# Unscreened universality class for superconductors with columnar disorder

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The phase transition in a model for vortex lines in high temperature superconductors with columnar defects, i.e., linearly correlated quenched random disorder, is studied with finite size scaling and Monte Carlo simulations. Previous studies of critical properties have mainly focused on the limit of strongly screened vortex line interactions. Here the opposite limit of weak screening is considered. The simulation results provide evidence for a distinct universality class, with values of the critical exponents that differ from the case of strong screening of the vortex interaction. In particular, scaling is anisotropic and characterized by a nontrivial value of the anisotropy exponent $\zeta=\nu_{\parallel}/\nu_{\perp}$. The exponents we find, $\zeta=1.25\pm0.1$, $\nu_{\perp}=1.0\pm0.1$, $z=1.95\pm0.1$, are similar to certain experimental results for $YBa_2Cu_3O_7$.

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Columnar defects have proven very effective at pinning vortex lines in high temperature superconductors (SC's), resulting in significant increases in the critical magnetic fields and currents.[1] The columnar defects are often produced as damage tracks by irradiating the sample with high-energy heavy ions. Such correlated disorder strongly affects the phase transition from the SC to normal state, and there is an ongoing theoretical and experimental effort to investigate the phase diagram and properties of the transition.[2–17] Below a critical temperature the vortices freeze into a glassy SC state, known as a Bose glass, which replaces the Abrikosov vortex lattice of the clean system. Upon heating the system undergoes a continuous phase transition to a resistive vortex liquid state. The transition is characterized by the critical exponents that have been measured in a number of experiments. The critical exponents are universal, i.e., they are common for all systems in the same universality class, and depend only on general features like the range of the interaction and distribution of the disorder. In spite of the fact that the screening length $\lambda$ in high temperature SC is quite large, most theoretical work on columnar defects has concentrated on strongly screened systems with short-range interactions.[18,3,4] In general, long-range interactions often change the universality class of a transition, compared to short-range interactions. It is, therefore, natural to study the universality class with long-range interactions. In this paper we present a systematic study of the SC transition in the systems with columnar disorder, an applied magnetic field, and weak screening, and compare the results with the experiments.

The superconducting phase transition with columnar disorder is also interesting from the perspective of quantum phase transitions in the systems with disorder.[2,3,18–20] In fact, statistical mechanics of vortex lines in three dimensions is closely analogous to world lines of quantum bosons in $(2+1)$ dimensions, where the vortex phase transition corresponds to the zero-temperature boson localization by substrate disorder. The quantum bosons have static disorder in imaginary time, corresponding to the columnar disorder for the vortex lines. The quantum dynamical exponent $\zeta$ (relating the diverging time and length scales at the transition) translates into an anisotropic scaling behavior for the vortex line problem, where correlations along the columnar defects diverge with a different rate than the perpendicular ones when approaching the Bose-glass transition.

The vortex phase transition with columnar disorder has previously been studied for the case of strongly screened vortex interactions.[3,4] There the correlation length exponent $\nu$, anisotropy exponent $\zeta$, and dynamic exponent $z$ take the values $\nu\approx1$, $\zeta=2$, $z\approx4.6$.[4] The result $\zeta=2$ follows if the compressibility is nonsingular through the transition, and is hence believed to be exact. The effect of long-range interactions in systems with correlated disorder has been considered in the context of the superconductor-insulator quantum phase transitions of disordered ultrathin films.[18–20] For long range planar $1/r$-interactions, which act only between the world line segments in the same $xy$ plane, and thus is completely local in the $z$ direction, the system is incompressible, with $\nu\approx1$ and $\zeta=1$.[18] Early simulations[5,21] of more realistic models (from the vortex point of view) show evidence for a nontrivial value of the anisotropy exponent close to 1. Some experiments[14,15] have been interpreted as belonging to an incompressible universality class with $\zeta\approx1$.

