RAPID COMMUNICATIONS

PHYSICAL REVIEW B **67**, 100508(R) (2003)

Josephson-phase qubit without tunneling

M. H. S. Amin,* A. Yu. Smirnov,† and Alec Maassen van den Brink‡

D-Wave Systems Inc., 320-1985 W. Broadway, Vancouver, British Columbia, Canada V6J 4Y3

(Received 12 December 2002; published 31 March 2003)

We show that a complete set of one-bit gates can be realized by coupling the two logical states of a phase qubit to a third level (at higher energy) using microwave pulses. Thus, one can achieve coherent control without invoking any tunneling between the qubit levels. We propose two implementations, using rf-SQUIDs and *d*-wave Josephson junctions.

DOI: 10.1103/PhysRevB.67.100508 PACS number(s): 03.67.Lx, 85.25.Cp, 85.25.Dq, 74.72.-h

In the field of Josephson qubits,[1] phase qubits enjoy continued attention. This is partly due to their tolerance to decohering background-charge noise compared to charge qubits. Most phase-based designs rely on the tunnel splitting Δ to flip the state, i.e., to perform a σ_x operation. This has several disadvantages. First, Δ is exponentially sensitive to the device parameters. This makes manufacturing spread especially severe, hampering scalability. Second, it is hard to stop the evolution, so one may need to, e.g., refocus.[2] One can in principle switch off Δ using a compound (Bloch-transistor) junction, but this considerably increases the parameter sensitivity even further.[3] Also, for many systems Δ is too small to be useful or even observable. Conversely, current-biased ''large-junction'' qubits[4,5] avoid the reliance on tunneling at the price of a large spacing between the logical levels, leading to a strong always-on σ_z evolution.

Recently, in Ref. 6 it has been proposed to flip the state of a qubit by two consecutive microwave pulses. The first pulse excites the qubit from, say, $|0\rangle$ to a higher[7] auxiliary state $|2\rangle$ through a Rabi oscillation. The next takes the qubit back to the logical space, but now to $|1\rangle$, addressing the first disadvantage above. However, this pulse sequence would carry $|1\rangle$ to $|2\rangle$ instead of the desired $|0\rangle$; *a fortiori*, it thus does not map a general (superposition) qubit state to another, hence is not a valid gate operation. Even if this would be remedied [by, e.g., preceding (following up) the sequence with an extra $|1\rangle \leftrightarrow |2\rangle$ ($|0\rangle \leftrightarrow |2\rangle$) pulse], the method's state selectivity relies on a bias between $|0\rangle$ and $|1\rangle$, so the second disadvantage is overcome at best partially (the bias can be removed during idle periods, but not during gate action); also, the inflexible restriction to bit-flip gates only remains.

Simultaneously, Ref. 8 has given a largely[9] correct proposal of using an auxiliary state to implement some gates for a different class of qubits. The coupling to the third state does not involve microwaves, and the resulting lack of tunability seems to limit the proposal to a discrete set of gates. In this paper, we resolve the abovementioned problems by showing that a general quantum gate *can* be realized with Rabi pulses alone, without using tunneling.

Consider a general system with a bistable potential (Fig. 1). The lowest levels in the left and right wells are taken as the logical states $|0\rangle$, $|1\rangle$. Unlike most other phase-qubit designs, we choose our parameters so as to make Δ smaller than all relevant energy scales, in particular the decoherence rate: $\Delta \ll 1/\tau_\varphi$ ($\hbar=1$). Then, one can consider $|0\rangle$, $|1\rangle$ as energy eigenstates.

We induce transitions to a higher state $|2\rangle$ by applying microwaves with frequencies near the energy differences $E_2-E_{0,1}$ (Fig. 1). The system then undergoes Rabi oscillations, starting from the logical space. After half a Rabi period ($t_c=\pi/\Omega_R$), the probability of finding the system in $|2\rangle$ will be zero again. The qubit wave function, however, will in general have changed: if, e.g., the system starts from $|0\rangle$, it will end up in a superposition. Thus, a matrix element has been created between $|0\rangle$ and $|1\rangle$, equivalent to a σ_x term in the reduced Hamiltonian.

