

# Influence of mutual drag of the carrier-phonon system on the thermopower and transverse Nernst-Ettingshausen effect

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The thermopower and Nernst-Ettingshausen (NE) effect in degenerate semiconductors and semimetals placed in high electric and magnetic fields are calculated by taking into account the heating of both electrons and phonons as well as their thermal and mutual drags. The magnetic and electric field dependences of the thermoelectric power and the transverse NE voltage are found in analytical forms. It is shown that in weak and high transverse magnetic fields, the electronic and phonon parts of the NE coefficients change their sign for some scattering mechanisms.

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## I. INTRODUCTION

The theoretical and experimental interest in thermoelectric power in bulk and recent low-dimensional systems has been intensified.<sup>1–13</sup> A relatively long survey of literature and some common misunderstandings in the field of thermoelectric power ( $\alpha$ ) and Nernst-Ettingshausen (NE) effect under different transport conditions<sup>3,7–9,14–22</sup> are given in our recent paper.<sup>23</sup>

Lei<sup>11</sup> showed in 1994 that the diffusion component of  $\alpha$  may be negative at a low lattice temperature range and high electric field while the phonon drag component is still positive. Such a result was also obtained by Babaev and Gassymov<sup>19</sup> in 1977. They theoretically investigated the NE effect and  $\alpha$  in semiconductors at high electric and nonquantizing magnetic fields by solving the coupled system of kinetic equations for electrons and phonons. The electron and phonon heating, and the phonon drag were taken into account. It was shown that when the temperature gradient of hot electrons is produced only by the lattice temperature gradient, the electronic parts of the thermoelectric and NE fields reverse their sign. In the case of phonon heating and  $T_p = T_e \gg T$ , both electronic and phonon parts of the thermoelectric and thermomagnetic fields reverse their sign for all cases considered. Here  $T_e$ ,  $T_p$ , and  $T$  are the temperature of electrons, phonons, and lattice, respectively.

The NE effect and  $\alpha$  in II–VI semiconductors have been investigated with increasing interest.<sup>24–27</sup> The earlier investigations of the magnetic field ( $H$ ) dependence of the longitudinal NE effect in HgSe (Refs. 28 and 29) and lead chalcogenides (Refs. 30 and 31) in the region of higher temperatures ( $T \geq 77$  K) demonstrated that the thermoelectromotive force exhibits saturation in the region of strong magnetic fields irrespective of the dominant scattering mechanism of charge carriers in the conduction band. However, the longitudinal NE effect in iron-doped HgSe samples at low temperatures ( $20 \leq T \leq 60$  K) has a maximum in the plot of  $\Delta\alpha(H) = |\alpha(H) - \alpha(0)|$ . Here  $\Delta\alpha(H)$  first increases quadratically with increasing  $H$  for  $\Omega\tau < 1$ , then passes through a maximum at  $H = H_m$ , and finally decreases as the field increases further (here  $\Omega = eH/mc$  is the cyclotron fre-

quency and  $\tau$  is the electron relaxation time). Another unusual fact is the sign reversal of the transverse NE coefficient  $Q_{\perp}(H)$  with increasing  $H$  in the range  $\Omega\tau > 1$ .<sup>26,27</sup> The experiments in gallium-doped HgSe revealed that at low temperatures the NE coefficients change sign with increasing gallium concentration or applied magnetic field strength. These unusual features of the NE effect may be attributed to the effect of mutual drag which can be observed in semiconductors with a high concentration of conduction electrons.<sup>32</sup>

In the absence of external magnetic field, the  $\alpha$  of hot electrons, taking into account the heating of phonons and the thermal drag, is considered in Ref. 18. In that paper, the deformation potential of interaction between electrons and phonons is considered. The transverse NE effect and  $\alpha$  of hot electrons in nondegenerate semiconductors are studied in Ref. 33 without taking into account the effect of phonon drag and their heating and in Ref. 19 by taking into account the thermal drag only in transverse magnetic field. However, these studies did not consider the mutual drag of charge carriers and phonons.

There are some investigations considering the electron-phonon drag and transport phenomena in semiconductors.<sup>34–37</sup> In Ref. 34, the electron–optical-phonon drag and the size effect are mainly considered. References 35 and 36 also considered the size effect in finite semiconductors under the conditions of mutual drag. The electric current and electron and phonon parts of the thermal fluxes are obtained in general forms for the degenerate statistics of electrons in Ref. 37. Gurevich and Mashkevich list the procedure for determining the distribution function of electrons. However, the list is not complete, because they obtain only general expressions for electric current and electron and phonon parts of thermal fluxes, but they did not find the external electric field dependence of the effective electron and phonon temperatures. Therefore, they did not find the thermoelectric coefficients and their external electric field dependence.

In the present paper, the NE effect and  $\alpha$  in degenerate semiconductors and semimetals placed in high external electric and longitudinal and transverse magnetic fields are investigated by taking into account the heating of electrons and phonons as well as the thermal and mutual drags of charge

carriers and phonons. The spectrum of charge carriers is assumed to be parabolic, e.g.,  $\varepsilon = p^2/2m$ . Consideration is made for both deformation ( $d-$ ) and piezoelectric ( $p-$ ) interaction potentials of electrons with phonons.

The organization of the paper is as follows: The system of equations of the problem and their solutions are given in Sec. II, the energy balance equations and their solutions for different scattering mechanisms are investigated in detail in Sec. III, and the thermopower in a longitudinal magnetic field is presented in Sec. IV. Section V concentrates on the thermopower and NE effect in transverse magnetic field. Finally, the conclusion is given in Sec. VI.

