Residual resistivity near a two-dimensional metamagnetic quantum critical point

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The behavior of the residual (impurity-dominated) resistivity is computed for a material near a two-dimensional quantum critical point characterized by a divergent $q=0$ susceptibility. A singular renormalization of the amplitude for back scattering of an electron off of a single impurity is found. When the correlation length of the quantum critical point exceeds the mean free path, the singular renormalization is found to convert the familiar ''Altshuler-Aronov'' logarithmic correction to the conductivity into a squared-logarithmic form. Impurities can induce unconventional Friedel oscillations, which may be observable in scanning tunneling microscope experiments. Possible connections to the metamagnetic quantum critical end point recently proposed for the material $Sr_3Ru_2O_7$ are discussed.

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I. INTRODUCTION

The interplay of quantum mechanics, disorder and reduced dimensionality is a central question in condensed-matter physics. The issue becomes particularly interesting for materials near a *quantum* critical point (i.e., one produced at temperature $T=0$ by variation of a control parameter such as pressure or chemical composition). Near such a critical point, quantum fluctuations may become particularly strong, and may interact with randomness in an important way. In this paper, we consider the heretofore little studied question of the effect of quantum critical fluctuations on the residual resistivity of a metal. We consider two-dimensional systems in which the criticality involves long-wavelength fluctuations and a conventional gradient expansion of the action exists, and find that the critical fluctuations lead to a singular, and possibly divergent, renormalization of the amplitude for an electron to backscatter off of an isolated impurity atom. For a system with a nonvanishing density of impurities, we find that this physics leads to a strengthening of the ''Altshuler-Aronov''[1] correction to the conductivity from $\ln(1/T)$ to $\ln^2(1/T)$.

Renormalization of some electron-impurity vertices is expected in materials near density wave transitions. For example, near a charge-density wave transition, an impurity will produce density fluctuations whose component at the ordering wave vector will diverge as the transition is approached (for an experimental demonstration, see Ref. 2). However, we show here that long-wavelength spin fluctuations can drastically increase backscattering by spinless impurities. Our work is related to a previous analysis, by Altshuler, Ioffe, Larkin, and Millis, of the staggered susceptibility of a model of electrons interacting with gauge fluctuations.[3]

Our work is motivated by recent experiments on the bilayer ruthenate, $Sr_3Ru_2O_7$, which apparently can, at ambient pressure, be tuned through a quantum critical point by variation of applied magnetic field.[4–6] $Sr_3Ru_2O_7$ has a layered crystal structure and highly anisotropic conductivity properties characterized by a very low in-plane (ab) resistivity ($\rho_{ab}\sim 5\ \mu\Omega$–cm as $T\to 0$) and a much higher out-of-plane (c axis) resistivity ($\rho_c\sim 10^3\ \mu\Omega$–cm as $T\to 0$),[7] so its electronic properties are to a good approximation two dimensional. At ambient pressure and no applied magnetic field the material is paramagnetic, but with a magnetic susceptibility χ which is strongly T dependent (for $T\gtrsim 10$ K) and at low T strongly enhanced above the band theory value.[7] As applied magnetic-field H is increased at low T ($T<2$ K) resistivity measurements indicate that the material passes near to a critical point.[4,5] In the vicinity of this critical point the differential magnetic-susceptibility $\chi_{\rm diff}=dM/dH$ becomes large and the residual resistivity $\lim_{T\to 0}\rho(T)$ exhibits a pronounced cusp.[5]

The natural interpretation is that the transition observed in $Sr_3Ru_2O_7$ is *metamagnetic* in nature. Metamagnetic materials exhibit a phase diagram in the field (H)-temperature (T) plane characterized by a line of first-order transitions across which the magnetization jumps. The first-order line ends in a critical point at temperature T_M and field H_M. In $Sr_3Ru_2O_7$, parameters are apparently such that at ambient pressure T_M is very near to $T=0$ (indeed for one field orientation it can be made to vanish exactly) so that one has a *quantum critical end point*.[4–6,8] A renormalization-group theory of quantum critical end points and their application to $Sr_3Ru_2O_7$ was given by Ref. 8. In this paper we extend this theory to include electron-impurity coupling and use it to calculate the residual resistivity. In addition we make a few remarks about the applicability of our calculations to more general classes of critical points.

