PHYSICAL REVIEW B **67**, 115125 (2003)

Conductivity in quasi-two-dimensional systems

K. Morawetz
Max-Planck-Institute for the Physics of Complex Systems, Noethnitzer Strasse 38, D-01187 Dresden, Germany

(Received 11 October 2002; published 27 March 2003)

The conductivity in quasi-two-dimensional systems is calculated using the quantum kinetic equation. Linearizing the Lenard-Balescu collision integral with the extension to include external field dependences allows one to calculate the conductivity with diagrams beyond the GW approximation including maximally crossed lines. Consequently the weak localization correction as an interference effect appears here from the field dependence of the collision integral (the latter dependence sometimes called intra-collisional field effect). It is shown that this weak localization correction has the same origin as the Debye-Onsager relaxation effect in plasma physics. The approximation is applied to a system of quasi-two-dimensional electrons in heterojunctions which interact with charged and neutral impurities and the low-temperature correction to the conductivity is calculated analytically. It turns out that the dynamical screening due to charged impurities leads to a linear temperature dependence, while the scattering from neutral impurities leads to the usual Fermi-liquid behavior. By considering an appropriate mass action law to determine the ratio of charged to neutral impurities we can describe the experimental metal-insulator transition at low temperatures as a Mott-Hubbard transition.

DOI: 10.1103/PhysRevB.67.115125 PACS number(s): 71.30.+h, 73.90.+f, 05.60.−k, 72.10.−d

I. INTRODUCTION

The low-temperature conductivity of quasi-two-dimensional systems such as metal oxide semiconductor field effect transistor (MOSFET) structures or heterojunctions reveals a surprising metal to insulator transition.[1] This critical review of the theoretical approaches has concluded that this phenomenon is insufficiently explained. The experimental data shows a pronounced transition from insulating behavior at low densities to a metallic behavior at high densities. The generic feature of the metal-insulator transition is the rapid change from insulating to conducting behavior when the density is increased very slightly at low temperatures. These density driven metal-insulator transitions are usually referred to as Mott transitions.[2] The characteristic feature of the Mott-Hubbbard transition is that an increase in the effective mass is directly responsible for increasing resistivity $\rho = m/e^2 n\tau$ while the Anderson scenario would assume a vanishing relaxation time $n\tau$. Measurements of the effective mass[3] seem to support a Mott-Hubbard transition rather than the Anderson transition quantitatively explained in Ref. 4.

In this paper we want to return to the original idea of the Mott transition in that a bound state is resolved with increasing density due to pressure ionization. We will show that a quantitative description of the experimental results can be achieved if one calculates the interplay of weak localization and trapping due to charged impurities as well as the scattering with neutral impurities on the same theoretical footing.

Weak localization as a quantum interference effect is intensively studied in the literature.[5–9] The maximally crossed diagrams lead to a diffusive pole which allows one to extract weak localization corrections to the conductivity[8,10–12] (and citations therein). This paper is devoted to an alternative route to weak localization. We will linearize the quantum kinetic equation derived at a lower level of approximation but with external fields in order to create higher order diagrams. The main aim is to show that weak localization has the same origin as the interference effect known from the Debye-Onsager relaxation effect in plasma physics.

In the following part of the introduction we will outline the model we want to use. This will give a summary about the kinetic approach adopted in this paper. The many-body approximation to be applied in this paper is also specified and it is clarified how higher order diagrams are generated by linearization, which is presented in detail in Appendix A. In the second section we calculate the electrical conductivity from the kinetic equation approach and present results for the relaxation time and the relaxation function. This relaxation function summarizes the quantum corrections to the conductivity due to interference effects. The Lenard-Balescu kinetic equation used as a starting point in this section is derived in Appendix B. The third section discusses the resulting conductivity formulas and shows that the relaxation function is identical to the one calculated for the weak-localization corrections. For comparison with experiment we include scattering from neutral impurities in addition to the scattering with charged impurities which are worked out in detail in Appendix C. The fourth section summarizes the results and gives an outlook. In Appendix D we discuss the polarization function in quasi-two-dimensional systems. The additional Appendixes E and F present calculations of the integrals used during the paper.

A. Outline of the model

As a model, we assume a quasi two-dimensional Coulomb potential, where the field lines are three dimensional but the motions of the particles are restricted to two dimensions. The effective potential takes then the form $V_{ab} = 4\pi e_a e_b \hbar/\sqrt{q_x^2+q_y^2}$, where, for instance, $e_a = e$ for electrons and $e_b = -Ze$ for charged ions. Our approach is conveniently based on the kinetic equation for the one-particle distribution $f_a(k,t)$ normalized to the area density

$$s\int \frac{d\mathbf{k}}{(2\pi\hbar)^2} f_a(\mathbf{k},t) = n_a, \tag{1}$$

where the spin degeneracy is denoted by s. The kinetic equation for that distribution function is

$$\partial_t f_a + e_a \mathbf{E}\cdot\nabla_{\mathbf{k}} f_a = \sum_b \mathcal{I}[f_a, f_b, \mathbf{E}], \tag{2}$$

where correlations are covered by the corresponding collision integral $\mathcal{I}$. This collision integral is explicitly field dependent due to the distortion of two-particle correlations. We consider the conductivity in a system described by a local equilibrium distribution

$$f_a(\mathbf{k},t) = \left(e^{\frac{[\mathbf{k}-\mathbf{p}_a(t)]^2-\mu_a}{2m_aT_a}} + 1 \right)^{-1} \tag{3}$$

with the mean mass-motion of the charged particle $\mathbf{p}_a(t)$. The center of mass motion is at rest, which means that the total sum of currents $\Sigma_b \mathbf{j}_b = \Sigma_b n_b e_b \mathbf{p}_b / m_b = 0$ with density n_b and mass m_b. In the following we will restrict ourselves to a two-component system. The generalization towards multicomponent systems is straightforward.

From the collision integral (2) we have two sources of linear response: A term proportional to the current $\mathbf{j} = -e_a n_a \mathbf{p}_a / m_a$ and a term proportional to the field. By multiplying Eq. (2) with k and integrating, the balance equation for the momentum reads

$$\partial_t(n_a\mathbf{p}_a) - n_a e_a \mathbf{E} = \int \frac{dk}{(2\pi\hbar)^2}\mathbf{k}\mathcal{I} = -n_a e_a \mathbf{E}\frac{\delta E}{E} + n_a \mathbf{p}_a \tau^{-1} \tag{4}$$

such that the current balance takes the form

$$\partial_t \mathbf{j}_a - \frac{n_a e_a^2}{m_a}\left(1 - \frac{\delta E}{E}\right)\mathbf{E} = -\tau^{-1}\mathbf{j}_a \tag{5}$$

and the stationary conductivity $\mathbf{j} = \sigma \mathbf{E}$ is

$$\sigma = \frac{n_a e_a^2}{m_a}\tau(1 - \delta E/E). \tag{6}$$

According to the distortion of the Fermi function (3) we have a linear response

$$f_a' - f_a = -\mathbf{p}_a(\partial_{\mathbf{k}_a'} - \partial_{\mathbf{k}_a}) f_0 \tag{7}$$

and we can represent the current relaxation time and the relaxation effect for a scattering with impurities (C4) with the potential V_s as

$$\begin{pmatrix} \dfrac{1}{\tau_i} \\ \dfrac{\delta E_i}{E} \end{pmatrix} = -\frac{2n_i}{n_a\hbar}\int \frac{dkdq}{(2\pi\hbar)^4}\partial_{\epsilon_k} f_0(\epsilon_k) V_s^2(q) q^2 \cos^2(q,E) \times \begin{pmatrix} \pi\delta(\epsilon_k - \epsilon_{k-q}) \\ \dfrac{\hbar}{2}\dfrac{\mathcal{P}'}{\epsilon_k - \epsilon_{k-q}} \end{pmatrix}. \tag{8}$$

The relaxation time τ is coming from the term proportional to the current in the collision integral. This is due to on-shell scattering represented by the delta function in Eq. (8). In addition to this the conductivity becomes renormalized by the explicit field dependence of the collision integral $\delta E/E$ which is an interference effect and corresponds to an off-shell scattering as expressed by the derivative of the principal value in Eq. (8). The latter effect has been the subject of various investigations for nondegenerate plasmas and is known as Debye-Onsager relaxation effect. The field dependence and quantum form is discussed in Ref. 13. For a plasma system this Debye-Onsager relaxation effect[14–20] was first derived within the theory of electrolytes.[21–25] (Debye derived a limiting law of electrical conductivity[21] which stated that the external electric field E on a single charge $Z = 1$ is diminished in an electrolyte solution by the amount $\delta E/E = \kappa e^2/6T$ where e is the elementary charge, E the electric field strength, T is the temperature of the plasma, and κ is the inverse screening radius of the screening cloud. This law is interpreted as a deceleration force which is caused by the deformed screening cloud surrounding the charge. Later it was shown by Onsager[22] that this result has to be corrected to $\delta E/E = \kappa e^2/3(2+\sqrt{2})T$ if the dynamics of ions $(Z=1)$ is considered. The linear response theory reproduce this Onsager result.[17,18,20] The kinetic theory leads to the Onsager result if asymmetric screening[13] is applied while the symmetric treatment leads to the Debye result.[19,20,26])

The theoretical calculations of conductivity in reduced dimensions is the topic of intensive investigations. These concern rigid two-dimensional electron systems[27] and quasi-two-dimensional systems.[28] In the latter study a three-dimensional system was considered, where the particles can only scatter in two dimensions leading to a cylindrical Fermi surface. The Born approximation and contact interaction result in a resistivity which has a leading low-temperature behavior as $1/\sigma \propto aT^2(1 + b\ln T)$. We will show that the Coulomb interaction with the dynamical screening results in a linear order as the leading term. This has been repeatedly reported in the literature both from an experimental and theoretical point of view. Numerical calculations of Coulomb scattering rates from impurities predict a linear temperature dependence of the mobility in silicon inversion layers.[29,30] This was attributed to the collisional level broadening in the screening function. Related results have been obtained in Ref. 31, where a significant suppression of the temperature dependence of the screening function was found. An analytical investigation of screening in quasi-two-dimensional systems was given in Ref. 32 where a linear temperature term in the conductivity was reported.

