t-J ring with an Anderson impurity: A model for a quantum dot

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(Received 19 September 2002; published 19 March 2003)

A quantum wire of finite length L with correlated electrons coupled to a quantum dot is studied. The wire is parametrized in terms of the supersymmetric t-J model and the dot is represented by an Anderson-like impurity. The model is integrable and we discuss the properties of the finite chain by solving the Bethe *Ansatz* equations. For a finite ring the energy spectrum is discrete and has to be obtained numerically. As a function of the coupling of the dot to the wire we discuss the ground state and low-energy excitations, the Aharonov-Bohm oscillations and the discontinuities of the magnetization of the system.

DOI: 10.1103/PhysRevB.67.115113 PACS number(s): 75.20.Hr, 71.27.+a

I. INTRODUCTION

Quantum dots offer the possibility to study the Kondo effect at the level of an *artificial* magnetic impurity,[1] in which the energy levels, the Coulomb interactions, and the contacts to metallic wires (hybridization) can be tuned by external parameters.[2] In the Kondo problem[3,4] the spin of a magnetic impurity is screened by the spins of itinerant electrons at low temperatures leading to a Fermi-liquid fixed point, while for energies much larger than the Kondo temperature the impurity spin remains unscreened.

Anderson's model, on the other hand, describes the local moment formation and the subsequent screening of the spin.[4,5] Due to the hybridization of the localized and conduction electrons, the valence of the impurity is usually noninteger, ranging from close to zero (nonmagnetic impurity), through the crossover region known as the mixed-valence regime, to the magnetic or Kondo case (integer valence).

A finite size ring has a discrete energy excitation spectrum. The gaps separating the low-lying excitations from the ground state determine the exponential activations of the low-temperature thermodynamic properties. As a consequence of the discrete spectrum the magnetization as a function of field has jumps at critical fields that depend on the energy gaps. The thermodynamics of Kondo and Anderson impurities in a finite size host of noninteracting electrons has been studied via the Bethe *Ansatz* in Refs. 6 and 7.

In this paper we consider a *finite* ring with correlated electrons parametrized by the supersymmetric t-J model. In contrast to previous studies,[6,7] the present situation refers to a lattice model of interacting electrons and an Anderson-like impurity can be placed on a link of the chain without destroying the integrability.[8,9] The impurity interacts with both lattice sites joined by the link and can more realistically represent a quantum dot than a Kondo impurity with a contact exchange. The quantum dot corresponds to a hybrid between a dot embedded into the ring and a side-branch to it.[10] Two partial waves (forward and backward moving electrons) are involved in the interaction of the impurity with the host, although the integrability imposes some restrictions on this interaction. The two partial waves allow one to investigate interference effects through the dot, such as Aharonov-Bohm persistent currents, in addition to thermodynamic properties. Due to the finite size of the ring the energy eigenvalues depend on two parameters which can be varied independently, namely, the length of the ring and the Kondo coupling. The energy eigenvalues are always discrete leading to a sawtoothlike pattern of the Aharonov-Bohm oscillations. In the Appendix we discuss the possibility to generate a sinusoidal pattern in addition to the sawtooth.

The remainder of the paper is organized as follows. In Sec. II we present the model. The Bethe *Ansatz* equations diagonalizing the model are introduced in Sec. III. In Sec. IV we study the ground state and the low-energy excitations. In Secs. V and VI we briefly discuss Aharonov-Bohm oscillations and the $T=0$ magnetization as a function of field, respectively. Conclusions follow in Sec. VII.

II. MODEL

In this paper we study a quantum ring of strongly correlated electrons coupled to a quantum dot, represented by an Anderson-like impurity. The correlated electrons are described in terms of the one-dimensional supersymmetric t-J model in a ring configuration of finite length L. The Hamiltonian for such a quantum wire is then (see Ref. 11)

$$H=\sum_i H_{i,i+1}=-t\sum_{i\sigma} P(c^\dagger_{i+1\sigma}c_{i\sigma}+c^\dagger_{i\sigma}c_{i+1\sigma})P + J\sum_{i\sigma\sigma'}(\mathbf{S}_i\cdot\mathbf{S}_{i+1}-\tfrac{1}{4}n_in_{i+1}), \qquad (1)$$

where $c^\dagger_{i\sigma}$ ($c_{i\sigma}$) create (annihilate) an electron of spin σ at the site i, P is a projector that prevents the double occupancy of every site, and $\mathbf{S}_i=\Sigma_{\sigma\sigma'}c^\dagger_{i\sigma}\mathbf{s}_{\sigma\sigma'}c_{i\sigma'}$ and $n_i=\Sigma_\sigma c^\dagger_{i\sigma}c_{i\sigma}$ are the spin and particle number operators at the site i. The model is integrable for $t=2J$ for all band fillings.[12–14] In order to close the ring we consider the site $L+1$ identical to the first site, i.e., the ring consists of L sites.

