\omega\gamma / (\hbar v_F g)$  and  $l_{\perp}^2 = g(\hbar v_F)^2 / (2t_{\perp}^2)$ . The length  $l_{\perp}$  exceeds the amplitude coherence length by factor  $\sqrt{g/2}(\Delta/t_{\perp})$  which can be estimated as 10. The driving force in the right-hand side is proportional to the applied electric field  $\bar{E}$  and has the form

$$F = \frac{e\bar{E}}{i\hbar\omega\gamma} 2\pi^3 \delta(\omega) \delta(\mathbf{q}_{\perp}).$$

Note that the decay length for the perturbations of the phase along the chains,  $L = 1/|k|$ , depends on  $\mathbf{q}_{\perp}$ . This length is relatively short, of the order of  $l_{\perp}$ , for perturbations strongly varying from chain to chain (i.e. at  $q_{\perp}^2 S \sim 1$ ). Nevertheless,  $l_{\perp} \gg \xi$ . For perturbations averaged over the sample cross section, i.e., those with  $q_{\perp} = 0$ , the decaying length  $L(q_{\perp} = 0)$  is much larger and diverges at small frequencies. If the distance between the contacts is smaller than  $L(q_{\perp} = 0)$ , one should expect that the average values  $\langle \theta_+ \rangle$  and  $\langle \theta_- \rangle$  would change synchronously. Equations (21) have synchronously changing solutions which obey the condition  $\langle \theta_+ \rangle = \pi - \langle \theta_- \rangle$ . Below we will concentrate on such a case of synchronous oscillations of phase-slips near both current contacts. Taking the inverse Fourier transform, we find that the time dependence of  $\theta \equiv \langle \theta_- \rangle$  for the synchronously changing solutions is described by an equation

$$\cos \theta = \int_{-\infty}^t \frac{dt_1}{\sqrt{t-t_1}} \left[ \frac{\partial \theta(t_1)}{\partial t_1} - f \right] \left[ 1 - \exp\left(-\frac{\ell^2}{t-t_1}\right) \right], \quad (22)$$

where dimensionless time  $t$  is measured in units of  $2\hbar v_F \gamma g / (\pi e^2 V_{\infty})$ , and

$$V_{\infty} = \frac{\Delta}{e} \langle \cos \delta\theta_{n+} + \cos \delta\theta_{n-} \rangle, \quad f = \frac{\bar{E}}{E_0}, \quad \ell = \frac{a}{a_0}, \quad (23)$$

$$E_0 = \frac{\pi e V_{\infty}^2}{\hbar v_F g}, \quad a_0 = \frac{\hbar v_F g}{2\sqrt{\pi e V_{\infty}}}.$$

Here  $f$  and  $\ell$  are the dimensionless driving electric field and the half-distance between the contacts, respectively. It is shown below that  $V_{\infty}$  determines the phase-slip voltage in case of infinite spacing between the contacts and relatively low applied electric field,  $\bar{E} < E_0$ .

As noted above, the phase-slip voltage is determined by the difference of the shifts of the chemical potential  $\mu$  at the current contacts [see Eq. (10)]. According to Eqs. (13), (19), and (20) the phase-slip voltage is equal to

$$V_{PS}(t) = (\Delta/e) \langle \cos \theta_{n-} - \cos \theta_{n+} \rangle = V_{\infty} \cos \theta. \quad (24)$$

Below we concentrate on calculation of the dc component of the phase-slip-voltage. Equation (24) shows that the maximum possible value of  $V_{PS}$  is equal to the energy gap  $2\Delta$  in the quasiparticle spectrum, as obtained in Ref. 17. However, fluctuations decrease this value.

Thus the average phase-slip dynamics is described by  $\theta$  obeying Eq. (22) and depends on the parameter  $V_{\infty}$  deter-

mined by fluctuations of the phase variations across the chord solitons. Fluctuations  $\delta\theta$  are described by general equations (21). Fluctuations  $\delta\theta_n$  at different chains are more or less independent, and typical wave vectors for the fluctuations can be estimated as  $q_{\perp} \sim 1/s$ . As noted above, the characteristic length for variations in the direction along the chain direction is quite short for such values of  $q_{\perp}$ ,  $L \sim l_{\perp}$ . If the distance between the contacts exceeds  $l_{\perp}$ , typical values of  $ka$  in equations for  $\delta\theta_n$  are large and  $\tanh ak \approx \coth ak \approx 1$ . Then the interaction between fluctuating parts of the phase variations across chord solitons at  $x = -a$  and  $x = a$ , presented by the second terms in the left-hand sides of Eqs. (21), can be neglected. Therefore, the strength of fluctuations and, hence, their effect on the phase-slip voltage can be found from an equation which does not depend explicitly on the contact spacing if the latter exceeds the length  $l_{\perp}$ .

