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Electron spin relaxation in semiconductors and semiconductor structures

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We suggest an approach to the problem of a free electron spin evolution in a semiconductor with arbitrary anisotropy or quantum structure in a magnetic field. The developed approach utilizes quantum kinetic equations for average spin components. These equations represent the relaxation in terms of correlation functions for fluctuating effective fields responsible for spin relaxation. In a particular case when autocorrelation functions are dominant, the kinetic equations reduce to the Bloch equations. The developed formalism is applied to the problem of electron spin relaxation due to exchange scattering in a semimagnetic quantum well (QW) as well as to the spin relaxation in a QW due to the Dyakonov-Perel mechanism. The results permit us to separate the longitudinal T_1 and transversal T_2 relaxation times in a strong enough magnetic field and to trace the cases of undistinguished parameters T_1 and T_2 in zero and small magnetic fields. Some new predictions of the developed theory are discussed. Namely, we discuss the nonmonotonic behavior of spin relaxation caused by exchange scattering under an external magnetic field and new peculiarities of electron spin evolution caused by the presence of three relaxation times (rather than two) for the Dyakonov-Perel mechanism in a quantum well.

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I. INTRODUCTION

The achievements in the investigations of ferromagnetism of diluted magnetic semiconductors[1] (DMS's) and in spin injection technology[2] with the possibility to fabricate spin-controlled devices have sparked renewed interest in the study of the spin relaxation of electrons, holes, and excitons (see Ref. 3 and references therein). During recent years, the optical picosecond technique has acquired the ability to observe the electron and hole spin kinetics in semiconductor quantum structures over a wide range of temperatures and magnetic fields.[4,5] In spite of intensive work spin relaxation processes are still not fully understood in these structures. One of the reasons is the deficiencies in the present theories. If, for example, the theory of the giant spin splitting of the electron states in DMS's has a good base in terms of the molecular exchange field,[6] spin relaxation theory has, by now, some gaps between phenomenological and microscopic approaches to this problem. The exception is the precession mechanism of relaxation[7] that cannot be readily confronted with other mechanisms or applied to the case of localized electrons.

The phenomenological description of carrier or exciton spin relaxation in semiconductors commonly starts from the Hamiltonian H_d without the dissipation terms responsible for spin relaxation. For a particle with spin S (electron, exciton, etc.), the number of equations for average spin values $A_I=\langle S_X^k S_Y^l S_Z^m\rangle$ (subscripts denote Cartesian components; k,l,m satisfy the condition $1\leqslant k+l+m\leqslant 2S$; the symbol I stands for all indexes on the right-hand side of A_I) is finite and is determined by the equations of motion ($\hbar=1$)

$$\frac{dA_I}{dt}=-i[A_I,H_d]. \tag{1}$$

The system (1) describes the evolution of the spin system without relaxation. To take it into consideration we should include the relaxation part R_I to Eq. (1). Generally speaking, the relaxation of each A_I to the thermal equilibrium value A_I^0 might depend on the full set of spin averages. Therefore the relaxation part reads

$$R_I=-\sum_{I'}\frac{(A_{I'}-A_{I'}^0)}{\tau_{I,I'}}. \tag{2}$$

Sometimes, the number of relaxation parameters $\tau_{I,I'}^{-1}$ can be reduced due to the presence of specific physical mechanisms (see, for example, Ref. 8). Nevertheless, it should be mentioned that a general procedure (for the estimation of the magnitudes of all relaxation parameters) allowing us to reduce the numbers of relaxation terms (2) is absent.

Another problem arises when one tries to relate the parameters $\tau_{I,J}^{-1}$ to the spin-flip rate W (this is the aim of the majority of microscopic calculations based on different known relaxation mechanisms). To clarify this problem we consider Bloch equations for the average spin components S_μ in a magnetic field directed along the OZ axis:

$$\frac{d}{dt}\begin{pmatrix} S_X\\ S_Y\\ S_Z\end{pmatrix}=\omega\begin{pmatrix} -S_Y\\ S_X\\ 0\end{pmatrix}-\begin{pmatrix} S_X/T_2\\ S_Y/T_2\\ (S_Z-S_Z^0)/T_1\end{pmatrix}. \tag{3}$$

Equation (3) looks like Eqs. (1) and (2), where S_Z^0 is a thermal equilibrium value of S_Z and ω is Zeeman splitting in a magnetic field. Equation (3) involves two relaxation times, the longitudinal T_1 and transversal T_2. In the framework of the phenomenological equation (3) it is not apparent how the parameters T_1 and T_2 having different physical meanings are related to the spin-flip probability W. Moreover, the transformation from the case $T_1=T_2$ at zero magnetic field ($\omega=0$) to the case $T_1\neq T_2$ at $\omega\neq 0$ cannot be traced both with the help of Eq. (3) and by microscopic calculations of the spin-flip rate W.

The present work is an attempt to fill the gap between the phenomenological approach to spin relaxation and micro-

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scopic calculations of the spin-flip rates in the case of electron relaxation with spin 1/2 in a semiconductor. This means that here we develop a universal procedure for the calculation of microscopic parameters $\tau_{I,I'}$ [Eq. (2)] that can be applied for any relaxation mechanism both for localized and for delocalized electrons. To do that, we derive the kinetic equations for electron spin components from first principles. First, we derive the quantum kinetic equations for the density matrix of spin 1/2 in terms of correlation functions of the dissipative subsystem. Despite the presence of some symmetry of correlation parameters, the kinetic equations can be only reduced to the form that remains, nevertheless, more complicated than Bloch equations. The next section presents an analysis of the corresponding correlation functions in terms of the relaxation time approximation. In subsequent sections the developed theory is applied to electron exchange scattering on the magnetic ions in a semimagnetic quantum well (QW). Then, the Dyakonov-Perel (DP) mechanism is considered. New peculiarities related to the magnetic anisotropy and modification of relaxation time magnitudes in a magnetic field are discussed. We discuss also possible applications of the results obtained here.

II. BASIC EQUATION FOR THE DENSITY MATRIX

There are several approaches to the derivation of kinetic equation for small subsystems (like spin) interacting with a large heat bath. One of them, the projection operator formalism (Ref. 9), had first been considered in Refs. 10 and 11 which adjusted the general Zwanzig's approach to magnetic relaxation of the localized electrons with an arbitrary discrete energy spectrum. Meanwhile, some important limitations (namely, the requirement of nondegeneracy of the spin energy spectrum and rather large spin level splitting) do not allow us to apply these theories to the above situations. The present work is free of the aforementioned limitations since it is focused on the evolution equation of the two-level system (or spin 1/2) only rather than on the complete problem of magnetic relaxation for a multilevel system.