The main task in this paper is to examine the superconducting phase transition with columnar disorder in the limit where the screening length is very long, such that any effects of screening of the vortex interaction can be effectively neglected. In other words, we set $\lambda\rightarrow\infty$, and thus examine a no-screening fixed point for the superconducting phase transition. The screening length in the high temperature superconductor is typically $\lambda\sim1000$ Å, so for most of the phase diagram the assumption of no screening is reasonable. We stress that one possibility is that the true asymptotic critical behavior is effectively described by a fixed point with a screened interaction. However, even if this is the case, we expect that the scaling properties upon approaching the transition is controlled by the unscreened model in a large region of the parameter space, before an eventual crossover to the true critical regime. In comparison, this occurs in clean systems without an applied field.[22] Our model also assumes no fluctuations in the amplitude of the SC order parameter (London approximation), which is valid in a large region of the phase diagram well below $H_{c2}$.[6] The interaction is taken to be a fully isotropic, three-dimensional (3D) long- range interaction between the vortex lines, which should apply for fairly isotropic systems. The form of the interaction makes our model *different* from most of the previously studied Boson-like models, with a planar interaction between the vortex lines. We consider only the case when both the magnetic field and all the columnar defects are aligned in the $z$ direction. The anisotropy due to the field and the columnar disorder allows for the possibility of an anisotropic correlation volume, where the correlation length in different directions can diverge with different exponents. In addition we consider dynamical aspects of the problem, which do not have a counterpart in the quantum boson problem, and compute the dynamical exponent $z$. This assumes that Monte Carlo (MC) dynamics for vortex lines can be equated with real dynamics, which should apply close to the transition where the dynamics is slow and overdamped.[23] Our values for $\nu,\zeta,z$ differ from those of the previously studied models, which suggests that the present model belongs to a new, distinct universality class.

Our starting point is the Ginzburg-Landau theory for the superconducting complex scalar order parameter field, $\psi(\mathbf{r})=|\psi|\exp i\theta(\mathbf{r})$,

$$H=\int d^d\mathbf{r}\left[\left|\left(\nabla-\frac{2\pi i}{\Phi_0}\mathbf{A}\right)\psi\right|^2+\alpha|\psi|^2+\frac{1}{2}\beta|\psi|^4+\frac{\mathbf{B}^2}{8\pi}-\frac{\mathbf{B}\cdot\mathbf{H}}{4\pi}\right], \tag{1}$$

where $\Phi_0$ is the magnetic flux quantum, $\mathbf{B}=\boldsymbol{\nabla}\times\mathbf{A}$ is the magnetic flux density, $\mathbf{A}$ is the magnetic vector potential, and $\mathbf{H}$ is the applied magnetic field. By fairly standard manipulations[25] this model can be transformed into a model involving only the vortex degrees of freedom. In the London approximation, i.e., neglecting fluctuations in the amplitude of $\psi(\mathbf{r})$, a discretized form of the vortex Hamiltonian reads

$$H=\frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'}V(\mathbf{r}-\mathbf{r}')\mathbf{q}(\mathbf{r})\cdot\mathbf{q}(\mathbf{r}')+\frac{1}{2}\sum_{\mathbf{r}}D(\mathbf{r}_\perp)q_z(\mathbf{r})^2, \tag{2}$$

where the sums over $\mathbf{r}$ run through all sites on a simple cubic lattice with $\Omega=L\times L\times L_z$ sites and periodic boundary conditions in all three directions. The vortex line variables are specified by an integer vector field $\mathbf{q}(\mathbf{r})$, such that the $\mu=x,y,z$ component is the vorticity on the link from the site $\mathbf{r}$ to $\mathbf{r}+\mathbf{e}_\mu$. The partition function is $Z=\mathrm{Tr}\exp(-H/T)$, where $T$ is the temperature, and Tr denotes the sum over all possible integers $q_\mu$, subject to the constraint that the discrete divergence $\nabla\cdot\mathbf{q}=0$ on all sites, i.e., the vortex lines have no free ends. The vortex-vortex interaction is given by

$$V(\mathbf{r})=\frac{K}{\Omega}\sum_{\mathbf{k}}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sum_\mu(2-2\cos k_\mu)+\lambda^{-2}}, \tag{3}$$