Here we do not solve the complicated problem of fluctuations of the phase difference across the chord soliton. Instead we roughly estimate reduction of the phase-slip voltage by thermal fluctuations. So consider Eqs. (21) in the limit  $a \rightarrow \infty$  for the fluctuating part of the phase  $\delta\theta$ . In the chain-number representation we obtain

$$\alpha \sum_m Q_m \delta\theta_{n-m} - [\cos(\langle \theta \rangle + \delta\theta_n) - \langle \cos(\langle \theta \rangle + \delta\theta_n) \rangle] = \delta\mu_n / \Delta, \quad (25)$$

where  $\alpha = t_{\perp} / (\sqrt{2g}\Delta)$ , and  $Q_m$  is the inverse Fourier transform of  $(q_{\perp} s)$ . Operator  $Q$  decays at large  $m$ ,  $Q_m \propto \sin \pi m / m^{3/2}$ , while at  $m=0$   $Q_0 = 1$ . Therefore, for crude estimates we can replace the sum over  $m$  in Eq. (25) by the term with  $m=0$ , which gives

$$\alpha \delta\theta_n - \cos(\langle \theta \rangle + \delta\theta_n) = \delta\mu_n / \Delta. \quad (26)$$

Now we estimate the fluctuations of the chemical potential in the right-hand part of Eq. (25). To do this one can apply the fluctuation-dissipation theorem to Eq. (15) for the CDW phase and to relate phase fluctuations to fluctuations of the chemical potential by means of Eq. (14). Upon integration of the spectral density of fluctuations over frequencies we find

$$\langle \delta\mu^2 \rangle_{\mathbf{q}} \sim \frac{2\pi T \hbar^2 v_F^2 g^2 q_{\parallel}^2 S}{\hbar v_F (1+g) q_{\parallel}^2 + J q_{\perp}^2}. \quad (27)$$

Integration of this equation over  $\mathbf{q}$  yields the mean square fluctuation of the chemical potential. In the integral over  $q_{\perp}$  the upper limit is of the order of  $1/\sqrt{s}$ , and in the integration over  $q_{\parallel}$  one needs to take into account the additional factor in the renormalized longitudinal elastic coefficient mentioned in the discussion after Eq. (16). Note that characteristic wave lengths of the fluctuations along the chain direction that give the largest contribution to the integral are of the order of  $g\xi \gg \xi$ . So, after integration, we obtain

$$\langle \delta\mu^2 \rangle \sim \sqrt{g} T \Delta. \quad (28)$$

For typical experimental temperatures<sup>15</sup> above 100 K this estimate implies that mean-square fluctuations of the chemical potential are of the order of  $\Delta$ . Hence the mean magnitude of the random force in the right-hand side of Eq. (26) is of the order of unity. It is difficult to find the solution for this case. Therefore, to make a crude estimate, we consider the case of a large right-hand side and extend the obtained result to the case of interest. In this limit the cosine terms in Eq. (26) can be neglected since they cannot be larger than unity, and one finds

$$\langle \delta\theta^2 \rangle \sim \frac{\langle \delta\mu^2 \rangle}{\alpha^2 \Delta^2} = \frac{T}{T_0}. \quad (29)$$

Hence, according to Eqs. (23) and (24) the parameter  $V_\infty$  can be estimated as

$$V_\infty = \frac{2\Delta}{e} \exp\left(-\frac{T}{T_0}\right), \quad T_0 \sim \frac{t_\perp^2}{\sqrt{g^3} \Delta}. \quad (30)$$

The characteristic temperature  $T_0$  is much smaller than both the CDW amplitude  $\Delta$  and the interchain transfer integral  $t_\perp$ , and may be of the order of few tens of K. Our crude estimate [Eqs. (29) and (30)] qualitatively agrees with the experimental data.<sup>15</sup>

Now we briefly discuss the phase-slip process in materials in which the Fermi surface is destroyed by the Peierls transition completely. As noted above, in such materials the problem is complicated by the nonlinearity of the screening by single-electron excitations thermally excited over the gap  $\Delta$ . However, if the temperatures is high enough so that the typical shift of the chemical potential which is of the order of  $V_{PS}$  is smaller than the temperature,  $V_{PS} \ll T$ , then relatively simple Eqs. (17) and (18) are valid. These equations resemble Eqs. (15) and (14) describing the case of NbSe<sub>3</sub>, the main difference is that the parameter  $g$  in Eqs. (15) and (14) is substituted for  $1/N_Q$  in Eqs. (17) and (18). Therefore, for the case of semiconducting CDW materials one can obtain equations similar to Eqs. (22)–(30), with  $g$  replaced by the temperature-dependent parameter  $1/N_Q$ . This leads to a different temperature dependence of the characteristic voltage,  $V_\infty$ , the characteristic field,  $E_0$ , and characteristic spacing between current contacts,  $a_0$ . As the temperature decreases  $V_{PS}$  increases, and eventually becomes larger than temperature  $T$ . Therefore, the screening becomes nonlinear, and the results based on the linearized equations for the quasiparticle density cannot be directly applied. Thus the temperature dependence of the phase-slip voltage in semiconducting CDW conductors is more complicated due to the screening by thermally excited quasiparticles.