Let the electron spins represent a *small part* of the total system so that all other variables are related to the bath at thermal equilibrium. The corresponding Hamiltonian reads

$$H=H_S^0+H_L+H_{SL}. \tag{4}$$

Here H_S^0, H_L, and H_{SL} are, respectively, the Hamiltonians of electron spin, dissipative subsystem (assumed to be at thermal equilibrium with a bath at a temperature T), and their interaction. According to the projection operator method[9] the density matrix of the system (4) is expressed in terms of the main part and the rest:

$$\rho=f\sigma+\eta, \tag{5}$$

where

$$\sigma=\mathrm{Tr}_L\rho,$$

$$f=\frac{\exp(-\beta H_L)}{\mathrm{Tr}_L\exp(-\beta H_L)},\qquad \beta=\frac{1}{T},$$

$$\eta=P\rho\equiv(1-f\,\mathrm{Tr}_L)\rho.$$

We are looking for the kinetic equation for the reduced density matrix σ with the Hamiltonian

$$H=H_S+H_L+V, \tag{6}$$

where

$$H_S=H_S^0+\langle H_{SL}\rangle,$$

$$V=H_{SL}-\langle H_{SL}\rangle, \tag{7}$$

so that $\langle V\rangle=0$ and $\langle\cdots\rangle=\mathrm{Tr}_L f\cdots$.

It can be shown[11] that exact form of the equation for σ is

$$\frac{d\sigma}{dt}=-i\mathcal{H}_S\sigma-\mathcal{C}(\sigma)-i\mathcal{D}(\rho(0)),$$

with

$$\mathcal{C}(\sigma)=\mathrm{Tr}_L\mathcal{V}\int_0^t\mathcal{S}(t,t')\mathcal{V}f\sigma(t')dt',$$

$$\mathcal{D}(\rho(0))=\mathrm{Tr}_L\mathcal{V}\mathcal{S}(t,0)P\rho(0),$$

$$\mathcal{S}(t,t')=\exp\left\{-i\int_{t'}^t(\mathcal{H}_S(\tau)+\mathcal{H}_L+P\mathcal{V})d\tau\right\}. \tag{8}$$

Here the script letters mean the Liouville operators introduced as $\mathcal{H}_S\sigma=[H_S,\sigma]$, etc. The term $\mathcal{D}(\rho(0))$ vanishes for the initial condition $\rho(0)=f\sigma(0)$.

The expansion of Eqs. (8) up to second order in the operator of interaction yields the kinetic equation for σ:

$$\frac{d\sigma(t)}{dt}=-i\mathcal{H}_S\sigma-\mathrm{Tr}_L\int_0^t\mathcal{V}(t,t)\mathcal{V}(t,t')f\sigma(t)dt', \tag{9}$$

where

$$V(t,t')=\exp\left\{-i\int_{t'}^t H_S(\tau)d\tau\right\}V(t)\exp\left\{i\int_{t'}^t H_S(\tau)d\tau\right\}, \tag{10}$$

$$V(t)=\exp\{iH_Lt\}V\exp\{-iH_Lt\}. \tag{11}$$

We consider the reduced density matrix $\sigma(t)$ at the long enough time scale (as compared to the times of attainment of a thermal equilibrium of dissipative subsystem). Thus, the upper limit of integration in Eq. (9) can be extended to infinity. Under this assumption we obtain a Markovian equation for $\sigma(t)$, provided that the spin relaxation times T_1, T_2 are sufficiently longer than the correlation times τ of the dissipative system responsible for spin relaxation.

To rewrite Eq. (9) in matrix form, we should use some base. As the base, here we use the eigenvectors of the spin operator s_Z. For $s=1/2$, the most general form of interaction is[12]

$$V=\mathbf{s}\boldsymbol{\Omega}, \tag{12}$$

while electron spin Hamiltonian can be written as

$$H_S = \omega s_Z, \tag{13}$$

where ω is the spin splitting in a magnetic field having regard to renormalization (7) and OZ is an axis of quantization. The operator $\mathbf{\Omega}$ represents the fluctuating effective magnetic field giving rise to spin-flip processes. The interaction in the form of Eq. (12) has been already used in Ref. 12 [see Eq. (3.35) therein] for a description of the DP mechanism.

Substitution of Eq. (12) for Eq. (9) (at $t\to\infty$) permits us to express the collision integral in Eq. (9) in terms of the Fourier transformation of correlation functions. Their number can be reduced if we take into account the spectral properties of correlation functions and the following identities (valid for any operators A and B):

$$\langle A(\tau)B\rangle_\omega = e^{-\beta\omega}\langle B(\tau)A\rangle_{-\omega} = e^{-\beta\omega}\langle BA(\tau)\rangle_\omega, \tag{14}$$

where the definitions of $A(t)$ and $B(t)$ are similar to Eq. (11). The definition of the Fourier transformation of the correlation function is

$$\langle A(\tau)B\rangle_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\langle A(\tau)B\rangle e^{i\omega\tau}d\tau. \tag{15}$$

It is convenient to introduce the average values of spin components instead of the density matrix σ:

$$\langle s_X\rangle \equiv \mathbf{X} = \mathrm{Tr}\,\sigma s_X,$$

$$\langle s_Y\rangle \equiv \mathbf{Y} = \mathrm{Tr}\,\sigma s_Y,$$

$$\langle s_Z\rangle \equiv \mathbf{Z} = \mathrm{Tr}\,\sigma s_Z. \tag{16}$$

Multiplying from the right Eq. (9) by the operator s_X, then by s_Y and by s_Z, and taking into account the explicit form of the interaction operator for spin 1/2 [Eq. (12)], one can calculate the corresponding traces over the spin states. The result can also be expressed via $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$, which yields the following closed system of equations for components $\langle \mathbf{s}\rangle$:

$$\frac{d}{dt}\begin{pmatrix}\mathbf{X}\\ \mathbf{Y}\\ \mathbf{Z}\end{pmatrix} = \omega\begin{pmatrix}-\mathbf{Y}\\ \mathbf{X}\\ 0\end{pmatrix} - \mathbf{\Gamma}\begin{pmatrix}\mathbf{X}\\ \mathbf{Y}\\ \mathbf{Z}-\mathbf{Z}_0\end{pmatrix}. \tag{17}$$

Here the relaxation matrix $\mathbf{\Gamma} = \|\Gamma_{\mu\nu}\|$ can be expressed via the correlation functions

$$\gamma_{\mu\nu}(\omega) = \langle \Omega_\mu(\tau)\Omega_\nu\rangle_\omega, \tag{18}$$

where $\mu,\nu = X,Y,Z$ are Cartesian indexes. No additional assumptions about the energy spectrum of the dynamic subsystem have been made here. This distinguishes substantially our equation (17) from those obtained earlier (Refs. 10 and 11). We recall here that this result appears due to the reduction of the general problem of relaxation of the system with an arbitrary discrete spectrum to the special case of spin-1/2 relaxation. Thus, Eq. (17) describes the relaxation of spin 1/2 to its equilibrium state $\mathbf{X}(t)\overset{t\to\infty}{\longrightarrow}\mathbf{X}_0$, $\mathbf{Y}(t)\overset{t\to\infty}{\longrightarrow}\mathbf{Y}_0$, and $\mathbf{Z}(t)\overset{t\to\infty}{\longrightarrow}\mathbf{Z}_0$, with $\mathbf{X}_0 = \mathbf{Y}_0 = 0$ and

$$\mathbf{Z}_0 = -\frac{1}{2}\tanh\frac{\beta\omega}{2}. \tag{19}$$

The general form of the relaxation matrix $\mathbf{\Gamma} = \mathbf{\Gamma}(\gamma_{\mu\nu})$ is quite complex. However, the symmetry properties of the correlation functions $\gamma_{\mu\nu}$ allow us to simplify significantly the expression for $\mathbf{\Gamma}(\gamma_{\mu\nu})$. First, we consider the case of symmetric correlation functions (18), $\gamma_{\mu\nu}(\omega) = \gamma_{\nu\mu}(\omega)$. The symmetry properties of the correlation function permit us to obtain