In this paper we want to investigate the effect of Coulomb screening on the conductivity. We will derive exact analytical results which show that due to dynamical screening the leading low-temperature contribution to the conductivity is linear. In contrast, the static screening leads to a quadratic temperature dependence typical for the Fermi liquid.

B. Many body approximation used in this paper

For the calculation of the conductivity we want to account for quantum interference effects such as weak localization. This means we have to include maximally crossed diagrams

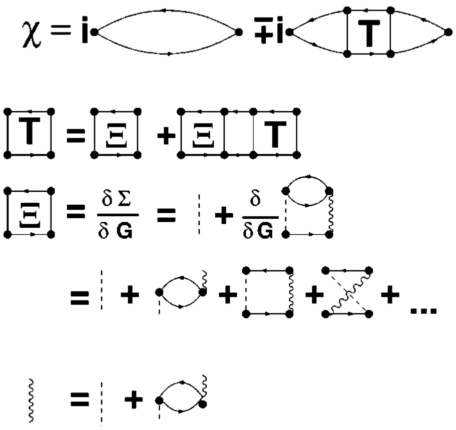


FIG. 1. The density response function (first line) in terms of the particle-hole T matrix. The latter one can be expressed in the second line as a sum of irreducible graphs which are given in terms of the self-energy. The latter one is used in screened potential $\mathcal{V}=V+V\Pi\mathcal{V}$ (last line) approximation.

at least.[5,7,12] In addition to the direct calculation of diagrams, we can employ the philosophy of variational techniques. In Ref. 33 it was described how one can use the variation of nonequilibrium Green's functions with respect to an auxiliary external field to create higher order diagrams in the response function. This makes use of the known variation technique summarized in Appendix A.

The response function can be given in terms of the particle hole T matrix which in turn can be represented by the sum of irreducible graphs Ξ according to Fig. 1. In turn, Ξ can be expressed as a variation of the self-energy Σ with respect to the Green's function (bare line) G. For the dynamical screened approximation used here we give the corresponding results in Fig. 1. One sees that in principle maximally crossed diagrams are accounted for.

Instead of the variation of the nonequilibrium Green's function we can use a proper reduction of the latter ones towards a kinetic equation. The linear response obtained from this kinetic equation including all external field effects is accounting then for higher order diagrams in a convenient way. As such, we obtain weak localization effects by proper linearization of the collision integral.

II. KINETIC EQUATION AND CONDUCTIVITY

The kinetic equation corresponding to the dynamical screened approximation is the quantum Lenard-Balescu equation, which has been derived for high fields in Refs. 13 and 34. A sketch of the derivation is in Appendix B

$$\frac{\partial}{\partial t}f_a+e\mathbf{E}\frac{\partial}{\partial \mathbf{k}_a}f_a=I_a^{\rm in}(\mathbf{k},t)-I_a^{\rm out}(\mathbf{k},t). \tag{9}$$

The collision-in integral is [$I^{\rm out}$ is given by $f\leftrightarrow 1-f$ and $L^{<}\leftrightarrow L^{>}$]

$$\begin{aligned}I_a^{\rm in}(\mathbf{k},t)=2\sum_b s_b\int\frac{d\mathbf{q}}{(2\pi\hbar)^2}V_{ab}^2(\mathbf{q})\int_0^\infty d\tau\\ \times\int\frac{d\omega}{2\pi}\cos\left[(\epsilon_{k-q}^a-\epsilon_k^a-\hbar\omega)\frac{\tau}{\hbar}+\frac{e_a\mathbf{E}\cdot\mathbf{q}\tau^2}{2m_a\hbar}\right]\\ \times f_a(\mathbf{k}-\mathbf{q}-e_a\mathbf{E}\tau,t-\tau)[1-f_a(\mathbf{k}-e_a\mathbf{E}\tau,t-\tau)]\\ \times\frac{\Pi_{bb}^{<}\left(\mathbf{q},\omega,t-\frac{1}{2}\tau\right)}{|\mathcal{E}\left(\mathbf{q},\omega,t-\frac{1}{2}\tau\right)|^2}\end{aligned} \tag{10}$$

with the free density fluctuation (B4)

$$\begin{aligned}\Pi_{bb}^{<}(\mathbf{q},\omega,t)=-2\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}\int_0^\infty d\tau\\ \times\cos\left[(\hbar\omega-\epsilon_p^b+\epsilon_{p+q}^b)\frac{\tau}{\hbar}+\frac{e_b\mathbf{E}\cdot\mathbf{q}\tau^2}{2m_b\hbar}\right]\\ \times f_b\left(\mathbf{p}+\mathbf{q},t-\frac{1}{2}\tau\right)\left[1-f_b\left(\mathbf{p},t-\frac{1}{2}\tau\right)\right].\end{aligned} \tag{11}$$

Here we use the sum over species explicitly.

The nested form of the Lenard-Balescu collision integral (10) is computationally advantageous. It tells that the collision of particle a with momentum k on a particle b with momentum p into $(a,k-q)$ and $(b,p+q)$ can be represented equivalently as a collision of the particle a with k on a hole a with $k-q$ by a dynamic plasmon emission which is considered as a particle-hole fluctuation of particles b. For static screening $\mathcal{E}(\mathbf{q},0,t)$, Eq. (10) reduces to the kinetic equation for statically screened Coulomb potentials in high electric fields (C1).[34,35]

Now we calculate the frequency integral in Eqs. (9) and (10) analytically using the identity (E1). We summarize the result of the frequency integration in the momentum dependent function $W(q)$ which takes the explicit form (E10) for quasi-two-dimensional systems.

Performing the balance equation for the current (4) we obtain in linear response the interference term or relaxation function

$$\begin{aligned}e_an_a\frac{\delta E}{E}=\frac{2e_a}{m_a\hbar^4}\sum_b s_b\int\frac{dq}{(2\pi\hbar)^2}W(q)q^2\cos^2(q,E)\\ \times\int_0^\infty d\tau\tau^2I_s(a,\tau)\left(-\frac{1}{2}\Pi_{bb}^{<}(q,0)\right)-(a\leftrightarrow b)\end{aligned} \tag{12}$$

and the relaxation time as

$$\tau^{-1}=\sum_b\left(R(a,b)-\frac{e_am_b}{e_bm_a}R(b,a)\right) \tag{13}$$

with

$$n_a R(a,b) = -\frac{4s_b}{m_a \hbar^4}\int \frac{dq}{(2\pi\hbar)^2} W(q) q^2 \cos^2(q,E) \times \int_0^\infty d\tau \tau I_s(a,\tau)\left(-\frac{1}{2}\Pi^<_{bb}(q,0)\right). \tag{14}$$

Here the static free density fluctuation reads

$$\Pi^>_{bb}(q,0) = \Pi^<_{bb}(q,0) = 2\pi\int \frac{dq}{(2\pi\hbar)^2} f_p^b (1-f_p^b)\delta(\epsilon_p - \epsilon_{p+q}) \tag{15}$$

and the integral

$$I_s(a,\tau) = \int \frac{dk}{(2\pi\hbar)^2} f_a(k)[1-f_a(k-q)]\sin(\epsilon_{k-q}-\epsilon_k)\frac{\tau}{\hbar} \tag{16}$$

will be calculated in Appendix F.

We remark that the interference effect (12) vanishes for identical scattering partners, e.g., electron-electron correlations. When calculating the explicit form of the relaxation time and relaxation function we employ charge neutrality $e_a n_a + e_b n_b = 0$ and restrict ourselves to the case of single charge ions $e_b = -e$. The case of higher charged ions is also available but is more involved. The generalization to systems with additional particles species is straightforward.

A. Relaxation time by charged impurities

Introducing the dimensionless integration variable $q = 2p_{fa}y$ we obtain the relaxation time to lowest order in temperature

$$\tau^{-1} = \sum_b \frac{m_a e_a^2 p_{fa} s_b}{\pi\hbar^4 n_a}\kappa_a\left(T_b - \frac{e_a n_a T_a}{e_b n_b}\right)\frac{\xi}{\xi+1} \times \int_0^1 dy \frac{y\kappa_a}{(y+\kappa_a)}\frac{1}{\sqrt{1-y^2}}, \tag{17}$$

where we have introduced the abbreviation $\xi = m_b^2 e_b^2 / m_a^2 e_a^2$. Since the momentum integration $y \leqslant 1$ is restricted by the low-temperature expansions to values below $2p_{fa}$ the low-temperature expansion of the inverse screening length (E11) becomes a constant

$$\kappa = \sum_c 2\pi e_c^2 \partial_\mu n_c = \frac{e_a^2 m_a s_a}{\hbar^2}\left(1 + \left|\frac{e_b s_b}{e_a s_a}\right|\sqrt{\xi}\right). \tag{18}$$

We distinguish here between the temperature of electrons T_a and the temperature of the ions T_b which could mime nonequilibrium effects.

