Partially spin-polarized quantum Hall effect in the filling factor range 1/3 < ν < 2/5

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The residual interaction between composite fermions (CF's) can express itself through higher order fractional Hall effect. With the help of diagonalization in a truncated composite-fermion basis of low-energy many-body states, we predict that quantum Hall effect with partial spin polarization is possible at several fractions between $\nu = 1/3$ and $\nu = 2/5$. The estimated excitation gaps are approximately two orders of magnitude smaller than the gap at $\nu = 1/3$, confirming that the inter-CF interaction is extremely weak in higher CF levels.

In recent years, the physics arising from the interactions between composite fermions has come into focus.[1–6] The model of noninteracting composite fermions explains the fractional quantization of Hall resistance[7] at $R_H = h/fe^2$ with

$$f = \frac{n}{2pn \pm 1} \tag{1}$$

as the integral quantum Hall effect[8] of composite fermions.[9] (Particle-hole symmetry in the lowest Landau level implies fractional Hall effect also at $1 - f$ or $2 - f$, for fully or partially spin-polarized systems, respectively.) The weak residual interaction between composite fermions is often masked by disorder or temperature, much as the fractional quantum Hall effect (FQHE), a manifestation of interelectron interactions, is absent in low mobility samples or at high temperatures. However, with improvements in experimental conditions, the physics originating from the interaction between composite fermions is beginning to emerge.[1,2]

A possible manifestation of the interaction between composite fermions will be the appearance of higher order FQHE states in between the above fractions. The possible fractions can be derived straightforwardly.[3,4,9] At the *nonintegral* values of the Composite-Fermion (CF) filling factor given by

$$\nu^* = n + \frac{m}{2p'm \pm 1} \tag{2}$$

the composite fermions in the topmost partially filled level may capture, as a result of the interaction between them, $2p'$ additional vortices to transform into higher-order composite fermions and fill m CF levels to exhibit new incompressible states, which will produce FQHE at $\nu = \nu^*/(2p\nu^* \pm 1)$ between the fractions $\nu = n/(2pn \pm 1)$ and $\nu = (n+1)/[2p(n+1) \pm 1]$. The situation is analogous to the appearance of FQHE of electrons in partially filled *electronic* Landau levels.

Pan *et al.*[1] have reported FQHE at $\nu = 4/11$, $\nu = 5/13$, and $\nu = 6/17$ in the filling factor range $2/5 > \nu > 1/3$. The $\nu = 4/11$ minimum is seen in magnetic fields as high as $B = 33$ T, pointing to a fully polarized QHE state. The simplest fractions in the above scenario are $\nu^* = 1 + 1/3$ and $\nu^* = 1 + 2/3$, which produce higher order FQHE at $\nu = 4/11$ and $\nu = 5/13$, and $\nu = 6/17$ originates from $\nu^* = 1 + 1/5$.

While it is encouraging that the desired fractions are obtained, more detailed theoretical investigations do not find new FQHE states for these fractions for *fully* spin-polarized composite fermions for an idealized model neglecting disorder, transverse thickness, and Landau-level mixing;[3,5,6] the residual interaction between composite fermions in *higher* CF levels does not appear to be sufficiently strongly repulsive at short distances to cause additional vortices to bind to composite fermions. It is not known at the present which of the neglected effects is responsible for the discrepancy.

While only the fully spin-polarized states are possible at sufficiently high magnetic fields, where a spin in the wrong direction costs a prohibitively high energy, FQHE states at the fractions of Eq. (1) with partial polarizations have been observed experimentally, and transitions between differently polarized states have been studied as a function of the Zeeman energy.[10] These studies are satisfactorily described by the composite-fermion theory including spin.[11,12] Composite fermions with spin are analogous to electrons with spin at ν^*, with the same Zeeman energy but with an effective cyclotron energy. While the Zeeman energy is very small compared to the cyclotron energy for electrons (in GaAs), the two are comparable for composite fermions, thus producing a richer variety of states.[10] This raises the question of whether FQHE states with partial spin polarization are possible at fractions like $\nu = 4/11$, which is the subject of this paper. It is stressed that the present work does not purport to be an explanation for the observations in Ref. 1 but a prediction for sufficiently low magnetic fields.

