Theory of the longitudinal vortex-shaking effect in superconducting strips

Grigorii P. Mikitik[1,2] and Ernst Helmut Brandt[1]
[1]*Max-Planck-Institut für Metallforschung, D-70506 Stuttgart, Germany*
[2]*B. Verkin Institute for Low Temperature Physics & Engineering, Ukrainian Academy of Sciences, Kharkov 61103, Ukraine*
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We show that for a thin superconducting strip placed in a perpendicular dc magnetic field—the typical geometry of experiments with high-T_c superconductors—the application of a weak ac magnetic field along the strip (i.e., perpendicular to the dc field and *parallel* to the circulating critical currents) generates a dc voltage in the strip, which causes the critical currents and irreversible magnetic moment to relax completely. This relaxation process is not due to thermally activated flux creep but to the drift of vortices towards the center of the strip under the influence of the ac field. This *longitudinal* vortex-shaking theory supplements our previous theory of *transverse* vortex shaking where the ac field was perpendicular to the irreversible currents. Together, both theories clarify the nature of the vortex-shaking effect in real superconducting samples of finite length.

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I. INTRODUCTION

Experimental investigation of the equilibrium properties of type-II superconductors can be performed only in the reversible region of the magnetic-field (H) versus temperature (T) plane. But often strong flux-line pinning prevents the measurement of equilibrium properties. In this context, Willemin *et al.*[1] recently made an interesting observation. Their experiments revealed that application of an additional small oscillating magnetic field *perpendicular* to the main dc field leads to a fast decay of the currents circulating in the critical state of various high-T_c superconductors. This effect dramatically extends the observable reversible domain in the H-T plane. The relaxation of the irreversible magnetization in these experiments was approximately exponential in time, and thus was obviously different from thermally activated flux creep, which would lead to a logarithmic time law. Using this vortex-shaking process, the melting transition of the vortex lattice in $YBa_2Cu_3O_{7-\delta}$ crystals was detected[2] at temperatures very close to the critical temperature T_c, where the melting could not be investigated before. With the same shaking method, it was discovered[3] that the order-disorder transition in the vortex lattice of $Bi_2Sr_2CaCu_2O_8$ is of the first order at low temperatures, where vortex pinning usually masks the corresponding jump in the equilibrium magnetization. Thus, the vortex-shaking process opens new possibilities in the experimental investigation of H-T phase diagrams of superconductors.

In our previous paper,[4] we gave a quantitative explanation of the *transverse* vortex-shaking effect for an infinitely long thin strip when the ac magnetic field h_{ac} lies in the plane of the strip and is *perpendicular* not only to the dc magnetic field (which is normal to the plane) but also to the currents circulating in the superconductor. In this situation, the self-field of the currents adds to h_{ac} on the upper (or lower) plane of the strip and subtracts from it on the opposite plane. As a result, the ac field tilts the vortices asymmetrically relative to the central plane of the strip, forcing them to "walk" towards the center of the strip, thus causing relaxation of the critical state, Fig. 1. We note that this result was obtained in the framework of a quasistatic approach based on the standard critical state concept and thus is valid even when thermally activated flux creep is negligible. However, it is clear that the vortex tilt across the width of the strip disappears if the ac magnetic field still lies in the plane of the strip but is *parallel* to the circulating currents, i.e., parallel to the length of the strip. In this case the above consideration fails. In real samples (which have, e.g., the shape of rectangular platelets), the applied ac field is perpendicular to the circulating currents in some regions of the superconductor and parallel in some others. Thus, the explanation of the vortex-shaking effect in real superconducting samples should combine both the transverse and longitudinal vortex shaking and requires further work.

In this paper we show that a vortex in the critical state moves towards the center of the strip even when it undergoes periodic tilt *along* the strip. On the basis of this somewhat surprising finding, we develop a quantitative theory of the *longitudinal* vortex-shaking effect in a strip when the ac field points along the strip, i.e., *parallel* to the circulating currents. Specifically, we shall consider the following situation: A thin superconducting strip fills the space $|x|\leqslant w$, $|y|<\infty$, $|z|\leqslant d/2$ with $d\ll w$; the constant and homogeneous external magnetic field H_a is directed along z, while the ac magnetic field $h_{ac}=h\cos\omega t$ is applied along y, i.e., perpendicular to H_a and parallel to the currents circulating in the sample (Fig. 1). We also make here the usual Bean assumption that the critical current density $j_{c\perp}$ perpendicular to the local induction **B** does not depend on B. The field H_a is assumed to be sufficiently large to exceed both the field of full penetration for the strip,[5] $H_p=(j_{c\perp}d/\pi)\ln(2ew/d)$, and the lower critical field H_{c1}, and so we put $B=\mu_0 H$. Besides this, to explain the physics with the least mathematical complication, we also assume that $h\gg J_c/2$; this allows us to neglect the influence of the self-field of the currents on the shape of the vortices in the strip. Here $J_c=dj_{c\perp}$ is the critical value of the sheet current J (i.e., the current density integrated over the thickness d) flowing in the strip.

Below we shall use the quasistatic approximation that is valid at sufficiently small frequency ω. Within this approximation, the description of the vortex-shaking effect in the strip reduces to solving a two-dimensional critical state prob-