The integral in Eq. (17) can be easily calculated

$$R = \tau_{ab}^{-1}\frac{m_a}{n_a e_a^2}\frac{e_a^2}{h} = \frac{8s_b}{s_a^3}\left(\frac{T_a}{\epsilon_{fa}} + \frac{T_b}{\epsilon_{fa}}\right)\frac{\xi}{1+\xi}\kappa'_a\kappa_a \times \left(\frac{\pi}{2} + \frac{\kappa'_a}{\sqrt{1-\kappa_a'^2}}\ln\frac{\kappa'_a}{(1+\sqrt{1-\kappa_a'^2})}\right), \tag{19}$$

where we will use the abbreviation

$$\kappa_a = \frac{\hbar\kappa}{2p_{fa}} = \frac{e_a^2 m_a s_a}{2\hbar p_{fa}} = \sqrt{\frac{s_a^3 e_a^4 m_a^2}{16\pi\hbar^4 n_a}} \equiv \sqrt{\frac{\hbar}{\tau_0\epsilon_f}} \tag{20}$$

with $\tau_0^{-1} = me^4 s^2/8/\hbar^3$ during the paper. Furthermore, we distinguish in Eq. (20) between the inverse screening length where ions are included κ'_a and where they are neglected κ_a

$$\kappa'_a = \kappa_a\left(1 + \sqrt{\xi}\frac{s_b}{s_a}\right). \tag{21}$$

We expand the above result for large and small κ_a which corresponds also to the small and large density limits

$$R = \frac{8s_b}{s_a^3}\left(\frac{T_a}{\epsilon_{fa}} + \frac{T_b}{\epsilon_{fa}}\right)\frac{\xi}{1+\xi}\kappa_a\begin{cases} 1 - \dfrac{\pi}{4\kappa'_a} + \dfrac{2}{3\kappa_a'^2} + o(\kappa_a'^{-3}), \\ \dfrac{\pi}{2}\kappa_a + \kappa_a^2\ln\dfrac{\kappa_a}{2} + o(\kappa_a^3). \end{cases} \tag{22}$$

It is interesting to investigate the limit of large ion masses, $\xi \to \infty$ and $T_b = 0$, which would correspond to the charged impurity limit. One gets from Eq. (22) for large κ_a

$$R = \frac{8s_b}{s_a^3}\frac{T_a}{\epsilon_{fa}}\kappa_a. \tag{23}$$

If we compare this with the neutral electron-impurity scattering result (C10), we see significant differences. While the statically screened result shows a Fermi-liquid behavior of const and T^2 terms, the dynamically screened result (23) leads to a linear temperature dependence.

B. Relaxation function by charged impurities

Interference effects from the relaxation function (12) can be calculated analogously

$$\frac{\delta E}{E} = \frac{e_a m_a^2 s_b}{2\pi p_{fa}\hbar^3 n_a}(e_b T_a - e_a T_b)\frac{\xi}{\xi+1} \times \int_0^\infty dy \frac{\kappa'_a}{y(y+\kappa'_a)}\partial_y^2\sqrt{y^2-1}\,\Theta(y-1\pm\eta). \tag{24}$$

The small η has been introduced to perform the principal value integration $\mathcal{P}$ according to Eq. (F9). It should be noted that Θ functions of the denominator and numerator cancel exactly and no restriction on y integration remains.

We have now to carefully consider the structure

$$\mathcal{I}_\pm = \int_0^\infty dy f_y \frac{\partial^2}{\partial y^2}[g_y\Theta(y-1\pm\eta)] \times \lim_{\eta\to 0}\left(\int_{1\mp\eta}^\infty dy f_y g''_y + (f_y g'_y - f'_y g_y)\big|_{y=1\mp\eta}\right) \tag{25}$$

with $f_y = \kappa'_a/y/(y+\kappa_a)$ and $g_y = \sqrt{y^2-1}$. Performing the integral one sees that the divergent contribution at $\eta \to 0$ is cancelled exactly by the $fg' - f'g$ term. We obtain

$$\mathcal{I}_{\pm}=\frac{\kappa_a'}{1-\kappa_a'^2}\pm\frac{\pi}{2}+\frac{\text{arcosh}\,\kappa_a'}{(\kappa_a'^2-1)^{3/2}}. \tag{26}$$

The principal value in Eq. (17) is calculated from $(\mathcal{I}_{+}+\mathcal{I}_{-})/2$ and we obtain (with charge $e_b=-e_a$)

$$\frac{\delta E}{E}=-\frac{2s_b}{s_a^2}\left(\frac{T_a}{\epsilon_{fa}}+\frac{T_b}{\epsilon_{fa}}\right)\kappa_a\frac{\xi}{1+\xi}\left(\frac{\kappa_a'}{1-\kappa_a'^2}+\frac{\text{arcosh}\,\kappa_a'}{(\kappa_a'^2-1)^{3/2}}\right). \tag{27}$$

The low density (large κ_a) expansion as well as the high density (small κ_a) expansion read

$$\frac{\delta E}{E}=-\frac{2s_b}{s_a^2}\left(\frac{T_a}{\epsilon_{fa}}+\frac{T_b}{\epsilon_{fa}}\right)\kappa_a\frac{\xi}{1+\xi}\times\begin{cases}-\dfrac{1}{\kappa_a'}+(\ln(2\kappa_a')-1)\dfrac{1}{\kappa_a'^3}+o(\kappa_a'^{-4}),\\[2ex] -\dfrac{\pi}{2}+2\kappa_a'-\dfrac{3\pi}{4}\kappa_a'^2+o(\kappa_a'^3).\end{cases} \tag{28}$$

It is clear that for the impurity limit with infinite masses $\xi\to\infty$ we have from Eqs. (21) and (28)

$$\left(\frac{\delta E}{E}\right)_i=0 \tag{29}$$

in agreement with the physical picture that if the ions cannot move the screening cloud cannot deform during the motion of the electrons and cannot induce a relaxation effect. This is different if the charged impurities do not contribute to the screening, $\kappa_a=\kappa_a'$ in Eq. (21), and we obtain a finite result. This case is anticipated here since the neutral static relaxation function already lead to finite results (C14).

III. DISCUSSION

For further progress we use the expansions for large κ_a or low densities. Collecting Eqs. (22) and (28) we obtain the conductivity in an analogous form to the Bloch-Grüneisen formula

$$\sigma=\frac{e^2}{h}\frac{1-\dfrac{\delta E}{E}(n)}{R(n)}, \tag{30}$$

where the dynamic parts comes from the scattering from charged impurities (22), and (28)

$$R=\frac{8s_b}{s_a^3}\left(\frac{T_a}{\epsilon_{fa}}+\frac{T_b}{\epsilon_{fa}}\right)\kappa_a\left(1-\frac{\pi}{4\kappa_a}+\frac{2}{3\kappa_a^2}+o(\kappa_a^{-3})\right),$$

$$\frac{\delta E}{E}=\frac{2s_b}{s_a^2}\left(\frac{T_a}{\epsilon_{fa}}+\frac{T_b}{\epsilon_{fa}}\right)\left(1-[\ln(2\kappa_a)-1]\frac{1}{\kappa_a^2}+o(\kappa_a^{-4})\right). \tag{31}$$

We find that both the relaxation time as well as the relaxation function have a linear temperature dependence for temperatures lower than the Fermi energy. This is in agreement with the experimental and theoretical works mentioned in the Introduction.

A. Relation to weak localization

The low density or weak potential limit (30) is now interesting to discuss. Using Eq. (20) we can rewrite Eq. (30) into a net relaxation effect $[s_a=s_b=2,T_a=T_b]$

$$\begin{aligned}\frac{\sigma}{\sigma_0}&=1-\frac{\delta E}{E}=1-2\frac{T}{\epsilon_f}\left[1+\frac{\tau_0\epsilon_f}{2\hbar}\ln\left(\frac{\tau_0\epsilon_f}{4\hbar}\right)\right]\\ &=1-2\frac{T}{\epsilon_f}-\frac{\hbar}{\tau\epsilon_f}\ln\left(\frac{\hbar}{4\tau T}\right),\end{aligned} \tag{32}$$

where we have introduced the temperature dependent relaxation time $1/\tau=\epsilon_f T\tau_0/\hbar^2=8\epsilon_f\hbar T/me^4s^2$. This is precisely the localization correction to the conductivity.[6,7] Therefore we understand now the physical meaning of the relaxation function at low temperatures. Please note that it vanishes here for small temperatures in contrast to the static result (C14).

B. Comparison with experiments

In order to describe realistic experiments, we extend our model with scattering from static neutral impurities (treated in Appendix C). In addition to Eq. (30) we have static contributions coming from the scattering from neutral impurities (C10),(C14) as

$$R_i=\frac{2^{3/2}s_in_i}{s_an_a}\left(\frac{a_0}{2r_0}\right)^2\left\{1-\frac{\pi}{2\kappa_i}-\frac{\pi^2}{24}\frac{T^2}{e_{fa}^2}\frac{\pi^3}{16\kappa_i}+o(\kappa_i^{-4})\right\},$$

$$\frac{\delta E_i}{\delta E}=\frac{n_i}{n_a}\left(\frac{a_0}{2r_0}\right)^2\left\{1+\frac{4-3\ln 2\kappa_i}{\kappa_i^2}+o(\kappa_i^{-4})\right\}. \tag{33}$$

We can safely use the large κ_a limit since typical densities of Ref. 36 are $7\times10^{10}\ \text{cm}^{-2}$ and we have

$$\kappa_a=\frac{284.9}{\sqrt{n_a/7\times10^{-10}\ \text{cm}^{-2}}}\frac{m^*}{m} \tag{34}$$

which is a large parameter.