## II. THEORY

Consider a degenerate semiconductor or semimetal with fully ionized impurities placed in high electric and nonquantizing magnetic fields. We assume that there are temperature gradients of both electrons ( $\nabla T_e$ ) and long-wavelength (LW) phonons interacting with electrons ( $\nabla T_{ph}$ ). The gradients may be realized by the gradient of heating electric field ( $\nabla E$ ): for example, by placing one end of the specimen to the wave guide with heating electromagnetic wave or by producing lattice temperature gradient ( $\nabla T$ ).

If the frequency of interelectronic collisions  $\nu_{ee}$  is much bigger than that of electron-phonon collisions for the energy transfer  $\nu_e$ , i.e.,  $\nu_{ee} \gg \nu_e$ , then the isotropic part of the distribution function of electrons has the form of a Fermi one with effective temperature of electrons,  $T_e$ ,

$$f_0(\varepsilon) = \left[ 1 + \exp\left(\frac{\zeta(T_e) - \varepsilon}{T_e}\right) \right]^{-1}, \quad (1)$$

where  $\zeta(T_e)$  is the chemical potential and  $\varepsilon$  is the energy of charge carriers. Note that  $T_e$  is in energy units.

We assume that in the lattice there is a thermal reservoir of short-wavelength (SW) phonons for LW phonons interacting with electrons.<sup>38</sup> The maximum momentum of LW phonons interacting with electrons satisfies the condition:  $q_{max} \approx 2p_0 < T/s \equiv q_T$ , where  $T$  is the lattice (reservoir) temperature,  $q_T$  is the momentum of thermal phonons,  $p_0$  is the momentum of electrons in the Fermi level, and  $s$  is the velocity of sound in the crystal. As it is shown in Ref. 38, under these conditions LW phonons are heated as well. Therefore, we assume that the isotropic part of the distribution function of phonons has the form

$$N_0(q) = \left[ \exp\left(\frac{\hbar\omega_q}{T_{ph}}\right) - 1 \right]^{-1} \approx \frac{T_{ph}}{\hbar\omega_q}. \quad (2)$$

In accordance with Ref. 38, the distribution function of phonons has the form of Eq. (2) only in two cases. In the first case the frequency of LW phonon-electron collisions  $\beta_e$  is much smaller than the frequency of LW-phonon-SW-phonon collisions  $\beta_{ph}$ . In this case  $T_{ph} = T$  if

$$\frac{N(T_e)}{N(T)} \frac{\beta_e}{\beta_{ph}} \approx \frac{T_e}{T} \frac{\beta_e}{\beta_{ph}} \ll 1. \quad (3)$$

In the second case  $\beta_e \gg \beta_{ph}, \beta_b^{(\varepsilon)}$ , where  $\beta_b^{(\varepsilon)}$  is the collision frequency of phonons with crystal boundaries connected with energy transfer to outside. In this case, the temperature of LW phonons becomes equal to the temperature of electrons ( $T_{ph} = T_e$ ), and LW phonons are in nonequilibrium state.

In high external fields electrons and phonons are essentially in a nonequilibrium and anisotropic state. Therefore, the distribution function of electrons  $f(\varepsilon)$  and that of phonons  $N(\mathbf{q})$  are, as usual, in the form

$$f(\varepsilon) = f_0(\varepsilon) + \frac{\mathbf{f}_1(\varepsilon) \cdot \mathbf{p}}{p}, \quad N(\mathbf{q}) = N_0(q) + \frac{\mathbf{N}_1(q) \cdot \mathbf{q}}{q}, \quad (4)$$

where  $\mathbf{f}_1(\varepsilon)$  and  $\mathbf{N}_1(q)$  are the antisymmetric parts of the distribution functions of electrons and phonons, respectively.

In the present paper, we assume that the so-called ‘‘diffusion approximation’’ for electrons and phonons applies. Therefore,  $|\mathbf{f}_1(\varepsilon)| \ll f_0(\varepsilon)$  and  $|\mathbf{N}_1(q)| \ll N_0(q)$ . The isotropic and anisotropic parts of the distribution functions of electrons and phonons are obtained from the coupled system of Boltzmann equations, which form the main equations of the problem:

$$\begin{aligned} & \frac{p}{3m} [\nabla \mathbf{f}_1(\varepsilon)] - \frac{2e}{3p} \frac{\partial}{\partial \varepsilon} [\varepsilon \mathbf{E} \cdot \mathbf{f}_1(\varepsilon)] \\ & = \frac{m}{2\pi^2 \hbar^3 p} \frac{\partial}{\partial \varepsilon} \left[ \int_0^{2p} dq \hbar \omega_q W_q q \left\{ \hbar \omega_q N_0(q) \left( \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) \right. \right. \\ & \quad \left. \left. + f_0(\varepsilon) [1 - f_0(\varepsilon)] \right\} \right], \end{aligned} \quad (5)$$

$$\begin{aligned} & \frac{p}{m} [\nabla f_0(\varepsilon)] - \frac{e \mathbf{E} p}{m} \left( \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) + \nu(\varepsilon) \mathbf{f}_1(\varepsilon) - \Omega [\mathbf{h} \cdot \mathbf{f}_1(\varepsilon)] \\ & = - \frac{4\pi}{(2\pi\hbar)^3} \frac{1}{p^2} \left( \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right) \int_0^{2p} dq W_q q^2 \hbar \omega_q \mathbf{N}_1(q), \end{aligned} \quad (6)$$

$$s \nabla N_0(q) + \beta(q) \mathbf{N}_1(q) = \frac{4\pi m W_q N_0(q)}{(2\pi\hbar)^3} \int_{\varepsilon(q/2)}^{\infty} dp \mathbf{f}_1(\varepsilon), \quad (7)$$

$$\begin{aligned} & \frac{s}{3} \nabla \mathbf{N}_1(q) + \beta_0(q) N_0(q) - [(\beta_{ph} + \beta_b^{(\varepsilon)}) N(q, T) \\ & \quad + \beta_e N(q, T_e)] = 0. \end{aligned} \quad (8)$$