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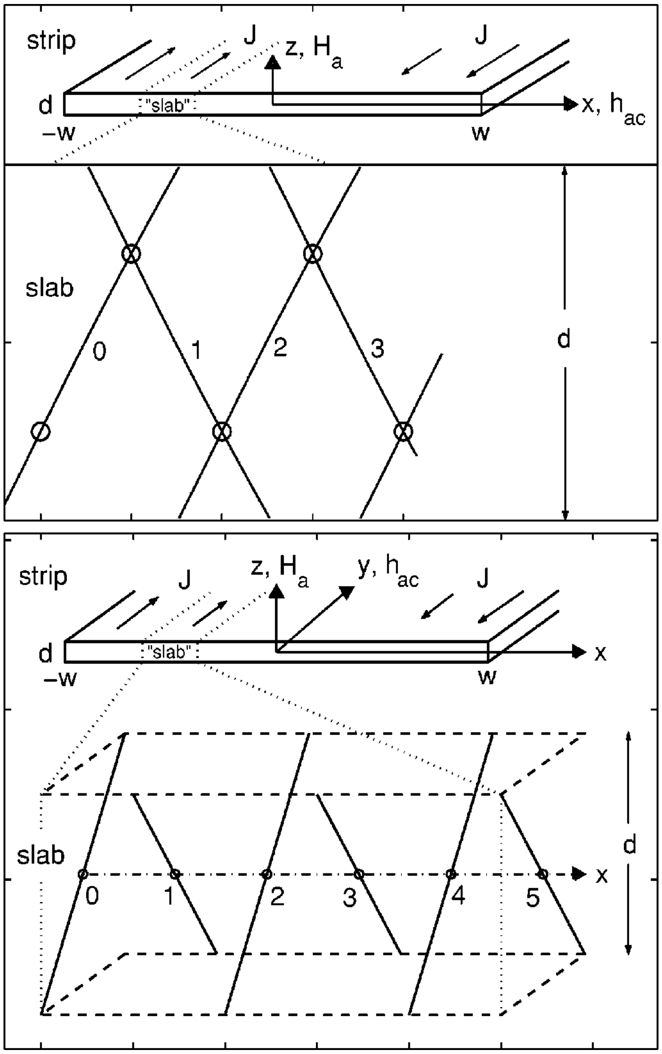

FIG. 1. Visualization of the two geometries of transverse (top) and longitudinal (bottom) vortex shaking in a long thin strip in a perpendicular dc magnetic field $H_a \gg J_c = j_{c\perp} d$. When the in-plane ac field $h_{ac} = h\cos\omega t$ with amplitude $h \gg J_c/2$ is applied, the sheet current J relaxes from its maximum value J_c to zero in both geometries since the vortices drift towards the center of the strip. Shown is one vortex at subsequent times $t\omega/\pi = 0, 1, 2, \ldots$. Top: If h_{ac} is transverse to J, the vortices "walk" in the xz plane as described in Ref. 4 (the vortices are straight since here $J_c \ll h_{ac}$, H_a is assumed). Bottom: If h_{ac} is parallel to J, the vortices periodically tilt in the yz plane and at the same time move along x as described in the text.

lem. However, the smallness of the parameter d/w enables us to simplify this problem by application of the approach of Ref. 6. Within this approach (which was also used in our previous paper[4]), we *split the problem into two simpler problems*: A one-dimensional problem across the thickness of the sample, and a problem for the infinitely thin strip. Namely, we first interpret a small section of the strip around an arbitrary point x (see Fig. 1) as an "infinite" slab of thickness d placed in a perpendicular dc magnetic field $H_z(x)$, in a parallel ac field $h\cos\omega t$, and carrying a sheet current $J_y(x)$. The resulting dc electric field E_y obtained for the slab, we then use as the local electric field $E_y(x)$ for an infinitely thin strip, to calculate the temporal evolution of the sheet current $J_y(x)$

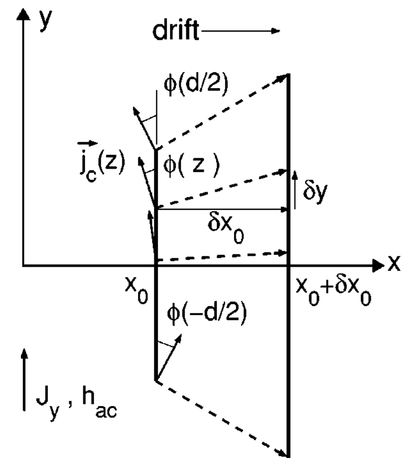

FIG. 2. Scheme of the vortex shift in a slab when the longitudinal ac magnetic field is increased from h_{ac} to $h_{ac} + \delta h$. Shown is the projection on the xy plane of a vortex (bold solid line) which tilts more away from the z axis and at the same time shifts from $x = x_0$ to $x = x_0 + \delta x_0$. The projected shifts of some vortex line elements are shown as dashed arrows with components δx_0 and $\delta y(z)$, $\delta y(z) = -\delta y(-z)$. These arrows are along the Lorentz force, which is perpendicular to the local current density $\mathbf{j}_c(z)$ (solid arrows) lying in the xy planes at the angle $\varphi(z)$ to the y axis.

and induction $B_z(x)$ in this strip by the method used in Refs. 7 and 8.

II. VORTEX DRIFT IN AN AC FIELD

We begin our analysis with the consideration of the above-mentioned longitudinal shaking mechanism in a small region of the strip (near $x = x_0$) which we approximate by an infinite slab in the x-y plane. Let a constant and homogeneous sheet current J_y flow in the slab placed in a constant and homogeneous external magnetic field H_z perpendicular to the plane of the slab. The applied ac magnetic field $h_{ac} = h\cos\omega t$ is in the y direction. The shaking mechanism is explained in Fig. 2: In order to tilt a vortex from z towards the y direction, the critical current density $\mathbf{j}_c(z) = [j_{cx}(z), j_{cy}(z), 0]$ must have a nonzero x component which is antisymmetric in z, $j_{cx}(z) = -j_{cx}(-z)$. Thus, $\mathbf{j}_c(z) = j_c(z)[-\sin\varphi(z), \cos\varphi(z), 0]$ flows at an angle $\varphi(z)$ to the y axis. Since the Lorentz force applied to an infinitesimal element of the vortex is normal to $\mathbf{j}_c(z)$, and the element shifts along this force, the tilt along y is always accompanied by a small shift along x. This shift is independent of the sign of h_{ac}, and hence, under the influence of an ac field, the vortex will drift in the direction of $[\mathbf{J}_y \times \mathbf{H}_z]$ by an oscillating screwlike motion.