Following the standard approach, we will neglect in this study the effects of finite thickness and Landau-level mixing. The most reliable method is exact numerical diagonalization, which is not an option for this problem because of the rather large Hilbert space. In this paper, we carry out diagonalization in a truncated low-energy CF basis of many-body states.[3] We will concentrate here on the filling factor range $2/5 > \nu > 1/3$, that is, $2 > \nu^* > 1$. The FQHE is most likely at $\nu^* = 1 + m/(2m+1)$, which correspond to electron filling factors $\nu = (3m+1)/(8m+3)$. The positive and negative integral values for m produce $\nu = 4/11, 7/19, \ldots$, and $\nu = 5/13, 8/21, \ldots$. It will be assumed that the up-spin composite fermions fill one level completely, and the down-spin composite fermions have filling factor $\nu_\downarrow^* = m/(2m+1)$ in the lowest spin-reversed band, giving the total CF filling $\nu^* = 1 + \nu_\downarrow^*$. For our truncated basis, we consider wave functions of the form

$$\Psi^{\alpha}_{(3m+1)/(8m+3)}=\Phi_1^2[\Phi_{1,\uparrow}\Phi^{\alpha}_{m/(2m+1),\downarrow}]. \quad (3)$$

Here, $\Phi_{1,\uparrow}$ is the fully occupied up-spin lowest Landau-level band. $\Phi^{\alpha}_{m/(2m+1),\downarrow}$ are various orthogonal wave functions (labeled by α) at filling $\nu^*_{\downarrow}=m/(2m+1)$ in the down-spin band, obtained by exact diagonalization at $\nu^*_{\downarrow}$. (Coupling to higher Landau levels is neglected.) The fully antisymmetric function Φ_1 is one filled Landau level of "spinless" electrons; the Jastrow factor Φ_1^2, as always, converts, through attachment of two vortices to each electron, the $\nu^*=1+m/(2m+1)$ wave function of electrons in square brackets to the $\nu^*=1+m/(2m+1)$ wave function of composite fermions, which is identified with a basis function for interacting electrons at $\nu=(3m+1)/(8m+3)$. (The spin part of the wave function is not explicitly shown. The full wave function is obtained by multiplying by the spin part $u_1 \ldots u_{N_{\uparrow}}d_{N_{\uparrow}+1} \ldots d_N$, where $N_{\uparrow}=N-N_{\downarrow}$ is the number of up-spin electrons, followed by antisymmetrization.) We will study below $m=1$ and $m=2$ ($\nu=4/11$ and $\nu=7/19$). The states of the form given in Eq. (3) obviously do not exhaust the entire Hilbert space at ν, as they neglect the mixing between the Landau levels of composite fermions, but we believe that they span the low-energy Hilbert space. If the system is incompressible, the ground state at ν is likely to be well described by

$$\Psi^{gr}_{\nu}=\Phi_1^2[\Phi_{1,\uparrow}\Phi^{gr}_{\nu^*_{\downarrow},\downarrow}], \quad (4)$$

where $\Phi^{gr}_{\nu^*_{\downarrow},\downarrow}$ is the Coulomb ground state at $\nu^*_{\downarrow}$. For $\nu=(3m+1)/(8m+3)$, the state at $\nu^*_{\downarrow}=m/(2m+1)$ is accurately given by the standard wave function $\mathcal{P}\Phi^2_{1,\downarrow}\Phi_{m,\downarrow}$, where $\mathcal{P}$ denotes projection into the lowest Landau level.

It is noteworthy that no assumption is made regarding the nature of the state in the reversed-spin sector, and the calculation can, in principle, give either a compressible or an incompressible ground state. Indeed, a similar study for *fully* spin-polarized systems at many fractions like $\nu=4/11$ failed

TABLE I. The parameters for the systems studied in this work. Q and q^* are the monopole strengths for electrons and composite fermions; ν and ν^* are filling factors for electrons and composite fermions; N is the total number of composite fermions and $N_{\downarrow}$ is the number of composite fermions in the reversed-spin sector. The last column shows the overlap of the "ground state" at $\nu=4/11$ and $\nu=7/19$ determined by diagonalization in the truncated basis (see text for details) with the wave function derived from the $\nu^*_{\downarrow}=1/3$ or $\nu^*_{\downarrow}=2/5$ incompressible state according to Eq. (4). The overlap is 100% when there is only a single uniform basis state.