$$\mathbf{\Gamma}^{(s)} = \pi\begin{pmatrix}\gamma_{zz}^0 + n\gamma_{yy} & -n\gamma_{xy} & -n\gamma_{xz}\\ -n\gamma_{xy} & \gamma_{zz}^0 + n\gamma_{xx} & -n\gamma_{yz}\\ -n\gamma_{xz} & -n\gamma_{yz} & n(\gamma_{xx}+\gamma_{yy})\end{pmatrix}, \tag{20}$$

where $n \equiv n(\omega) = (1+e^{\beta\omega})/2$, $\gamma_{\mu\nu} \equiv \gamma_{\mu\nu}(\omega)$, and $\gamma_{\mu\nu}^0 \equiv \gamma_{\mu\nu}(0)$. Notice that Eqs. (17) and (20) are, generally speaking, derived for the case of an anisotropic medium and arbitrary electron spin splitting ω.

In the case of zero spin splitting ($\omega = 0$) and isotropic medium, the relaxation matrix $\mathbf{\Gamma}$, Eq. (20), depends on two components only, $\gamma_d^0 = \gamma_{\mu\mu}^0$ and $\gamma_n^0 = \gamma_{\mu\nu}^0$ ($\mu\neq\nu$). In this case, Eq. (20) is similar to that obtained in Ref. 12 in terms of a fluctuating internal magnetic field.

In the case of strong enough spin splitting ω, one can expect that $\gamma_{\mu\nu}(\omega) \ll \gamma_{zz}(0)$, because the $\gamma_{\mu\nu}(\omega)$ is damped with a factor of about inverse correlation time. If we introduce $T_1^{-1} = \pi n[\gamma_{xx}(\omega)+\gamma_{yy}(\omega)]$ and $T_2^{-1} = \pi\gamma_{zz}(0)$ and omit the off-diagonal components of Γ, the latter inequality permits us to reduce Eq. (17) to the form of a Bloch equation (3).

The second important case corresponds to the antisymmetric coefficients $\gamma_{\mu\nu}(\omega) = -\gamma_{\nu\mu}(\omega)$ for $\nu\neq\mu$. In this case we can obtain the relaxation matrix in the form of the sum of two parts:

$$\mathbf{\Gamma}^{(a)} = \mathbf{\Gamma}^{(1)} + \mathbf{\Gamma}^{(2)}, \tag{21}$$

where $\mathbf{\Gamma}^{(1)} = \|\mathbf{\Gamma}_{\mu,\nu}^{(s)}\delta_{\mu,\nu}\|$ is the diagonal part of the matrix $\mathbf{\Gamma}^{(s)}$, while

$$\mathbf{\Gamma}^{(2)} = i\pi n\begin{pmatrix}\gamma_{xy} & 0 & \gamma_{yz}\\ 0 & \gamma_{xy} & \gamma_{zx}\\ 0 & 0 & \gamma_{xy}\end{pmatrix}. \tag{22}$$

In the limit of zero magnetic field, the coefficients $\gamma_{\mu\nu}(\omega\to 0)$ vanish at $\nu\neq\mu$; thus Eq. (21) transforms into matrix (20) with zero off-diagonal components. The spectral properties (14) with respect to the definition of $n(\omega)$ permit us to observe that both the relaxation matrix (21) and matrix (20) are even functions of the electron spin splitting ω: $\mathbf{\Gamma}(\omega) = \mathbf{\Gamma}(-\omega)$.[13]

Equations (17) with definitions (20) and (21) are the main result of the theoretical background of this work. In the derivation of kinetic equations (17) we assume neither any specific properties of the heat bath with Hamiltonian H_L. Nor do we assume any specific interaction mechanism H_{SL}. Below we shall show how Eq. (17) can be used in the particular cases corresponding to Eqs. (20) and (21).

III. CORRELATION TIME APPROXIMATION

The technique of Green's functions,[14] being a very powerful and general tool for the analysis of correlation functions in solids, sometimes lacks a clear physical meaning. At the same time, the correlation functions can be evaluated pretty good within the exponential decay approximation in terms of the appropriate correlation times τ_p.

To link Eq. (17) with real physical cases, we first consider the electron spin-flip scattering by quasiparticles with quantum number q and continuous energy spectrum ε_q. The corresponding Hamiltonian (in terms of creation $a_q^\dagger$ and annihilation a_q operators) reads $H_L=\Sigma_q \varepsilon_q a_q^\dagger a_q + H_{int}$, where the operator H_{int} describes quasiparticle scattering. The expansion of interaction H_{SL} over quasiparticle variables gives the following form of the effective magnetic field:

$$\Omega_\mu = \sum_p V_\mu^p Q_p, \tag{23}$$

where Q_p is a combination of operators $a_q^\dagger, a_{q'}, \ldots,$ while p is a set of indexes $(q,q',\ldots)$; the matrix elements V_μ^p depend on the specific mechanism of spin relaxation.

First of all, let us consider the case $H_{int}\rightarrow 0$. This means that $\Omega_\mu(t)=\Sigma_p V_\mu^p Q_p \exp(i\omega_p t)$ and

$$\langle \Omega_\mu(t)\Omega_\nu\rangle = \sum_{p_1,p_2} V_\mu^{p_1} V_\nu^{p_2} \langle Q_{p_1} Q_{p_2}\rangle \exp(i\omega_p t), \tag{24}$$

where ω_p is an energy difference between the states linked by operator Q_p; for instance, $\omega_p=\varepsilon_q-\varepsilon_{q'}$, if $Q_p=a_q^\dagger a_{q'}$. In the approximation (24), we find immediately

$$\gamma_{\mu\nu}(\omega) = \sum_{p_1,p_2} V_\mu^{p_1} V_\nu^{p_2} \langle Q_{p_1} Q_{p_2}\rangle \delta(\omega_{p_1}+\omega). \tag{25}$$

Here the δ function provides the conservation of energy in relaxation processes. The term $n[\gamma_{xx}(\omega)+\gamma_{yy}(\omega)]$ in Eq. (20) with respect to Eq. (25) corresponds just to the Fermi golden rule for the electron spin transition probability.

To account for the influence of quasiparticle diffusion on spin relaxation we assume that the approximation of the correlation time τ_p holds. Mathematically this means that the correlation function $\langle Q_{p_1}(t) Q_{p_2}\rangle$ acquires the damping factor $\exp(-|t|/\tau_{p_1})$, which leads to the spectral dependence of Eq. (18) in the form

$$\gamma_{\mu\nu}(\omega) = \sum_{p_1,p_2} V_\mu^{p_1} V_\nu^{p_2} \langle Q_{p_1} Q_{p_2}\rangle \frac{1}{\pi} \frac{\tau_{p_1}^{-1}}{(\omega_{p_1}+\omega)^2+\tau_{p_1}^{-2}}. \tag{26}$$

Two typical cases should be distinguished in the subsequent consideration. First, let the inequality $\langle(\omega_p+\omega)^2\rangle \gg \langle \tau_p^{-2}\rangle$ take place. This means that the summation in the Eq. (26) can be performed in the limit $\tau_{p_1}^{-1}\rightarrow 0$, which reduces Eq. (26) to Eq. (25). This situation frequently occurs for different cases of electron spin-flip scattering, if the average electron energy $\langle \varepsilon_q\rangle$ (which is about thermal energy T for the nondegenerate case or Fermi energy ε_F for the degenerate case) exceeds its relaxation rate $\langle \tau_p^{-1}\rangle$.