In Eqs. (5)–(8),  $e$  is the absolute value of the electronic charge,  $m$  is the effective mass of electrons,  $\hbar\omega_q$  and  $q$  are the energy and the quasimomentum of phonons, respectively, and  $N(q, T)$  and  $N(q, T_e)$  are the equilibrium Planck distribution functions with temperatures  $T$  and  $T_e$ .  $W_q$  is the quantity from which the scattering probability of electrons by acoustical phonons is obtained. It is defined as

$$W_q = W_0 q^t. \quad (9)$$

For  $d$  interaction  $t=1$  and  $W_0 \approx 2\pi G^2/\rho s \hbar$ , where  $G$  is the deformation potential constant. On the other hand, for  $p$  interaction  $t=-1$  and  $W_0 = (4\pi)^3 \hbar e^2 \Sigma^2 / \rho s \epsilon_0^2$ , where  $\Sigma$ ,  $\rho$ , and  $\epsilon_0$  are the piezoelectric module, density, and the dielectric constant of the crystal, respectively.

The collision frequency of electrons with phonons  $\nu_{ph}(\epsilon)$  is

$$\nu_{ph}(\epsilon) = \frac{mW_0}{2\pi^2 \hbar^3 p^3} \int_0^{2p} dq q^{3+t} \left[ N_0(q) + \frac{1}{2} \right]. \quad (10)$$

The total collision frequency of phonons is defined as

$$\beta(q) = \beta_0(q) + \beta_b(q) = \beta_e(q) + \beta_{ph}(q) + \beta_b^{(e)}(q) + \beta_b^{(p)}(q), \quad (11)$$

where the indices mean the collision frequency of phonons with electrons ( $e$ ), phonons ( $ph$ ), and crystal boundaries ( $b$ ) for the energy or momentum transfer to outside. The total collision frequencies of phonons  $\beta(q)$  and electrons  $\nu(\epsilon)$  may be given in the form

$$\beta(q) = \beta(T) \left( \frac{qs}{T} \right)^k, \quad \nu(\epsilon) = \tilde{\nu} \left( \frac{\epsilon}{\zeta_0} \right)^r, \quad \tilde{\nu} = \nu_0 \Theta_{e,ph}^\ell, \quad (12)$$

where  $\zeta_0 = \epsilon_F$  is the Fermi energy,  $\nu_0 = \nu(\zeta_0)$  and  $\Theta_{e,ph} = T_{e,ph}/T$  is the dimensionless temperature of electrons and phonons. For the scattering of electrons by the impurity ions  $r = -3/2$ ,  $\ell = 0$ ; by the deformation potential of acoustical phonons ( $d$  interaction)  $r = 1/2$ ,  $\ell = 1$ ; by the piezoelectric potential of acoustical phonons ( $p$  interaction)  $r = -1/2$ ,  $\ell = 1$ ; and  $k = 0, 1, t$  for scattering of LW phonons by crystal boundaries, by SW phonons and electrons, respectively.

By neglecting the first term in Eq. (8), we obtain Eq. (2). By using Eqs. (1) and (6), for  $\mathbf{f}_1(\epsilon)$  we have

$$\begin{aligned} \mathbf{f}_1(\epsilon) - \frac{\Omega}{\nu(\epsilon)} [\mathbf{h} \cdot \mathbf{f}_1(\epsilon)] - \frac{p}{m\nu(\epsilon)} \left( e\mathbf{E}' + \left[ \frac{\epsilon - \zeta}{T_e} \right] \nabla T_e \right) \frac{\partial f_0(\epsilon)}{\partial \epsilon} \\ = \frac{m^2}{2\pi^3 \hbar^3 p^3(\epsilon)} \left( \frac{\partial f_0(\epsilon)}{\partial \epsilon} \right) \int_0^{2p} dq W_q \hbar \omega_q \frac{q^2}{\beta(q)} \left\{ s \nabla N_0(q) \right. \\ \left. - \frac{mW_q N_0(q)}{2\pi^2 \hbar^3} \int_{\epsilon(q/2)}^\infty dp \mathbf{f}_1(\epsilon) \right\}. \quad (13) \end{aligned}$$

The first term on the right-hand side of Eq. (13) is in accordance with the thermal drag and the second term with the mutual drag. Equation (13) is the integral equation for  $\mathbf{f}_1(\epsilon)$ , but if we assume as usual

$$\mathbf{f}_1(\epsilon) = p\mathbf{V}(\epsilon) \left[ -\frac{\partial f_0(\epsilon)}{\partial \epsilon} \right], \quad (14)$$

then for the case of degenerate electrons, this equation becomes an algebraic one. In Eq. (14),  $\mathbf{V}(\epsilon)$  is the drift velocity of electrons. Replacing the integral

$$\int_{\epsilon(q/2)}^\infty d\epsilon V(\epsilon) \left( -\frac{\partial f_0(\epsilon)}{\partial \epsilon} \right)$$

by  $V(\zeta_0)$ , we obtain the following equation for  $\mathbf{V}(\epsilon)$ :