The impurity is introduced without destroying the integrability of the model.[8,9] The impurity is placed on a link, e.g.,

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the one joining the sites 1 and L, and interacts with both of these sites. The general form of the impurity Hamiltonian is[15,16]

$$H_{\rm imp}=(1+\epsilon^2)^{-1}(H_{L,{\rm imp}}+H_{{\rm imp},1}+\epsilon^2 H_{1,L} + i\epsilon[H_{{\rm imp},L}+H_{1,{\rm imp}},H_{1,L}]), \quad (2)$$

where the square bracket denotes a commutator and ϵ is a parameter. Note that here a spin 1/2 impurity is assumed and $H_{{\rm imp},i}$ has the same form as $H_{i,i+1}$, i.e., it is a graded permutator consisting of fermionic and bosonic components. The integrability of this impurity model as a function of ϵ has been discussed in several papers.[8,9,15,16] The impurity can as well be generalized to arbitrary spin S, but here we limit ouselves to discuss the $S=1/2$ case. Note that the impurity interacts with two sites with both partial waves (right and left moving particles).

The hybridization impurity partially localizes an itinerant electron. The valence of the quantum dot then depends on the parameter ϵ, the number of electrons in the system N_e and the length of the ring L. For $S=1/2$ and $\epsilon=0$, $H_{\rm imp}$ just corresponds to adding one more site to the host (between sites L and 1). For general ϵ the three-site terms of the impurity Hamiltonian violate the T and P symmetries separately, while their product PT is of course invariant. The three-site terms can be avoided by placing the quantum dot at the open end of the host chain.[15–17] This considerably simplifies the impurity Hamiltonian, since one of the neighboring host sites is absent.

Finite size effects of correlated electron hosts containing impurities have been studied before,[16] but always for a very large system, where the mesoscopic contributions are of the order of L^{-1}. The mesoscopic contributions describe the long wavelength excitations about the Fermi points which constitute the conformal towers. These finite size corrections to the energy yield the pattern of Aharonov-Bohm-Casher oscillations and the asymptotic of correlation functions for long times and/or long distances. In this paper we consider a quantum correlated electron ring that is sufficiently short such that the individual energy eigenvalues are discrete and cannot be represented by a pseudocontinuous variable. The energy levels have then to be determined numerically.

Previous investigations of small size systems of other models within the framework of Bethe's *Ansatz* are the following: (1) A thorough study of short Heisenberg chains by comparing exact diagonalizations with the Bethe *Ansatz* was performed by Karbach and Müller,[18] (2) all the energy levels and the thermodynamics for an exchange Kondo impurity placed at the center of a small metallic sphere with three and five electrons in s states were obtained in Ref. 6, and (3) the charge and spin gaps for an Anderson impurity in a small metallic sphere were also studied.[7] In contrast to these previous studies we now consider a correlated electron host and an impurity interacting with two (rather than one) partial waves. Impurities in small boxes have also been investigated with approximate methods, e.g., within the noncrossing diagram approximation the Kondo resonance of an Anderson impurity in an ultrasmall metallic grain was found to be strongly dependent on the relative magnitude of T_K and the mean level spacing in the grain and the parity of the number of electrons.[19] An Anderson impurity like model in a finite size system was also studied perturbatively by Büttiker and Stafford[10] in the context of tunneling into a quantum dot.

The coupling of a quantum dot to a quantum wire can either be metallic or capacitive. In the former the electrons have to move through the quantum dot, i.e., the quantum dot is embedded into the conductor, and a Coulomb blockade may arise if the Fermi level in the conductor is off resonance with the discrete energy levels in the dot. The quantum dot can also form a side-branch to a small metallic ring, i.e., the coupling is capacitive and the electrons do not have to pass through the dot in order to continue along the wire. Our impurity corresponds to a hybrid of these two situations with fine-tuned parameters such that the model remains integrable.