We now formalize this qualitative consideration, using a quasistatic approach. Since the critical current density $\mathbf{j}_c(z)$ at a given depth in general is not normal to the magnetic field, its amplitude may exceed $j_{c\perp}$ (but its component perpendicular to $\mathbf{H}$ equals $j_{c\perp}$):[6]

$$j_c(z) = \frac{j_{c\perp}}{[1 - \sin^2\theta\cos^2\varphi]^{1/2}}. \tag{1}$$

Here θ is the angle of the magnetic-field tilt, $\sin\theta = h_{ac}/(H_z^2 + h_{ac}^2)^{1/2}$, while the denominator is the sine of the angle between $\mathbf{j}_c(z)$ and the total magnetic field $\mathbf{H}$ (the vector sum of the external and the ac fields). A vortex at an arbitrary moment of time, t, has the shape of a straight line tilted at an angle θ away from the z axis:

$$x = x_0(t);$$

$$y = y_0 + \frac{h_{ac}}{H_z} z, \tag{2}$$

where $x_0(t)$ and y_0 are the coordinates of the vortex in the plane $z=0$. It is clear from symmetry that $y_0 = \text{const}$, and without any loss of generality we shall put $y_0 = 0$ below. Let the ac field change by δh. The Lorentz force applied to an infinitesimal element of the vortex is proportional to $[\mathbf{j}_c(z) \times \mathbf{H}]$ and hence is directed along the vector

$$(\cos\varphi, \sin\varphi, -(h_{ac}/H_z)\sin\varphi).$$

Then, the shift of the element can be written as

$$\delta x = \delta x_0;$$

$$\delta y = \tan\varphi\, \delta x_0;$$

$$\delta z = -(h_{ac}/H_z)\tan\varphi\, \delta x_0, \tag{3}$$

where δx_0 is the shift of the vortex in the plane $z=0$. Since the shifted element belongs to the vortex with a changed value $h_{ac} + \delta h$, it follows from Eq. (2) that

$$\delta y = \frac{\delta h}{H_z} z + \frac{h_{ac}}{H_z} \delta z.$$

Inserting Eqs. (3) into this expression, we obtain the formula

$$H_z \left(\frac{\delta x_0}{\delta h} \right) \left(1 + \frac{h_{ac}^2}{H_z^2} \right) \tan\varphi = z, \tag{4}$$

which enables one to find the function $\varphi(z)$ if the ratio $(\delta x_0/\delta h) \equiv x_0'$ is known. This ratio can be expressed in terms of the sheet current J_y,

$$J_y = 2\int_0^{d/2} j_c(z) \cos\varphi(z)\, dz, \tag{5}$$

as follows: Inserting Eqs. (1) and (4) into formula (5), integrating, and defining $v = 2|x_0'|(H_z^2 + h_{ab}^2)^{1/2}/d$, we arrive at

$$\tilde{J}_y \cos\theta = v \ln \frac{1 + (1+v^2)^{1/2}}{v}, \tag{6}$$

where $\tilde{J}_y \equiv J_y/J_c$ and $\cos\theta = H_z/(H_z^2 + h_{ac}^2)^{1/2}$.

Let the solution of Eq. (6) for v be (Fig. 3)

$$v = g(\tilde{J}_y \cos\theta).$$

Then, expressing x_0' via v and integrating the x_0' over h_{ac}, one can calculate the shift of the vortex during one cycle of the ac field:

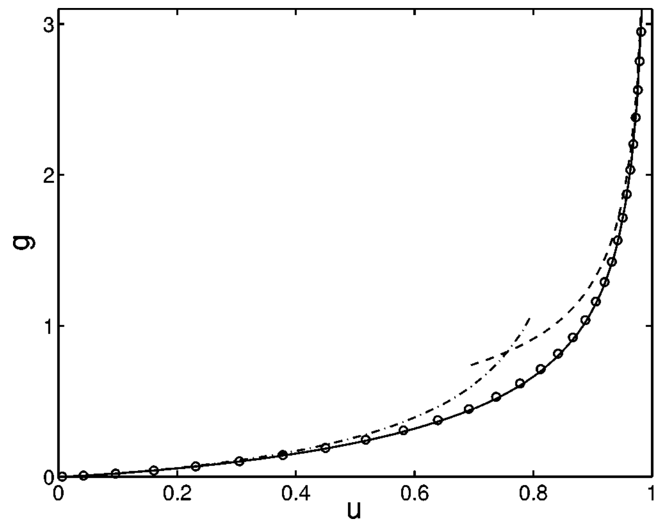

FIG. 3. The function $g(u)$ defined by Eq. (6) (circles). Also shown is the approximation (11) (solid line) and the limits (10) (dash-dotted line) and (9) (dashed line).

$$\Delta x_0 = 2d \int_0^h \frac{g(\tilde{J}_y \cos\theta)}{(H_z^2 + h_{ac}^2)^{1/2}} dh_{ac}. \tag{7}$$

The dc electric field generated by this shift of the vortex lines is $E_y = (\omega/2\pi)\mu_0 H_z \Delta x_0$, and hence we arrive at

$$E_y = \frac{\mu_0 \omega d}{\pi} H_z \int_0^{\theta_m} \frac{g(\tilde{J}_y \cos\theta)}{\cos\theta} d\theta, \tag{8}$$

where θ_m is the maximum tilt angle of the total magnetic field $\mathbf{H}$, $\cos\theta_m = H_z/(H_z^2 + h^2)^{1/2}$.