ν	ν^*	N	Q	$N_{\downarrow}$	q^*	Overlap (%)
$\frac{4}{11}$	$1+\frac{1}{3}$	6	6.5	2	1.5	100
		10	12.0	3	3.0	100
		14	17.5	4	4.5	90.1
		18	23.0	5	6.0	98.8
$\frac{7}{19}$	$1+\frac{2}{5}$	11	13.0	4	3.0	100
		18	22.5	6	5.5	99.0

to yield an incompressible ground state,[3] contrary to what one might have naively expected.

An earlier study[4] began with the *assumption* of the partially spin-polarized ground state

$$\Psi^{gr}_{\nu=4/11}=\Phi_1^2\Phi_{1,\uparrow}[\Phi_{1,\downarrow}]^3 \quad (5)$$

which is derived from Laughlin's wave function[13] $[\Phi_{1,\downarrow}]^3$ in the spin-reversed sector,[14] and considered a trial wave function for its neutral excitation containing a pair of CF particle-hole pair in the reversed-spin sector. It was found that the energy of the excitation remains positive for all wave vectors, indicating that the assumed ground-state wave function is stable against excitations. While this study did not eliminate FQHE at $\nu=4/11$, it did not test whether the ground state is necessarily incompressible, and if so, whether it is well described by the trial wave function in Eq. (4). The present study provides a more rigorous (though still not conclusive) test for partially polarized QHE at $\nu=4/11$.

In the following discussion we will employ the spherical geometry,[15,16] where we consider N electrons moving on the surface of a sphere at the presence of a magnetic monopole with strength Q at the center. The magnitude of the radial magnetic field B is given by $2Q\phi_0/4\pi R^2$, where $\phi_0=he/c$ is the flux quantum, R is the radius of the sphere, and Q is either an integer or a half integer due to the Dirac quantization condition. The composite-fermion theory maps the system of interacting electrons at Q to the CF system at $q^*=Q-p(N-1)$. It is convenient to label the wave function by the monopole strength; for example, the wave function at Q is obtained from the electron wave function Φ_{q*} by $\Psi_Q=\mathcal{P}[\Phi_1^{2p}\Phi_{q*}]$.

The CF theory fixes the relation between N and Q as follows. The effective q^* is determined by requiring that $N_{\downarrow}$ electrons have filling $\nu^*_{\downarrow}=m/(2m+1)$: $q^*=N_{\downarrow}(2m+1)/2m-(m+2)/2$. With $N_{\uparrow}=2q^*+1$ and $Q=q^*-(N-1)$, we get

$$Q=\frac{8m+3}{6m+2}N-\frac{m^2+10m+3}{6m+2}. \quad (6)$$

Therefore, at $\nu=4/11$, where the effective filling $\nu^*=1+1/3$ with $m=1$, the relation is

$$Q_{4/11}=(11N-14)/8. \quad (7)$$

Similarly, for $\nu=7/19$, where $\nu^*=1+2/5$ and $m=2$,

$$Q_{7/19}=(19N-27)/14. \quad (8)$$

Note that both relations give the desired filling factors in the thermodynamic limits: $\nu=\lim_{N\to\infty}N/2Q$. For a given particle number N, the pair (N,Q) is the only input in the numerical calculation. Table I gives the systems we have studied below.

The energy spectrum is calculated numerically by the Monte Carlo (MC) method, following Ref. 3. Because the low-energy basis states from exact diagonalization are not necessarily orthonormal for a given L, we use the standard Gram-Schmidt procedure to obtain an orthonormal basis. Some technical details ought to be mentioned here. The Me-
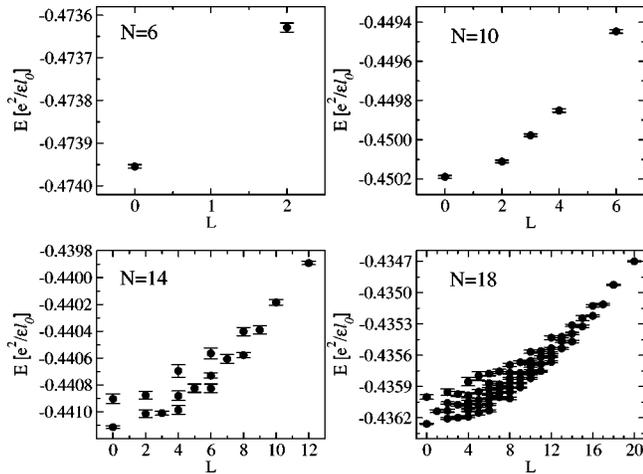
FIG. 1. The energy spectrum at $\nu=4/11$ for $N=6$, 10, 14, and 18 particles. It is assumed that the state has partial spin polarization. The single-particle energy E includes the interaction with the uniform, positively charged background. The quantity $l_0=\sqrt{\hbar c/eB}$ is the magnetic length at ν, and ϵ is the dielectric constant of the host semiconductor. The error bars show the statistical uncertainty in the Monte Carlo simulation.