The rest of this paper is organized as follows. Section II presents the critical theory, Sec. III gives the analysis of the renormalization of the Born approximation amplitude for an electron to scatter off of an isolated impurity, Sec. IV considers the single impurity problem beyond the Born approximation and applies the results to $Sr_3Ru_2O_7$, Sec. V treats the extension to a nonvanishing density of impurities and Sec. VI is a conclusion and discussion of implications for other experiments.

II. METAMAGNETIC QUANTUM CRITICALITY

Reference 8 employed the standard approach of Hertz[9] and Millis,[10] which involves integrating out electronic de-

grees of freedom to obtain an effective-field theory with overdamped bosonic excitations, which is then analyzed. This approach is presently the subject of debate.[11] On the experimental side, the theory is apparently inconsistent with data obtained on a variety of materials with antiferromagnetic critical points, and on the theoretical side the validity of integrating out gapless electronic degrees has been questioned,[12–14] both for two-dimensional antiferromagnets and for ferromagnetic transitions involving an order parameter with continuous symmetry and time-reversal invariance in the disordered phase. For a more general discussion see Ref. 15. However, the assumptions made in Refs. 12 and 13 do not apply to metamagnetic transitions, which involve an Ising order parameter and an explicitly broken time-reversal symmetry, nor do the Fermi-surface nesting complications of antiferromagnetic transitions[14] apply here, because the order parameter is centered at $q=0$.

The order parameter for a metamagnetic critical point occurring at a field H_M is the difference of the local magnetization density $M(\mathbf{x},\tau)$ (which we write here in space and imaginary time) from the average value M^* produced by the field H_M at $T=0$. ($M^*\approx 0.5\mu_B$ for $Sr_3Ru_2O_7$.) It is convenient to measure the field and magnetization with respect to H_M, M^*, and to normalize the field to an energy scale E_0 related to the energy per site of a hypothetical fully polarized state and the magnetization fluctuations per Ru to a magnetic scale M_{sat} via

$$\phi(\mathbf{x},\tau)=\frac{M(\mathbf{x},\tau)-M^*}{M_{sat}}, \tag{1}$$

$$h=\frac{(H-H_M)M_{sat}}{E_0}, \tag{2}$$

where M_{sat} is a hypothetical high-field saturation magnetization, chosen here to be $M_{sat}=2/\mu_B$. In a clean (nondisordered) system the action describing the fluctuations of ϕ was argued[8] to be ($\beta=1/T$),

$$S=\frac{E_0}{2}\int \frac{d^2x}{a^2}\int_0^\beta d\tau\left[2h\phi+\xi_0^2(\nabla\phi)^2+\frac{1}{2}\phi^4\right]+S_{dyn} \tag{3}$$

with $\phi(\mathbf{q},\nu)=E_0a^{-2}\int d^2x\int_0^\beta d\tau e^{i\mathbf{q}\cdot\mathbf{x}-i\nu\tau}\phi(\mathbf{x},\tau)$ and, in a clean system,

$$S_{dyn}=\frac{a^2}{2E_0}\int \frac{d^2q}{(2\pi)^2}T\sum_{\nu_n}\frac{|\nu_n|}{vq}|\phi(\mathbf{q},i\nu_n)|^2, \tag{4}$$

where $\nu_n=2\pi nT$. For $Sr_3Ru_2O_7$, $E_0\approx 7000$ K;[8] a value for ξ_0 has not been established. A natural guess would be something of the order of the in-plane lattice constant $a\approx 4$ Å. However in many ferromagnetic materials band theory suggests that the momentum dependence of the magnetic polarizability is very weak, implying $\xi_0<a$; while if the considerations of Ref. 12 are relevant, a considerably larger value might be appropriate. A value for v has also not yet been established; the natural expectation is that it is of order the planar-band Fermi velocity $v_F\approx 1$ eV–Å. The theory is above its upper critical dimension so the nonlinear term is a ''dangerously'' irrelevant operator and may be treated by standard means.[8,10]