where we choose units such that $K=1$, and consider the limit of no screening, corresponding to $\lambda=\infty$. Randomness is included in the vortex model (2) in the second term in the form of a random core energy, which corresponds roughly to a random $\alpha(\mathbf{r})$ in Eq. (1). In real systems, each columnar defect gives approximately the same pinning energy, while they are located at random positions. On the lattice it is more practical to instead insert one column on each link of the lattice in the $z$ direction, but with a random pinning energy. We use a uniform distribution of random pinning energies in the interval $0\leqslant D(\mathbf{r}_\perp)\leqslant 0.8$, where each $D(\mathbf{r}_\perp)$ is constant in the $z$ direction to model correlated disorder. We model the net applied magnetic field as a fixed number of $L^2/4$ vortex lines penetrating the system in the $z$ direction, corresponding to quarter filling, i.e., one vortex on every fourth link in the $z$ direction. To test for universality of our critical exponents, we also studied the case of $L^2/2$ vortex lines penetrating the system in the $z$ direction and disorder strength $0\leqslant D(\mathbf{r}_\perp)\leqslant 1.0$.

The MC trial moves consist of attempts to insert vortex line loops of random orientation on randomly selected plaquettes of the lattice. One MC sweep consists of one attempt on average to insert a loop on every plaquette, which we define as one MC time step, $\Delta t=1$. The attempts are accepted with probability $1/(1+\exp\Delta E/T)$, where $\Delta E$ is the energy change for inserting the loop. The initial vortex configuration is taken to be a regular lattice of straight lines. To approach equilibrium we discard about $2\times10^4$ MC sweeps ($6.5\times10^4$ for the resistivity) before any measurements are taken, followed by equally many sweeps for collecting data. To verify that the warmup time is long enough, we tried to vary the warmup time between 5000 and $10^5$ sweeps for a single temperature, close to $T_c$, which gave no significant differences in the final results. The results were averaged over up to 1000–2000 samples of the disorder potential. For the simulation of static quantities, we use an exchange MC algorithm to speed up convergence.[24] Note that the exchange method can not be used for dynamic quantities, since it involves large nonlocal moves in phase space. In the following we denote thermal averages by $\langle\cdots\rangle$ and disorder averages by $[\cdots]$.

In the simulation, the helicity modulus and the rms current is obtained by the following procedure.[25] An extra term $H_Q=(K/2\Omega)\mathbf{Q}^2$ is included in the Hamiltonian, where $Q_\mu$ is the total projected area of vortex loops added during the simulation.[25] The helicity modulus in the direction $\mu$ ($\mu=x,z$) is then given by

$$Y_\mu=1-(K/\Omega T)[\langle Q_\mu^2\rangle-\langle Q_\mu^\alpha\rangle\langle Q_\mu^\beta\rangle], \tag{4}$$

and the rms current density is given by $J_\mu=(K/\Omega)[\langle Q_\mu^\alpha\rangle\times\langle Q_\mu^\beta\rangle]^{1/2}$, where we use two different replicas in our simulations, denoted $\alpha$ and $\beta$, to avoid any bias in the expectation values. We also calculate the linear resistivity $\rho$, in which case $H_Q$ is not included in $H$, by evaluating the Kubo formula[26]

$$R_\mu=\frac{1}{2T}\sum_{t=-t_0}^{t_0}[\langle V_\mu(t)V_\mu(0)\rangle], \tag{5}$$

where $t$ is the MC time and $t_0\rightarrow\infty$, and the voltage is $V_\mu\sim\Delta Q_\mu$. $\Delta Q_\mu$ is the net change in the projected vortex loop area during a sweep. In practice the summation time $t_0$ is chosen large enough that the resistivity is independent of $t_0$.