#### IV. PHASE-SLIP DYNAMICS AND PHASE-SLIP VOLTAGE

In this section we consider the solution of Eq. (22) for the phase-slip phase  $\theta$ . According to Eq. (24), time averaging  $\cos \theta$  determines the phase-slip voltage when phase slips at different current contacts take place synchronously on the average.

To solve Eq. (22) analytically is rather difficult. There-

fore, we numerically solved the exact equation and analytically solved a model equation obtained from Eq. (22) by substitution of cosine function for the sawtooth function. Such a substitution converts the nonlinear equation (22) into a linear equation with a driving force induced by the discontinuities of the sawtooth function. Such a substitution allows one to solve the model equation exactly by means of the Fourier transformation. Both the solution of the model equation and the numerical solution of the exact equation give similar results.

First we discuss general properties of Eq. (22), and of its solutions. The equation resembles an equation of motion of an overdamped pendulum with retardation in damping. It contains two dimensionless parameters  $f$  and  $\ell$ , describing the applied electric field and the distance between the contacts measured in units of  $E_0$  and  $a_0$ , respectively [see Eq. (23)]. Using the explicit expression [Eq. (16)] for the renormalization parameter  $g$  obtained within our model and a typical value<sup>15</sup> of the phase-slip voltage in NbSe<sub>3</sub>,  $V_\infty \approx 1$  mV, the units can be estimated as

$$E_0 = \frac{\pi V_\infty^2}{2e} \sim 10 \text{ V/cm}, \quad a_0 = \frac{2e}{\sqrt{\pi} V_\infty} \sim 1 \text{ } \mu\text{m}. \quad (31)$$

At  $f\ell \ll 1/(2\sqrt{\pi})$ , which in dimensional units corresponds to  $2a\bar{E} \ll V_\infty$ , Eq. (22) has a stationary solution. This means that at voltages smaller than the critical value the phase slips are absent and the CDW current is equal to zero. At larger voltages the phase  $\theta$  starts to increase linearly with time and acquires also an oscillating component. Both the frequency of the oscillations that determines the CDW current and the time average value of  $\cos \theta$  that determines the dc component of the phase-slip voltage depend on the dimensionless parameters  $f$  and  $\ell$ . This dependence is the most simple in the following limiting cases.

In the limit  $\ell \rightarrow \infty$  and  $f \ll 1$  the solution has the form of a steplike function, so that most of time  $\theta$  is close to  $(2n+1)\pi$  where  $n$  is an integer and exhibits periodic jumps by  $2\pi$ . The average  $\cos \theta$  in this regime is close to  $-1$ , and  $V_{PS}$  is equal to  $V_\infty$ . Thus  $V_\infty$  determines the phase-slip voltage in long samples with independent phase slips at the opposite current contacts.

If both characteristic frequencies and intercontact distance are small enough, so that the dimensionless frequency  $\omega$  satisfies the condition  $\omega \ell^2 \ll 1$ , Eq. (22) is reduced to a standard equation of motion of an overdamped pendulum:

$$2\sqrt{\pi}\ell \left( \frac{\partial \theta}{\partial t} - f \right) - \cos \theta = 0. \quad (32)$$

The solution of this equation, which describes many physical models including resistively shunted Josephson junctions, is well known. In particular, for the frequency of the oscillations which determines the CDW current, and for the time-averaged value  $\cos \theta$  which determines the dc component of the phase-slip voltage, in this limit one has

$$\omega = \sqrt{f^2 - 1/(4\pi\ell^2)}, \quad \overline{\cos \theta} = \sqrt{4\pi\ell^2 f^2 - 1} - 2\sqrt{\pi}\ell f. \quad (33)$$



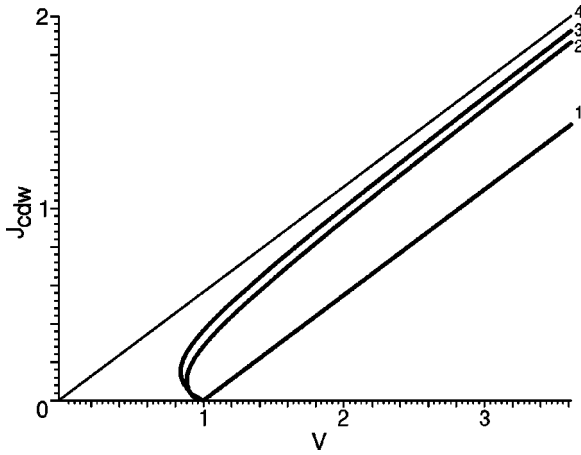


FIG. 2. Dependence of the CDW current (in arbitrary units) on total voltage (in units of  $V_\infty$ ) for different values of the distance between the current contacts. Curve 1, 2, and 3 correspond to an infinite distance, and to distances  $a=4a_0$  and  $a=a_0$ , respectively. The respective phase-slip voltages  $V_{PS}$  are determined by difference of the voltages between the thick curves and straight line 4.