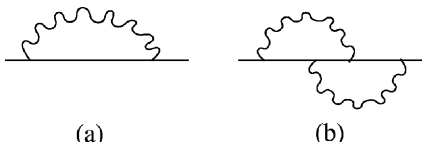

FIG. 1. Diagrams representing (a) the lowest-order self-energy, (b) an example of higher-order corrections. The wavy line represents the bosonic fluctuations.

We have written a two-dimensional theory. At some scale a crossover to three-dimensional behavior will occur, but present neutron experiments have so far been unable to observe any correlations along the c axis.[16] Because the transition takes place in a magnetic field, the fluctuating field has Ising symmetry and time-reversal invariance is explicitly broken. Neither ''precession'' terms in the dynamics nor the anomalous $|q|$ momentum dependence proposed in Ref. 12 will occur.

The action given in Eq. (3) describes the critical fluctuations. In the absence of critical fluctuations we take the action for electrons to be

$$S_{el}=\frac{a^2}{E_0^2}T\sum_{\omega_n}\int\frac{d^2p}{(2\pi)^2}\psi_\alpha^\dagger(\mathbf{p},i\omega_n)(-i\omega_n+\xi_\mathbf{p})\psi_\alpha(\mathbf{p},i\omega_n), \tag{5}$$

where ψ_α represents the electrons with the ''spin'' index α, $\xi_\mathbf{p}$ is the electron dispersion and $\omega_n=(2n+1)\pi T$. The coupling between the electrons and the critical fluctuations can be written as

$$S_{\phi-\psi}=g\int\frac{d^2x}{a^2}\int E_0 d\tau\psi_\alpha\sigma^z_{\alpha\beta}\psi_\beta\phi, \tag{6}$$

where σ^z is the z component of the Pauli matrix.

To obtain the propagator D describing the metamagnetic fluctuations we expand the action about the mean-field value ϕ_0 given by $\partial S/\partial\phi=0$ and read off the quadratic term, obtaining

$$D(\mathbf{q},i\nu_n)=\frac{1}{\frac{|\nu_n|}{vq}+\xi_0^2q^2+h^{2/3}}. \tag{7}$$

The coupling constant g may be determined because the damping term in S_{dyn} arises from processes in which a critical fluctuation decays into one particle-hole pair[9,10] and is

$$g^2=\frac{4\pi^2}{a^2E_0}\frac{v_F^2}{vS_F} \tag{8}$$

with S_F the length of the Fermi surface in two dimensions ($S_F=2\pi p_F$ for a circular Fermi surface).

Using these definitions we find that the one-loop self-energy at $T=0$, shown in Fig. 1(a), is

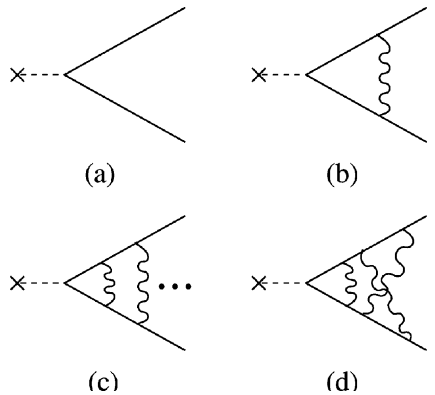

FIG. 2. Diagrams corresponding to the renormalization of the impurity scattering amplitude. (a) the bare vertex, (b) the lowest-order correction, (c) the sum of the ladder diagrams, and (d) an example of higher-order corrections.