Now we have two unknown fit parameters in the theory. These are the energy level $E_b=E_c-E_D$ of the impurities determining the ratio of impurity density to electron densities[37,4]

$$\frac{n_i}{n_a}=n_a\left(\frac{2\pi\hbar^2}{m_aT}\right)e^{\beta E_b} \tag{35}$$

and the ratio of the scattering length to the scattering range of neutral impurities $\zeta=(a_0/r_0)^2\epsilon_{fa}$. From Eq. (30) we can write the final conductivity formula in the form

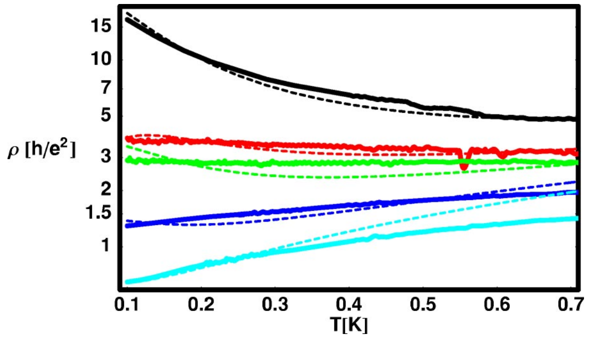


FIG. 2. The conductivity versus temperature according to Eq. (36) as dashed lines. The experimental curves are the solid lines (Ref. 36). From top to bottom they correspond to densities of $n_a = 6.85, 7.17, 7.25, 7.57, 7.85 \times 10^{10}$ cm^{-2}.

$$\frac{\sigma}{\sigma_0} = 1 - \frac{\zeta}{2T} e^{E_b/T} - \frac{T}{2\epsilon_{fa}} + \frac{T}{2\epsilon_{fa}\kappa_a^2} \ln \frac{2\kappa_a}{e},$$

$$\sigma_0 = \frac{e^2}{h}\left[\frac{\zeta}{\sqrt{2}T} e^{E_b/T} + \frac{T}{\epsilon_{fa}}\left(\kappa_a - \frac{\pi}{4}\right)\right], \qquad (36)$$

where we have used the spin degeneracy of the heavy impurities $s_b = 1$ and the temperature $T_b = 0$. The best fits to the experimental results[36] are plotted in Fig. 2.

We see in Fig. 2 a clear insulator to metal transition for low temperatures when the density is increased very slightly. The fitting formula (36) works quite well at all experimental densities for low temperatures, however, the formula fails for higher temperatures. This is because we used the low temperature Sommerfeld expansion and the Fermi energy is 1.9 K$\times n_a / 7 \times 10^{10}$ cm^{-2} in this case, such that at 0.8 K we expect deviations from the leading low-temperature behavior. Despite this imperfect agreement with the data, it is quite satisfying that the metal to insulating transition can be described completely by the scattering with charged and neutral donor impurities supplemented by a mass action law. This strongly favors the Mott-Hubbard transition picture.

The best fit parameter are shown in Fig. 3. We see that the effective binding energy of the electrons to the impurities as well as the scattering strength decrease rapidly when passing through the critical density. This is in agreement with the picture that we have pressure ionization, i.e., a crossing between continuum and bound state levels. Therefore, this underlines the Hubbard-Mott transition which we have seen already in the increase of the effective mass[3] as described in Ref. 4.

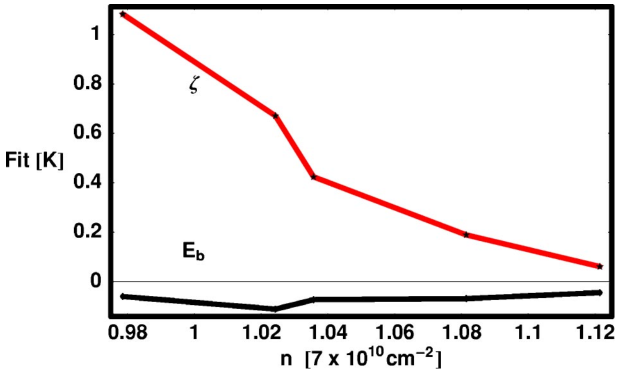


FIG. 3. The fit parameter E_b and ζ from Eq. (36) versus density.

A remark concerning the formula (36) should be made here. Instead of the trapping at charged impurities resulting in the mass action law (35), one could assume in principle any other trapping mechanism. In particular, we have shown that three-particle bound states can describe the experimental data as well.[38] The underlying conductivity formula is precisely Eq. (36) where the number of three-particle bound states diminishes the charged impurity density correspondingly. So the basic transport mechanisms outlined in this paper here remain the same. The only mechanism that is not possible to extract so far is that of actual trapping.

IV. SUMMARY AND OUTLOOK

Linearizing the Lenard-Balescu collision integral including all external field dependences allows one to derive a conductivity on the level of an infinite series of diagrams including maximally crossed lines. The field dependence of the collision integral yields an interference effect which is shown to describe just the weak localization corrections. This has the same formal origin as the Debye-Onsager relaxation effect in plasma and electrolyte systems.

For the low-temperature regime, it was possible to calculate the conductivity analytically. It was found that the dynamical screening by charged impurities leads to a linear temperature dependence of the conductivity, while neutral impurities give rise to the usual Fermi liquid behavior. This finding is general for any scattering of light particles from heavy particles. Therefore it might also be of use for scattering rates in high-T_c cuprates.[39]

The comparison with experiment is performed assuming an appropriate mass action law between the charged donor impurities and the neutral ones which are considered to be captured electrons. The experimental metal-insulator transition can be described quantitatively by fitting the effective binding energy and the unknown scattering strength.

As noted, a similar quality of description of the experimental data was achieved assuming a three particle clustering instead of trapping of electrons. The latter process, however, relies on the same transport picture as outlined in this paper. Only the composition of neutral and charged impurities is determined differently. So far we cannot determine which process is actually happening. In order to achieve this we must study the magnetic field dependence, which shows quite unique and remarkable features in the experiment. This is left for further work. To conclude, we suggest that the metal insulator transition found in experiments can be described within a Mott-Hubbard transition scenario in agreement with the effective mass measurements.[4]

ACKNOWLEDGMENTS

Many discussions with Peter Fulde, James Peter Hague, Rajesh Narayanan, Debanand Sa, and Nicolas Shannon are gratefully acknowledged. To Enver Nakhmedov I am espe-

cially indebted for suggesting to me the problem of metal to insulator transition.

APPENDIX A: VARIATIONAL TECHNIQUE OF LINEAR RESPONSE

Assuming in addition to the interaction potential $V_{11'}$ a coupling of an external potential $U_{11'}$ where the numbers sign cumulative indices such as space time, etc., coordinates, we can express the two particle Green function $G_{121'2'}=1/i^2\langle Ta_1a_2a_2^+a_1^+\rangle$ by a variation of the one particle Green function $G_{12}=1/i\langle Ta_1a_2^+\rangle$ with respect to the external potential[40,41] as

$$G_{121'2'}=G_{11'}G_{22'}\mp\frac{\delta G_{11'}}{\delta U_{2'2}},\tag{A1}$$

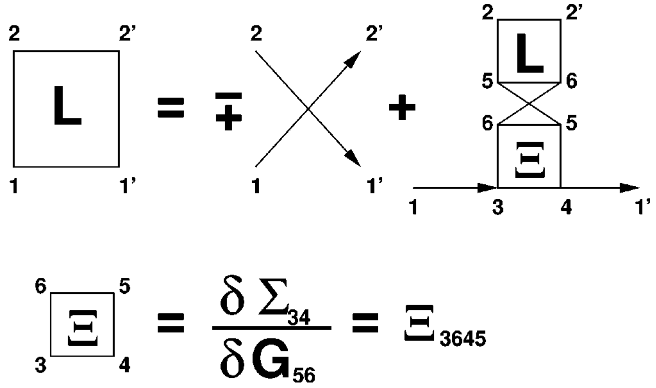
where the upper sign denotes the Fermi and the lower the Bose functions. Using the Dyson equation

$$G^{-1}=G_0^{-1}-\Sigma-U\tag{A2}$$

we can calculate the derivative in Eq. (A1) and with the help of the chain rule and $\delta G=-G\,\delta G^{-1}G$, one can express the fluctuation function as

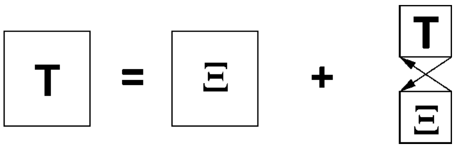
$$\begin{aligned}L_{121'2'}&=G_{121'2'}-G_{11'}G_{22'}\\&=\mp G_{12'}G_{21'}\mp G_{13}\frac{\delta\Sigma_{34}}{\delta U_{2'2}}G_{41'}\\&=\mp G_{12'}G_{21'}+G_{13}\frac{\delta\Sigma_{34}}{\delta G_{56}}L_{5262'}G_{41'}\,.\end{aligned}\tag{A3}$$

Double occurring indices are understood as integrated over. With the definition of the occurring vertex function we can express this graphically:



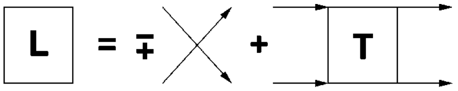
(A4)

Sometimes it is of advantage to express this density fluctuation function by the T matrix. Defining



(A5)

we can express



(A6)

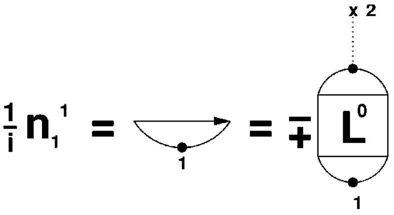
With the help of Eq. (A1), the density response to an external potential can be expressed in terms of the density fluctuation function L of Eq. (A3). Therefore we remark that the density is given by $iG_{11^+}=\langle a_1^+a_1\rangle=\langle\hat n_1\rangle=n_1$ and we have from Eqs. (A1) and (A3) for the response function χ

$$\chi_{12}=\frac{\delta n_1}{\delta U_{22}}=\mp iL_{121^+2^+}=\pm i\langle(\hat n_1-n_1)(\hat n_2-n_2)\rangle.\tag{A7}$$

The last identity follows from the definition of L and underlines the names density fluctuation function. We see now that the linear density variation due to an external potential can be expressed as

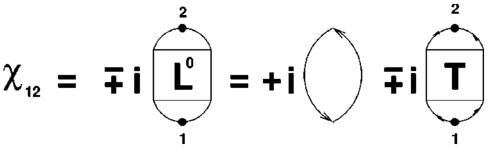
$$\frac{1}{i}n_1^1=\mp L^0_{121^+2^+}U_2=\frac{1}{i}\chi_{12}U_{22},\tag{A8}$$

where the upper index indicates the order of external field dependence. Graphically we can express it as



(A9)

where we will design the external field as a dotted line ending with a cross. According to Eqs. (A8) and (A9) we can express the first order response function as



(A10)

where we have used the definition of the T matrix in the u channel [Eq. (A5)]. Equations (A4) and (A10) constitute the bases for approximation used in this paper.