III. ELECTRIC FIELD AND CURRENT DENSITY IN A SLAB

The function $g(u)$ introduced above is defined in the interval $0 < u < 1$ and can be easily found numerically by plotting its inverse function (6): $u = g \ln(g^{-1} + \sqrt{1+g^{-2}}) = g\, \text{arcsinh}(1/g)$, see Fig. 3. At $1-u \ll 1$, the function $g(u)$ has the limiting form:

$$g(u) \approx \frac{1}{[6(1-u)]^{1/2}}. \tag{9}$$

At small $u \ll 1$ we find

$$g(u) \approx \frac{u}{\ln\left[\frac{2}{u}\ln\left(\frac{2}{u}\ln\frac{2}{u}\right)\right]}, \tag{10}$$

i.e., $g(u)$ is not strictly proportional to u. A very good approximation valid at all u is (see Fig. 3)

$$g(u) \approx u/\ln\frac{(1+g_0^2)^{1/2}+1}{g_0},$$

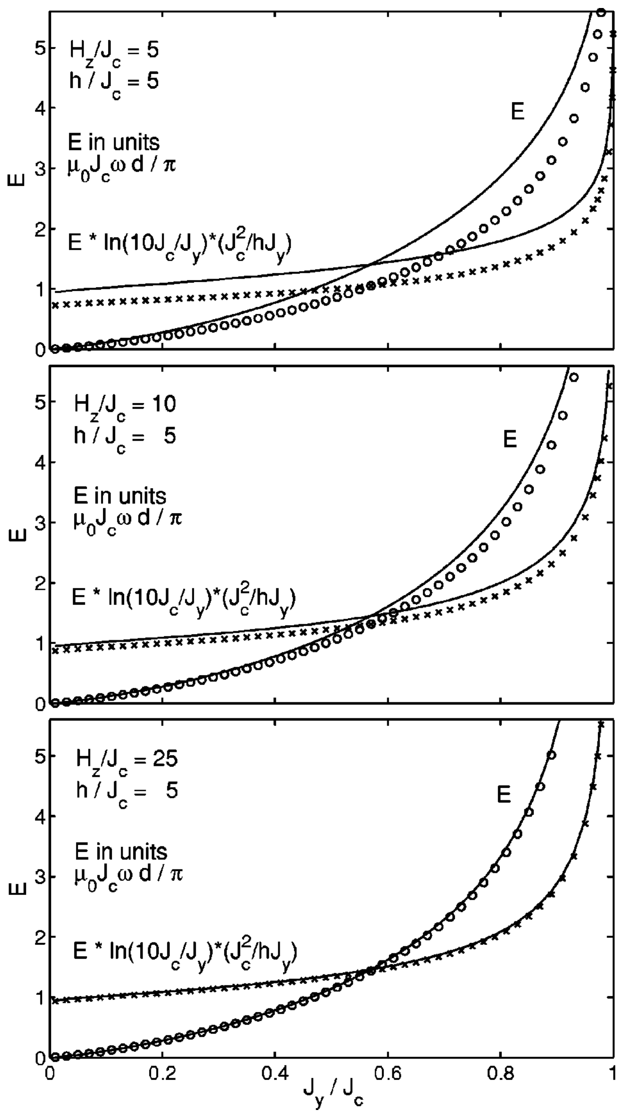

FIG. 4. The electric field E generated by longitudinal vortex shaking in units $\mu_0 J_c \omega d/\pi$ plotted versus $\tilde{J}_y = J_y/J_c$ for $h/J_c = 5$ and $H_z/J_c = 5$ (top), 10 (middle), and 25 (bottom). Shown is E and $E\cdot \ln(10J_c/J_y)(J_c^2/hJ_y)$ to enlarge the area near $E=0$. The circles and crosses mark the exact result, Eq. (8). The solid lines show the analytic approximation, Eq. (16).

$$g_0(u) = \frac{u+u^2}{[24(1-u)]^{1/2}}. \tag{11}$$

If desired, $g(u)$ can be obtained with arbitrary accuracy by iterating Eq. (11) a few times.

Equation (8) for the electric field is valid for all ratios h/H_z, see Fig. 4. We now consider the practically important case of a weak ac field, $h^2 \ll H_z^2$ (but still $h \gg J_c/2$). In this case Eq. (8) simplifies. If $q^2 \equiv (h/H_z)^2 [2(1-\tilde{J}_y)]^{-1} \ll 1$ (i.e., $\tilde{J}_y$ is not too close to unity), we obtain

$$E_y = \frac{\mu_0 \omega d}{\pi} h g(\tilde{J}_y), \tag{12}$$

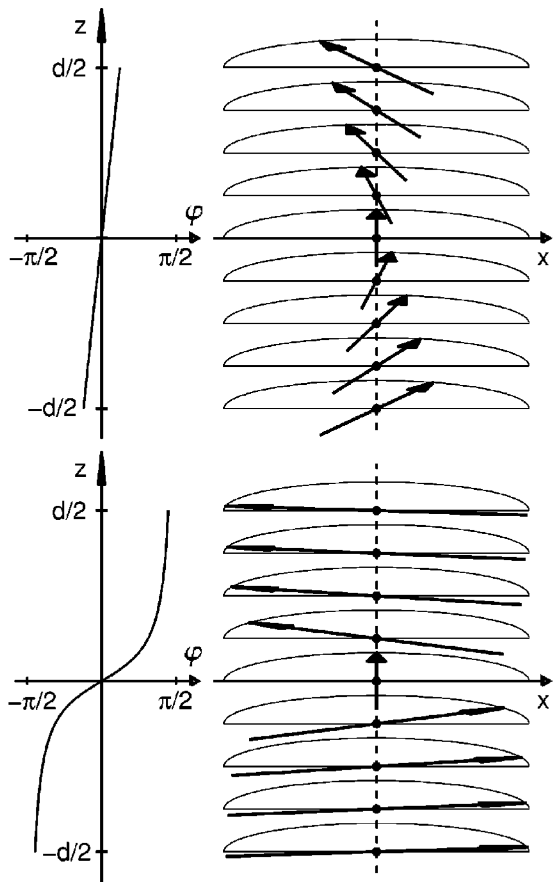

FIG. 5. Direction of the current density $\mathbf{j}_c(z)$ flowing in the xy planes of a slab as a function of the height z. The angle $\varphi(z)$ between $\mathbf{j}_c$ and the y axis given by Eq. (18) is shown at the left. Top: For $\tilde{J}_y = 0.975$, corresponding to the onset of relaxation in the strip. Bottom: For $\tilde{J}_y = 0.4$, corresponding to some later time.