Second, let the identity $\omega_p\equiv 0$ hold. In terms of the effective fluctuating magnetic field $\mathbf{\Omega}(t)$, this means that $\mathbf{\Omega}(t)$ is constant between two successive electron collisions so that the electron spin loses its initial orientation due to precession around the $\mathbf{\Omega}$ direction. The corresponding operator (23) contains terms with $Q_p=a_q^\dagger a_q$ and $\omega_p\equiv 0$. So Eq. (26) assumes the form

$$\gamma_{\mu\nu}(\omega) = \sum_{p_1,p_2} V_\mu^{p_1} V_\nu^{p_2} \langle Q_{p_1} Q_{p_2}\rangle \frac{1}{\pi} \frac{\tau_{p_1}}{(\tau_{p_1}\omega)^2+1}, \tag{27}$$

which cannot be reduced to integration with a δ function. The latter situation corresponds to a precession mechanism of spin relaxation (an example to be considered in Sec. V).

Comparison of Eqs. (27) and (25) suggests that spin-flip scattering and precession mechanisms of electron spin relaxation can be recognized by the spectral properties of the fluctuating magnetic field. Namely, spin-flip scattering relaxation has $\mathbf{\Omega}(t)$ with a spectral density of correlation function (25) proportional to the density of states of the dissipative subsystem (white noise). The precession mechanism is provoked by magnetic fields $\mathbf{\Omega}(t)$, which fluctuate like an exponential noise.

Physically, in the former case the time dependence of $\mathbf{\Omega}(t)$ can be easily understood as a series of δ functions with arguments corresponding to instants t_i of spin-dependent electron collisions with mean time lapse $\tau_{sf}=\langle t_{i+1}-t_i\rangle$: $\mathbf{\Omega}(t)=\Sigma_i \Delta\boldsymbol{\phi}_i \delta(t-t_i)$, where $\Delta\boldsymbol{\phi}_i$ are the angles of spin rotations accompanying spin-dependent collisions. Considering the relaxation as a Brownian motion of angle $\phi(t)$ between current and initial (at $t=0$) spin directions with diffusion coefficient $D=\langle \Delta\boldsymbol{\phi}_i^2\rangle/\tau_{sf}$, one can find the loss of spin polarization $\mathbf{Z}=\frac{1}{2}\langle \cos\phi\rangle$ by direct integration of $\cos\phi$ over the diffusion distribution $f(\phi,t)=(2\pi Dt)^{-1/2}\exp(-\phi^2/2Dt)$: $\int \cos\phi f(\phi,t)d\phi = e^{-Dt/2}\equiv e^{-t/T_s}$. Thus, the spin relaxation rate is expected to be $T_s^{-1}=\frac{1}{2}\langle \Delta\phi_i^2\rangle \tau_{sf}^{-1}$. In particular, the considered pattern of spin relaxation corresponds to the Elliott-Yafet mechanism[15,16] when some spin-flip probability $(\sim\langle \Delta\boldsymbol{\phi}_i^2\rangle)$ accompanies each electron momentum diffusion with relaxation time τ_p (which defines the parameter $\tau_{sf}^{-1}=\tau_p^{-1}$).

In the latter case, the $\mathbf{\Omega}(t)$ may be imagined as a series of flat steps over intervals between electron collisions, $\mathbf{\Omega}(t)=\Sigma_j \mathbf{\Omega}_j[\theta(t-t_j)-\theta(t-t_{j+1})]$; the sum is taken over successive collisions with electron momentum diffusion. The spin rotation during the interval $t_{j+1}-t_j$, when $\mathbf{\Omega}(t)=\mathbf{\Omega}_j$ is a constant, can be described as a precession of a free spin $\Delta\boldsymbol{\phi}_j=\mathbf{\Omega}_j(t_{j+1}-t_j)$, which defines the coefficient of spin diffusion $D=\langle \Delta\boldsymbol{\phi}_i^2\rangle/\tau_p=\langle \mathbf{\Omega}_i^2\rangle \tau_p$, where $\tau_p=\langle t_{j+1}-t_j\rangle$.

Thus, the spin relaxation rate $T_s^{-1}=\frac{1}{2}\langle\boldsymbol{\Omega}_i^2\rangle\tau_p$ is proportional to the momentum relaxation time. This result is a distinctive feature of the DP mechanism.

Note that both the above cases had been considered earlier separately (see Ref. 12 and references therein). Here we consider them in the framework of a unified approach. We should also mention that a more advanced theory based on the microscopic calculation of correlation functions (18) (in terms of Green's functions,[14] for example) provides a possibility to describe the spin relaxation beyond the approximation of the correlation time τ_p. Below, to discuss the effects of a magnetic field and electron energy spectrum anisotropy in QW's, we consider electron correlation functions in the approximation of the relaxation time τ_p.

IV. EXCHANGE ELECTRON SCATTERING ON MAGNETIC IONS IN QW'S

Consider the problem of electron spin exchange scattering on the magnetic ions in a two-dimensional (2D) quantum well (see Refs. 17 and 18). The carrier-ion interaction operator has the form of the following contact interaction:

$$H_{SL}=-\alpha\sum_j \mathbf{S}^j\mathbf{s}\delta(\mathbf{r}-\mathbf{R}_j), \tag{28}$$

where α is an exchange integral, $\mathbf{S}^j$ is the magnetic ion spin at the site $\mathbf{R}_j$, and $\mathbf{s}$ and $\mathbf{r}$ are electron spin and coordinate, respectively. The summation is performed over all magnetic impurities. The Hamiltonian of the dissipative subsystem includes the Zeeman energy of magnetic ions and the electron kinetic energy $\varepsilon_{\mathbf{k}}$ with in-plane wave vector $\mathbf{k}$. For simplicity, we restrict our consideration to the ground state of confined electrons only. So the Hamiltonian assumes the form

$$H_L=\sum_j \omega_0 S_Z^j+\sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}}a_{\mathbf{k},\sigma}^{\dagger}a_{\mathbf{k},\sigma}, \tag{29}$$

where ω_0 is the magnetic ion Zeeman splitting in a magnetic field directed along the OZ axis, $a_{\mathbf{k},\sigma}^{\dagger}$ and $a_{\mathbf{k},\sigma}$ are the creation and annihilation operators, and the spin index σ is introduced to normalize the chemical potential for the total numbers of electrons. In our case,

$$\sum_{\mathbf{k},\sigma}\langle a_{\mathbf{k},\sigma}^{\dagger}a_{\mathbf{k},\sigma}\rangle\equiv\sum_{\mathbf{k},\sigma} f_{\mathbf{k},\sigma}=1. \tag{30}$$