III. BETHE *ANSATZ* EQUATIONS

The model is diagonalized by two nested Bethe *Ansätze* in terms of two sets of rapidities. Within the quantum inverse scattering method there are two approaches that can be used: (i) Lai's method,[12,14] where the first set of Bethe equations refers to the charges and the second diagonalizes the spin degrees of freedom and (ii) the Sutherland approach[13] in terms of a graded algebra. Both approaches are technically similar and differ mainly in the assumption of the vacuum (reference) state. In method (i) the vacuum state corresponds to the empty lattice, while in (ii) the reference state is the state with an electron with up-spin at every site. Method (i) is physically very transparent but leads to a ground state with complex rapidities, which have a very simple form in the thermodynamic limit, but are difficult to determine numerically for finite L. Within approach (ii), on the other hand, the ground state consists of real rapidities which are relatively easy to determine with standard numerical procedures.

Within the Sutherland representation[13] the Bethe equations for periodic boundary conditions take the following form:[16]

$$\left(\frac{p_j-\epsilon-i/2}{p_j-\epsilon+i/2}\right)e^{ik_jL}=-\prod_{l=1}^{M_1}\frac{p_j-p_l-i}{p_j-p_l+i}\prod_{\alpha=1}^{M_2}\frac{p_j-\lambda_\alpha+i/2}{p_j-\lambda_\alpha-i/2},$$
$$j=1,\dots,M_1,$$
$$\prod_{j=1}^{M_1}\frac{\lambda_\alpha-p_j-i/2}{\lambda_\alpha-p_j+i/2}=1, \quad \alpha=1,\dots,M_2, \quad (3)$$

where $M_1=N_\downarrow+N_h$ is the number of down-spin electrons plus holes and $M_2=N_h$ is the number of holes. Note that $N_\uparrow+N_\downarrow+N_h=L+1$, where the additional site refers to the impurity. The rapidities p_j are related to the wave numbers k_j of down spin electrons and holes by $p_j=\frac{1}{2}\tan(k_j/2)$, and the λ_α rapidities represent the holes. The energy and the momentum of the system are given by

$$E=2(M_2-M_1)-2\sum_{j=1}^{M_1}\cos(k_j), \quad P=\sum_{j=1}^{M_1}k_j, \quad (4)$$

and the z projection of the magnetic moment of the system is $S^z = \frac{1}{2}[1 + L - 2M_1 + M_2]$. Only the first factor on the left-hand side of the first set of Eqs. (3) is caused by the impurity, while the energy, Eq. (4), depends only implicitly on the parameter ϵ of the impurity.

For $N_h = 0$ Eqs. (3) and (4) correspond to an antiferromagnetic Heisenberg spin 1/2 chain with an $S = 1/2$ impurity.[20] In this limit ($N_h = 0$) Eq. (3) is identical to the ones describing the magnetic degrees of freedom of the $s-d$ (Kondo) magnetic $S = 1/2$ impurity in a free electron host [with ϵ being inverse proportional to the s-d exchange coupling between the host and with $E_{\mathrm{Kondo}} = P + \mathrm{const}$ (Refs. 3–5)].

Taking the logarithm of Eq. (3) we obtain

$$2L\,\arctan(2p_j) + 2\,\arctan[2(p_j - \epsilon)] = 2\pi J_j + 2\sum_{l=1}^{M_1} \arctan(p_j - p_l) - 2\sum_{\alpha=1}^{M_2} \arctan[2(p_j - \lambda_\alpha)],$$

$$2\sum_{j=1}^{M_1} \arctan[2(\lambda_\alpha - p_j)] = 2\pi I_\alpha, \tag{5}$$

where the J_j and I_α are quantum numbers that arise from the multivaluedness of the logarithm. The J_j are integer (half-integer) if $M_1 - M_2$ is even (odd), and the I_α are integer (half-integer) if M_1 is even (odd). From Eqs. (5) we have that the total number of possible J_j values is $L - M_1 + M_2 + 3$ and the total number of possible I_α values is $M_1 + 1$. The low energy states are determined by two sets of quantum numbers, M_1 values for the J_j and M_2 values for the I_α. All quantum numbers within a set have to be different for the eigenfunctions to be linearly independent.