while if the parameter q^2 is of the order of or greater than unity [i.e., $1-\tilde{J}_y < (h/H_z)^2$], we find

$$E_y = \frac{\mu_0 \omega d}{\pi} \frac{H_z}{\sqrt{3}} \ln[(1+q^2)^{1/2} + q]. \tag{13}$$

It should be emphasized that these expressions for longitudinal vortex shaking noticeably differ from the transverse shaking formula:[4]

$$E_y = \frac{\mu_0 \omega d}{\pi} h \tilde{J}_y, \tag{14}$$

which gives the electric field when h_{ac} is directed along the x axis and under our assumption $h \gg J_c/2$. If we generalize the definition of the parameter q as follows,

$$q \equiv \sqrt{3}\, \frac{h}{H_z} g(\tilde{J}_y), \tag{15}$$

it is easy to verify that Eq. (13) reduces to Eq. (12) at $q^2 \ll 1$. Thus, formula (13) is a good interpolation formula for the electric field at all magnitudes of $\tilde{J}_y$ except very close to $\tilde{J}=1$, see below. This formula may be rewritten in explicit form as

$$E_y=\frac{\mu_0\omega d}{\pi}\frac{H_z}{\sqrt{3}}\text{arcsinh}\left[\sqrt{3}\,\frac{h}{H_z}g(\tilde{J}_y)\right], \tag{16}$$

with $\tilde{J}_y=J_y/J_c$ and $g(u)$ from Eq. (11). Figure 4 shows that this expression is an excellent approximation to the exact electric field, Eq. (8), for $h^2\ll H_z^2$ and leads to reasonable results even when $h\sim H_z$.

Let us now consider the distribution of $\mathbf{j}_c(z)$ in the critical state of the slab, Fig. 5. Again, we confine ourselves to the case, $h^2\ll H_z^2$. In this case the magnitude of $\mathbf{j}_c(z)$ remains almost constant, $j_c(z)\approx j_{c\perp}$, see Eq. (1). As to the directions of $\mathbf{j}_c(z)$, one obtains from Eq. (4)

$$\tan\varphi(z)=\frac{2z}{d}\frac{\cos\theta}{g(\tilde{J}_y\cos\theta)}\frac{x_0'}{|x_0'|}. \tag{17}$$

Note that the ratio x_0' sharply changes its sign when h_{ac} reaches its maximum or minimum value. At small $(h/H_z)^2$, we may put $\cos\theta\approx 1$ in this formula, and hence

$$\tan\varphi(z)\approx\frac{2z}{d}\frac{1}{g(\tilde{J}_y)}\frac{x_0'}{|x_0'|}. \tag{18}$$

At $1-\tilde{J}_y\ll 1$ (beginning of the relaxation) one obtains

$$\varphi(z)\approx\frac{2z}{d}\frac{x_0'}{|x_0'|}[6(1-\tilde{J}_y)]^{1/2}, \tag{19}$$

i.e., the direction of the current density deviates from the y axis very little. But if $\tilde{J}_y\ll 1$ (near the end of the relaxation) one arrives at

$$\tan\varphi(z)\approx\frac{2z}{d}\frac{x_0'}{|x_0'|}\frac{\ln(2\sqrt{24}/\tilde{J}_y)}{\tilde{J}_y}, \tag{20}$$

and the direction of $\mathbf{j}_c(z)$ is almost normal to the y axis except in the vicinity of the plane $z=0$, where a sharp turn of the directions occurs. In this case the small value of $\tilde{J}_y$ is due to the small x component of $\mathbf{j}_c(z)$ in almost the whole volume of the slab.

IV. VORTEX SHAKING IN A STRIP

We now consider the temporal evolution of the profiles $J_y(x)$ and $B_z(x)$ in the infinitely thin strip.[7] This evolution is caused by the slow drift of vortices toward the center of the strip and is given by the Maxwell equation

$$\frac{\partial B_z}{\partial t}=-\frac{\partial E_y}{\partial x}, \tag{21}$$

with E_y specified by the formulas obtained above. Since $E_y=v_xB_z$, where v_x is the average velocity of the vortices, the above equation simply expresses the conservation law of the vortex number inside the strip. On the other hand, the Biot-Savart law yields a relation between the sheet current J_y and B_z:

$$\frac{B_z(x)}{\mu_0}=H_a+\frac{1}{2\pi}\int_{-w}^{w}\frac{J_y(u)du}{u-x}. \tag{22}$$

Equations (21) and (22), together with the initial condition

$$J_y(x)|_{t=0}\approx -J_cx/|x|, \tag{23}$$

are sufficient to determine the temporal evolution of J_y, E_y, and B_z in the strip, i.e., the functions $J_y(x,t)$, $E_y(x,t)$, and $B_z(x,t)$.

Equations (21) and (22) can be reduced to a single equation in $J_y(x,t)$. Namely, taking the time derivative of Eq. (22) and inserting Eq. (21), we obtain

$$\frac{\partial E_y(x,t)}{\partial x}=-\frac{\mu_0}{2\pi}\int_{-w}^{w}\frac{\dot{J}_y(u)du}{u-x}$$

($\dot{J}_y=\partial J_y/\partial t$). Integrating this formula over x and using the symmetry $J_y(-x)=-J_y(x)$, we arrive at[8]

$$E_y(J_y(x,t))=\frac{\mu_0}{2\pi}\int_0^w\ln\left|\frac{x-u}{x+u}\right|\dot{J}_y(u,t)du, \tag{24}$$

with $E_y(J_y)$ given by formulas of the previous sections. This implicit equation for $J_y(x,t)$ is easily solved numerically by writing it as a matrix equation for the functions $J_i(t)=J_y(x_i,t)$ calculated at discrete values x_i of the coordinate x. Inverting this matrix, one obtains an explicit expression for the time derivatives $\dot{J}_i(t)$ as a function of all $J_i(t)$, and $J_y(x,t)$ is then obtained by simple time integration. Equivalently, the equation for $J_y(x,t)$ can be also represented in the explicit form[4]

$$\frac{\partial J_y(x,t)}{\partial t}=\frac{2}{\pi\mu_0}\int_{-w}^{w}\frac{du}{u-x}\left(\frac{w^2-u^2}{w^2-x^2}\right)^{1/2}\frac{\partial E_y(J)}{\partial u}. \tag{25}$$