More quantitatively, let us write the Hamiltonian as

$$H=H_0+V(t), \tag{1}$$

$$H_0=E_0|0\rangle\langle 0|+E_1|1\rangle\langle 1|, \tag{2}$$

where H_0 accounts for the uncoupled Josephson junction (with $E_2\equiv 0$ for convenience) and $V(t)$ for the microwaves. This simple model captures the physics outlined above in two regimes. *Regime (a)* corresponds to near-degenerate logical levels and one external frequency,

$$V(t)=Ve^{-i\omega t}+\text{H.c.} \tag{3a}$$

and $|\delta_j|\lesssim|V|$, with the detunings $\delta_j\equiv\omega+E_j$ ($j=0,1$) and $|V|$ the size of a typical matrix element of V [Fig. 1(a)]. On the other hand, in *regime (b)* the logical levels are well separated, $|E_1-E_0|\gg|V|$, and each is coupled to $|2\rangle$ by its own frequency:

$$V(t)=V_0e^{-i\omega_0 t}+V_1e^{-i\omega_1 t}+\text{H.c.} \tag{3b}$$

Again $|\delta_j|\lesssim|V|$, with now $\delta_j\equiv\omega_j+E_j$ [Fig. 1(b)].

We expand the wave function as $|\psi\rangle=\Sigma_{j=0}^{2}c_j|j\rangle$ and introduce $\tilde{c}_j=c_je^{-i\omega_j t}$ ($j=0,1$). In the rotating-wave approximation[10] (RWA), $\tilde{\psi}=(\tilde{c}_0,\tilde{c}_1,c_2)^{\mathrm{T}}$ obeys $i d_t\tilde{\psi}=\tilde{H}\tilde{\psi}$ with the time-independent[11]

$$\tilde{H}=\begin{pmatrix}\delta_0 & 0 & u_0^*\\ 0 & \delta_1 & u_1^*\\ u_0 & u_1 & 0\end{pmatrix}, \tag{4}$$

in terms of the only relevant matrix elements $u_j=\langle 2|V_j|j\rangle$. In particular, setting $\omega_0=\omega_1\equiv\omega$ and $V_0=V_1\equiv V$ in regime (a), Eq. (4) holds in both regimes.

A watershed now occurs between the case of equal detunings[12] $\delta_0=\delta_1$, which will shortly be reduced to the

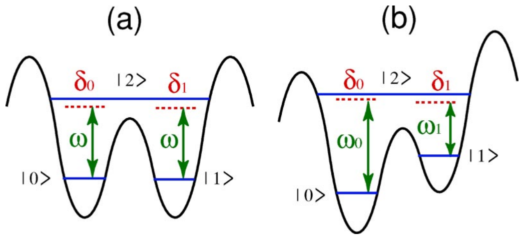

FIG. 1. (Color online) Gate operation by coupling the two logical states to a third level with microwaves. (a) (Near-)degenerate states and one pulse frequency. (b) Biased states and two frequencies.

standard Rabi problem, and the more complicated $\delta_0 \neq \delta_1$, which however does not correspond to a useful gate operation. Note that in regime (a), the former case is the simple one of degenerate qubit levels; this may be the preferred mode of operation in practice.

First taking $\delta_0 = \delta_1 = \delta$, we define the Rabi frequency

$$\Omega_{\mathrm{R}} = \sqrt{\delta^2/4 + |u_0^2| + |u_1^2|}, \tag{5}$$

and a mixing angle $0 < \eta < \pi$ by $\tan \eta = 2\sqrt{|u_0^2| + |u_1^2|}/\delta$. One readily finds the inert eigenfunction

$$\tilde{\psi}_0 = \frac{(u_1, -u_0, 0)^{\mathrm{T}}}{\Omega_{\mathrm{R}} \sin \eta} \tag{6}$$

obeying $\tilde{H}\tilde{\psi}_0 = \tilde{\nu}_0 \tilde{\psi}_0$ with $\tilde{\nu}_0 = \delta$, which is decoupled from $|2\rangle$ by destructive interference of the microwaves.[13] In the complementary 2×2 space, simple algebra yields the rest of the spectrum as $\tilde{\nu}_\pm = \Omega_{\mathrm{R}}(\cos \eta \pm 1)$,

$$\tilde{\psi}_+ = \left(\frac{u_0^*}{2\Omega_{\mathrm{R}} \sin \eta/2}, \frac{u_1^*}{2\Omega_{\mathrm{R}} \sin \eta/2}, \sin \eta/2 \right)^{\mathrm{T}}, \tag{7}$$

$$\tilde{\psi}_- = \left(\frac{u_0^*}{2\Omega_{\mathrm{R}} \cos \eta/2}, \frac{u_1^*}{2\Omega_{\mathrm{R}} \cos \eta/2}, -\cos \eta/2 \right)^{\mathrm{T}}. \tag{8}$$