APPENDIX B: DYNAMICAL SCREENED APPROXIMATION IN NONEQUILIBRIUM

The self-energy is given in terms of the dynamical potential $\mathcal{V}$ according to Fig. 1

$$\Sigma_a^<(\mathbf{k},t,t')=\int\frac{d\mathbf{q}}{(2\pi\hbar)^2}\mathcal{V}_{aa}^<(\mathbf{q},t,t')G_a^<(\mathbf{k}-\mathbf{q},t,t'),\tag{B1}$$

where the dynamical potential is expressed within Coulomb potentials $V_{ab}(\mathbf{q})$

$$\mathcal{V}^{<}_{aa}(\mathbf{q},t,t')=\sum_{dc} V_{ad}(\mathbf{q})\mathcal{L}^{<}_{dc}(\mathbf{q},t,t')V_{ca}(\mathbf{q}) \tag{B2}$$

via the density-density fluctuation

$$\mathcal{L}^{<}_{ab}(\mathbf{q},t,t') =\delta_{ab}\int d\bar{t}\,d\bar{\bar{t}}\,(\mathcal{E}^{r})^{-1}(\mathbf{q},t,\bar{t})\Pi^{<}_{aa}(\mathbf{q},\bar{t},\bar{\bar{t}})(\mathcal{E}^{a})^{-1}(\mathbf{q},\bar{\bar{t}},t'). \tag{B3}$$

Here Π is the free density fluctuation or polarization function

$$\Pi^{<}_{aa}(\mathbf{q},t,t')=\int \frac{d\mathbf{p}}{(2\pi\hbar)^2}G^{<}_{a}(\mathbf{p},t,t')G^{>}_{a}(\mathbf{p}-\mathbf{q},t',t) \tag{B4}$$

and $\mathcal{E}^{r/a}$ the retarded/ advanced dielectric function

$$\mathcal{E}^{r/a}(\mathbf{q},t,t')=\delta(t-t')\pm i\Theta[\pm(t-t')]\sum_{b} V_{bb}(\mathbf{q})\times[\Pi^{>}(\mathbf{q},t,t')-\Pi^{<}(\mathbf{q},t,t')]. \tag{B5}$$

One easily convince oneself that this set of equations (B1)–(B5) is gauge invariant. The correlation or Green's function can be related to the Wigner distribution f_a by the GKB ansatz[42]

$$G^{<}(\mathbf{k},\tau,t)=\exp\left\{-\frac{i}{\hbar}\left(\epsilon_k\tau+\frac{e^2E^2}{24m}\tau^3\right)\right\}\times f\left(\mathbf{k}-\frac{e\mathbf{E}|\tau|}{2},t-\frac{|\tau|}{2}\right) \tag{B6}$$

and analogously for $G^{>}$ by replacing $f\leftrightarrow(1-f)$.

With the help of the gauge invariant formulation of Green's function, we can write the kinetic equation for the Wigner function $f(\mathbf{p},t)=G^{<}(\mathbf{p,R},t,\tau=0)$ finally[35]

$$\begin{aligned}\frac{\partial}{\partial t}f(\mathbf{k},t)+e\mathbf{E}\cdot\nabla_{\mathbf{k}}f(\mathbf{k},t) &=\int_0^{t-t_0}d\tau\Bigg[\left\{G^{>}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,\tau,t-\frac{\tau}{2}\right),\right.\\ &\left.\Sigma^{<}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,-\tau,t-\frac{\tau}{2}\right)\right\}_{+}\\ &-\left\{G^{<}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,\tau,t-\frac{\tau}{2}\right),\right.\\ &\left.\Sigma^{>}\left(\mathbf{k}-\frac{e\mathbf{E}}{2}\tau,-\tau,t-\frac{\tau}{2}\right)\right\}_{+}\Bigg].\end{aligned} \tag{B7}$$

This kinetic equation is exact in time convolutions. This is necessary because gradient expansions in time are connected with linearization in electric fields and consequently fail.[43] The gradient approximation in space has been applied assuming slow varying processes in space and we have dropped all R dependence for simplicity. Introducing Eq. (B1) into the equation for the Wigner function (B7) one obtains the kinetic equation (9) with the explicit form of collision integral (10).

APPENDIX C: STATICALLY SCREENED OR FINITE RANGE IMPURITY SCATTERING

Using the static approximation for the dielectric function $\mathcal{E}(\mathbf{q},0,t)$ in Eq. (10), the kinetic equation for statically screened Coulomb potentials in high electric fields appears[35,44]

$$\frac{\partial}{\partial t}f_a+e\mathbf{E}\partial\mathbf{k}_a f_a=\sum_b I_{ab}$$

$$\begin{aligned}I_{ab}=\frac{2s_b}{\hbar^2}\int \frac{d\mathbf{k}'_a d\mathbf{k}_b d\mathbf{k}'_b}{(2\pi\hbar)^4}\delta(\mathbf{k}_a+\mathbf{k}_b-\mathbf{k}'_a-\mathbf{k}'_b)V_s^2(\mathbf{k}_a-\mathbf{k}'_a,t)\\ \times\int_0^{\infty}d\tau\cos\left\{(\epsilon_a+\epsilon_b-\epsilon'_a-\epsilon'_b)\frac{\tau}{\hbar}\right.\\ \left.-\frac{\mathbf{E}\tau^2}{2\hbar}\left(\frac{e_a\mathbf{k}_a}{m_a}+\frac{e_b\mathbf{k}_b}{m_b}-\frac{e_a\mathbf{k}'_a}{m_a}-\frac{e_b\mathbf{k}'_b}{m_b}\right)\right\}\\ \times\{f_{a'}f_{b'}(1-f_a)(1-f_b)-f_af_b(1-f_{a'})(1-f_{b'})\}\end{aligned} \tag{C1}$$

describing the scattering of particles a (electrons) with other species b with the distribution function $f_b=f_b(k_b-e_bE\tau,T-\tau)$. The potential turns out to be the static Debye one

$$V_s(q)=\frac{2\pi e_a e_b\hbar}{(q+\hbar\kappa)} \tag{C2}$$

with the static screening length κ given by

$$\kappa=\sum_c 2\pi e_c^2\partial_\mu n_c \tag{C3}$$

and the chemical potential μ.

We will now use this statically screened result in order to describe the scattering neutral impurities if we use the range of potential $r_0=1/\kappa$ and replace the charges by the scattering strength $g_{ab}=e_ae_b$.

The calculation of the impurity scattering in quasi two dimensions is now analogously to the Brooks-Hearing result for three dimensions and starts from the Born collision integral (C1) which takes for infinite heavy ions $[m_b/m_a\to\infty]$

$$\begin{aligned} I_{ab}(k_a)=&\frac{2s_b}{\hbar^2}\int\frac{d\mathbf{k}_a' d\mathbf{k}_b d\mathbf{k}_b'}{(2\pi\hbar)^4}\delta(\mathbf{k}_b-\mathbf{k}_b')V_s^2(\mathbf{k}_a-\mathbf{k}_a',t)\\ &\times\int_0^\infty d\tau\cos\left\{(\epsilon_a-\epsilon_a')\frac{\tau}{\hbar}-\frac{\mathbf{E}\tau^2}{2\hbar}\left(\frac{e_a\mathbf{k}_a}{m_a}-\frac{e_a\mathbf{k}_a'}{m_a}\right)\right\}\\ &\times f_b\{f_{a'}-f_a\}. \end{aligned} \tag{C4}$$

We assume parabolic bands $\epsilon=k^2/2m$.

The relaxation function $\delta E/E$ would correspond to linearization of the cos function in Eq. (C4) with respect to the field while the relaxation time is obtained taking into account the linearization with respect to the momentum p_a of the displaced distributions (3). Cross terms such as p_aE are already of second order response. The result can be written in the concise form (8).

1. Relaxation time by neutral impurity scattering

In the following we give an explicit calculation. Employing the Yukawa or Debye potential (C2) one obtains for the relaxation part

$$I_{ab}^R(k_a)=-p_a\frac{\partial f_0}{\partial_{k_a}}\frac{2\pi m_a}{\hbar}g_{ab}^2\int_0^\infty d\alpha\frac{\cos(\alpha-\phi)-\cos\phi}{\left(2k_a\left|\sin\frac{\alpha}{2}\right|+\hbar\kappa\right)^2} \tag{C5}$$

with the angle between k_a and the field direction p_a denoted by ϕ.