$$\begin{aligned} \mathbf{V}(\epsilon) = -\frac{\nu(\epsilon)}{m[\Omega^2 + \nu^2(\epsilon)]} \left\{ \left[ \mathbf{F} + \frac{\Omega}{\nu(\epsilon)} [\mathbf{h} \cdot \mathbf{F}] \right. \right. \\ \left. \left. + \mathbf{h} \cdot [\mathbf{h} \cdot \mathbf{F}] \frac{\Omega^2}{\nu^2(\epsilon)} \right] - m\nu(\epsilon)\gamma(\epsilon) \left[ \mathbf{V}_0 + \frac{\Omega}{\nu(\epsilon)} [\mathbf{h} \cdot \mathbf{V}_0] \right. \right. \\ \left. \left. + \frac{\Omega^2}{\nu^2(\epsilon)} \mathbf{h} \cdot (\mathbf{h} \cdot \mathbf{V}_0) \right] \right\}, \quad (15) \end{aligned}$$

where

$$\mathbf{F} = e\mathbf{E}' + \left( \frac{\epsilon - \zeta}{T_e} \right) \nabla T_e + A_{kt} \left( \frac{\epsilon}{T} \right)^{(t-k)/2} \nabla T_{ph}, \quad (16)$$

$$A_{kt} = \frac{2^{3+3(t-k)/2} \beta_e(T)}{3+t-k} \frac{\beta_e(T)}{\beta(T)} \left( \frac{ms^2}{T} \right)^{(t-k)/2}, \quad (17)$$

$$\mathbf{E}' = \mathbf{E} + \mathbf{E}_T + \frac{1}{e} \nabla \zeta(T_e), \quad (18)$$

where  $\mathbf{E}$  is the external field,  $\mathbf{E}_T$  is the thermoelectric field, and  $V_0 = V(\zeta_0)$ .

The expression characterizing the mutual drag of charge carriers and phonons is

$$\gamma(\epsilon) = \frac{3+t}{2^{3+t}} \frac{\nu_{ph}(\epsilon)}{\nu(\epsilon)} \frac{1}{p^{3+t}} \int_0^{2p} dq \frac{\beta_e(q)}{\beta(q)} q^{2+t}. \quad (19)$$

The mutual drag is strong as  $\gamma \rightarrow 1$ . This means that electrons and phonons are scattered preferably by each other, i.e.,  $\nu(\epsilon) \approx \nu_{ph}(\epsilon)$  and  $\beta(q) \approx \beta_e(q)$ . In fact, there are other scattering mechanisms of electrons and phonons, because in the present work we assume the diffusion approximation  $\gamma(\epsilon)$  must be smaller than 1.

To obtain  $\mathbf{V}(\zeta) \equiv \mathbf{V}_0(\zeta)$  from Eq. (13) by the accuracy of the second approximation on degeneracy, we get the following relation for the electrical current:

$$\begin{aligned} \mathbf{J} = \sigma_{11} \mathbf{E}' + \sigma_{12} [\mathbf{h} \cdot \mathbf{E}'] + \sigma_{13} \mathbf{h} \cdot [\mathbf{h} \cdot \mathbf{E}'] + \beta_{11}^{(e)} \nabla T_e \\ + \beta_{12}^{(e)} [\mathbf{h} \nabla T_e] + \beta_{13}^{(e)} \mathbf{h} \cdot [\mathbf{h} \nabla T_e] + \beta_{11}^{(ph)} \nabla T_{ph} \\ + \beta_{12}^{(ph)} [\mathbf{h} \nabla T_{ph}] + \beta_{13}^{(ph)} \mathbf{h} \cdot [\mathbf{h} \nabla T_{ph}], \quad (20) \end{aligned}$$

where

$$\sigma_{1i} = \int_0^\infty d\epsilon a(\epsilon) [1 + b(\epsilon) g_i(\epsilon)],$$

$$\beta_{1i}^{(e)} = \frac{1}{e} \int_0^\infty d\epsilon a(\epsilon) \left[ \frac{\epsilon - \zeta}{T_e} + g_i(\epsilon) d_0(\epsilon) \right],$$

$$\beta_{1i}^{(ph)} = \frac{A_{kt}}{e} \int_0^\infty d\epsilon a(\epsilon) \left\{ \left( \frac{\epsilon}{T} \right)^{(t-k)/2} + \left( \frac{\zeta_0}{T} \right)^{(t-k)/2} b(\epsilon) g_i(\epsilon) \right\} \quad (21)$$

and

$$a(\varepsilon) = \frac{2^{3/2} m^{1/2} e^2}{3 \pi^2 \hbar^3} \left( \frac{\Omega}{\nu(\varepsilon)} \right)^{i-1} \frac{\varepsilon^{3/2} \nu(\varepsilon)}{\Omega^2 + \nu^2(\varepsilon)} \left( - \frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \right),$$

$$d_0(\varepsilon) = \frac{\pi^2 T_e}{12 \zeta_0} b(\varepsilon), \quad b(\varepsilon) = \frac{\gamma(\varepsilon) \nu(\varepsilon)}{\Omega^2 + \tilde{\nu}^2(\varepsilon) (1 - \gamma_0)^2},$$

$$g_1(\varepsilon) = \tilde{\nu}(1 - \gamma_0) - \frac{\Omega^2}{\nu(\varepsilon)}, \quad g_2(\varepsilon) = \nu(\varepsilon) + \tilde{\nu}(1 - \gamma_0),$$

$$g_3(\varepsilon) = \nu(\varepsilon) \left[ 1 + \frac{\tilde{\nu}(1 - \gamma_0)}{\nu(\varepsilon)} + \frac{\Omega^2 + \nu^2(\varepsilon)}{\nu(\varepsilon) \tilde{\nu}(1 - \gamma_0)} \right],$$

$$\tilde{\nu} = \nu(\zeta_0, \Theta_{ph}), \quad \gamma_0 = \gamma(\zeta_0) \Theta_{ph}^{1-\ell}. \quad (22)$$

### III. ENERGY BALANCE EQUATIONS AND THEIR SOLUTIONS

To define the total thermopower and thermomagnetic effects as a function of  $E$ ,  $H$ , and  $\gamma_0$ , we must start from the energy balance equation obtained in Ref. 38. We consider two different cases.