IV. GROUND STATE AND LOW-ENERGY EXCITATIONS

We now obtain the ground state energy for a small system corresponding to $L = 16$, $N_h = 7$, and $N_\uparrow = N_\downarrow = 5$. In this case $M_1 = 12$ and $M_2 = 7$, such that the J_j are half-integers and the I_α are integers. For the ground state we have then $J_j = -\frac{11}{2}, -\frac{9}{2}, \ldots, \frac{11}{2}$, and $I_\alpha = -3, -2, \ldots, 3$. The ground state energy is a symmetric function of ϵ and is shown in Fig. 1 as the solid curve (a). The ground state energy increases monotonically with $|\epsilon|$. For a large system, i.e., in the thermodynamic limit, we can extract the impurity contribution to the energy,[8,9] but that is not possible for a nanoscale system where the impurity has a strong feedback on the host.[6,7]

Low-energy excited states are obtained by introducing ''holes'' into the distributions of quantum numbers. Excitations among the hole states are shown in Fig. 1 curves (b) (long-dashed) and (c) (dotted). They correspond to the same set of J_j as the ground state, but $I_\alpha = -3, \ldots, 2, 4$ (''hole'' at 3) and these I_α with reversed signs (''hole'' at -3). The energies of these two states are, of course, identical for $\epsilon = 0$ and lie about $2\pi/L$ above the ground state. The energy difference between the two excited states is the consequence of the chirality due to the impurity.

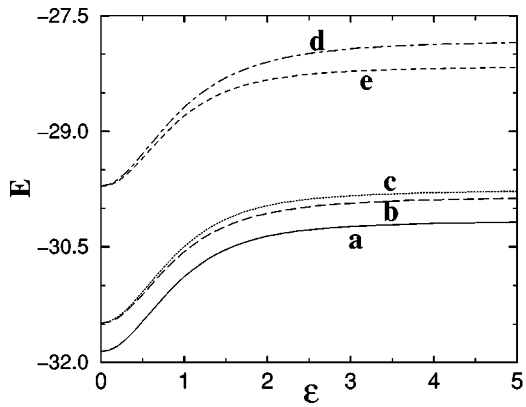

FIG. 1. Energy as a function of the impurity parameter ϵ for low-lying eigenstates for $L = 16$ and $N_h = 7$. The impurity has a net chirality. The sets of quantum numbers for each state are given in the text. (a) The ground state (solid line) corresponds to $N_\uparrow = N_\downarrow = 5$ ($M_1 = 12$ and $M_2 = 7$). The lowest energy hole or charge excitations [(b) (long-dashed) and (c) (dotted)] obtained by introducing a ''hole'' into the set of hole quantum numbers. (d) (dash-dotted) and (e) (dashed) shows the lowest energy spin-flip excitations ($M_1 = 11$, $M_2 = 7$). The energy differences between the states (b),(c) and (d),(e) are the consequence of the chirality of the impurity.

Spin-flip states have a considerably larger energy. The spin-flip excitations of lowest energy are displayed in Fig. 1 as curves (d) (dash-dotted) and (e) (dashed). If a spin is flipped $N_\downarrow$ is decreased by one unit to 4, such that now $M_1 = 11$ and $M_2 = 7$. Consequently now the J_j are the set of integers $-5, \ldots, 5$ and the I_α are given by the set of half-integers $-\frac{5}{2}, -\frac{3}{2}, \ldots, \frac{7}{2}$ for curve (d) and the same set of I_α with reversed sign for curve (e). Note that flipping a spin does not only change the magnetization but also raises the energy considerably. In addition to these two types of excitations there are also spin-non-flip (singlet) excitations, which do not change the magnetization, consisting of a pair of complex conjugated p_j rapidities. Their energy is difficult to calculate and is of the order of the spin-flip excitations. The energy difference between curves (b) and (c) and curves (d) and (e) in Fig. 1 is a consequence of the chirality of the impurity. Since the impurity breaks the T and P symmetries, the energies of otherwise equivalent states are different.