Next we consider the anisotropic finite size scaling relations used to extract critical properties from the MC data.[3,4]

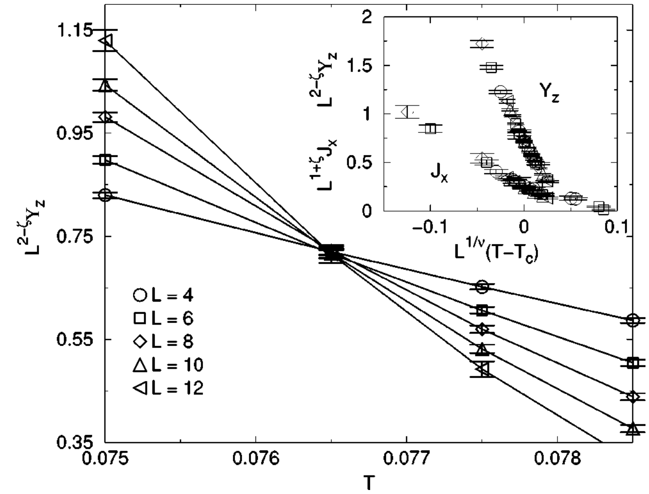


FIG. 1. MC data for the helicity modulus in the $z$ direction vs temperature for different system sizes $L$. $T_c \approx 0.0765$ is estimated as the temperature where all curves intersect. Inset: Finite size scaling collapse of the data for fillings $f=1/2$ and $f=1/4$, and for two disorder strengths (see text) using $\nu=1.0$, $\zeta=1.25$.

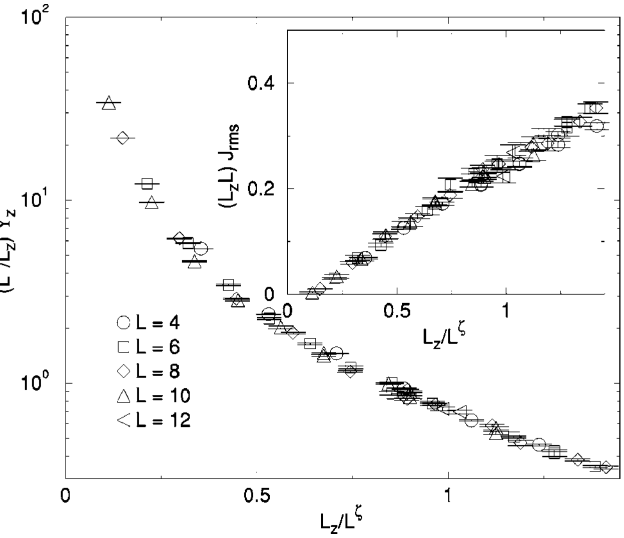


FIG. 2. Finite size scaling collapse according to Eqs. (6) and (7), of the data for the helicity modulus in the $z$ direction (main part) and the rms current in perpendicular directions (inset) at $T=T_c$. The best collapse is obtained for $T_c \approx 0.0765$, $\zeta \approx 1.25$.

At the transition temperature $T_c$ the correlation length in the $xy$ planes, $\xi$, in the $z$ direction, $\xi_z$, and the correlation time, $\tau$, are assumed to diverge as $\xi \sim |T-T_c|^{-\nu}$, $\xi_z \sim \xi^\zeta$, and $\tau \sim \xi^z$. The anisotropic finite size scaling ansatz[4] for the helicity modulus in the $z$ direction is

$$Y_z = L^{1-d+\zeta} f_z(L^{1/\nu}(T-T_c), L_z/L^\zeta), \tag{6}$$

where $d=3$ is the spatial dimensionality, and $f_z$ is a scaling function (all scaling functions will from now onwards be suppressed). Similarly, for the current density in the $x$ direction, we have

$$J_x \sim L^{2-d-\zeta}. \tag{7}$$

The linear resistivity is given by $\rho = E/J$, where $E$ is the electric field and $J$ is the current density, and scales as

$$\rho_x \sim L^{d-3+\zeta-z}, \quad \rho_z \sim L^{d-1-\zeta-z}. \tag{8}$$

In the simulations we consider a whole range of system sizes $L_z$ for each $L$, since the anisotropy exponent $\zeta$ that enters the aspect ratio, $L_z \sim L^\zeta$, is *a priori* unknown. For nonintegral values of $L_z$, we simulate two systems with nearest integer $L_z$ and interpolate the results using linear interpolation. For the resistivity, both linear and logarithmic interpolations were tested and agree within error bars. Once we locate the correct value for $\zeta$, the argument $L_z/L^\zeta$ in the scaling functions can be made constant by setting $L_z \propto L^\zeta$.