(a) LW phonons are not heated and electrons transfer their energy gained from the external field to the reservoir of SW phonons, which has the equilibrium state at the lattice temperature  $T$ . Then, the energy balance equation has the form

$$\frac{(eE)^2 \nu_0 (1 - \gamma_0)}{\Omega^2 + \nu_0^2 (1 - \gamma_0)^2} = \frac{32^{2+t}}{(3+t) \pi^2 \hbar^3} m^3 s T p'_0 W_0 [\Theta_e - 1], \quad (23)$$

where  $\gamma_0 = \gamma(\zeta_0)$ .

(b) LW phonons are heated and they transfer their energy gained from electrons to the reservoir of SW phonons,

$$\frac{(eE)^2 \tilde{\nu}(1 - \gamma_0)}{\Omega^2 + \tilde{\nu}^2 (1 - \gamma_0)^2} = 6 m s p_0 \beta_{ph}(T) [\Theta_e - 1], \quad (24)$$

in this case  $\gamma_0 = \gamma(\zeta, \Theta_{ph})$ .

We consider now the dependences of  $\Theta_e$  on  $E$ ,  $H$ , and  $T$ , which are obtained by solving Eqs. (23) and (24). The solution of Eq. (23) for the arbitrary scattering mechanisms, degree of electron heating, and the ratio  $\Omega/\nu_0$  is

$$\Theta_e = 1 + \left( \frac{E}{E_i} \right)^2 \frac{(1 - \gamma_0)}{1 + (\nu_0/\Omega)^2 (1 - \gamma_0)^2}, \quad (25)$$

where  $E_i = (32^{2+t} m^3 s T p'_0 W_0 \Omega^2 / (3+t) \pi^2 \hbar^3 e^2 \nu_0)^{1/2}$ . We may consider  $\Theta_e$  in two limits:  $\Omega \gg \nu_0$  and  $\Omega \ll \nu_0$ . In the first limit

$$\Theta_e = 1 + \left( \frac{E}{E_i} \right)^2 (1 - \gamma_0), \quad (26)$$

and in the second limit

$$\Theta_e = 1 + \left( \frac{E}{E_j} \right)^2 (1 - \gamma_0)^{-1}, \quad (27)$$

where  $E_j = E_i \nu_0 / \Omega$ .

From Eq. (26), for different scattering mechanisms  $E_i$  take the forms

$$\text{for } r=1/2, r=-1/2: \quad E_1^2 = 3 \left( \frac{Hs}{c} \right)^2,$$

$$\text{for CI/DA:} \quad E_2^2 = \frac{9\pi}{2} \left( \frac{\varepsilon_0 G H}{c e^2} \right)^2 \frac{T n^{1/3}}{F \rho},$$

$$\text{for CI/PA:} \quad E_3^2 = 3 \left( \frac{4\pi \Sigma H}{c e} \right)^2 \frac{T}{F \rho n^{1/3}}.$$

Hereafter, CI/DA (CI/PA) means that energy of electrons is scattered by the deformation acoustical, (piezoacoustical) phonons, and momentum of electrons by the charged impurity ions (CI). Similarly, from Eq. (27),

$$\text{for DA:} \quad E_4^2 = 3 \left( \frac{m G^2 T n^{1/3}}{\rho e s \hbar^3} \right)^2,$$

$$\text{for PA:} \quad E_5^2 = \frac{1}{3} \left( \frac{32 \pi m^2 e \Sigma^2 T}{\varepsilon_0^2 \hbar^3 \rho s n^{1/3}} \right)^2,$$

$$\text{for CI/DA:} \quad E_6^2 = \frac{2}{3} \left( \frac{e m^2 G}{\varepsilon_0 \hbar^3} \right)^2 \frac{F T n^{1/3}}{\rho},$$

$$\text{for CI/PA:} \quad E_7^2 = \frac{1}{3} \left( \frac{8 e^2 m^2 \Sigma^2}{\varepsilon_0^2 \hbar^3} \right)^2 \frac{F T}{\rho n^{1/3}}.$$

In the case of phonon heating ( $T_{ph} = T_e$ ) if  $E \perp H$  and  $\Omega \gg \nu$ , then for DA and PA scattering mechanisms of electrons by phonons one finds  $\Theta_e$  as

$$\Theta_e = \left[ 1 - \left( \frac{E}{E_i} \right)^2 (1 - \gamma_0) \right]^{-1}, \quad (28)$$

where the characteristic fields  $E_i$  are

$$\text{for DA:} \quad E_8^2 = \frac{3}{2} \left( \frac{H T}{c s m G} \right)^2,$$

$$\text{for PA:} \quad E_9^2 = 3 \left( \frac{\varepsilon_0 H T^2 n^{1/3}}{8 c s m e \Sigma} \right)^2 \rho.$$

If the momentum of electrons is scattered from the impurity ions, regardless of the scattering of energy from either DA or PA phonons, we find

$$\Theta_e = \frac{1 + (E/E_{10})^2}{1 + \gamma_0 (E/E_{10})^2}, \quad E_{10}^2 = \frac{9\pi}{4} \left( \frac{\varepsilon_0 H T^2}{c s^2 m e^2} \right)^2 \frac{T n^{1/3}}{F \rho}. \quad (29)$$

As is seen from Eq. (29), under the condition of mutual drag the electron temperature is finite, i.e., because  $\gamma_0 (E/E_{10})^2 \gg 1$  and  $\Theta_e \leq 1/\gamma_0 = \text{const.}$