The most important conclusion of this section is that for the finite t-J ring the energy levels are discrete and that charge excitations have a much smaller gap than the spin-flip energies. For low temperature properties this means that both the specific heat and the magnetic susceptibility are exponentially activated, but the activation gap for the specific heat is much smaller than the spin-gap of the susceptibility. The energies depend on two parameters, namely, L^{-1} (which determines the average spacing between the levels in the host) and ϵ, the coupling between the dot and the host. The variation of the ground state and excitation energies is maximum when the two parameters give rise to comparable effects.

V. PERSISTENT CURRENTS

The external magnetic and electric fluxes threading the ring yield a change in the momentum $\Delta P = N_\uparrow \vartheta_\uparrow + N_\downarrow \vartheta_\downarrow$,

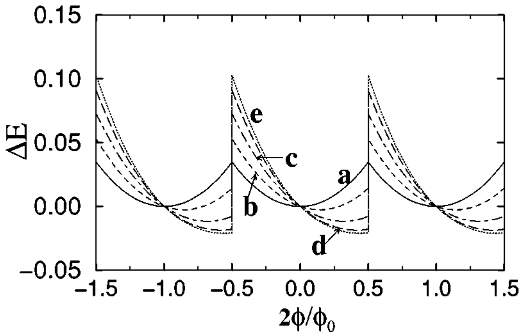

FIG. 2. Aharonov-Bohm oscillations in the ground state energy, $\Delta E = E(\epsilon, 2\Phi/\Phi_0) - E(\epsilon, 0)$, (for $L=16$, $N_h=7$, and $N_\uparrow = N_\downarrow = 5$) as a function of the flux $2\Phi/\Phi_0$ threading the ring. The values of $E(\epsilon,0)$ can be read off from Fig. 1(a). The period of oscillation is one-half the unit magnetic flux because the electrons travel in pairs. The curves correspond to different values of ϵ: (a) $\epsilon=0$ (solid), (b) $\epsilon=0.75$ (dashed), (c) $\epsilon=1.5$ (dash-dotted), (d) $\epsilon=2.25$ (long-dashed), and (e) $\epsilon=3$ (dotted). The discontinuities are the consequence of the chirality of the impurity.

where $\vartheta_\sigma = \Phi/\Phi_0 + \sigma F/F_0$ are the Aharonov-Bohm-Casher phase shifts. Here Φ is the magnetic flux, $\Phi_0 = hc/e$ is unit magnetic flux, $F = 4\pi\tau$ is the electric flux generated by a string passing through the center of the ring with linear charge density τ, $F_0 = hc/\mu_B$ is the unit electric flux, and μ_B is the Bohr magneton. The fluxes give rise to charge and spin persistent currents of Aharonov-Bohm-Casher type[21,22] in a closed ring configuration.

The situation is simplest for $N_\uparrow = N_\downarrow = N_p$ for which $\Delta P = (\vartheta_\uparrow + \vartheta_\downarrow) N_p = \vartheta_p N_p$, where N_p and ϑ_p are the number of paired electrons and their Aharonov-Bohm phase shift, respectively. The energy is now necessarily periodic in $2\Phi/\Phi_0$. Since $N_p = M_1 - M_2$, for periodic boundary conditions the Aharonov-Bohm phases are introduced into the BAE by shifts in the quantum numbers, i.e., $J_j \rightarrow J_j + \{\{\vartheta_p\}\}$ and $I_\alpha \rightarrow I_\alpha - \{\{\vartheta_p\}\}$, where $\{\{a\}\}$ denotes the fractional part of a to the nearest half-integer. The expression of the energy remains unchanged, Eq. (4). In a finite ring these shifts change the values of the rapidities and hence also the energy in a periodic manner. The periodicity interferes with the chirality due to the impurity, so that the oscillation pattern will in general be asymmetric.

In Fig. 2 we show the Aharonov-Bohm oscillations [as given by $\Delta E = E(\epsilon, 2\Phi/\Phi_0) - E(\epsilon, 0)$] for five values of ϵ and the same set of quantum numbers as for the ground state in Fig. 1(a). The energy value without flux can be found from Fig. 1(a). For $\epsilon=0$ the impurity has no chirality and is just one more site in the chain. The Aharonov-Bohm oscillations of the energy are then parabolic and symmetric. For $\epsilon \neq 0$ the oscillations are still parabolic but with a displacement due to the chirality of the impurity. Consequently, the energy has a mesoscopic discontinuity when $2\Phi/\Phi_0$ is a half-integer, which of course has no physical consequences. The period of the Aharonov-Bohm oscillations is always twice the unit magnetic flux, as expected for paired charges traveling together. The persistent current is obtained by differentiating the energy with respect to the flux, which for a parabolic energy pattern always yields a saw-tooth shaped current.