In terms of Eqs. (6)–(8), it is trivial to compute the evolution over half a Rabi period $\tilde{U}(t_{\mathrm{c}}) = \exp\{-i\tilde{H}t_{\mathrm{c}}\}$, decomposing into a reduced gate action $\tilde{U}_{\mathrm{r}}$ in the logical space and a trivial phase for $|2\rangle$ [cf. above Eq. (1) and Figs. 2(a), 2(b)]. Only the former concerns us here,

$$\tilde{U}_{\mathrm{r}} = \frac{1}{\Omega_{\mathrm{R}}^2 \sin^2 \eta} \begin{pmatrix} \zeta|u_0^2| + \zeta^2|u_1^2| & u_0^* u_1(\zeta - \zeta^2) \\ u_0 u_1^*(\zeta - \zeta^2) & \zeta^2|u_0^2| + \zeta|u_1^2| \end{pmatrix}, \tag{9}$$

a central result, with $\zeta = -e^{-\pi i \cos \eta}$ running through the unit circle with detuning. Clearly, $\tilde{U}_{\mathrm{r}}$ is unitary, overcoming the problem[6] mentioned in the introduction. The repeated evolution $\tilde{U}(nt_{\mathrm{c}})_{\mathrm{r}} = \tilde{U}_{\mathrm{r}}^n$ follows by simply putting $\zeta \mapsto \zeta^n$ in Eq. (9); hence, the only advantage of taking $n > 1$ seems to lie in accessing $\zeta^n \approx 1$ without large detuning.

Let us demonstrate that already in its two simplest limits, Eq. (9) is flexible enough to lead to universal computing; contrast Refs. 6 and 8. For unbiased systems with symmetric potential [cf. Eqs. (12) and (13) below] and $u_0 = u_1$,

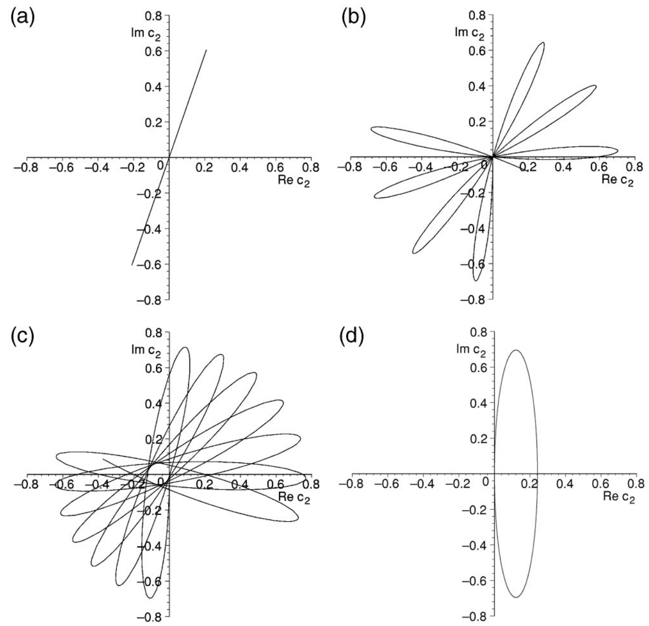

FIG. 2. Evolution of $c_2(t) \equiv \langle 2|\tilde{U}(t)|0\rangle$ for (a) $\delta_0 = \delta_1 = 0$, $u_0 = e^{-i/3}$, $u_1 = 1.2$, $0 \leqslant t \leqslant 5\pi$; (b) $\delta_0 = \delta_1 = 0.25$, $u_0 = u_1 = 1$, $0 \leqslant t \leqslant 5\pi$; (c) $\delta_0 = 0.25$, $\delta_1 = 0$, $u_0 = u_1 = 1$, $0 \leqslant t \leqslant 10\pi$; (d) $-\delta_0 = \delta_1 = 0.25$, $u_0 = 1$, $u_1 = e^{i/3}$, $0 \leqslant t \leqslant 5\pi$.

$$\tilde{U}_{\mathrm{r,sy}}\left(\frac{\delta}{\Omega_{\mathrm{R}}}\right) = \exp\left\{ i\left[\frac{\pi}{2} - \frac{3\pi\delta}{4\Omega_{\mathrm{R}}} + \left(\frac{\pi}{2} + \frac{\pi\delta}{4\Omega_{\mathrm{R}}} \right)\sigma_x \right] \right\}. \tag{10}$$