The solution of Eq. (22) can be easily found also in the limit of large electric field,  $f \rightarrow \infty$ :

$$\omega = f - \frac{1}{4\pi\ell\sqrt{2f}}, \quad \overline{\cos\theta} = \sqrt{\frac{1}{8\pi f}}. \quad (34)$$

Thus the phase-slip voltage decreases with increasing electric field even in case when the distance between the contacts is large [cf. Eq. (24)]. This happens at large electric fields  $\bar{E} \gg E_0$  [see Eq. (31)] when the driving force exerted on the CDW is much larger than the threshold force related to the potential barrier producing the phase-slip voltage (also note the analogy with a pendulum mentioned above).

The general tendency of the solution is a decrease of the phase-slip voltage with  $f$  increasing and  $\ell$  decreasing. This tendency also persists if the dimensionless parameters are of the order unity. The results of numerical calculations of the CDW current as a function of the total voltage  $V = 2\bar{E}a + V_{PS}$  are shown in Fig. 2. At small voltages the curves have a region of negative differential conductivity originating from a rapid increase of the CDW current (i.e., of the frequency  $\omega$ ) when  $\bar{E}$  exceeds the critical value [cf. Eq. (33)]. In real experimental conditions such a dependence is expected to be easily washed out by effects of pinning, and by a nonuniform distribution of the current density.

## V. CONCLUSION

A theoretical model of the current conversion in mesoscopic CDW conductors is presented. The model involves a microscopic treatment of the phase-slip process which consists of a periodic local suppression of the CDW amplitude at conducting chains. The cores are described as dynamical chord solitons in which the CDW order parameter varies at distances of the order of the amplitude coherence length. This variation is accompanied by quasiclassic perturbations

of the phase decaying at large distances. The electric field induced by deformations of the CDW greatly enhances the decaying length [Eqs. (23) and (31)] of such perturbations. If the distance between current contacts is smaller than or comparable to this characteristic length, the phase slips near different current contacts interact and the interaction decreases the phase-slip voltage. Phase slips are stimulated by fluctuations. The crude estimate of suppression of  $V_{PS}$  by thermal fluctuations [Eq. (30)] suggests a strong decrease of the phase-slip voltage. It was found that the phase-slip voltage may decrease also if the applied electric field greatly exceeds the threshold force related to the potential barrier preventing the phase slippage.

In this study the inertial term in the equation of motion for the CDW phase was neglected. This is valid if the CDW current is small enough, and corresponds to frequencies which can be estimated as

$$\omega \ll \frac{1}{\tau} \frac{m}{m^*},$$

where  $\tau$  is the electron momentum scattering time. This condition is fulfilled at current densities used in experiments of Ref. 15. However, at larger frequencies the inertial term modifies the decay length, and hence Eq. (22), describing the average phase-slip dynamics. Therefore, the results of this study need modifications at large frequencies for which the inertial term becomes important.

As mentioned above, the phase-slip dynamics changes as the distance between the current contacts becomes comparable to the decay length of the CDW phase perturbations that are enhanced by the long-range Coulomb effects. The latter depend on the distribution of the charge density between the CDW and single-particle excitations. Therefore, in order to be closer to the experimentally studied case of  $\text{NbSe}_3$  the decay length was calculated within the model of a CDW conductor consisting of both normal-metal and CDW-state conducting chains. The possibility of a direct quantitative comparison of our theoretical results with the experimental data is limited because we considered a simplified one-dimensional distribution of the current density. In the real experimental conditions<sup>15</sup> side contacts are used, and the samples have comparable transverse and longitudinal sizes. In this case the current distribution is rather complicated. In particular, due to a large anisotropy of the CDW compounds the current injected into a sample perpendicularly to the conducting chains flows near the contacts in different directions, including the direction opposite to another contact. Therefore, effective sizes of regions with more or less uniform current density along the chains are distributed in a wide range starting from the intercontact distance up to the length of the order of the sample thickness multiplied by the square root of the large anisotropy factor of the conductivity. Therefore, it is difficult to make a quantitative comparison of the theory with the experimental data. However, the theoretical values of the contact separation for which the phase-slip voltage starts to decrease and the phase-slip voltage itself are in a qualitative agreement with the experimental data.

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