If  $E \perp H$  and  $\Omega \ll \nu$ , for DA and PA scattering mechanisms  $\Theta_e$  becomes

$$\Theta_e = \frac{1}{2} \left\{ 1 + \left[ 1 + 4 \left( \frac{E}{E_i} \right)^2 (1 - \gamma_0)^{-1} \right]^{1/2} \right\}, \quad (30)$$

where the characteristic fields  $E_i$  are

$$\text{for DA: } E_{11}^2 = \frac{3}{2} \left( \frac{mGT^3 n^{1/3}}{e s^3 \hbar^3 \rho} \right)^2,$$

$$\text{for PA: } E_{12}^2 = 3 \left( \frac{4m\Sigma T^3}{\varepsilon_0 s^3 \hbar^3 \rho} \right)^2.$$

For both CI/DA and CI/PA scattering mechanisms the critical field is the same, and  $\Theta_e$  is found to be as

$$\Theta_e = \frac{1}{2\gamma_0} \left\{ (1 + \gamma_0) - \left[ (1 - \gamma_0)^2 - 4\gamma_0 \left( \frac{E}{E_{13}} \right)^2 \right]^{1/2} \right\},$$

$$E_{13}^2 = \frac{1}{\pi} \left( \frac{emT^2}{\varepsilon_0 \hbar^3 s^2} \right)^2 \frac{FTn^{1/3}}{\rho}. \quad (31)$$

Finally, in the absence of mutual drag ( $\gamma_0 \rightarrow 0$ ), Eq. (31) gives

$$\Theta_e = \left( \frac{E}{E_{13}} \right)^2. \quad (32)$$

#### IV. THERMOPOWER IN LONGITUDINAL MAGNETIC FIELD

We will first consider the case  $\mathbf{E} \perp \mathbf{H} \parallel \nabla T_{e,ph} \parallel \hat{\mathbf{z}}$ . From the  $J_z = 0$  condition, we have

$$E_{Tz} + \frac{1}{e} \nabla_z \zeta(T_e) = \alpha_e \nabla_z T_e + \alpha_{ph} \nabla_z T_{ph},$$

$$\alpha_{e,ph} = \frac{\beta_{11}^{(e,ph)} + \beta_{13}^{(e,ph)}}{\sigma_{11} + \sigma_{13}}, \quad (33)$$

where  $\alpha_e$  and  $\alpha_{ph}$  are the electron and phonon parts of the differential thermopower, respectively. By taking into account the fact that  $\gamma(\varepsilon) = \gamma_0(\varepsilon/\zeta_0)^{t-k/2-r}$ , we find

$$\alpha_e = -\frac{1}{e} \frac{\pi^2}{6} \left[ 3 - 2r - \gamma_0 \left( \frac{5}{2} - r \right) \right] \frac{T}{\zeta_0} \Theta_e,$$

$$\alpha_{ph} = -\frac{1}{e} A_{kr} \left( \frac{\zeta_0}{T} \right)^{(t-k)/2}. \quad (34)$$

The thermopower is given by

$$V = \int_0^{L_z} dz (\alpha_e \nabla_z T_e + \alpha_{ph} \nabla_z T_{ph}) = V_e + V_{ph}, \quad (35)$$

where  $L_z$  is the size of the specimen in the  $z$  direction. As follows from Eqs. (34) and (35) in a weak longitudinal magnetic field ( $\Omega \ll \nu_0[1 - \gamma_0]$ ), in the absence of phonon heat-

ing the electronic part of the total thermopower  $V_e$  is proportional to  $E_0^4/(1 - \gamma_0)^2$ , and the phonon part  $V_{ph}$ , in general, does not depend on  $\gamma_0$ .

At high magnetic field ( $\Omega \gg \nu_0[1 - \gamma_0]$ ),  $V_e$  is proportional to  $(E_0/H)^4(1 - \gamma_0)^2$ , with  $E_0$  being the heating electric field intensity at the end of the specimen where electrons are highly heated. Therefore, with increasing  $\gamma_0$ , at weak magnetic field  $V_e$  grows as  $\sim (1 - \gamma_0)^{-2}$ , and at high magnetic field  $V_e$  decreases as  $\sim (1 - \gamma_0)^2$ .

In the case of strong heating of LW phonons and for the scattering of momentum and energy of electrons by acoustical phonons, at weak magnetic fields, from Eqs. (24), (34), and (35), we have

$$V_{ph} \sim \Theta_e \sim \frac{E_0}{1 - \gamma_0}, \quad V_e = \left( \frac{E_0}{1 - \gamma_0} \right)^2,$$

and at high magnetic fields,

$$V_{ph} \sim \left[ 1 - \frac{E_0^2}{E_{01}^2} (1 - \gamma_0) \right]^{-1}, \quad V_e \sim \left[ 1 - \frac{E_0^2}{E_{01}^2} (1 - \gamma_0) \right]^{-2},$$

$$E_0^2(1 - \gamma_0) < E_{01}^2 = \frac{6sp_0\beta_{ph}H^2}{(mc^2\nu_0)^2}.$$

In the calculation for the dependences of  $V_e$  and the NE voltage ( $U$ ) on  $E$  and  $H$  in the transverse magnetic fields, it is necessary to assume that  $\nabla T_{e,ph}$  is constant along the specimen; i.e., at one end of the specimen electrons are heated strongly by the electric field ( $\Theta_e \gg 1$ ), yet at the other end their temperature is  $T$ .