The current relaxation time is now obtained by

$$n_a\mathbf{p}_a\tau_{ei}^{-1}=\int\frac{dk_a}{(2\pi\hbar)^2}\mathbf{k}_aI_{ab}(k_a) \tag{C6}$$

from which one gets $[\kappa_p=\hbar/2r_0p]$

$$\tau_i^{-1}=\frac{m_ag_{ab}^2}{2^{3/2}\hbar^3}\frac{n_bs_b}{n_a}\int_0^\infty dp\,\partial_pf_0\left[\frac{1}{\kappa_p^2-1}+\frac{\ln\left(\frac{1+\sqrt{1-\kappa_p^2}}{\kappa_p}\right)}{(\kappa_p^2-1)^{3/2}}\right]. \tag{C7}$$

This current relaxation time in the low-temperature Sommerfeld expansion leads to $[\kappa_i=\hbar/2r_0p_{fa}]$

$$\begin{aligned} \tau_i^{-1}=&\frac{m_ag_{ab}^2}{2^{3/2}\hbar^3}\frac{n_bs_b}{n_a}\left(\frac{1}{\kappa_i^2-1}+\frac{\ln\left(\frac{1+\sqrt{1-\kappa_i^2}}{\kappa_i}\right)}{(\kappa_i^2-1)^{3/2}}\right.\\ &\left.+\frac{\pi^2}{24}\frac{T^2}{\epsilon_f^2}\partial_{\kappa_i}^2\left[\frac{1}{\kappa_i^2-1}+\kappa_i^4\frac{\ln\left(\frac{1+\sqrt{1-\kappa_i^2}}{\kappa_i}\right)}{(\kappa_i^2-1)^{3/2}}\right]\right). \end{aligned} \tag{C8}$$

The second temperature correction is negative and diminishes the positive first part. As long as $T<\epsilon_f$ the net relaxation time is positive and continously falling to zero for $\kappa_i\to\infty$.

The high density expansion reads

$$\begin{aligned} R_i=\tau_i^{-1}\frac{m_a}{n_ae_a^2}\frac{e_a^2}{h}=&\frac{2^{3/2}s_bn_b}{s_an_a}\left(\frac{m_ag_{ab}r_0}{\hbar^2}\right)^2\kappa_i'^2(1+\sqrt{\xi})^2\\ &\times\left\{-1-\ln\frac{\kappa_i}{2}-\left(5+6\ln\frac{\kappa_i}{2}\right)\frac{\kappa_i^2}{4}\right.\\ &\left.-\frac{\pi^2}{24}\frac{T^2}{e_f^2}\left[2+\left(19+12\ln\frac{\kappa_i}{2}\right)\kappa_i^2\right]+o(\kappa_i^3)\right\} \end{aligned} \tag{C9}$$

and the low density expansion or short range expansion

$$\begin{aligned} R_i=&\frac{2^{3/2}s_bn_b}{s_an_a}\left(\frac{m_ag_{ab}r_0}{\hbar^2}\right)^2\kappa_i'^2(1+\sqrt{\xi})^2\\ &\times\left\{\frac{1}{\kappa_i^2}-\frac{\pi}{2\kappa_i^3}-\frac{\pi^2}{24}\frac{T^2}{\epsilon_f^2}\frac{\pi^3}{16\kappa_i^3}+o(\kappa_i^{-4})\right\}. \end{aligned} \tag{C10}$$

2. Relaxation effect by neutral impurity scattering

The relaxation function is now obtained if we linearize Eq. (C4) with respect to the external field. We obtain

$$\begin{aligned} \int&\frac{dk_a}{(2\pi\hbar)^2}\mathbf{k}_aI_{ab}^{\delta E}(k_a)\\ &=-\mathbf{E}\frac{n_ie}{m\hbar^3}\int\frac{dkdq}{(2\pi\hbar)^4}f_0(\epsilon_k)V_s^2(q)q^2\\ &\qquad\times\cos^2(q,E)\int_0^\infty d\tau\tau^2\sin\left(\frac{q^2+2\mathbf{k}\cdot\mathbf{q}}{2m\hbar}\tau\right) \end{aligned} \tag{C11}$$

from which one gets

$$\begin{aligned} \frac{\delta E}{E}&=\frac{n_i}{4\pi\hbar^5mn_a}\int_0^\infty dqq^3V_s^2(q)\int_0^\infty d\tau\tau^2I_s\\ &=\frac{n_i}{n_a}\kappa_a^2\left(\frac{mg_{ab}r_0}{\hbar^2}\right)^2\int_0^\infty\frac{dy}{(y+\kappa_a)^2}\partial_y^2\sqrt{y^2-1}\Theta(y-1), \end{aligned} \tag{C12}$$

where we have used Eq. (F9) for the last line. Employing the same regularization due to the principal value (25) we end up with

$$\frac{\delta E}{E}=\frac{n_i}{n_a}\left(\frac{mg_{ab}r_0}{\hbar^2}\right)^2\left\{\frac{2\kappa_a^2+\kappa_a^4}{(\kappa_a^2-1)^2}+\frac{3\kappa_a^3\ln(\kappa_a-\sqrt{\kappa_a^2-1})}{(\kappa_a^2-1)^{5/2}}\right\}. \tag{C13}$$

The T^2 dependent term could be given analogously. The needed low density or short range expansion now reads

$$\frac{\delta E_i}{E}=\frac{n_i}{n_a}\left(\frac{m g_{ab} r_0}{\hbar^2}\right)^2\left\{1+\frac{4-3\,\ln 2\,\kappa_a}{\kappa_a^2}\right\}+o(\kappa^{-4}). \tag{C14}$$

APPENDIX D: POLARIZATION FUNCTION IN 2D

Here we discuss the properties of low-temperature polarization function

$$\Pi(\omega,q)=\int\frac{dp}{(2\pi\hbar)^2}\times\frac{f[(p+q/2)^2/2m]-f[(p-q/2)^2/2m]}{\frac{p\cdot q}{m}-\omega-i0}. \tag{D1}$$

The imaginary part is easily rewritten as

$$\begin{aligned}\mathrm{Im}\Pi(\omega,q)&=\pi\int_0^\infty\frac{dpp}{(2\pi\hbar)^2}f\left(\frac{p^2}{2m}\right)\int_0^{2\pi}d\phi\\&\quad\times\left\{\delta\left(\frac{pq}{m}-\frac{q^2}{2m}-\omega\right)-\delta\left(\frac{pq}{m}+\frac{q^2}{2m}-\omega\right)\right\}\\&=\frac{m^{3/2}}{2^{3/2}\pi\hbar^2 q}\int_0^\infty\frac{d\epsilon}{\sqrt{\epsilon}}\left[f\left(\epsilon+\frac{(q/2+m\omega\hbar/q)^2}{2m}\right)\right.\\&\quad\left.-f\left(\epsilon+\frac{(q/2-m\omega\hbar/q)^2}{2m}\right)\right].\end{aligned} \tag{D2}$$

The energy shifts in the distribution function we absorb into an effective chemical potential which should be positive in order to obtain nonzero contribution at Sommerfeld expansion

$$\mu_{\mathrm{eff}}=\epsilon_f-\frac{\left(\pm\frac{q}{2}+\frac{m\omega\hbar}{q}\right)^2}{2m}\geqslant 0. \tag{D3}$$

The low-temperature Sommerfeld expansion reads than $f_0(\epsilon)=n[(\epsilon-\mu_{\mathrm{eff}})/T]$

$$\begin{aligned}\int_0^\infty\frac{d\epsilon}{\sqrt{\epsilon}}n\left(\frac{\epsilon-\mu_{\mathrm{eff}}}{T}\right)&=2\int_{\mu_{\mathrm{eff}}/T}^\infty dx n(x)\partial_x\sqrt{Tx+\mu_{\mathrm{eff}}}\\&=2\int_{\mu_{\mathrm{eff}}/T}^\infty dx n(x)[1-n(x)]\sqrt{Tx+\mu_{\mathrm{eff}}}\\&=2\sqrt{\mu_{\mathrm{eff}}}\left(1-\frac{\pi^2T^2}{24\mu_{\mathrm{eff}}}\right)\\&=2\left(1-\frac{\pi^2T^2}{12}\frac{\partial}{\partial\epsilon_f}\right)\sqrt{\mu_{\mathrm{eff}}}.\end{aligned} \tag{D4}$$

Therefore it is enough to know the zero temperature result since the T^2 correction are given simply by derivatives with respect to ϵ_f.

Introducing dimensionless coordinates as in[28]

$$x=\frac{q}{2p_f},\quad x_0=\frac{\hbar\omega}{4\epsilon_f}, \tag{D5}$$

we get finally for the imaginary part of the polarization function

$$\begin{aligned}\mathrm{Im}\Pi(\omega,q)&=\frac{m}{4\pi\hbar^2 x}\left\{\Theta(x-|x_0+x^2|)\sqrt{1-\left(x+\frac{x_0}{x}\right)^2}\right.\\&\quad\left.-\Theta(x-|x_0-x^2|)\sqrt{1-\left(x-\frac{x_0}{x}\right)^2}\right\}\end{aligned} \tag{D6}$$

which is of course the result given in Ref. 28. The corresponding real part is given by the Hilbert transform according to Eq. (D1)

$$\mathrm{Re}\Pi(\omega,q)=-2\int\frac{d\omega'}{2\pi}\frac{\mathrm{Im}\Pi(\omega',q)}{\omega-\omega'}. \tag{D7}$$

Using the integral

$$F(a)=\int_{-1}^1\sqrt{\frac{1-z^2}{a-z}}=\pi\begin{cases}a & 1\geqslant|a|\\ a-\mathrm{sgn}(a)\sqrt{a^2-1}\end{cases} \tag{D8}$$

we obtain

$$\mathrm{Re}\Pi=-\frac{m}{4\pi^2\hbar^2 x}\left\{F\left(\frac{x_0}{x}+x\right)-F\left(\frac{x_0}{x}-x\right)\right\}. \tag{D9}$$

The real and imaginary part is plotted in Fig. 4.

APPENDIX E: INTEGRALS OVER DIELECTRIC FUNCTIONS

In order to perform the frequency integration in (9) we use a very useful relation, which has been given in[45]

$$\int\frac{d\omega}{2\pi}\frac{H(\omega)}{\omega}\mathrm{Im}\mathcal{E}^{-1}(q,\omega)=\frac{H(0)}{2}\mathrm{Re}\left(1-\frac{1}{\mathcal{E}(q,0)}\right). \tag{E1}$$

For the integration of Eq. (9) we set $H(\omega)=\cos(\omega\tau+A)\cos(\omega\tau'+B)\omega/\mathrm{Im}\mathcal{E}$ with A and B are the remaining content of the cosine functions of Eqs. (10) and (11).