The general case is well-understood within Lai's representation[12,14] for the Bethe's *Ansatz* in the very large L limit. The ground state consists of two sets of states, namely spin-paired electrons and unpaired electrons. The former yield oscillations in the persistent charge current of frequency $\Phi_0/2$, while the unpaired states carry only a simple charge and give rise to a period of Φ_0. The persistent charge current is then in general a superposition of oscillations with two periods. Only unpaired electron states carry a spin and yield Aharonov-Casher oscillations with a period of F_0. The starting point in Lai's representation is the physical vacuum (no electrons) and electrons are added by the Bethe wave functions. Unfortunately, the solutions of the BAE in this representation are complex numbers for the rapidities, which are difficult to obtain accurately for a system with small L. This method is therefore not very practical for small L.

The Bethe vacuum state within the elegant Sutherland representation is the ferromagnetic state with one electron per site. The Bethe states introduce spin-flips and holes and the system is integrable because of the underlying supersymmetric BFF algebra. Within this representation the ground state and low energy excitations have only real rapidities, which is a very desirable feature for systems of small size. Here the vacuum state already carries charge and spin, and the analysis of the persistent currents is more difficult. Additional complications arise from the BFF supersymmetry. The description of the continuum of electron-hole excitations is a nontrivial issue in this representation already for the t-J model in the thermodynamic limit without impurity.[23]

For a system with discrete energy levels, the level crossings as a consequence of the magnetic flux, always yield a saw-tooth-like persistent current. It is, however, possible to obtain sinusoidal oscillations as demonstrated by the simple model presented in the Appendix. The following paragraph briefly describes how these oscillations arise.

The impurity and the sites 1 and L form a triangle with interactions along the three sides. Due to the conditions imposed by the integrability, the phase shifts from site 1 to L through the impurity or along the link are necessarily the same. In the Appendix we present the exact solution of a simple model for charged spinless fermions, which in the presence of a magnetic field threading the ring, yields two periods of oscillation for the charge persistent current. While the gas of charges in the ring gives rise to the usual parabolic pattern for the energy, the triangle at the impurity causes sinusoidal oscillations with a different frequency. For this simple model the persistent charge current is then a superposition of a sawtooth with a sinusoidal oscillation.

VI. MAGNETIZATION

The magnetization S_z of the system is a function of the number of electrons with up and down spin of the state. Depending on the number of electrons the magnetization is either a half-integer or an integer. For the particular situation considered here, S_z is integer and varies from 0 to 5 in integer steps as a function of increasing magnetic field. For given S_z the state of minimal energy corresponds to real rapidities and the quantum numbers form a compact set with-

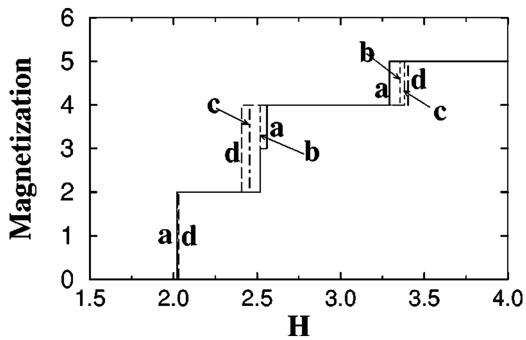

FIG. 3. Zero-temperature magnetization as a function of field for $L=16$ and $N_h=7$. The four values of ϵ correspond to (a) $\epsilon=0$ (solid), (b) $\epsilon=0.75$ (dashed), (c) $\epsilon=1.5$ (dash-dotted), and (d) $\epsilon=3$ (long-dashed). For this example the magnetization value 1 is not stable and $S_z=3$ is stable only for a very small range of fields for small ϵ as a consequence of the change in parity of the rapidities with S_z in the *t-J* model.