In the derivation of E_y (Sec. II), we have assumed that J_y and H_z do not change during one cycle of the ac field. This approximation is justified when the relaxation time of the profiles $J_y(x)$ and $H_z(x)$ considerably exceeds the period $2\pi/\omega$ of the ac field. Equation (25) shows that this condition is fulfilled if the sheet current J_y is not too close to J_c, and if the strip is sufficiently wide. Indeed, the right-hand side of the equation is proportional to $(d/w)\omega h$; this becomes evident if one introduces the dimensionless length x/w and takes into account formula (12). Thus, the decrease of $\tilde{J}_y=J_y/J_c$ during one cycle is determined by the parameter $h/(wj_{c\perp})$, and the above formulas are applicable when $wj_{c\perp}\gg h\gg dj_{c\perp}/2$. At the beginning of the relaxation when J_y is very close to J_c, the right-hand side of Eq. (25) contains H_z instead of h, see formula (13). Hence, if $H_a>wj_{c\perp}$, our assumption fails at this initial stage. However, the duration of this stage is so small at $h^2\ll H_a^2$ that one may neglect it in calculations.

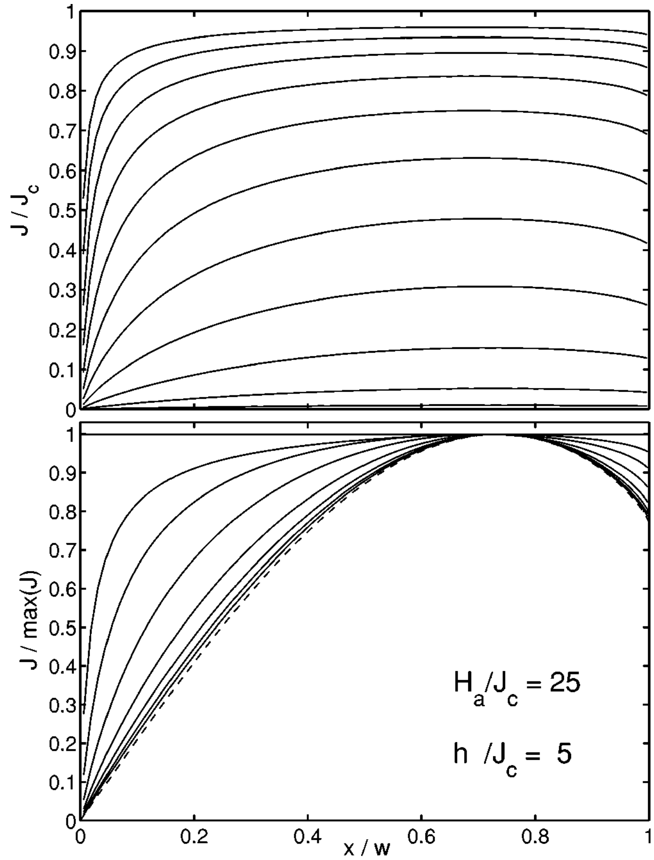

FIG. 6. Profiles of the sheet current $J(x,t)$ for a strip with $H_a/J_c=25$ and $h/J_c=5$. Shown are the profiles at various times t in units $t_1=(\pi/\omega)(w/d)(J_c/h)$, from top to bottom. Upper panel: $t=0$, 0.005, 0.01, 0.02, . . . ,1.28, 2.56, and 5.12. Lower panel: $t=0$, 0.025, 0.1, 0.4, 1.6, 6.4, 25.6, and 102.4. To show how the shape changes, the profiles in the lower panel are normalized to unity maximum value. The limiting profile, obtained for a strip with linear $E(J)$, is shown as the dashed line.

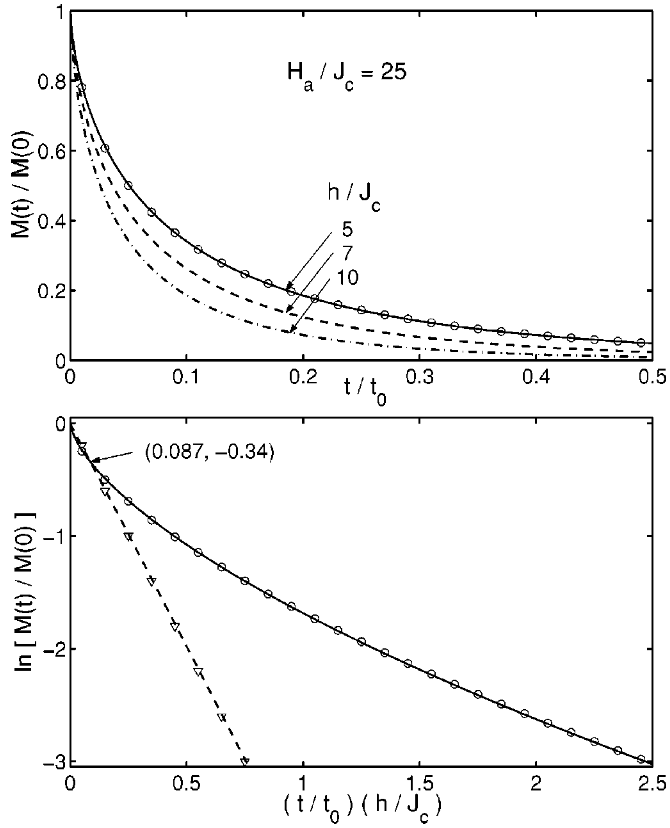

FIG. 7. Time dependence of the magnetic moment $M(t)$ of the strip relaxing during longitudinal shaking at various amplitudes $h \gg J_c/2$ of the ac magnetic field. Top: $h/J_c=5$, 7, and 10, and $H_a/J_c=25$. The time unit is $t_0=\pi w/(\omega d)$. Bottom: The same data, when plotted versus the reduced time $(t/t_0)(h/J_c)=t/t_1$, collapse into one universal curve (solid line), which is not exactly exponential even at large times. The dashed line shows the magnetic moment relaxing during transverse shaking with $E(J)$ from Eq. (14). This ohmic relaxation ($E\propto J$) is exactly exponential (after a short transient time). The curves cross at the point (0.087, -0.34). The circles and triangles show the analytical expressions (26) and (27).