One can also drive at resonance $\delta = 0$, but with arbitrary $|u_0/u_1|$ [in regime (b)]. Setting $(|u_0^2| - |u_1^2|)/(|u_0^2| + |u_1^2|) = \cos \xi$, one has $2u_0^* u_1/(|u_0^2| + |u_1^2|) = e^{i\gamma} \sin \xi$, and

$$\tilde{U}_{\mathrm{r,res}}(\xi) = i e^{i(\pi/4 + \gamma/2)\sigma_z} e^{i\xi\sigma_x} e^{i(\pi/4 - \gamma/2)\sigma_z}. \tag{11}$$

Of course, one always has the phase shifts $e^{i\chi\sigma_z}$ available, by applying a small bias but no microwaves. Thus, the equivalence above Eq. (1) is quantitative: adding either Eq. (10) or Eq. (11) suffices to generate all one-bit gates. For $u_0 = u_1$, $\delta = 0$, both of the above reduce to a quantum NOT $\tilde{U}_{\mathrm{r}} \propto \sigma_x$; in general, $[\tilde{U}_{\mathrm{r}}, \sigma_z] \neq 0$ unless $u_0 u_1 = 0$.

In the "laboratory frame" $\psi_{\mathrm{r}} = (c_0, c_1)^{\mathrm{T}}$, $U_{\mathrm{r}} \propto e^{i(\omega_0 - \omega_1)t_{\mathrm{c}}\sigma_z/2}\tilde{U}_{\mathrm{r}} = e^{i(E_1 - E_0)t_{\mathrm{c}}\sigma_z/2}\tilde{U}_{\mathrm{r}}$ [$= \tilde{U}_{\mathrm{r}}$ in regime (a)]. For this specific form, it is assumed that the gate operation starts at $t = 0$; this fixes the phases of $V_{0,1}$ in Eq. (3b).

The effective operation rate Ω_{R} in Eq. (5) depends on *intra*well matrix elements u_j, between wave functions having an overlap of $O(1)$. For reasonable microwave powers, one thus expects a speedup compared to conventional designs relying on a small Δ. Indeed, the analysis of Ref. 6 applies, showing that the number of operations achievable in τ_φ is increased by an order of magnitude. If anything, the present situation is slightly more favorable still, since our gate operation is a *one*-step process.

Generalizing the above to $\delta_0 \neq \delta_1$ would lead to tedious cubic equations. Fortunately however, this is unnecessary since the crucial decomposition of $\tilde{U}(t)$ then does not generally occur for any finite t.[14] To see this, start, e.g., from $|0\rangle$ and plot $c_2(t)$ by diagonalizing a few instances of Eq. (4) numerically. The locus of c_2 will evolve in a daisylike pattern [Fig. 2(c)], without returning to the origin like it does periodically for equal detunings [Figs. 2(a) and 2(b)]. These numerics can be supplemented with an expansion in $\delta_0 - \delta_1$, the case $\delta_0 = \delta_1 = 0$ being a particularly simple zeroth-order problem.

Some idealizations have been made in the above: H as in Eqs. (1) and (2) is a low-dimensional approximation and the effective Eq. (4) follows only in the RWA. The pertinent errors typically are $\sim|V|/|\Delta E|$, where ΔE can be the distance $E_2 - E'$ (positive or negative) to an ignored level E' or $E_1 - E_0$ in regime (b), etc. These can be reduced using a narrow-band, low-power source, but only under the condition $\Omega_R \tau_\varphi \gg 1$ of fast gate operation. The issue is well understood, and techniques such as pulse-shaping exist to counteract off-resonant (including counter-rotating) errors,[15] in addition to general quantum error-correction methods. The same holds for timing errors.

We now propose two exemplary implementations.

(a) SQUIDs. One can use any SQUID qubit, such as the three-junction[16] or the usual rf-SQUID. The latter consists of a superconducting ring interrupted by a junction with Josephson energy E_J. The free energy is

$$\mathcal{U}(\phi) = \frac{(\Phi_0 \phi/2\pi - \Phi_e)^2}{2L} - E_J \cos\phi, \tag{12}$$

with ϕ the phase difference across the junction and $\Phi_0 = \pi/e$ the flux quantum. When the external flux $\Phi_e = \Phi_0/2$ and the ring inductance $L > \Phi_0^2/4\pi^2 E_J$, $\mathcal{U}$ will have the bistable shape of Fig. 1(a). The states $|0\rangle$ and $|1\rangle$ correspond to opposite directions of persistent current.