out leaving holes. For the case considered in Fig. 1 we have $M_2=N_h=7$ and $M_1=\frac{1}{2}(L+1+M_2)-S_z=12-S_z$. The free energy at $T=0$ is $E-HS_z$ and the state with lowest free energy is the ground state at the given field. Hence, the magnetization as a function of field has steps and plateaus (of integer units) when there is a crossover of ground states, as shown in Fig. 3 for four values of ϵ. For the present example, states with odd values of S_z are energetically less favorable than those with even S_z. Consequently, the first step in the magnetization is a jump of two units, so that $S_z=1$ is skipped, and only for $\epsilon=0$ there is a small range of fields for which $S_z=3$ is stable. $S_z=5$ is the saturation magnetization for which all spins are aligned. The critical fields (fields at which S_z is discontinuous) all decrease with increasing $|\epsilon|$.

The energy differences between states with even and odd values of S_z arise from the fact that the sets of quantum numbers $\{J_j\}$ and $\{I_\alpha\}$ alternate between integers and half-integers with increasing S_z, and hence the energy values alternate with S_z. With increasing system size and number of electrons the energy differences between even and odd S_z values is expected to decrease. In this case the number of jumps increases and all the S_z values are allowed. In the thermodynamic limit the magnetization divided by the number of electrons (or the saturation magnetization) becomes a continuous function of the magnetic field. Note that for the finite system the impurity contribution to the magnetization is strongly coupled to that of the ring and there is no simple way to separate them (in contrast to the large L limit, where it is the contribution of order L^{-1}). On the other hand, it is possible to define an impurity energy as $E_{\rm imp}=E(\epsilon)-E(0)$. For $\epsilon=0$ the impurity is just one more link in the ring. $E_{\rm imp}$ then represents the energy difference of replacing one site by the impurity.

VII. CONCLUSIONS

In summary, we have studied some properties of a magnetic hybridization impurity in a finite ring. The strongly correlated electron host is described by the supersymmetric *t-J* model. The Hamiltonian of the host and the impurity is integrable and is diagonalized in terms of nested Bethe *Ansätze*. We used the Sutherland representation for which the ground state and low energy excitations correspond to real charge and spin rapidities. We studied the ground state energy, charge and spin-flip excitations as a function of the impurity rapidity. The energies are a function of two parameters, namely, the length of the ring L which is related to the energy spacing in the ring and the ring-dot coupling ϵ. The gap in the charge excitations is smaller than that of the spin-flip excitations. The former is the leading activation energy for the specific heat, while the latter determines the low-temperature dependence of the magnetic susceptibility.

We investigated the pattern of oscillations due to a magnetic flux threading the ring for the situation where $N_\uparrow=N_\downarrow$. The energy shows Aharonov-Bohm oscillations that are piecewise parabolic with a periodicity of $\Phi_0/2$, and correspond to a sawtoothlike persistent charge current. The pattern depends on the chirality of the magnetic impurity, which introduces asymmetries and discontinuities in the energy. The period of $\Phi_0/2$ arises from the fact that electrons travel as paired spin singlets. The integrable model only yields a sawtooth pattern for the persistent current.

In the Appendix we present a simple exactly solvable model for Aharonov-Bohm oscillations of charged spinless fermions on a ring with an impurity. The pattern of oscillations is a superposition of two types of oscillations: While the ring yields a sawtoothlike persistent current of periodicity Φ_0, the loop formed by the impurity and the two neighboring lattice sites gives rise to sinusoidal oscillations with a periodicity that depends on the flux threading the triangle. Also in this case the impurity can be considered a hybrid quantum dot, i.e., simultaneously embedded into and a side-branch to the ring.

The zero-temperature magnetization displays steps as a function of field. The critical fields at which the magnetization jumps depend on the impurity parameter ϵ. The sets of quantum numbers describing the ground state for a given S_z alternate between integer and half-integer depending on the parity of S_z. Thus, depending on the size of the system, the ground state energy does not necessarily increase monotonically with S_z. Consequently, there are situations where S_z jumps in two units, or the range of fields for some steps is rather small. This effect is not present in a host without correlations.[6] In the thermodynamic limit the steps in the magnetization, normalized to the saturation value, become very small and asymptotically the magnetization is a continuous and monotonically increasing function of the external field. While for a small system, such as the one considered here, it is not possible to separate the magnetization effects of the ring and the impurity, this is not a difficulty for large systems where the feedback of the impurity onto the host can be neglected.