The numerical method of solving Eq. (24) is well elaborated,[7,8] and we use it to analyze the evolution of the sheet current $J(x,t)$. In Fig. 6 the profiles of the sheet current $J_y(x,t)$ in the strip are shown at various times. When $h^2 \ll H_a^2$, formula (12) is practically valid at any $\tilde{J}_y$, and the relaxation depends only on the combination $\omega(d/w)(h/J_c)$. Hence, if the time is measured in units of $t_1=(\pi w j_{c\perp}/\omega h)$, all profiles for various ω, h, and various specimens coincide at equal times. But the profiles do not have a time-independent universal spatial shape here, as has occurred with transverse vortex shaking[4] at long times due to the fact that $E\propto J$, Eq. (14). However, at large times, when $\tilde{J}_y \ll 1$, the voltage-current characteristic becomes almost linear [see Eq. (10)], and the profiles tend to the universal shape obtained for the linear $E_y(J_y)$.

In Fig. 7 we show the time dependence of the magnetic moment[9] per unit length of the strip, $M(t)=\int_{-w}^{w}J(x,t)x\,dx$, at various amplitudes h of the ac magnetic field. Again, when $h^2 \ll H_a^2$, the rate of the magnetic-moment decay is proportional to the combination $\omega(d/w)(h/J_c)$. In other words, all dependences $M(t)$ should map onto one universal curve if one uses the time unit $t_1=\pi w j_{c\perp}/(\omega h)$. Figure 7 (bottom) shows that this coincidence occurs within line thickness for $h/H_a \leq 0.4$. But in contrast to the transverse vortex-shaking effect,[4] this universal dependence is not exactly exponential in time since E_y is not strictly proportional to J_y ("ohmic") even at small J_y [compare Eqs. (10) and (12) with Eq. (14)]. The deviation from an exponential law is only weak over each decade in $M(t)$ but is such that the relaxation becomes slower at larger times. An excellent fit within line thickness of both M and $\ln M$ up to times $t/t_1 \approx 8$, corresponding to $M(t)/M(0) \geq 10^{-3}$, is

$$M(t)\approx M(0)\exp[-1.68(t/t_1)^{0.64}], \tag{26}$$

while for transverse shaking Ref. 4 yields at $h \gg J_c$

$$M(t)\approx M(0)\exp[-4.01t/t_1]. \tag{27}$$

Comparing the relaxation of $M(t)$ during longitudinal and transverse vortex shaking, one can see in Fig. 7 that initially

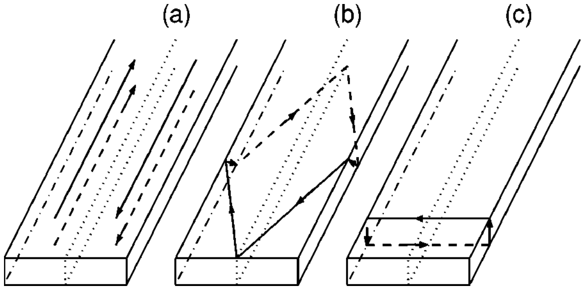

FIG. 8. Current stream lines at the surface of the strip. (a) Onset of the relaxation: longitudinal currents caused by the applied dc field. (b) During the relaxation: long current loops standing still during most of each period, but stretching and flipping periodically at the moments when $dh_{ac}/dt=0$. (c) End of the relaxation: short current loops caused by the ac field. Inside the strip the current density is $\mathbf{j}_c(z)=j_c(z)[-\sin\varphi(z),\cos\varphi(z),0]$, with magnitude (1) and angle (17). Thus, in (a) $\mathbf{j}_c(z)=-j_{c\perp}\,\mathrm{sgn}(x)\hat{\mathbf{y}}$, in (b) the angle $\varphi(z)$ varies continuously across z, see Fig. 5, and in (c) $\mathbf{j}_c(z)=-j_{c\perp}\,\mathrm{sgn}(z)\hat{\mathbf{x}}$.

the longitudinal shaking yields faster relaxation, but at short times $t/t_1\approx 0.087$ the two curves cross at $M(t)/M(0)\approx 0.71$, and at longer times the relaxation by longitudinal shaking is slower.

Knowing the function $J_y(x,t)$ in the infinitely thin strip and the distribution of $\mathbf{j}_c$ over the thickness of the slab, Eq. (17), one can construct the two-dimensional distribution of critical current density over the cross section of a strip with finite thickness d and the evolution of this distribution in time. Figure 8 shows stream lines of the current density at the surface of the strip at the beginning, at an intermediate moment, and at the end of the relaxation process. It is seen that the longitudinal currents flowing in the initial state of the strip evolve into a long loop which gradually shortens its size in the y direction, and eventually the currents circulate only in the x-z plane. In other words, the initial critical state across the width of the strip (caused by H_a) relaxes, and the critical state only across the thickness of the superconductor (caused by h_{ac}) remains.

V. LARGE AC MAGNETIC FIELDS

We now consider the case in which the magnitude of the ac magnetic field is large, $h>j_{c\perp}w$, and thus our above assumption that J_y is constant during one cycle of the ac field fails. Although in this situation the decay of J_y is fast and takes only a few periods of the ac field, $2\pi/\omega$, the quasistatic approximation (Bean model) is still valid if ω is small, thus formulas (1)–(6) remain true. One should only modify the expression for the electric field E_y.