Now we can perform the renormalization procedure (7) with operator (28) and represent the interactions H_{SL}, Eq. (12), and V, Eq. (7), in second-quantized form with

$$\Omega_\mu=-\frac{\alpha}{S_0}\sum_{\substack{\mathbf{k},\mathbf{k}',\sigma\\ \mathbf{k}\neq\mathbf{k}'}}\sum_j |\psi_\perp(\mathbf{R}_j)|^2 e^{i(\mathbf{k}-\mathbf{k}')\mathbf{R}_j}Q_{\mu,\mathbf{k},\mathbf{k}',\sigma},$$

$$Q_{\mu,\mathbf{k},\mathbf{k}',\sigma}=S_\mu^j a_{\mathbf{k},\sigma}^{\dagger}a_{\mathbf{k}',\sigma},\ \mu=X,Y,Z. \tag{31}$$

Here S_0 is the area of a sample and $\psi_\perp(\mathbf{R}_j)$ is a perpendicular (to the plane of a structure) component of the confinement wave function. The term with $\mathbf{k}=\mathbf{k}'$ is excluded from Eq. (31) due to the renormalization procedure (7). This procedure yields the following Hamiltonian of the dynamic subsystem:

$$H_S\equiv\omega s_Z=(\omega_e+G_{eZ})s_Z. \tag{32}$$

Equation (32) defines the electron spin splitting ω in a total field, consisting of an external magnetic field (with Zeeman energy ω_e) and exchange molecular field:

$$\mathbf{G}_e=-\frac{\alpha}{S_0}\sum_j\langle\mathbf{S}^j\rangle|\psi(z_j')|^2$$

$$=-\alpha\langle\mathbf{S}\rangle\int_{-\infty}^{\infty}n_m(z')|\psi(z')|^2dz'. \tag{33}$$

Here $n_m(z')$ is a local concentration of magnetic ions assumed to be the function of coordinate z' directed along the growth axis OZ'. In the case of magnetic QW's with width L_w and nonmagnetic barriers, the integration should be performed between $-L_w/2$ and $L_w/2$, while $n_m(z)$ should be replaced by the average concentration n_m in the QW. Moreover, in the case of bulk samples or infinitely high barriers Eq. (33) reproduces the well-known result for an exchange molecular field, $\mathbf{G}_e=-\alpha n_m\langle\mathbf{S}\rangle$ (Ref. 6).

Let the relaxation time of the electron momentum be shorter than the spin relaxation time and longer than the reciprocal value $\hbar/T$ of the average electron kinetic energy T. As discussed above, these conditions (which are usually true for semiconductors) lead to Eq. (25) for correlation functions. The spin operators S_X and S_Y are conveniently split into two parts: $S_X=(S_++S_-)/2$ and $S_Y=(S_+-S_-)/2i$. With respect to the definition (29) one can find $Q_{\pm,\mathbf{k},\mathbf{k}',\sigma}(t)=S_\pm^j a_{\mathbf{k},\sigma}^{\dagger}a_{\mathbf{k}',\sigma}e^{i(\pm\omega_0+\varepsilon_{\mathbf{k}_1}-\varepsilon_{\mathbf{k}_2})t}$ so that corresponding correlation functions are

$$\gamma_{\pm,\mp}(\omega)=\frac{\alpha^2}{S_0^2}\langle S_\pm S_\mp\rangle\sum_{\mathbf{k}_1,\mathbf{k}_2,\sigma}\sum_j|\psi(z_j)|^4 f_{\mathbf{k}_1,\sigma}(1-f_{\mathbf{k}_2,\sigma})$$

$$\times\delta(\pm\omega_0+\varepsilon_{\mathbf{k}_1}-\varepsilon_{\mathbf{k}_2}+\omega). \tag{34}$$

Equation (34) allows us to find $\gamma_{xx}(\omega)=\gamma_{yy}(\omega)=[\gamma_{+,-}(\omega)+\gamma_{-,+}(\omega)]/4$ as well as $\gamma_{xy}(\omega)=-\gamma_{yx}(\omega)=i[\gamma_{+,-}(\omega)-\gamma_{-,+}(\omega)]/4$. The latter equality shows that the exchange scattering corresponds to the relaxation matrix in the form (21). Thus, according to the definition (21) and identity (14), we find

$$n\gamma_{xx}(\omega)=\frac{1}{8}[\gamma_{+,-}(\omega)+\gamma_{-,+}(\omega)+\gamma_{+,-}(-\omega)$$

$$+\gamma_{-,+}(-\omega)], \tag{35}$$

$$n\gamma_{xy}(\omega)=\frac{i}{8}[\gamma_{+,-}(\omega)-\gamma_{-,+}(\omega)-\gamma_{+,-}(-\omega)$$

$$+\gamma_{-,+}(-\omega)]. \tag{36}$$

Using Eq. (34) and the identity $\langle S_\pm S_\mp\rangle=\langle S_X^2\rangle+\langle S_Y^2\rangle\pm2\langle S_Z\rangle$ we can derive the relaxation parameters in the form

$$\pi n\gamma_{xx}(\omega)=\frac{\pi\alpha^2}{4S_0^2}\sum_{\mathbf{k},\mathbf{k}',\sigma}^{(\mathbf{k}\neq\mathbf{k}')}\sum_j|\psi(z_j)|^4\Big\{\langle S_X^2\rangle[f(1-f')+f'(1-f)][\delta(\omega+\varepsilon-\varepsilon'+\omega_0)+\delta(\omega+\varepsilon-\varepsilon'-\omega_0)]+\frac{\langle S_Z\rangle}{2}(f-f')[\delta(\omega+\varepsilon-\varepsilon'+\omega_0)-\delta(\omega+\varepsilon-\varepsilon'-\omega_0)]\Big\},\qquad(37)$$

$$\pi n\gamma_{xy}(\omega)=i\frac{\pi\alpha^2}{4S_0^2}\sum_{\mathbf{k},\mathbf{k}',\sigma}^{(\mathbf{k}\neq\mathbf{k}')}\sum_j|\psi(z_j)|^4\Big\{\langle S_X^2\rangle[f(1-f')+f'(1-f)][\delta(\omega+\varepsilon-\varepsilon'+\omega_0)-\delta(\omega+\varepsilon-\varepsilon'-\omega_0)]+\frac{\langle S_Z\rangle}{2}(f'-f)[\delta(\omega+\varepsilon-\varepsilon'+\omega_0)+\delta(\omega+\varepsilon-\varepsilon'-\omega_0)]\Big\},\qquad(38)$$

where $\varepsilon=\varepsilon_{\mathbf{k},\boldsymbol{\sigma}},\varepsilon'=\varepsilon_{\mathbf{k}',\sigma}$ and $f=f_{\mathbf{k},\sigma},f'=f_{\mathbf{k}',\sigma}$ are Fermi-Dirac distribution functions. While deriving the Eqs. (37) and (38), we assume that neither 2D structure stress nor internal magnetic field affects the transversal magnetization (which means $\langle S_X^2\rangle=\langle S_Y^2\rangle$). Similarly, we find

$$\gamma_{zz}^0=\frac{\alpha^2}{S_0^2}\langle S_Z^2\rangle\sum_{\mathbf{k},\mathbf{k}',\sigma}^{(\mathbf{k}\neq\mathbf{k}')}\sum_j|\psi(z_j)|^4f(1-f')\,\delta(\varepsilon-\varepsilon'),\qquad(39)$$

and $\gamma_{xz}(\omega)=\gamma_{yz}(\omega)=0$ because $\langle S_\pm S_Z\rangle=0$. It is evident from Eqs. (37), (38), and (39) that $\gamma_{xx}(\omega)\to\gamma_{zz}^0$ and $\gamma_{xy}(\omega)\to0$ in zero magnetic field ($\omega,\omega_0\to0$).