$$\Sigma_1(\mathbf{p},i\omega)=\frac{2iv_F}{S_F\xi_0^2}\int_0^\infty dx\ x\ln\left(\frac{\frac{\omega\xi_0}{v}+x^3+h^{2/3}x}{x^3+h^{2/3}x}\right). \quad (9)$$

Notice that as $\omega\to 0$ at fixed $h\neq 0$,

$$\Sigma_1(\mathbf{p},i\omega)=\frac{2v_F}{vS_F\xi_0}\frac{i\omega}{h^{1/3}}$$

while as $h\to 0$ at fixed ω we have

$$\Sigma_1(\mathbf{p},i\omega)=\frac{i\ sgn(\omega)|\omega|^{2/3}2\pi(v/\xi_0)^{1/3}}{\sqrt{3}S_F\xi_0(v/v_F)}.$$

In spatial dimension $d>2$ the fact that the theory is above the upper critical dimension guarantees that a one-loop (Migdal-like) approximation yields an asymptotically exact approximation to the self-energy, but in $d=2$ this is not the case.[17–19] While higher corrections such as those shown in Fig. 1(b) do not change the powers, they do introduce a dependence on momentum $\delta p=p-p_F$ and induce a dependence on $\omega/[\omega^*(h)]$ with $\omega^*(h)=vh/\xi_0$. The scaling function for the self-energy has only been computed in large-N and small-N expansions, where N is the number of order-parameter components. Unfortunately, in the metamagnetic problem of interest here $N=1$.

III. ELECTRON-IMPURITY VERTEX: BORN APPROXIMATION

We now consider the effect of critical fluctuations upon the amplitude, $A_{\mathbf{p}_1,\mathbf{p}_2}$ for an electron initially in a state of momentum $\mathbf{p}_1$ to scatter off of an isolated impurity atom into a state of momentum $\mathbf{p}_2$. The basic electron-impurity vertex is shown in Fig. 2(a); we assume that this is structureless; corresponding to scattering amplitude independent of $\mathbf{p}_{1,2}$. The first correction due to critical fluctuations is shown in Fig. 2(b), and is found to be divergent when $\mathbf{p}_{1,2}$ are on the Fermi surface and are such that the Fermi surface tangents at these two points are parallel. For a circular Fermi surface this condition corresponds to $\mathbf{p}_1+\mathbf{p}_2=0$ and $|\mathbf{p}_1|=p_F$. In this case, choosing the x coordinate to be parallel to $\mathbf{p}_1$ and $\mathbf{p}_{1,2}$ to be on the Fermi surface we have $\varepsilon_{\mathbf{p}_1+\mathbf{q}}=v_F(q_x+q_y^2/2q_0)$ and $\varepsilon_{\mathbf{p}_2+\mathbf{q}}=v_F(-q_x+q_y^2/2q_0)$ with q_0 a quantity of the order of p_F parametrizing the curvature of the Fermi surface. If $\mathbf{p}_2=-\mathbf{p}_1+\mathbf{p}$ with $|\mathbf{p}|\ll|\mathbf{p}_1|$, we find that the correction shown in Fig. 2(b) is

$$A_1(\mathbf{p},h)=A_0 I\left(\frac{2\pi q_0}{S_F}\right)\ln\left(\frac{1}{\max[h^{2/3},(p_y\xi_0)^2]}\right) \quad (10)$$

with

$$I(b)=\frac{2b}{\pi}\int_0^\infty dy\frac{\left(\frac{2}{\sqrt{3}}b\right)y}{(1+y^3)\left[\left(\frac{2}{\sqrt{3}}b\right)^2+y^4\right]}$$

and A_0 is the bare scattering amplitude; $I(1)\approx 0.23$, $I(10)\approx 0.5$, and $I(b)$ is an increasing function of b. Notice that if $\pi-\theta$ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$, then $p_y\xi_0\propto\theta$.