The x component of the velocity of a vortex line when the ac field changes by δh is

$$v_x(t)=(\delta x_0/\delta h)\cdot(dh_{ac}/dt)=\omega h|\cos\omega t||x_0'|,$$

where $|x_0'|=(d/2)\cos\theta g(\tilde{J}_y\cos\theta)/H_z$ [see also Eq. (6)]; $\cos\theta=H_z/(H_z^2+h^2\sin^2\omega t)^{1/2}$ is a function of time t. Here we choose h_{ac} in the form $h_{ac}=h\sin\omega t$, to start with the value

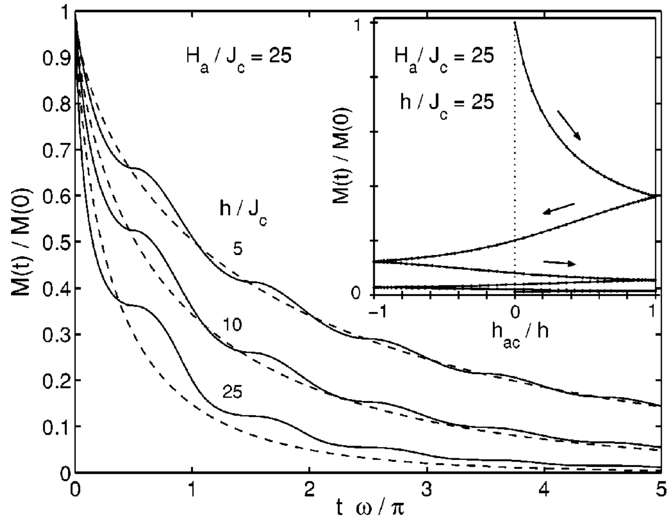

FIG. 9. Relaxation of the magnetic moment $M(t)$ of the strip obtained from the time-averaged electric field, Eq. (16) (dashed lines, see also Fig. 7), and from the exact $E_y(t)$, Eq. (28) (solid lines), plotted versus $t\cdot(\omega/\pi)$ for $w/d=20$, $H_a/J_c=25$, and three amplitudes $h/J_c=5$, 10, and 25. For the latter large amplitude the inset shows $M(t)$ plotted versus the ac field $h_{ac}(t)=h\sin(\omega t)$.

$\theta=0$ at $t=0$. Inserting this $v_x(t)$ into the formula for the electric field $E_y=v_xB_z$, we arrive at

$$E_y(t)=\frac{\mu_0 h\omega d}{2}|\cos\omega t|\cos\theta g(\tilde{J}_y\cos\theta). \tag{28}$$

Formula (28) gives the instantaneous value of the electric field E_y in the slab. In the limiting case considered above, when the sheet current J_y and the field H_z are approximately constant during one cycle of the ac field, the averaging of Eq. (28) over one cycle leads to expression (8).

To describe the longitudinal vortex-shaking effect in the strip at $h>j_{c\perp}w$, one can use the equations of Sec. IV with E_y given by formula (28), which replaces the expression (8) or its approximation, Eq. (16). Figure 9 shows the relaxation of the magnetic moment $M(t)$ of the strip obtained both from the exact $E_y(t)$, Eq. (28), and from approximation (16), for $w/d=20$. It can be seen that at not too large relative amplitudes $h/wj_{c\perp}=(h/J_c)(d/w)\ll 1$ the exact $M(t)$ slightly oscillates about the smooth $M(t)$ of Fig. 7. At large amplitudes $h=H_a>wj_{c\perp}\gg J_c$, the magnetic moment relaxes practically completely within a few periods of the ac field, and it oscillates about a curve which now deviates from the smooth $M(t)$ obtained using Eq. (16). Interestingly, this theoretical $M(t)$ (see inset to Fig. 9) qualitatively resembles the experimental data[10–13] obtained earlier for different geometries of the superconductor.

VI. CONCLUSIONS

In the vortex-shaking effect the applied ac magnetic field h_{ac} is perpendicular to the external dc magnetic field H_a, or almost perpendicular.[1–3] We consider this effect in an infinitely long thin strip of thickness d, with the dc field perpendicular to the plane of the strip, and distinguish between the longitudinal and the transverse effects since they have different physical origin. We classify these effects according to the direction of the ac field relative to the direction of the currents circulating in the critical state of the strip. In the transverse effect the ac field lies in the plane of the strip and is perpendicular to the currents (i.e. to the length of the strip), while in the longitudinal effect the ac field is along the currents. The theory of the transverse effect was given in our previous paper.[4] Here we have presented the theory of the *longitudinal* effect, when the ac field is along the strip, under the assumption that the amplitudes of H_a and h_{ac} considerably exceed the critical value of the sheet current, $J_c = d j_{c\perp}$. Note that if both the dc and ac magnetic fields are normal to the strip plane, no relaxation should occur, except when an additional transport current is applied to the strip.[14]

We have shown that periodic tilt of vortices along the strip is accompanied by a drift of the vortices towards the center of the sample. This drift generates an electric field that eventually leads to the complete relaxation of the critical state across the width of the strip. Strictly speaking, the relaxation of the currents and of the irreversible magnetic moment of the strip during longitudinal shaking is not described by an exponential law (as it occurs in the transverse effect) but is well approximated by a stretched exponential, Eq. (26). Shortly after its onset, the relaxation is faster than that with transverse shaking, but then it becomes noticeably slower than this. Interestingly, in the longitudinal effect, the relaxation not only causes a redistribution of currents over the cross section of the strip, but it also changes their direction (eventually by $\pi/2$), see Fig. 7, while in the transverse effect the direction of the currents remains unchanged. The obtained results together with those of Ref. 4 will enable one to describe the vortex-shaking effect in real superconducting samples of finite length, where both types of shaking occur.

ACKNOWLEDGMENTS

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