It should be noted that Eqs. (37), (38), and (39) can be obtained in terms of the Green's functions formalism (see Ref. 14) with Hamiltonian (29) under the assumption that the motions of a magnetic ion spin and electron are not correlated: $\langle S_\mu^j a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k}',\sigma}S_\nu^j a_{\mathbf{k}_1,\sigma}^\dagger a_{\mathbf{k}_1',\sigma}\rangle=\langle S_\mu^j S_\nu^j\rangle\langle a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k}',\sigma}a_{\mathbf{k}_1,\sigma}^\dagger a_{\mathbf{k}_1',\sigma}\rangle$. Contrary to the above approach, the equations for Green's functions can easily incorporate other terms in the Hamiltonian H_L of the dissipative subsystem as well as take into account magnetic-ion–electron correlations. This is beyond the scope of the present paper.

Now we are interested in the case of a nondegenerate electron gas. For the case of single-electron relaxation, Eq. (30) reads

$$\sum_\sigma f_{\mathbf{k},\sigma}=\frac{2\pi\hbar^2}{S_0mT}e^{-\varepsilon_{\mathbf{k}}/T},\qquad(40)$$

where m is the in-plane effective mass of the carriers. Beginning with Eq. (40), we restore $\hbar$ for convenience. Calculation of the sums in Eq. (39) yields

$$\frac{\pi}{\hbar}\gamma_{zz}^0=I\frac{\alpha^2mn_m}{2L_w\hbar^3}\langle S_Z^2\rangle,\qquad(41)$$

where the dimensionless parameter I reflects the "overlap" of the ψ function and magnetic ions:

$$I=L_w\int_{-\infty}^{\infty}\frac{n_m(z')}{n_m}|\psi(z')|^4dz'.\qquad(42)$$

In the case of an infinitely deep QW containing magnetic ions with concentration n_m, the integration in Eq. (42) gives $I=3/2$.

In the same manner we can find γ_{xx} and γ_{xy}. After some algebra,

$$\frac{\pi}{\hbar}n\gamma_{xx}=I\frac{\alpha^2mn_m}{2L_w\hbar^3}\left\{\langle S_X^2\rangle F\left(\frac{\omega}{T},\frac{\omega_0}{T}\right)+\frac{\langle S_Z\rangle}{2}\left[F_1\left(\frac{\omega}{T}+\frac{\omega_0}{T}\right)-F_1\left(\frac{\omega}{T}-\frac{\omega_0}{T}\right)\right]\right\},\qquad(43)$$

$$\frac{\pi}{\hbar}n\gamma_{xy}=iI\frac{\alpha^2mn_m}{2L_w\hbar^3}\left\{\langle S_X^2\rangle\frac{\exp\left(-\frac{|\omega+\omega_0|}{T}\right)-\exp\left(-\frac{|\omega-\omega_0|}{T}\right)}{4}+\frac{\langle S_Z\rangle}{2}\left[F_1\left(\frac{\omega}{T}+\frac{\omega_0}{T}\right)+F_1\left(\frac{\omega}{T}-\frac{\omega_0}{T}\right)\right]\right\},\qquad(44)$$

where we introduce the functions

$$F(x,y)=\frac{1}{4}[2+\exp(-|x+y|)+\exp(-|x-y|)],\qquad(45)$$

$$F_1(x)=\frac{1}{4}\operatorname{sgn}(x)[\exp(-|x|)-1].\qquad(46)$$

The function $F(x,y)$ decreases monotonically from 1 to 1/2, which reflects the suppression of electron spin fluctuations by a magnetic field.

Recall that ω and ω_0 are the electron spin splitting in an effective field and magnetic ion Zeeman splitting in an external magnetic field, respectively. In the case of equidistant

Zeeman energy spectra of the magnetic ions, the magnetic field dependences of spin square averages are

$$\langle S_X^2\rangle = \frac{1}{2}\coth\frac{\omega_0}{2T}\, b_S\left(\frac{\omega_0}{T}\right),$$

$$\langle S_Z^2\rangle = S(S+1) - 2\langle S_X^2\rangle, \tag{47}$$

where $b_S(x) = -\langle S_Z\rangle = SB_S(Sx)$ is nonnormalized Brillouin function:

$$b_S(x) = \left(S+\frac{1}{2}\right)\coth\left(S+\frac{1}{2}\right)x - \frac{1}{2}\coth\frac{x}{2}. \tag{48}$$

Equations (41), (43), and (44) together with Eqs. (47) and (48) allow us to compare the different terms in the relaxation matrix (21). Numerical analysis shows that the second term in Eq. (43) is negligible in comparison with the first one as well as $n|\gamma_{xy}| < \gamma_{zz}^0, n(\gamma_{xx}+\gamma_{xx})$. Thus, the kinetic equation (17) transforms into the Bloch equation (3) with the following relaxation parameters:

$$\frac{1}{T_1} = I\frac{\alpha^2 m}{L_w\hbar^3} n_m \langle S_X^2\rangle F\left(\frac{\omega}{T}, \frac{\omega_0}{T}\right), \tag{49}$$

$$\frac{1}{T_2} = I\frac{\alpha^2 m}{L_w\hbar^3} n_m \left\{\frac{\langle S_Z^2\rangle}{2} + \frac{\langle S_X^2\rangle}{2} F\left(\frac{\omega}{T}, \frac{\omega_0}{T}\right)\right\}. \tag{50}$$

The right-hand sides of Eqs. (49) and (50) become equal to each other at zero magnetic field as is expected. Moreover, as $\omega, \omega_0 \to 0$, Eqs. (49) and (50) transform just to the double rate W of spin-flip relaxation obtained with the help of Fermi's golden rule.[17,18]

With the magnetic field increasing, one can obtain, in the limit $\omega \gg T$,

$$\frac{T_1}{T_2} = 2S + \frac{1}{2}. \tag{51}$$

Equation (51) shows that the rate of phase relaxation T_2^{-1} can exceed the longitudinal relaxation rate T_1^{-1} by a factor about one order of magnitude in the case of $S=5/2$ (ions of Mn^{2+}) despite their relationships to the microscopic parameters in Eqs. (49) and (50) being identical.

Figure 1 shows the magnetic field dependence of the longitudinal and transversal relaxation times calculated with the help of Eqs. (49), (50), (47), and (48). One can see that the magnetic field suppresses the longitudinal relaxation while it enhances the transversal relaxation for $S>1/2$. In the limit of saturated magnetic fields, $\omega, \omega_0 \gg T$, the decreasing of T_2 reaches the factor $\frac{2}{3}(S+1)/(S+1/4)$. Note that the decreasing of the transversal relaxation time in a magnetic field was observed for Mn spins but has not been explained until now.[19]

Equations (49) and (50) do not manifest any spin relaxation anisotropy with respect to the growth axis OZ'. Nevertheless, the striking difference between the 3D and 2D cases can be expected in temperature dependences of relaxation times due to the different densities of states of 3D and 2D structures. For instance, in the 3D case the factor $\sqrt{T}$ is expected to appear in Eqs. (49) and (50) (see Refs. 20 and 21).