#### V. THERMOPOWER AND NERNST-ETTINGSHAUSEN EFFECT IN TRANSVERSE MAGNETIC FIELD

In general, the thermomagnetic effects are measured experimentally under the condition of  $\nabla_x T_{e,ph} = 0$ . We will direct the external fields  $\mathbf{E}$  and  $\mathbf{H}$  along the  $y$  axis and the temperature gradients along the  $z$  axis. Therefore, from Eq. (20) and the condition  $J_x = J_z = 0$ , for the transverse NE voltage, we obtain

$$E_{Tx} = -H(Q_e \nabla_z T_e + Q_{ph} \nabla_z T_{ph}),$$

$$Q_{e,ph} = \frac{1}{H} \frac{\sigma_{11}\beta_{12}^{(e,ph)} - \sigma_{12}\beta_{11}^{(e,ph)}}{\sigma_{11}^2 + \sigma_{12}^2}. \quad (36)$$

In this case the thermoelectric field  $E_{Tz}$  coincides with Eq. (33) by changing  $\alpha_{e,ph}$  as

$$\alpha_{e,ph} = -\frac{\sigma_{11}\beta_{11}^{(e,ph)} + \sigma_{12}\beta_{12}^{(e,ph)}}{\sigma_{11}^2 + \sigma_{12}^2}. \quad (37)$$

We would like to investigate Eqs. (35) and (36) by taking into account Eqs. (21) and (22) in the weak- and high-magnetic-field limits in the following subsections.

### A. Weak-magnetic-field case

If  $\tilde{\nu}^2 \gg \Omega^2$ , then for the electron part  $Q_e$  and phonon part  $Q_{ph}$  of the NE coefficients, we obtain

$$Q_e = -\frac{1}{e} \frac{\pi^2}{3} \frac{\mu_0}{c} \frac{T}{\zeta_0} \left\{ r + \gamma_0 \left( \frac{5}{4} - 2r \right) \right\} \Theta_e \Theta_{ph}^{-\ell},$$

$$Q_{ph} = -\frac{1}{e} \frac{\pi^2}{6} \frac{\mu_0}{c} (t-k) \left( \frac{T}{\zeta_0} \right)^{2+(k-t)/2}$$

$$\times \left\{ r + \gamma_0 \left( 1 - 2r + \frac{t-k}{4} \right) \right\} A_{kt} \Theta_e^2 \Theta_{ph}^{-\ell}, \quad (38)$$

where  $\mu_0 = e/mv_0$  is the mobility of ‘‘cold’’ electrons. As is seen from this equation, under the conditions of strong mutual drag for the parabolic spectrum of electrons ( $\beta_e \gg \beta_{ph}, \beta_{pb}$ , i.e.,  $\beta_e \approx \beta$ ), the phonon part of the NE coefficient  $Q_{ph} = 0$  or, more exactly,  $Q_{ph} \sim (\beta - \beta_e)/\beta = (\beta_{ph} + \beta_{pb})/\beta \ll 1$  (see also Refs. 20 and 21). Moreover, electrons and phonons form a system coupled by the mutual drag with common temperature  $T_e = T_{ph}$  and drift velocity  $v_e = v_{ph} = s$ . For this reason, there is only one thermomagnetic coefficient for the quasiparticle (electron dressed by phonon) coupled by the mutual drag. The quasiparticle has the electronic charge  $e$  and the mass of phonons  $M = T_e/s^2$  (see Refs. 39–42). However, since we assume the diffusion approximation,  $\gamma_0 < 1$  or  $u < s$ ,  $Q_{ph}$  is proportional to  $(\beta_{ph} + \beta_{pb})/\beta \neq 0$ . Only when  $\gamma_0 = 1$  or  $u = s$ , we have  $Q_{ph} = 0$ .

The expression of  $\alpha_{e,ph}$  at weak magnetic field coincides with Eq. (34). Therefore, here we give only the expressions denoting the change in  $\alpha_e$  and  $\alpha_{ph}$  in the weak magnetic field:

$$\Delta \alpha_e = -\frac{1}{e} \frac{\pi^2}{3} \left\{ \frac{7}{4} - 2r \right.$$

$$\left. + \left( \frac{1}{4} - r \right) \frac{\gamma_0(2 - \gamma_0)}{(1 - \gamma_0)^2} \right\} \frac{\Omega^2}{\nu_0^2} \frac{T}{\zeta_0} \Theta_e \Theta_{ph}^{-2\ell},$$

$$\Delta \alpha_{ph} = -\frac{1}{e} A_{kt} \left( \frac{\zeta_0}{T} \right)^{(t-k)/2} \frac{\Omega^2}{\nu_0^2 (1 - \gamma_0)^2} \Theta_{ph}^{-2\ell}. \quad (39)$$

In the case of scattering of electrons by deformation acoustic phonons ( $r = 1/2$ ) as  $\gamma_0 \rightarrow 1$ , the last term in square bracket in Eq. (39) is negative and much bigger than the other terms; hence,  $\Delta \alpha_e$  changes its sign. The NE voltage has the form

$$U = - \int_0^{L_x} dx H (Q_e \nabla_z T_e + Q_{ph} \nabla_z T_{ph}) = U_e + U_{ph}. \quad (40)$$

In the cases of the absence and presence of phonon heating, the energy balance equation in the transverse magnetic field has the form, respectively,

$$E^2 = E_{02}^2 (1 - \gamma_0) (\Theta_e - 1), \quad E_{02}^2 = \frac{32^{2+t} m^3 s \nu_0 p_0' T W_0}{(3+t) \pi \hbar^3 e^2}, \quad (41)$$

$$(eE)^2 = 6 \beta_{ph}(T) m s p_0 \tilde{\nu} (1 - \gamma_0) (\Theta_e - 1). \quad (42)$$