A very similar result was obtained in the context of the U(1) gauge theory of the "RVB" regime of the two-dimensional t-J model,[17] where the $2p_F$ spin susceptibility was considered. The U(1) gauge theory possesses an interaction (mediated by an internal gauge field) with a very similar mathematical structure to our interaction $D(\mathbf{q},i\nu_n)$ [Eq. (7)], except that in the U(1) problem gauge invariance dictates that in the RVB regime the mass (h in our notations) vanishes. Further, in the vertex computation two additional minus signs occur (but compensate each other), one from the fact that gauge interaction involves currents which are oppositely directed at momentum transfer $2p_F$ and one from the transverse nature of the gauge interaction.

Higher-order diagrams such as those shown in Fig. 2(c) may be evaluated similarly; we find that the leading behavior of the nth order term is

$$A_n=A_0\frac{1}{n!}\left[I\left(\frac{2\pi q_0}{S_F}\right)\ln\left(\frac{1}{\max[h^{2/3},\theta^2]}\right)\right]^n \quad (11)$$

so that finally we obtain

$$A(\theta,h)\sim A_0(\max[h^{2/3},\theta^2])^{-\psi} \quad (12)$$

with $\psi=I(S_F/2\pi q_0)$ at one-loop order.

We should further consider the effect of higher-order diagrams, such as those shown in Fig. 2(d). These do not change the basic power counting, and may, therefore, be expected not to alter the basic power law we have found; they will, however, change the numerical value of the exponent. According to the result of small-N expansion,[17] the effect of "crossed diagrams" is to increase the exponent. We conclude that the Born-approximation amplitude for an electron to back-scatter off of an impurity suffers a singular renormalization near criticality.

FIG. 3. Diagrams representing multiple scattering of electrons off of an impurity. The shaded triangle represents the renormalization of the Born scattering amplitude given by Fig. 2(c).

IV. BEYOND BORN APPROXIMATION; RESISTIVITY

The preceding section presented the renormalization of the leading-order term in the electron-impurity vertex. In this section we consider effects arising when the Born approximation is not justified, either because the initial scattering amplitude is not small or because the renormalizations increase an initially weak interaction beyond the regime of validity of the Born approximation. Corrections to the Born approximation result for the electron self-energy correspond to multiple scattering of the electron off of the same impurity, and are represented diagrammatically in Fig. 3. Within the ''noncrossing'' approximation used in the previous sections we find that the leading renormalization near criticality comes from the correcting each impurity vertex individually. The series for the self-energy may be summed by defining the T matrix which for incident electron energies very close to the Fermi surface becomes

$$T(\theta-\theta',h)=A(\theta-\theta',h) - i\int \frac{d\theta_1}{2\pi}N(\theta_1)A(\theta-\theta_1,h)T(\theta_1-\theta',h) \tag{13}$$

with $N_0=\int(d^2k)/(2\pi)^2\delta(\varepsilon_p)$ the single-spin Fermi-surface density of states. Assuming for simplicity a circular Fermi surface with density of state N_0, we may solve the equation by resolving T and A into their angular components $T_m=\int(d\theta/2\pi)T(\theta,h)e^{im\theta}$ and $A_m=\int(d\theta/2\pi)A(\theta,h)e^{im\theta}$ so that

$$T_m=\frac{A_m}{1+iA_mN_0}. \tag{14}$$

We note that the sign of T_m and, therefore, the sign of the angular momentum m channel phase-shift alternates with m, so that the Friedel sum rule is straightforwardly satisfied.