tropolis algorithm employed in our MC calculations has minimum statistical error when the weight function behaves approximately as the wave function. We find that it is crucial to use several weight functions with different angular momenta L in order to reduce the error to desired level. We divide the MC calculations in ten configurations, with the number of iterations on the order of 10^7 for each configuration. Such large number of steps are required to determine accurately the extremely small energy differences. To reduce the computation time for large systems, for example, $N=14$, 18 at $\nu=4/11$, we place each of the MC configuration on a single node (dual 1-GHz Intel Pentium III processor) of a PC cluster.

Figure 1 shows the low-energy spectrum at $\nu=4/11$ for $N=6$, 10, 14, and 18, for which there are $N_\downarrow=2$, 3, 4, and 5 composite fermions in the spin-reversed CF level. The dimension of the basis is the same as that of the lowest Landau-level Hilbert space of $N_\downarrow$ particles at q^*. In all cases, the ground state is a uniform state with $L=0$. From the analogy to $\nu=1/3$, it might be expected that the excitation spectrum contains a well defined branch of composite-fermion exciton,[17] containing a single multiplet at $L=2,\ldots,N_\downarrow$. This CF exciton branch is identifiable for $N=14$ but not at $N=18$. Nonetheless, there is a well defined gap in all cases. Figure 2 shows the N dependence of minimum energy needed for the creation of a CF exciton. There are substantial finite-size fluctuations in the value of the gap, because the number of spin reversed composite fermions is quite small, but we believe that our results indicate that the gap remains finite in the thermodynamic limit, producing incompressibility at 4/11. We will use the gap of the largest system studied as a rough estimate of the thermodynamic gap. The next fraction we consider is $\nu^*=1(\uparrow)+2/5(\downarrow)$, corresponding to a partially polarized state at $\nu=7/19$. As Fig. 3 shows, the state here is also incompressible. A ther-
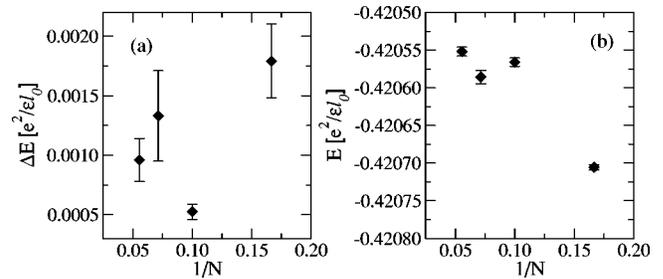
FIG. 2. Neutral excitation gap (ΔE) and the ground-state energy per particle (E) at $\nu=4/11$ as a function of N^{-1}, N being the number of composite fermions.

modynamic extrapolation for the gap is not possible for $\nu=7/19$, for we have only two results, but the gap for $N=18$ is taken as an estimate of the thermodynamic limit.

The gaps for $\nu=4/11$ and $\nu=7/19$ are estimated to be $\sim 0.001 e^2/\epsilon l_0$ and $\sim 0.0008 e^2/\epsilon l_0$, which are roughly two orders of magnitude smaller than the gaps at $\nu=1/3$ and $\nu=2/5$, $0.1 e^2/\epsilon l_0$, and $0.055 e^2/\epsilon l_0$, respectively (for the model neglecting transverse thickness). Here, $l_0=\sqrt{\hbar c/eB}$ is the magnetic length at ν, B is the external magnetic field, and ϵ is the dielectric constant of the host material. The gap value at $\nu=4/11$ is consistent with that quoted in Ref. 4. It is noted that the gaps are not affected by the Zeeman energy, so long as the partially polarized state is the ground state, because the low-energy excitations of these states do not involve any spin reversal. (The Zeeman energy is much higher, for typical experimental parameters, than the energy scales considered in this work, making spin-flip excitations irrelevant to the low-energy physics.) Figure 2 estimates the thermodynamic limit of the ground-state energy at $\nu=4/11$ to be $\sim -0.420\,54 e^2/\epsilon l_0$, which also is in good accord with the value calculated earlier.[4]

