Shortest path across a mesoscopic system

Liqun He,[1,2] Eugene Kogan,[2] and Dawei Luo[3]
[1]*Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, People's Republic of China*
[2]*Minerva Center and Jack and Pearl Resnick Institute of Advanced Technology, Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel*
[3]*Department of Mechanical Engineering, University of Kentucky, Lexington, Kentucky 40506-0108*
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We study distribution functions (DF's) of mesoscopic hopping conductance numerically by searching for the shortest path and the results are compared with analytical predictions. We have found that distributions obtained by choosing the chemical potentials randomly (for a fixed impurity configuration), which corresponds to a typical experimental situation, coincide with those obtained when both impurity configuration and chemical potential is chosen randomly, in agreement with the ergodicity hypothesis. The DF's obtained for one-dimensional (1D) systems were found to be quite close to the independent predictions of Mel'nikov *et al.* and Raikh and Ruzin. For $D=2$, the DF's both for a narrow system and a thin film look similar (and close to the 1D case), which means that the short 2D still lies in the narrow regime defined by Raikh and Ruzin. The distribution function for the conductance of the square sample is nearly Gaussian as predicted by both Altshuler *et al.* and Serota *et al.* Our results also hint that the puncture nature of 2D systems seems to be featured by the position of DF peak and the long tail might show the preference of conductance fluctuation.

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Mesoscopic conductance fluctuations in the insulating regime of small, disordered transistors were first observed by Pepper[1] in GaAs MESFET's and then studied in detail in Si MOSFET's by Fowler, Webb, and co-workers[2] in the early 1980's. Extremely strong random fluctuations, spanning several orders of magnitude, were observed at low temperatures in the conductances of narrow-channel devices as the gate voltage was varied. Lee[3] proposed a model in which electrons move by variable-range hopping (VRH) along a 1D chain, and the lognormal distribution for one-dimensional (1D) conductors was analytically predicted by Mel'nikov[4] and Abrikosov.[5] Serota, Kalia and Lee[6] and Yu and Serota[7] went on to study the ensemble distribution of the total chain resistance R and its dependence on the temperature T and the sample length L. In their ensemble, the random impurities are distributed uniformly in energy and position along the chain. In experiments a single device is generally used, so that the impurity configuration is fixed, and fluctuations are observed as a function of some variable external parameter such as the chemical potential. An ergodicity hypothesis is then invoked to the effect that the same ensemble is sampled in both cases, something that has been verified experimentally by Orlov *et al.*[8] Using the natural logarithm of the resistance, the authors of Ref. 6 obtained for the mean and standard deviation

$$\langle \ln \rho \rangle \sim \left(\frac{T_0}{T}\right)^{1/2} \left[\ln\left(\frac{2L}{\xi}\right)\right]^{1/2}, \tag{1}$$

$$s \equiv \langle (\ln \rho - \langle \ln \rho \rangle)^2 \rangle \sim \left(\frac{T_0}{T}\right)^{1/2} \left[\ln\left(\frac{2L}{\xi}\right)\right]^{-1/2}, \tag{2}$$

where ξ is the localization radius and T_0 is the characteristic temperature for Mott VRH: $T_0 = 1/k_B \rho \xi$ (ρ is the density of states at the Fermi energy). It can be seen that the size s of the fluctuations decreases extremely slowly with length, a result characteristic of 1D which was first pointed out by Kurkijarvi.[9] The explanation is simply that exceptionally large resistance elements, even though they may be statistically rare, dominate the overall resistance since they cannot be by-passed in this geometry. The averaging assumed in the derivation of Mott's hopping law for 1D does not occur and the total resistance takes on the activated form of the largest individual element.

The crossover from 1D to 2D were also studied.[10–14] The theoretical description of hopping conductivity in narrow 2D strips was given by Raikh and Ruzin (RR)[10,11] and Serota,[12] and numerical simulations from narrow 2D to square 2D at a certain temperature were also done by Xie and Das Sarma.[13] The conductance DF (on the metallic side) was first fully considered analytically in 2D and above by Altshuler, Kravtsov, and Lerner,[14] where they have predicted the Gaussian distribution with long lognormal tails. Similar to the 1D case, these fluctuations are also of "geometrical" origin, arising the finite widths of channels. In a previous work,[15] He *et al.* did numerical simulations on all 2D cases by percolating the system, including the short 2D which has been studied experimentally by Hughes *et al.*[16] The results for the long 2D and square 2D are in accordance with the work of others. Due to the large size in width, the electrical network within the short 2D was thought to be composed of parallel resistors, and it was expected that there would be a long tail of DF towards the high conductance,[16] but obtained numerical DF for short 2D is of a long tail to the low conductance. To check the results further, we numerically simulated these samples by searching for the shortest path across system other than by percolating it. The shortest paths are punctures which short out less conductive paths in the 2D geometry. The aim of the present work is to report this study, and first of all, it is also started by replacing the transport

problem across a 2D mesoscopic system in Mott hopping regime with a random resistor network as was done previously.