The final result depends on the parameter $g_0=A_0N_0$ and on the exponent ψ. If $\psi<1/2$, then none of the A_m or T_m are divergent as $h\to 0$. The distance to criticality, parametrized by h, controls the number $m_{\max}$ of angular momentum channels for which A_m and T_m are non-negligible ($m_{\max}\sim h^{-1/3}$). On the other hand, if $\psi>1/2$, then in each angular momentum channel the amplitude A diverges as $h\to 0$ so that the resulting phase shift saturates at $\pi/2$ in each channel, and again the number of channels that are relevant diverges rather strongly as $h\to 0$.

The contribution of this scattering amplitude to the probability that an electron is scattered through an angle θ, $W(\theta)$, is given by $|T(\theta)|^2$. The impurity scattering rate revealed by the resistivity, $\Gamma_{tr}=\int(d\theta/2\pi)[1+\cos(\theta)]W(\theta)$ (note that we have defined angular coordinates so that $\theta=0$ corresponds to backscattering). Because we are concerned with a scattering amplitude strongly peaked about the back-scattering direction, the $\cos(\theta)$ factor is unimportant. Then Γ_{tr} is given by the imaginary part of the diagonal T-matrix $\Gamma_{tr}\sim \mathrm{Im}T(0;h)$ in the usual approximation. Therefore, we obtain

$$\rho_{res}\sim \mathrm{Im}[T_0]+2\sum_{m=1}^{\infty}\mathrm{Im}T_m. \tag{15}$$

As noted above, within this approximation the resistivity diverges as criticality is approached, but the approximation itself breaks down when the mean-free-path implied by Eq. (15) becomes smaller than the correlation length. The question of a divergent resistivity is further examined in Sec. VII.

V. APPLICATION TO DATA

In this section we attempt to relate calculations of the h dependence of ρ_{res} to data obtained on $Sr_3Ru_2O_7$. We first note that it is sensible to discuss the scattering from an isolated impurity only if the elastic mean-free-path l is greater than the correlation length $\xi=\xi_0h^{-1/3}$. The observed low-T resistivity of clean samples of $Sr_3Ru_2O_7$ is of the order of $5\mu\Omega$–cm corresponding to a $p_Fl\approx 250$, i.e., to an l of the order of 300–400 Å. The appropriate value of ξ_0 is not known at present, but if it is of the order of the lattice constant or smaller then almost all the available data are in the regime in which the calculation applies.

$Sr_3Ru_2O_7$ presents an interesting issue in modeling. If the estimates presented in Ref. 8 are correct, then even in zero field the material is close to a ferromagnetic critical point, so that renormalizations of impurity vertices could be substantial, even at vanishing applied field (provided, of course, that a sufficiently wide regime exists in which the $|\mathbf{q}|$ effects of Ref. 12 are not important). What is unambiguously measurable, however, are the effects caused by a varying magnetic field. Further, the parameters presented in Ref. 8 allow a quantitative determination of the parameter h above in terms of the applied magnetic field. We have used the parameters $u_{cc}=-3500$ K, $v_{cccc}=58\,000$ K, $r=100$ K defined and given below Eq. (6) of Ref. 8 to compute the ''mass'' in Eq. (7) for applied fields from 0 to 10 T, and have used this and Eq. (15) to calculate the field dependence of the residual resistivity for various choices of exponent ψ and initial scattering amplitude. Representative results are shown in Fig. 4. The values at applied field $H=0$ depend on the behavior of the theory at higher-energy scales, which is not known for the reasons given above. However, the change in behavior near the metamagnetic field (about 7 T for the parameters used here) should be reliable. We observe that there is some interplay between exponent and initial scattering strength but that sharpness and relative height of the resistivity peak depend most strongly on the exponent. Comparison to data[5] shows that our calculation is consistent with a moderate bare scattering amplitude and an exponent $\psi\approx 3/4$. We stress that the exponent estimate is obtained from the relative height of the resistivity peak, and not from a fit of the shape of the