Due to the quantization of the energy levels the system at low T is rather stable to electric and magnetic fields. Small changes in the bias voltage (chemical potential) do not affect the quantum dot, because the spin and charge sectors are decoupled. Due to the steps the magnetization is stable to fluctuations in the magnetic field. The magnetization can be changed with a magnetic tip, such that the system could be used as a magnetic storage device.

ACKNOWLEDGMENTS

P. S. acknowledges the support by the National Science Foundation under Grant No. DMR01-05431 and the Department of Energy under Grant No. DE-FG02-98ER45797.

APPENDIX

In this Appendix we present a solvable simple model of charged spinless fermions traveling on a ring with an impurity, which displays sawtoothlike and sinusoidal persistent currents. The impurity interacts with the two neighboring sites, in this case 1 and L. The Hamiltonian under consideration is

$$H=-t\sum_{i=1}^{L}(c_{i+1}^{\dagger}c_i+c_i^{\dagger}c_{i+1})+\epsilon c_{\rm imp}^{\dagger}c_{\rm imp} -t_R(c_1^{\dagger}c_{\rm imp}+c_{\rm imp}^{\dagger}c_i)-t_L(c_L^{\dagger}c_{\rm imp}+c_{\rm imp}^{\dagger}c_L), \quad (A1)$$

where $L+1\equiv 1$, $c_{\rm imp}^{\dagger}$ ($c_{\rm imp}$) creates (annihilates) a fermion on the impurity site, and t_L and t_R are hopping matrix elements from and to the impurity. Rewriting the Hamiltonian in reciprocal (k) space, i.e., using

$$c_j=L^{-1/2}\sum_{k=1}^{L}\exp(-ikj)c_k, \quad (A2)$$

we obtain

$$H=\sum_{k=1}^{L}\epsilon(k)c_k^{\dagger}c_k+\epsilon c_{\rm imp}^{\dagger}c_{\rm imp}-L^{-1/2}\sum_{k=1}^{L}(\tilde{t}_k c_k^{\dagger}c_{\rm imp}+\text{H.c.}), \quad (A3)$$

where $\tilde{t}_k=t_Re^{ik}+t_L$ and $\epsilon(k)=-2t\cos(k)$.

The exact expression for the single particle Green's function is

$$G_{k,k'}(z)=\frac{\delta_{k,k'}}{z-\epsilon(k)}+\frac{1}{z-\epsilon(k)}T_{k,k'}(z)\frac{1}{z-\epsilon(k')}, \quad (A4)$$

where $T_{k,k'}(z)$ is the t matrix due to the scattering off the impurity,

$$T_{k,k'}(z)=\frac{\tilde{t}_k}{\sqrt{L}}\left[z-\epsilon-\frac{1}{L}\sum_{p=1}^{L}\frac{\tilde{t}_p\tilde{t}_p^{*}}{z-\epsilon(p)}\right]^{-1}\frac{\tilde{t}_k'}{\sqrt{L}}. \quad (A5)$$

The magnetic flux threading the ring gives rise to phase shifts in the wave vectors, i.e.,

$$\epsilon(k)\rightarrow -2t\cos(k+2\pi\{\{\vartheta\}\}),$$
$$t_R\rightarrow t_Re^{-i2\pi f_R\vartheta},\quad t_L\rightarrow t_Le^{i2\pi f_L\vartheta}, \quad (A6)$$
$$\tilde{t}_k\tilde{t}_k^{*}=t_R^2+t_L^2+2t_Rt_L\cos[k+2\pi\vartheta(1-f_R-f_L)],$$

where again $\vartheta=\Phi/\Phi_0$. Here $f_R\vartheta$ and $f_L\vartheta$ are the phase shifts along the links connecting the impurity, which in general can be different from each other. Hence, unless $f_R+f_L=1$ there are two types of oscillations in the system: (i) the usual Aharonov-Bohm oscillations of parabolic shape and periodicity Φ_0 arising from the ring and (ii) the sinusoidal oscillations due to the impurity of periodicity $\Phi_0/(1-f_R-f_L)$. If $f_R+f_L=1$ the two paths of the triangle connecting the impurity to the link have the same phase shifts, so that no new oscillation arises.

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