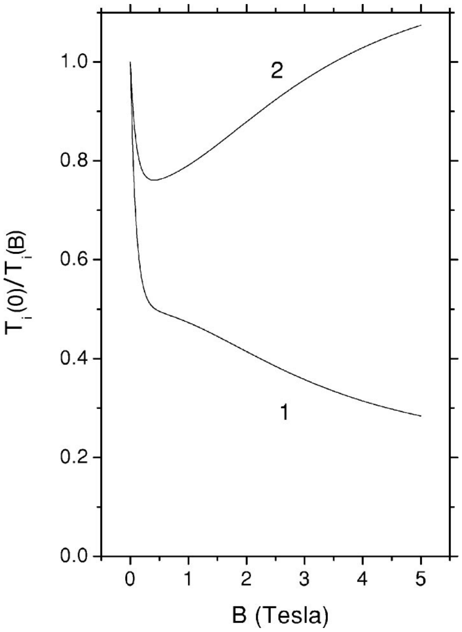

FIG. 1. Magnetic field dependences of the rates of longitudinal [Eq. (49), $i=1$] and transversal [Eq. (50), $i=2$] electron spin relaxation at $T=4$ K in a $\mathrm{Cd}_{1-x}\mathrm{Mn}_x\mathrm{Te}$ QW with $x=0.017$ and $L_w=8$ nm.

At the end of this section we estimate the electron spin relaxation time of the exciton composed of electrons and holes with effective masses $m_e=0.096m_0$ and $m_h=0.25m_0$ (so $m=0.346m_0$, where m_0 is the free electron mass) in $\mathrm{Cd}_{1-x}\mathrm{Mn}_x\mathrm{Te}$ QW's with $\alpha=1.5\times10^{-23}$ eV cm^3, $x=0.017$, and $L_w=8$ nm. At zero magnetic field one can find $T_2=5$ ps, which is quite close to the experimental value observed recently via the time-resolved magneto-optic Kerr effect.[19]

V. DYAKONOV-PEREL MECHANISM

It has been shown in a number of works[12,22–24] that a fluctuating effective magnetic field accompanying the electron momentum scattering in semiconductors with a zinc-blende lattice results in quite efficient spin relaxation. In this section we show how the Dyakonov-Perel mechanism can be incorporated into the above developed formalism in the case of electron spin relaxation in a QW.

The Hamiltonian of the dissipative subsystem is assumed to have the form

$$H_L = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\mathbf{k}',\sigma} V_{\mathbf{k},\mathbf{k}'} a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k}',\sigma}. \tag{52}$$

The second term in Eq. (52) has a symbolic meaning: we assume $V_{\mathbf{k},\mathbf{k}'}$ to be an operator responsible for electron scattering by impurities, phonons, or electron-electron collisions.

This electron scattering changes randomly an effective magnetic field $\mathbf{B}=\mathbf{b}(\mathbf{k})/g_e\mu$ that corresponds to band spin splitting

$$H_{int}=\sum_{\mathbf{k},\sigma}\mathbf{s}\mathbf{b}(\mathbf{k})a^{\dagger}_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma}. \tag{53}$$

Therefore, the operator of this field,

$$\Omega_{\mu}=\sum_{\mathbf{k},\sigma}b_{\mu}(\mathbf{k})a^{\dagger}_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma}, \tag{54}$$

can be expressed in terms of Eq. (23) with

$$V^{p}_{\mu}=b_{\mu}(\mathbf{k}),Q_p=a^{\dagger}_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma},p\equiv\{\mathbf{k},\sigma\}. \tag{55}$$

The specific form of the function $b_{\mu}(\mathbf{k})$ is determined by a band Hamiltonian (see, for instance, Ref. 12) and depends on the dimensionality of the system under consideration.[23,24]

The following calculations depend on the specific mechanism of an electron momentum relaxation.[12] The corresponding formalism in terms of Green's functions is presented in Ref. 14. For simplicity, here we consider the correlation functions (18) in relaxation time approximation (27). With respect to the fact that for the nondegenerate case $\langle Q_{p_1}Q_{p_2}\rangle=f_{p_1}(1-f_{p_1})\delta_{p_1p_2}=f_{p_1}\delta_{p_1p_2}$, Eq. (27) takes the form

$$\frac{\pi}{\hbar}n\gamma_{\mu\nu}=\frac{1}{\hbar}\sum_{p}b_{\mu}(\mathbf{k})b_{\nu}(\mathbf{k})f_p\frac{\tau_p}{\tau_p^2(\mathbf{k})\omega^2+1}. \tag{56}$$

Consider now the electron spin splitting in a QW caused by k^3 terms in a band Hamiltonian of bulk crystal.[22] If the external magnetic field is directed along the axis of QW growth, we have[23]

$$b_{\mu}(\mathbf{k})=r_{\alpha}\kappa_{\mu}, \tag{57}$$

with

$$r_{\alpha}=\frac{\alpha_0\hbar^3}{m^{3/2}(2E_g)^{1/2}}, \tag{58}$$

$$\kappa_x=-k_xq^2,\quad \kappa_y=k_yq^2,\quad \kappa_z=0, \tag{59}$$

where m is the in-plane effective mass, α_0 is a constant, and q^2 is defined via the electron ψ function confined in a QW,

$$q^2=\left\langle\psi\left|-\frac{\partial^2}{\partial z^2}\right|\psi\right\rangle. \tag{60}$$

Note that it is assumed that $q\gg k$ in Eqs. (57), (58), and (59).

We see that only two components of relaxation parameters, γ_{xx} and γ_{yy} have nonzero values. Calculations with the help of Eq. (40) lead to the following expression for the transversal relaxation time:

$$\frac{1}{T_2}=\frac{\pi}{\hbar}n\gamma_{xx}=\frac{\pi}{\hbar}n\gamma_{yy}=\frac{1}{T_0}\frac{1}{1+(\omega\tau_p)^2}, \tag{61}$$

where T_0 corresponds to the relaxation time at zero field,

$$\frac{1}{T_0}=\frac{\alpha_0^2\hbar^2q^4T}{2m^2E_g}\tau_p, \tag{62}$$

$$\tau_p^{-1}=\langle\tau_p^{-1}(\mathbf{k})\rangle. \tag{63}$$

The denominator on right-hand side of Eq. (61) reflects the dependence of T_2 on electron spin splitting but under the assumption that the cyclotron frequency ω_c is sufficiently small, $\omega_c\tau_p\ll 1$. Note that approximation (63) is not necessary for calculations of T_1 and T_2. For more accurate integration (see Refs. 12 and 7), the dependence $\tau_p=\tau_p(\mathbf{k})$ can be taken into account.