Imagine that a particle is about to transport a conductive network. In the principle of lowest energy, it prefers to the path of the smallest overall resistance among all possible paths. The path of lowest energy cost is equivalent to the shortest path of graph theory, which is just what the nature of puncture of the system means. It means that we could also approximate the resistance of the system by looking for the shortest path, as well as by percolating the system as usual. The shortest paths among percolation systems have been studied considerably recently[17–21] in terms of minimal path or optimal path between a pair of sites within the same cluster, and the study on the scaling form of the probability of the shortest path with regard to their Euclidean distance and the cluster mass (M_B)[19] has shown that the average conductance of the percolation backbone is strongly correlated with the shortest path, and it decreases with increasing minimal path. This means the shortest path determines the average conductance in nature.

A network is set up by resistors ρ_{ij} between sites i and j

$$\ln \rho_{ij} = 2\alpha d + (|E_i - \mu| + |E_j - \mu| + |E_i - E_j|)/2kT. \quad (3)$$

Here α is the inverse localization length; d is the distance of two localized sites; E_i and E_j are energies of sites i,j; μ is the chemical potential; and T is the temperature. Energy is chosen randomly from a uniform distribution in the range $-0.5 \sim +0.5$. Thus the mesoscopic system is reduced to a random resistor network (RRN). To percolate the network, the resistors joining electrodes are selected in ascending order until the first percolation path connects the reservoirs. The resistance of the percolation path is taken to be the resistance of the entire system.

To solve it by the shortest path, the Dijkstra algorithm[22] is applied, which is used to search for shortest paths to all nodes from a single source in a fully connected graph. In the graph, the RRN is partially connected and there are more than one source node except for the 1D case. So, some modifications on this algorithm have been done in the simulation.

In the calculation, positions of impurities are uniformly distributed over the system, their energies are distributed evenly between $-0.5 \sim +0.5$, and the gate voltage μ is randomly chosen. Thus we can consider the chemical potential distributions (for a fixed impurity configuration) and the ensemble distributions (for a fixed chemical potential). For a 1D system of $L=1000$, chemical potential $\mu=0$, and temperature $T=0.001$, we found that the profile of their individual resistances is similar to that along the percolating path (Fig. 1 of Ref. 15). This means that it is also the single largest hop along the shortest path that controls the overall conductance of the system. With increasing temperature, the sizes of fluctuation of individual resistors along the shortest path become close.

As has been done previously,[15] we also validate the approach by obtaining the DF's of 1D systems first. The size of the 1D system is 1000 in length and 50 in localization length at a temperature of $T=0.001$. The chemical potential range is $\mu=-0.1 \sim +0.1$. The results are shown in Fig. 1. According to the RR theory for the 1D case,[23] the conductance is predicted as 9.49, while we see from Eqs. (1) and (2) that the expectation occurs at about 8.59 and the standard deviation is around 3.92. The former is in good agreement with the numerical results of the chemical distribution function and the latter predicted by Ref. 6 shows good agreement with the results of the ensemble distribution function. As found in Ref. 15, with the increase of temperature, the fluctuation of 1D resistance also becomes small.

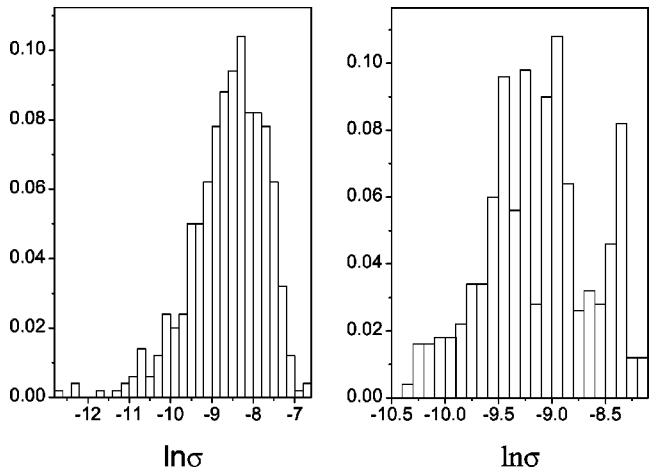

FIG. 1. The conductance of 1D system for $T=0.001$: (a) ensemble distribution function, (b) chemical potential distribution function. The histograms in both figures are the numerical result.