Lets first prove the relation (E1). We consider the following integral including the dielectric function

$$\begin{aligned}I&=\int\frac{d\omega}{2\pi}\frac{H(\omega)}{\omega}\mathrm{Im}\mathcal{E}^{-1}(\omega)\\&=\int\frac{d\omega}{4\pi i}\left(\frac{1}{\omega+i\eta}+\frac{1}{\omega-i\eta}\right)H(\omega)(f^--f^+),\end{aligned} \tag{E2}$$

where $f^+=1-1/\mathcal{E}$ and $f^-=(f^+)^*$. In the following we will assume that the function $H(\omega)$ is analytical. Since $f^\pm(\omega)$ has no poles in the lower/upper half plane and vanishes with $\sim\omega^{-2}$ for large ω we have the identity

$$\int\frac{d\omega}{2\pi i}H(\omega)\frac{f^\pm(\omega)}{(\omega\pm i\eta)}=\mp f^\pm(0)H(0) \tag{E3}$$

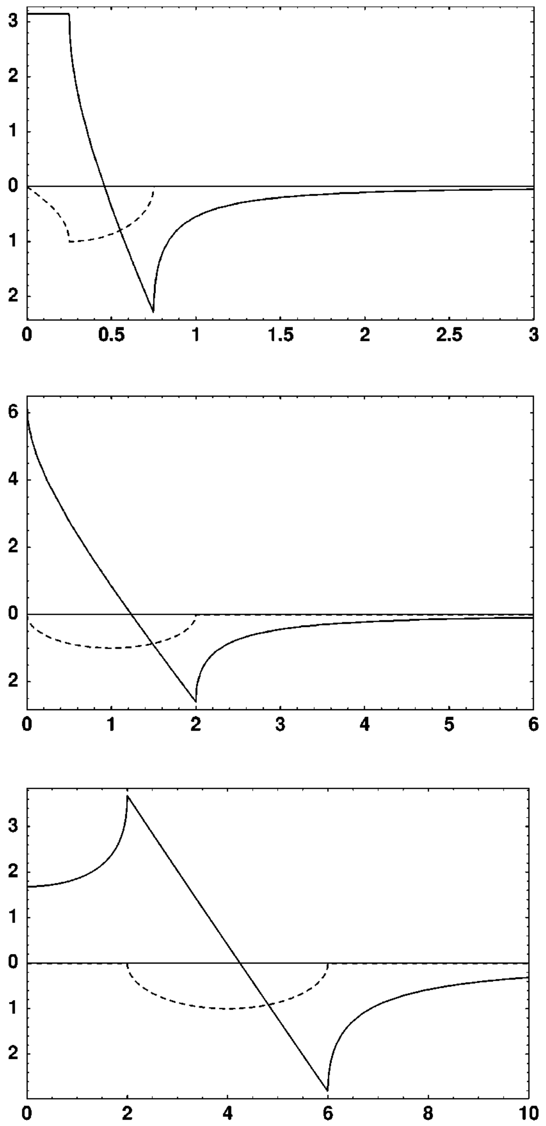


FIG. 4. The real (solid) and imaginary (dashed) part of the polarization $4\pi\hbar^2/m\times\Pi$ versus x_0 for $x=0.5$ (above), $x=1$ (middle) and $x=2$ (below) according to Eq. (D5).

and all other combinations of signs vanish. With the help of the relation (E3) we compute easily for Eq. (E2)

$$I=\frac{1}{2}H(0)\mathrm{Re}\left(1-\frac{1}{\mathcal{E}(0)}\right) \tag{E4}$$

which proves relation (E1).

1. Regularization of integration

We are now going to give explicit forms including the dielectric function $\mathcal{E}=1-V(q)\Pi(q,\omega)$ where the polarization function Π was discussed in the previous chapter. The forms appearing throughout the paper are

$$I=\int\frac{d\omega}{2\pi}\frac{h(\omega)}{|\mathcal{E}(\omega)|^2}=-\int\frac{d\omega}{2\pi}\frac{\omega h(\omega)/\mathrm{Im}\mathcal{E}(\omega)}{\omega}\mathrm{Im}\mathcal{E}^{-1}(\omega) \tag{E5}$$

such that we can apply Eq. (E4) with $H(\omega)=\omega h(\omega)/\mathrm{Im}\mathcal{E}(\omega)$. In the case where $\mathrm{Im}\mathcal{E}=0$ appears an ambiguity which we have to remove. We add an infinitesimal small classical process to the particle-hole fluctuation which modifies the polarization in the following way:

$$\delta\mathrm{Im}\Pi=\eta\omega e^{-c\omega^2}\approx\eta[\omega+o(\omega^3)]. \tag{E6}$$

This will make the imaginary part of the polarization nonzero everywhere and will not introduce additional poles in the upper half plane. Therefore we can apply the integration (E4). The corresponding real part according to Eq. (E4) will be $\propto\eta$ and drop out in the final form (E4).

2. Specific forms

Now we write down the required forms for Eq. (E4). One gets the static result from Eqs. (D6) and (D9) at low temperatures[28]

$$\Pi(0,q)=-\frac{\partial}{\partial\mu}n\begin{cases}1, & q<2p_f,\\[4pt] 1-\sqrt{1-\left(\dfrac{2p_f}{q}\right)^2}, & q>2p_f\end{cases} \tag{E7}$$

with the chemical potential μ and from Eq. (D2)

$$\mathrm{Im}\Pi(\omega,q)=-\frac{m^2\omega\hbar\Theta(2p_f-q)}{\pi\hbar^2q\sqrt{4p_f^2-q^2}}\left(1+\frac{8\pi^2T^2m^2}{(4p_f^2-q^2)^2}\right)+o(\omega^2,T^4). \tag{E8}$$

The region where $\mathrm{Im}\Pi\neq0$ correspond exactly to the upper case of Eq. (E7). Using this expansion we obtain

$$H(0)=\lim_{\omega\to0}\frac{\hbar\omega}{\mathrm{Im}\mathcal{E}^R}=-\frac{\hbar q^2}{\sum_b\dfrac{m_b^2e_b^2}{\sqrt{4p_{fb}^2-q^2}}\left(1+\dfrac{8\pi^2m_b^2T_b^2}{(4p_{fb}^2-q^2)^2}\right)}, \tag{E9}$$

where we have to keep in mind that the above procedure in calculating the frequency integral works only for finite $\mathrm{Im}\Pi$ or finite particle-hole fluctuations. According to Eq. (D2) this restricts the later q integration to values smaller than $2p_{fb}$, respectively.

We get finally for Eq. (E1)

$$\begin{aligned}W(q)&=V(q)^2\frac{1}{2}\lim_{\omega\to0}\frac{\hbar\omega}{\mathrm{Im}\mathcal{E}^R}\mathrm{Re}\left(1-\frac{1}{\mathcal{E}^R(0,q)}\right)\\&=-\frac{2\pi e_a^2e_b^2\hbar^4\kappa(q)}{q+\hbar\kappa(q)}\\&\quad\times\frac{1}{\sum_b\dfrac{m_b^2e_b^2}{\sqrt{4p_{fb}^2-q^2}}\left(1+\dfrac{8\pi^2m_b^2T_b^2}{(4p_{fb}^2-q^2)^2}\right)},\end{aligned} \tag{E10}$$

where the screening length is from Eq. (E7)

$$\kappa=\sum_b 2\pi e_b^2\partial_\mu n_b\begin{cases}1, & q<2p_{fb},\\ 1-\sqrt{1-\left(\dfrac{2p_{fb}}{q}\right)^2}, & q>2p_{fb}\end{cases} \tag{E11}$$

and since $n=(s/4\pi\hbar^2)p_f^2+o(\mathrm{e}^{-\epsilon_f/T})$ one has $\partial_\mu n=m_b s_b/2\pi\hbar^2$.

APPENDIX F: LOW-TEMPERATURE EXPANSION OF INTEGRALS

The integrals occurring in Eqs. (12), (14), and (16) will now be calculated. Using $\int_0^\infty d\tau\cos x\tau=\pi\delta(x)$ we can write for Eqs. (B4) and (15)

$$\begin{aligned}-\frac{1}{2}\Pi^<(q,\omega=0)&=\int\frac{dp}{(2\pi\hbar)^2}f_b(p)(1-f_b(p+q))\\&\quad\times\int_0^\infty d\tau'\cos(\epsilon_{p+q}-\epsilon_p)\frac{\tau'}{\hbar}\\&=\int\frac{dp}{(2\pi\hbar)^2}f_b(p)[1-f_b(p)]\pi\hbar\,\delta(\epsilon_{p+q}-\epsilon_p)\\&=\frac{m}{4\pi\hbar q}\int_0^\infty dp f_b(p)(1-f_b(p))\int_{-1}^1\frac{dx}{\sqrt{1-x^2}}\\&\quad\times\left[\delta\left(x+\frac{q}{2p}\right)+\delta\left(x-\frac{q}{2p}\right)\right]\\&=\frac{m_b^2T_b}{\pi\hbar q}\int_{q^2/8m_bT_b-\epsilon_{fb}/T_b}^\infty dx\frac{n(x)[1-n(x)]}{\sqrt{8m_b(T_bx+\epsilon_{fb})-q^2}},\end{aligned}\tag{F1}$$

where we used $x=p^2/2m_bT_b-\epsilon_{fb}/T_b$ and $n(x)=1/(\mathrm{e}^x+1)$ and ϵ_{fb} is the Fermi energy. The last integral is only non-zero for negative lower integration limits implying $2p_{fb}>q$. Expanding the square root in terms of the temperature T_b we obtain finally

$$\begin{aligned}-\frac{1}{2}\Pi^<(q,\omega=0)&=-\frac{1}{2}\Pi^>(q,\omega=0)\\&=\frac{m_b^2T_b\Theta(2p_{fb}-q)}{\pi\hbar q\sqrt{4p_{fb}^2-q^2}}\left(1+\frac{8\pi^2m_b^2T_b^2}{(4p_{fb}^2-q^2)^2}\right).\end{aligned}\tag{F2}$$