The smallness of the gaps for the higher order FQHE states confirms that the interaction between the composite fermions in higher CF levels is exceedingly weak compared to the Coulomb interaction between *electrons* that governs
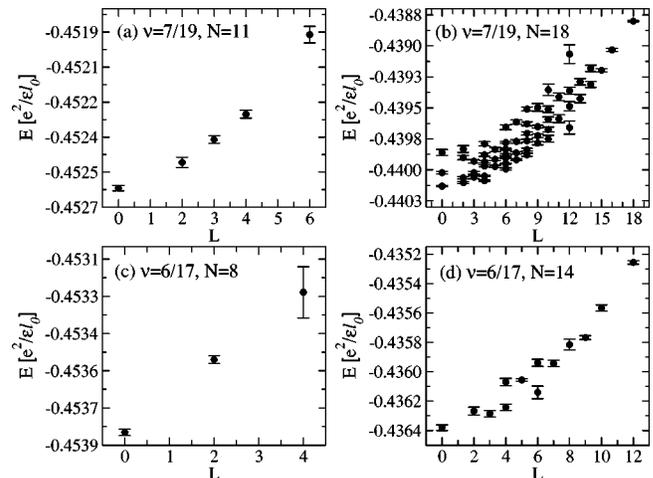
FIG. 3. The energy spectrum at $\nu=7/19$ for $N=11$ and $N=18$ and at $\nu=6/17$ for $N=8$ and $N=14$ for a partially spin-polarized system.

the gaps at $\nu = 1/3$ and $2/5$. It is remarkable that the composite-fermion theory is capable of capturing such subtle quantitative physics, and that experiments have come to a stage where higher order FQHE states are now beginning to reveal themselves.

Table I gives the overlaps of the ground state with the wave function of Eq. (4). The overlaps are fairly large, confirming that the trial wave function of Eq. (4) describes the ground state effectively; in other words, the physics of the FQHE at $\nu = 4/11$ and $\nu = 7/19$ is related to the $\nu_{\downarrow} = 1/3$ and $\nu_{\downarrow} = 2/5$ FQHE in the spin-reversed sector.

We have also investigated the possibility of partially polarized QHE at $\nu = 6/17$, which maps into $\nu^* = 1(\uparrow) + 1/5(\downarrow)$ of composite fermions. The theoretical spectra for 8 and 14 particles at $Q_{6/17} = (17N - 22)/12$, shown in Fig. 3, provide an indication of an incompressible state here as well. Surprisingly, the gap for $N = 14$ is $\sim 0.0014 e^2 / \epsilon l_0$, which is of the same order as the $\nu = 4/11$ gap. However, the system sizes are effectively very small, as can be seen by the fact that there are only one or two basis functions at each angular momentum, which prevents us from making a more reliable assertion regarding the presence of incompressibility at $\nu = 6/17$. At the moment, we are unable to get sufficient accuracy at the next particle number ($N = 20$).

Our study thus predicts that partially polarized higher order FQHE at fractions of the form $\nu = (3m+1)/(8m+3)$ should be possible in an appropriate range of Zeeman energy and temperature. The temperature scale set by the gap is on the order of 150 mK (at $B = 10$ T) for GaAs, which is an upper limit because corrections due to finite thickness and disorder are expected to suppress the gap substantially. It is at present not possible to ascertain theoretically the relevant Zeeman energy range, for lack of a quantitative understanding of the fully polarized states at these filling factors. The modifications due to finite transverse thickness, not included above, are also of relevance.

Similar considerations may also be useful for the QHE-like features seen previously at $\nu = 7/11$ in very low-density samples.[18] The relevance of our results to the Raman experiment[2] in the filling factor range $2/5 \geqslant \nu \geqslant 1/3$, where the level structure of composite fermions is observed, also deserves further investigation.

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