We determine the critical temperature $T_c$ for the phase transition and the critical exponents from the MC data for the helicity modulus in Eq. (4), by fits to the finite size scaling form in Eq. (6). Figure 1 shows our estimate of $T_c$, using the scaling form of the helicity modulus in the $z$ direction. By selecting the value for $\zeta$ that gives the best common intersection point, we obtain $\zeta = 1.25 \pm 0.1$ and $T_c = 0.0765$, where the error bar on $\zeta$ is estimated by the interval outside which scaling gets considerably worse. The rms current density in the $x$ direction, $J_x$, can also be used to calculate $T_c$. The result agrees, within the error bars, with the result from $Y_z$.

Right at $T_c$ it is possible to obtain a data collapse when plotting various quantities as a function of $L_z/L^\zeta$. Figure 2 shows such plots for $(L^2/L_z)Y_z$ and $(L_zL)J_{\mathrm{rms}}$. Adjusting $T_c$ and $\zeta$ until the best collapse is obtained gives identical estimates, within the error bars, as the previous method. This clearly demonstrates that the scaling is anisotropic.

The inset of Fig. 1 shows a determination of the correlation length exponent $\nu$, using finite size scaling, for data from two different fillings, $f=1/2$ and $f=1/4$, and two different disorder strengths. We use fits to Eq. (6) to obtain a data collapse for different system sizes ($L=4-8$ for $f=1/2$, $L=8-12$ for $f=1/4$) over an entire temperature interval around $T_c$. Both $T_c$ and $\nu$ were free parameters in the fit resulting in $\nu = 1.0 \pm 0.1$, $T_c = 0.0765$ for $f=1/4$ and $T_c = 0.0747$ for $f=1/2$. The data collapse to a common universal scaling function indicates that the results are indeed universal.

Next we study the critical MC dynamics of the model. We calculate the resistivity $\rho$ in the $x$ and $z$ directions from the Kubo formula in Eq. (5) at $T=T_c=0.0765$. This gives us both a useful consistency test of the value $\zeta \approx 1.25$ found above, and a value for the dynamic exponent $z$. To verify that the summation time in Eq. (5) is long enough, we plot $\rho$ vs $t_0$ in the inset of Fig. 3. We also note that the correlation time is at most $\sim 10^3$ sweeps, which is much less than the equilibration times used in the simulations. Figure 3 shows the resistivity in the $x$ and $z$ directions as a function of system size. We calculate $\zeta$ and $z$ by making a power law fit of the data points in the figure to Eq. (8). This gives $\zeta = 1.3 \pm 0.1$, $z = 1.95 \pm 0.1$, where the error bars are estimated by the bootstrap method,[27] in good agreement with $\zeta \approx 1.25$ found above.

Finally we will compare our findings with some other results. The results for critical exponents, $\zeta = 1.25 \pm 0.1$, $\nu = 1.0 \pm 0.1$, $z = 1.95 \pm 0.1$, imply that the linear resistivity