It follows from Eq. (38) that  $Q_e$  has two components. Then, by using Eqs. (38), (40), and (41) we obtain the first and the second components of the electron part of NE voltage as

$$U_e^I \sim (1 - \gamma_0)^{-2} E_0^4, \quad U_e^{II} \sim \gamma_0 (1 - \gamma_0)^{-2} E_0^4. \quad (43)$$

By analogy, we may obtain the phonon part as

$$U_{ph} \sim (1 - \gamma_0)^{-2} E_0^4. \quad (44)$$

It is interesting that in the absence of phonon heating both  $U_e$  and  $U_{ph}$  are proportional to  $H E_0^4$ ; i.e., they have the same dependence on the intensity of electric and magnetic fields for all scattering mechanisms of electrons in the case of strong electron heating,  $\Theta_e \gg 1$ . If the energy and momentum of electrons are transferred to phonons, in the case of strong electron and phonon heating, we have

$$U_e^I \sim E_0 H (1 - \gamma_0)^{-1/2}, \quad U_e^{II} \sim E_0 H \gamma_0 (1 - \gamma_0)^{1/2} \quad (45)$$

and

$$U_{ph}^I \sim E_0^2 H (1 - \gamma_0), \quad U_{ph}^{II} \sim E_0^2 H \gamma_0 (1 - \gamma_0)^{-1}. \quad (46)$$

Therefore, the mutual drag of electrons and phonons causes essential changes in the thermomagnetic behavior of semiconductors and semimetals. The strong phonon heating leads to an important contribution to these effects, because in this case  $\gamma_0 \sim \Theta_e$  for the scattering of momentum of electrons by impurity ions and energy by LW phonons (the thermal drag case).

### B. High-magnetic-field case

In the limit  $\Omega^2 \gg \tilde{\nu}^2$ , the thermomagnetic coefficients take the forms

$$\alpha_e = -\frac{1}{e} \frac{\pi^2}{2} \frac{T}{\zeta_0} \Theta_e; \quad (47)$$

$$Q_e = -\frac{1}{e} \frac{\pi^2}{3} \left( r + \frac{5}{4} \gamma_0 \right) \frac{c}{H^2 \mu_0} \frac{T}{\zeta_0} \Theta_e \Theta_{ph}^\ell,$$

$$Q_{ph} = -\frac{1}{e} \frac{\pi^2}{6} (t-k) \left\{ r + \gamma_0 \left( 1 + \frac{t-k}{4} \right) \right\}$$

$$\times \frac{c A_{kt}}{H^2 \mu_0} \left( \frac{T}{\zeta_0} \right)^{2+(k-t)/2} \Theta_e^2 \Theta_{ph}^\ell. \quad (48)$$

As it is seen from Eq. (48), for the case of weak mutual drag, both the electronic and phonon parts of the transverse NE coefficients change their sign for the scattering of electrons by the piezoacoustical phonons ( $r = -1/2$ ). Moreover, the

phonon part of NE coefficient changes its sign if LW phonons are scattered by SW phonons ( $k=1$ ), and electrons are scattered by piezo acoustical phonons ( $t=-1$ ).

For the case under consideration, the expression for  $\alpha_{ph}$  coincides with Eq. (34). As follows from Eqs. (38) and (48), at weak and high magnetic fields  $Q_{ph}=0$  for the scattering of LW phonons by electrons ( $k=t$ ) and by SW phonons ( $t=1$ ). From Eqs. (40)–(42) and (48), in the absence of phonon heating, we obtain

$$U_{e,ph}^I \sim H^{-1} E_0^4 (1 - \gamma_0)^{-2}, \quad U_{e,ph}^{II} \sim H^{-1} E_0^4 \gamma_0 (1 - \gamma_0)^{-2}. \quad (49)$$

In the case of strong phonon heating, for the scattering of energy and momentum of electrons by phonons, we find

$$U_e \sim H^{-1} E_0^3 (1 - \gamma_0)^{-3/2}, \quad U_{ph} \sim H^{-1} E_0^4 (1 - \gamma_0)^{-2}. \quad (50)$$

## VI. CONCLUSION

In the present work, we have shown that at weak longitudinal magnetic fields in the absence of phonon heating, the electron part of thermoelectric power  $V_e$  increases with increasing  $E_0$  and the degree of mutual drag of electrons and phonons  $\gamma_0$ . Nevertheless, the phonon part  $V_{ph}$  does not depend on  $\gamma_0$ . At longitudinal high magnetic fields,  $V_e$  in-

creases with increasing  $E_0$  and decreases with increasing  $H$  and  $\gamma_0$ . In the case of strong phonon heating, if the momentum and energy of electrons are transferred to acoustical phonons at weak magnetic fields,  $V_e$  and  $V_{ph}$  grow as  $E_0$  and  $\gamma_0$  increase. It has been shown that at high magnetic field for a given  $\gamma_0 < 1$ ,  $V_e$  and  $V_{ph}$  grow as  $H$  increases.

In a weak transverse magnetic field,  $V_e$  and  $V_{ph}$  are exactly the same as in the case of longitudinal magnetic field, and in the absence of phonon heating both the electron and phonon parts of the transverse NE voltage  $U_e$  and  $U_{ph}$  are proportional to  $HE_0^4$ . In the case of strong electron and phonon heating both  $U_e$  and  $U_{ph}$  grow as  $E$ ,  $H$ , and  $\gamma_0$  increase. At high magnetic field in the absence of phonon heating,  $U_e$  and  $U_{ph}$  grow with increasing  $E_0$  and  $\gamma_0$ , and decrease linearly with increasing  $H$ . It has also been shown that in weak and high transverse magnetic fields, both the electronic and phonon parts of the NE coefficients change their sign for some scattering mechanisms.

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