On the basis of the above validity, the 2D cases are numerically simulated by the shortest path, including cases of long 2D, square 2D, and short 2D. In our simulation on a 2D system of $W \times L$, parameters, such as localization length, temperature, etc., are the same as in the 1D case, while $W=100, L=1000$ for long 2D, $W=1000$ and $L=100$ for short 2D, and $W=L=1000$ for a square 2D system. Figure 2 shows the results.

According to Mott's law

$$\ln \sigma_c = -(T_0/T)^{1/3}, \quad (4)$$

the critical log conductance for a 2D system at $T=0.001$ is -1.77. The great difference between it and the numerical result $(\ln \sigma \approx -9)$ shows the size effect stemming from the high Ohmic regime on the conductance.[11] According to the RR theory for 2D,[11] the effective width of this long 2D system is $\omega = W/\xi(\ln \sigma_c)^2 = 0.64$, which means that it lies in the regime of optimal breaks and its resistance has to be

$$\ln(R/L) = 1/(2\xi\rho WT). \quad (5)$$

So the log resistance of long 2D provided by it is 7 which is closer to but smaller than our numerical result. Note that the prediction of RR theory for 1D is 9.49, which is a little greater than our numerical result. With the increase of the ratio of width to length, the magnitude of conductance substantially increases, which has been predicted theoretically.[11–14] The results reveal that the situation of long 2D is close to the 1D case, and the normal 2D is close to

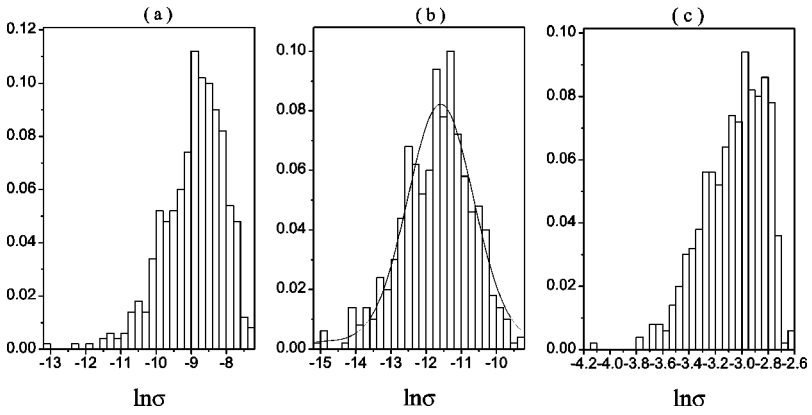

FIG. 2. The ensemble distribution function for the conductance of 2D system: (a) long 2D, (b) square 2D, (c) short 2D. Histograms are numerical results. The solid curve in (b) is the Gaussian fitting.

Gaussian. Just as discovered previously,[15] the DF of conductance across the short 2D system calculated in this paper is still close to that of the 1D case other than a mirror reflection of the latter as expected.[16] From $\ln(L/\xi) \ll (W/\xi)^{1/2}$, it seems that the short 2D still lies in the regime discussed in Ref. 11 and the conductance through the typical cross section is controlled by the inhomogeneous cuts within the infinite cluster.[11] From the fact that the DF peak of short 2D is shifted to the low end of resistance much more than these of other two cases, it seems to be more proper to explain the puncture nature of 2D systems by the position of the peak of DF while the long tail of DF shows the bias conductance fluctuations.

In conclusion, we have numerically studied the DF's of hopping conductance through 2D mesoscopic systems with various aspect ratios. The simulations are based on a search of the shortest path which is supposed to reflect the puncture nature of VRH system more directly, and the results are compared with analytical predictions. The results show that the shortest path provides the resistances across 2D systems closer to predictions than the percolation.[15] In our work, three cases of 2D systems, i.e., long 2D, square 2D, and short 2D, have been studied, and the obtained resistances of all samples are greater than the prediction of Mott's law due to the size effect. For the long 2D, the conductance obtained numerically is smaller than its corresponding RR prediction but is little greater than the RR prediction for the 1D case. The DF of the square sample is close to Gaussian.[14] Just as discovered previously for the short 2D system,[15] the shape of its DF is similar to that of the long 2D opposed to a mirror reflection of the latter. According to the RR theory,[11] the short 2D seems to be still a narrow one where the spare inhomogeneous cuts or breaks control the conductance. From the DF's in Fig. 2, it seems to be more reasonable to feature the puncture nature of 2D systems by the peak position of DF's whose long tails may show the preference of conductance fluctuation.

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