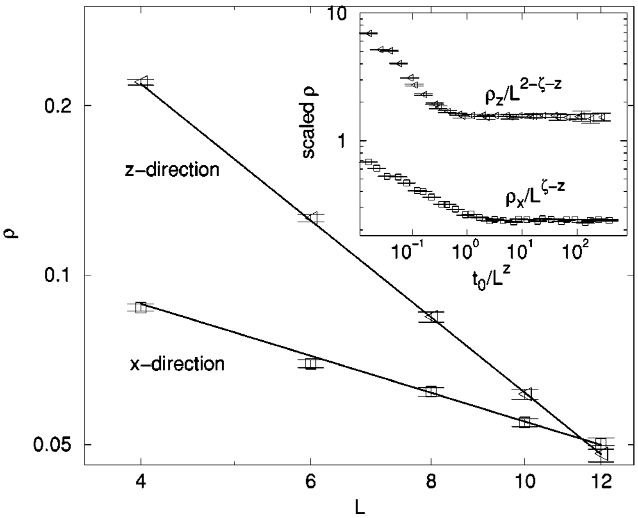


FIG. 3. MC data for resistivity in the $x$ and $z$ directions for different system sizes $L$ at $T=T_c$. The straight lines are power law fits to the data points, with exponents $z-\zeta=0.65$ and $z+\zeta-2=1.28$. Inset: Finite size scaling plot of the resistivity as a function of the summation time $t_0$.

scales as $\rho \sim |T-T_c|^s$ with $s_x=\nu(z+3-d-\zeta)\approx 0.7$, $s_z=\nu(z+1-d+\zeta)\approx 1.3$. For the nonlinear current-voltage characteristic at $T=T_c$, we have $E\sim J^p$ with $p_x=(1+z)/(d+\zeta-2)\approx 1.3$, $p_z=(\zeta+z)/(d-1)\approx 1.6$, for $d=3$. Table I shows a comparison between the simulation results and the results from a selection of transport experiments where the critical exponents studied in this paper have been measured. In the table we observe that several experiments agree very well with the previously known exponents for screened vortex interactions. Notably, however, the table also shows that the results from two of the experiments on $YBa_2Cu_3O_7$ agree quite well with our exponents for unscreened 3D interactions, which suggests that they may effectively belong to the new universality class considered here. Naively one may expect that the present results describe the transition when the screening length is much longer than the vortex spacing. For $YBa_2Cu_3O_7$, where λ ~1400 Å, this corresponds to $B\gtrsim 0.1$ T. The precise location of the crossover between weak and strong screening is unclear, as well as the precise role of anisotropy and other parameters, and would be interesting to investigate further both experimentally and theoretically.

TABLE I. Summary of a few selected experiments and simulations of high temperature superconductors with columnar disorder.

| Experiment simulation | $\nu$ | $\zeta$ | $z$ |
|---|---|---|---|
| (K,Ba)$BiO_3$ (Ref. 10) | $1.1\pm 0.1$ | $\approx 2$ | $5.3\pm 0.3$ |
| $Tl_2Ba_2CaCu_2O_8$ (Ref. 8) | $1.1\pm 0.2$ | $1.9\pm 0.2$ | $4.9\pm 0.2$ |
| $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_8$ (Ref. 11) | $1.04\pm 0.06$ | $\approx 2$ | $5.28\pm 0.05$ |
| $YBa_2Cu_3O_7$ (Ref. 14) | $\approx 1.0$ | $\approx 1.1$ | $\approx 2.2$ |
| $YBa_2Cu_3O_7$ (Ref. 15) | $0.9\pm 0.2$ | $1.2\pm 0.2$ | $2.3\pm 0.3$ |
| Simulations ($\lambda\to 0$) (Refs. 18 and 4) | $\approx 1.0$ | 2 | $4.6\pm 0.3$ |
| Present work ($\lambda\to\infty$) | $1.0\pm 0.1$ | $1.25\pm 0.1$ | $1.95\pm 0.1$ |

In summary, we obtained scaling properties and numerical results for the critical exponents that apply for the superconducting transition in the systems with columnar defects in the limit of a long screening length, and compared with the experiments. The critical exponents for this universality class differ considerably from the strongly screened case,[3,4] and also from the case of planar $1/r$-interactions.[18] In particular, the anisotropy exponent differs from the value $\zeta=1$ assigned to the universality class of incompressible dirty bosons with planar long-range interactions.[19,18,20] Further work is motivated in order to further clarify the origin of the scaling properties obtained in different experiments and when the different models apply. Experimental studies to look for a crossover between unscreened and screened scaling behavior would also be interesting.

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