The integrals (12), (14) can be tremendously simplified observing that Eq. (16) can be written

$$\begin{aligned}I_s(a,\tau)&=\int dkf_k(1-f_{k-q})\sin(\epsilon_{k-q}-\epsilon_k)\frac{\tau}{\hbar}\\&=\int dk(f_{k-q}-f_k)g_{\epsilon_{k-q}-\epsilon_k}\sin(\epsilon_{k-q}-\epsilon_k)\frac{\tau}{\hbar}\\&=\int dkf_k\left(g_{\epsilon_{k+q}-\epsilon_k}\sin(\epsilon_k-\epsilon_{k+q})\frac{\tau}{\hbar}\right.\\&\quad\left.-g_{\epsilon_k-\epsilon_{k-q}}\sin(\epsilon_{k-q}-\epsilon_k)\frac{\tau}{\hbar}\right)\\&=\int dkf_k\sin(\epsilon_k-\epsilon_{k-q})\frac{\tau}{\hbar}(g_{\epsilon_{k-q}-\epsilon_k}+g_{\epsilon_k-\epsilon_{k-q}})\\&=\int dkf_k\sin(\epsilon_{k-q}-\epsilon_k)\frac{\tau}{\hbar},\end{aligned}\tag{F3}$$

where $g_x=1/(\mathrm{e}^{x/T}-1)$ and we have used $k\to-k$ transformation in coming from the third to the fourth equality.

The time integral (14) can be now represented as a derivative of a δ function with respect to k. A partial integration leads than to

$$\begin{aligned}\int_0^\infty d\tau\tau I_s(a,\tau)&=\int\frac{dk}{(2\pi\hbar)^2}f_k\int_0^\infty d\tau\tau\sin(\epsilon_{k-q}-\epsilon_k)\frac{\tau}{\hbar}\\&=-\frac{m_a^2}{4\pi q^2}\int_{q/2}^\infty\frac{dk}{k}\partial_k[kf_a(k)]\\&\quad\times\int_{-1}^1\frac{dx}{x\sqrt{1-x^2}}\left[\delta\left(x-\frac{q}{2p}\right)-\delta\left(x+\frac{q}{2p}\right)\right]\\&=-\frac{m_a^2}{\pi q^3}\int_{q/2}^\infty dk\partial_k[kf_a(k)]\frac{k}{\sqrt{k^2-\dfrac{q^2}{4}}}\\&=\frac{m_a^{3/2}}{4\sqrt{2}\pi q}\int_{-a/T_a}^\infty dxn(x)[1-n(x)]\\&\quad\times\frac{\Theta(2p_{fa}-q)}{\sqrt{T_ax+a}}\end{aligned}\tag{F4}$$

with $a=\epsilon_{fa}-q^2/8m_a$. Expanding the argument in terms of T_a one arrives analogously to Eq. (F2) at the result

$$\begin{aligned}\int_0^\infty d\tau\tau I_s(a,\tau)&=\frac{m_a^2}{4\pi q\sqrt{4p_{fa}^2-q^2}}\left(1+\frac{8\pi^2m_a^2T_a^2}{(4p_{fa}^2-q^2)^2}\right)\\&\quad\times\Theta(2p_{fa}-q).\end{aligned}\tag{F5}$$

The integral (12) requires some more care. We use the principal value identity $\int_0^\infty d\tau \sin \tau x = \mathcal{P}/x$ and write

$$\int_0^\infty d\tau \tau^2 I_s(a,\tau) = -\frac{m_a^3 \hbar}{\pi^2 q^3}\mathcal{P}\int_0^\infty dk f_a(k) \partial_q^2 \int_0^{2\pi} \frac{d\phi}{\frac{q}{2k} - \cos\phi}. \tag{F6}$$

Using

$$\int_0^{2\pi} \frac{d\phi}{a - \cos\phi} = \frac{2\pi}{\sqrt{a^2-1}}\Theta(a-1) \tag{F7}$$

we can write after one partial integration

$$\int_0^\infty d\tau \tau^2 I_s(a,\tau) = \frac{m_a^3 \hbar \mathcal{P}}{\pi q^3} \partial_q^2 \int_{-\epsilon_{fa}/T}^{q^2/8m_a T_a - \epsilon_{fa}/T_a} dx n(x) \times [1-n(x)] \sqrt{q^2 - 8 m_a T_a x - 4 p_{fa}^2}. \tag{F8}$$

Expanding again the argument in terms of T_a we arrive at

$$\int_0^\infty d\tau \tau^2 I_s(a,\tau) = \frac{m_a^3 \hbar \mathcal{P}}{\pi q^3} \frac{\partial^2}{\partial q^2}\left[\sqrt{q^2 - 4p_{fa}^2} \times \left(1 - \frac{8\pi^2 m_a^2 T_a^2}{3(q^2 - p_{fa}^2)^2}\right)\Theta(q - 2p_{fa})\right]. \tag{F9}$$

---

[1] E. Abrahams, S.V. Kravchenko, and M.P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).

[2] F. Gebhard, *The Mott Metal-Insulator Transition* (Springer, Berlin, 1997).

[3] A.A. Shashkin, S.V. Kravchenko, V.T. Dolgopolov, and T.M. Klapwijk, Phys. Rev. B **66**, 073303 (2002).

[4] K. Morawetz, cond-mat/0210168 (unpublished).

[5] D. Vollhardt and P. Wölfle, Phys. Rev. B **22**, 4666 (1980).

[6] B.L. Altshuler and A.G. Aronov, in *Electron-electron Interactions in Disordered Conductors* (Elsevier Science, New York, 1985).

[7] T.R. Kirkpatrick and D. Belitz, Phys. Rev. B **34**, 2168 (1986).

[8] D. Belitz and T.R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[9] T. Jonckheere *et al.*, Phys. Rev. Lett. **85**, 4269 (2000).

[10] E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[11] S. Chakravarty and A. Schmid, Phys. Rep. **140**, 193 (1986).

[12] T. Dittrich *et al.*, *Quantum Transport and Dissipation* (Wiley-VCH, Weinheim, 1998).

[13] K. Morawetz, Phys. Rev. E **62**, 6135 (2000).

[14] B.B. Kadomtsev, Zh. Éksp. Teor. Fiz. **33**, 151 (1958) [Sov. Phys. JETP **33**, 117 (1958)].

[15] Y.L. Klimontovich and W. Ebeling, Zh. Eksp. Teor. Fiz. **63**, 904 (1972).

[16] W. Ebeling, Ann. Phys. (N.Y.) **33**, 5 (1976).

[17] W. Ebeling and G. Röpke, Ann. Phys. (Leipzig) **36**, 429 (1979).

[18] G. Röpke, Phys. Rev. A **38**, 3001 (1988).

[19] K. Morawetz and D. Kremp, Phys. Lett. A **173**, 317 (1993).

[20] A. Esser and G. Röpke, Phys. Rev. E **58**, 2446 (1998).

[21] P. Debye and E. Hückel, Z. Phys. **15**, 305 (1923).

[22] L. Onsager, Z. Phys. **8**, 277 (1927).

[23] H. Falkenhagen, *Elektrolyte* (S. Hirzel Verlag, Leipzig, 1953).

[24] H. Falkenhagen, W. Ebeling, and W.D. Kraeft, in *Ionic Interaction*, edited by Petrucci (Academic Press, New York, 1971), Chap. 1, p. 1.

[25] D. Kremp, D. Kraeft, and W. Ebeling, Ann. Phys. (Leipzig) **18**, 246 (1966).

[26] K. Morawetz, Plasma Phys. **37**, 195 (1997); **37**, 4(E) (1997).

[27] P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).

[28] C. Hodges, H. Smith, and J.W. Wilkins, Phys. Rev. B **4**, 302 (1971).

[29] F. Stern, Phys. Rev. Lett. **44**, 1469 (1980).

[30] S. DasSarma and E.H. Hwang, Phys. Rev. Lett. **83**, 164 (1999).

[31] S. DasSarma, Phys. Rev. B **33**, 5401 (1986).

[32] A. Gold and V.T. Dolgopolov, Phys. Rev. B **33**, 1076 (1986).

[33] N.H. Kwong and M. Bonitz, Phys. Rev. Lett. **84**, 1768 (2000).

[34] K. Morawetz, Phys. Rev. E **50**, 4625 (1994).

[35] A.P. Jauho and J.W. Wilkins, Phys. Rev. B **29**, 1919 (1984).

[36] S.V. Kravchenko and T.M. Klapwijk, Phys. Rev. Lett. **84**, 2909 (2000).

[37] H. Ibach and H. Lüth, *Solid State Physics: An Introduction to Principles of Materials Science* (Springer Verlag, Berlin, 1996).

[38] E. Nachmedov and K. Morawetz, Phys. Rev. B **66**, 195333 (2002).

[39] N.E. Hussey, Eur. Phys. J. B **31**, 495 (2003).

[40] L.P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).

[41] W.D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, *Quantum Statistics of Charged Particle Systems* (Akademie Verlag, Berlin, 1986).

[42] P. Lipavský, V. Špička, and B. Velický, Phys. Rev. B **34**, 6933 (1986).

[43] G.D. Mahan, Phys. Rep. **145**, 251 (1987).

[44] H. Haug and A.P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer, Berlin, 1996).

[45] Y.L. Klimontovich, *Kinetic Theory of Nonideal Gases and Nonideal Plasmas* (Academic Press, New York, 1975).