According to Eq. (20), the longitudinal relaxation rate is

$$\frac{1}{T_1}=\frac{\pi}{\hbar}n(\gamma_{xx}+\gamma_{yy})=\frac{2}{T_2}. \tag{64}$$

We can see that T_1 can be shorter than T_2 in 2D systems. The relation (64) between T_1 and T_2 is consistent with that obtained by other density matrix methods [see Eq. (A15) from Ref. 7].

Another important case corresponds to a magnetic field directed along the OZ axis in the QW plane while the OX axis is perpendicular to this plane. One can find

$$\kappa_x=0,\quad \kappa_y=-k_yq^2,\quad \kappa_z=k_zq^2. \tag{65}$$

In this case, three parameters describe the electron spin relaxation:

$$\frac{1}{T_{2Y}}=\frac{1}{T_0},$$

$$\frac{1}{T_{2X}}=\frac{1}{T_0}\left(1+\frac{1}{1+(\omega\tau_p)^2}\right),$$

$$\frac{1}{T_1}=\frac{1}{T_0}\frac{1}{1+(\omega\tau_p)^2}. \tag{66}$$

Obviously, in zero field $(\omega=0)$ Eqs. (66) are identical to Eqs. (61) and (64): the in-plane relaxation parameter T_1 coincides with the other one T_{2Y} and is twice as large as T_{2X}.

We see that transversal relaxation (61) can be suppressed significantly by strong enough electron spin splitting in a magnetic field perpendicular to the QW plane ($\mathbf{B}\|\mathbf{OZ}'$, where $\mathbf{OZ}'$ is a growth direction) due to the dynamical averaging effect.[25] When a magnetic field is parallel to the sample surface [$\mathbf{B}\perp\mathbf{OZ}'$, Eqs. (66)], the transversal relaxation time would decrease for no more than 2 times.

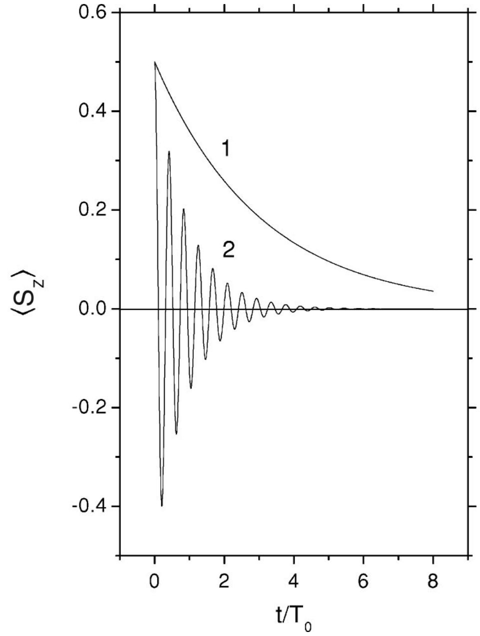

FIG. 2. Evolution of electron spin projection on the axis growth OZ' in a magnetic field directed along [Eq. (67), curve 1] and across [Eq. (68), curve 2] OZ'.

To illustrate the effect of magnetic anisotropy in QW's, let us consider a typical experimental situation[5,19] of normal to the QW plane spin relaxation. In the case $\mathbf{B}\|\mathbf{OZ}'$ [Eq. (64)], the electron spin deviation from its equilibrium value is described by a simple exponential law

$$2(Z-Z_0)=e^{-2\eta t}, \quad \eta=\frac{1}{1+(\omega\tau_p)^2}. \tag{67}$$

A more complicated case corresponds to $\mathbf{B}\perp\mathbf{OZ}'$. The solution of the kinetic equations (17) with relaxation parameters (66) gives rise to the evolution equation for the normal (to QW) spin component

$$2X=e^{-(1+\eta/2)t}\left(\cos\tilde{\omega}t-\frac{\eta}{2}\,\frac{\sin\tilde{\omega}t}{\tilde{\omega}}\right), \quad \tilde{\omega}=\sqrt{\omega^2-\eta^2/4}. \tag{68}$$

In Eqs. (67) and (68) the time and frequency are in units of T_0 and T_0^{-1} [Eq. (62)], respectively. In the zero-field case, $\omega\rightarrow 0$, Eqs. (67) and (68) are reduced to the same dependence e^{-2t}. The difference between these two cases becomes important as a magnetic field increases (Fig. 2).

Note that we consider only one reason for the magnetic anisotropy of the DP mechanism in QW's. In the case of Landau quantization ($\mathbf{B}\|\mathbf{OZ}'$) when the cyclotron frequency ω_c exceeds the rate of electron scattering τ_p^{-1}, the plane waves cease to be an adequate representation of the electron wave functions. This situation had been considered in detail in Ref. 25.

Actually, the typical scattering time of an electron in semiconductors is about $\tau_p\sim 10^{-12}$ s. Thus, to make the effect of magnetic field anisotropy visible at moderate magnetic fields we should use DMS structures where the inequality $\omega\tau_p>1$ can be reached due to giant spin splitting [Eqs. (32) and (33)].

VI. CONCLUSIONS

In this work, we develop the microscopic theory of electron spin evolution in semiconductors in terms of quantum kinetic equations for an arbitrary mechanism of spin relaxation. The relaxation term in these equations has been derived as a matrix of correlation functions, providing relatively short correlation times (as compared to spin relaxation times, $\tau\ll T$) of the dissipative subsystem. Our theory permits us to distinguish the relaxation processes of transversal and longitudinal (with respect to the direction of the external magnetic field) components and to capture the case of zero and small magnetic fields.

We show that spin relaxation processes due to exchange scattering in a semimagnetic QW reveal quite different behavior of the longitudinal and transversal spin relaxation times in a magnetic field. Specific calculations of the relaxation parameters show that only longitudinal relaxation can be described by flip-flop processes, suppressed in a magnetic field. On the other hand, transversal relaxation is due to effective exchange field fluctuations, which increase with magnetic field. Qualitatively, the latter effect has the same reason as the growth of the linewidth of spin-flip Raman scattering by shallow donors in DMS's.[26]

Actually, the peculiarities of 2D structures in the exchange scattering problem reveal themselves only via the electron density of states. A nontrivial manifestation of low-dimensional structures is predicted for the particular magnetic anisotropy of the DP mechanism. In this case a significant difference in transversal relaxation rates for parallel and perpendicular (to growth axis) magnetic field orientations has been found.

The relaxation mechanisms considered in this paper can be discerned by their different dependences on the QW width. As may be seen from Eqs. (49), (50) and Eq. (62), the spin relaxation rates are proportional to I/L_w or to q^4. In the case of deep enough QW's, this means proportionality to L_w^{-1} and L_w^{-4} for exchange scattering and the DP mechanism, respectively.

The proposed approach to relaxation can be extended to any two-level system described by a fictitious spin 1/2. From this standpoint, the analysis of heavy-hole spin relaxation in QW's in terms of Eqs. (17), (20), and (21) promises new insight into this problem. Namely, in our approach, relaxation in the case of an in-plane magnetic field would involve tree relaxation times—longitudinal T_1, transversal (along QW plane) $T_{2\|}$, and perpendicular $T_{2\perp}$—while the usual description of a heavy-hole relaxation involves only one relaxation time. Investigations of hole relaxation in QW's or uniaxial semiconductors will be reported elsewhere.

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