A deviation of Φ_e from $\Phi_0/2$ tilts $\mathcal{U}$ [Fig. 1(b)], generating a σ_z operation; applying an rf flux performs a σ_x-like gate, Eq. (9). To read out the qubit one should measure the SQUID-generated flux at $\Phi_e = \Phi_0/2$; its two directions correspond to the logical states.

(b) Current-biased d-wave junctions. In d-wave grain boundaries, the order parameter is oriented differently on the two sides of the junction. The resulting Josephson potential is intrinsically bistable,[2,17–19] realizing Fig. 1.

In general, the current-phase relation can have many harmonics. Here, we approximate $I(\phi) = I_1 \sin\phi - I_2 \sin 2\phi$, where I is the current through and ϕ the phase difference across the junction. The free energy thus is

$$\mathcal{U}(\phi) = -E_J\left[\cos\phi - \frac{\alpha}{4}\cos(2\phi)\right] - \frac{I_b}{2e}\phi, \tag{13}$$

where $E_J = I_1/2e$ is the Josephson energy corresponding to the first harmonic, $\alpha = 2I_2/I_1$, and I_b is the bias current. When $I_b = 0$, the minima of Eq. (13) are located at

$$\phi = \begin{cases} \pm \arccos(1/\alpha), & \alpha > 1; \\ 0, & \alpha \leqslant 1. \end{cases} \tag{14}$$

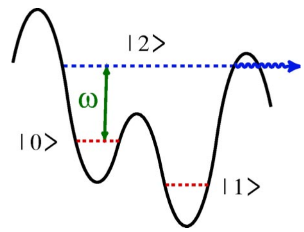

FIG. 3. (Color online) Qubit readout using microwave-assisted tunneling to the resistive state. Only the left state will tunnel out.

For $\alpha > 1$, $\mathcal{U}$ thus is doubly degenerate, with barrier height $\delta\mathcal{U} = E_J(\alpha + \alpha^{-1} - 2)/2$ between the minima.

A finite I_b removes the degeneracy; this can be used for the σ_z operation. The gate Eq. (9) can be performed using ac bias currents with appropriate frequencies, as discussed before. For readout, we apply an I_b such that one of the excited states has a high probability of tunneling to the continuum (Fig. 3). By selectively coupling one logical state to this excited level, we can determine the qubit state by measuring the junction voltage.[4]

Decoherence in d-wave qubits is a controversial subject but not central here, so we merely mention a few sources besides external noise (e.g., in I_b). The contribution of ungapped nodal quasiparticles is often overestimated: for a misoriented grain boundary, a node on one side always faces a gapped direction on the other, suppressing tunneling exponentially.[20] More problematic are midgap (Andreev) states. Still, since these are split at the qubit's working point, the decoherence due to them can be shown to be tolerable.[21]

As a sideline, a classic double-well system with a tunnel splitting is the NH_3 molecule. Taking a heavier central nucleus, one arrives at PH_3 and AsH_3 as instances of Fig. 1(a) on a much larger energy scale.[22]

In conclusion, it has been shown that microwave coupling via an auxiliary level suffices for coherent control of a Josephson-phase qubit. The advantages include comparative tolerance to device-parameter spread, ability to operate without refocusing, and speed. Charge-noise tolerance (cf. the first paragraph) should be excellent: without a need for ϕ-tunneling, the ratio of E_J to the charging energy E_C can (and should) be comparatively large. A finite E_C is needed only to ensure appreciable level spacings, as determined by the plasma frequency $\sim\sqrt{E_J E_C}$; suitable device parameters can be readily chosen. For full-fledged quantum computing, one should additionally describe the coupling of these qubits into a quantum register. While, e.g., tunable-bus proposals[23] have the promise of being able to couple any type of Josephson qubit, the detailed investigation is still in progress.

We thank A. Blais and A. M. Zagoskin for their remarks on the manuscript, J. P. Hilton for pointing out the three-pulse sequences in the second paragraph, M. F. H. Steininger for the molecular examples, and T. Hakioğlu, J. M. Martinis, and J. Siewert for discussions.

*Electronic address: amin@dwavesys.com
†Electronic address: anatoly@dwavesys.com
‡Corresponding author. Electronic address: alec@dwavesys.com

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