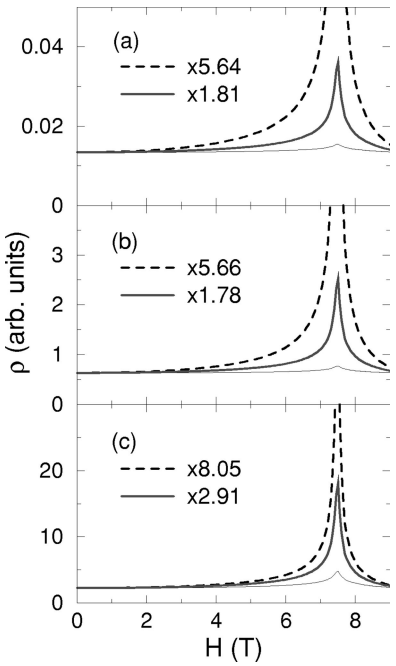

FIG. 4. Residual resistivity calculated for different exponents and initial scattering amplitudes and scaled to the $H=0$ values. Dashed lines, $\psi=1$; heavier solid lines, $\psi=0.75$; light solid lines, $\psi=0.5$. Three panels show the results of different initial scattering amplitude: (a) $g_0=0.5$, (well below unitarity limit for initial point-impurity scattering amplitude), (b) $g_0=1.0$ (roughly half way to unitarity limit), and (c) $g_0=5.0$, (close to unitarity limit).

$\rho(B)$ traces, and that the estimate is rough (although it seems that an exponent of 1/2 is clearly too small and of unity is clearly too large). We also note that the rather small resistivities observed in experimental systems suggest either that the initial scattering amplitude is weak or that the renormalizations associated with the nearby ferromagnetic quantum critical point are not large, perhaps for the reasons given in Ref. 12. Finally, we remark that the data in Ref. 5 exhibit additional B-dependence not accounted for in the present theory. We believe that the main source of this additional field dependence is a B^2 orbital magnetoresistance, but the issue deserves further investigation.

VI. FRIEDEL OSCILLATIONS

The physics which gives rise to the singular correction to the conductivity also gives rise to a singular behavior[3] in the susceptibility at wave-vector $q=2p_F$ which is proportional to $(2p_F-q)^{1-3\psi}$; this leads to a change in amplitude and distance dependence of the Friedel oscillations induced by an impurity in two dimensions. We find that if $\psi\leqslant 1/6$ the effects considered here are a subleading correction to the conventional behavior while if $1/6\leqslant\psi\leqslant 5/6$ the Friedel oscillations decay as $\cos(2p_FR)/R^{(5/2-3\psi)}$. For $\psi>5/6$ a more sophisticated treatment is needed. At nonzero field, the spin-up and spin-down Fermi surfaces are characterized by different Fermi wave vectors so we predict two superposed oscillations emerging from each impurity site, each decaying with a characteristic power. These oscillations should be observable in scanning tunnel microscope (STM) experiments. We note, however, that the effect discussed here is a long-wavelength, low-energy effect. It does not imply that the density modulations are greatly enhanced near to an impurity; only that they decay much less (or more) rapidly with distance than in a noncritical material.

It is also interesting to consider the situation at $H=0$, i.e., near to a 2D ferromagnetic transition, and indeed we note that for the $Sr_3Ru_2O_7$ parameters given in Ref. 8, $H=0$ corresponds to $h\sim 10^{-3}$ so one might expect the enhancements to be noticeable even at zero field. The Friedel oscillations from a nonmagnetic impurity would indeed be long ranged, however, it is interesting to note that the RKKY interactions are suppressed (in a system with Heisenberg symmetry) because the spin commutation relations lead to a minus sign in the renormalization of σ_z vertex by σ_x or σ_y fluctuations.

We also note that field-dependent STM studies might present an interesting test of the result[12] that for 2D Heisenberg materials the leading momentum dependences is $|\mathbf{q}|$. In this case, the renormalizations we have discussed would not exist. As H is increased, the effects that produce $|\mathbf{q}|$ are believed to be cut off, and the unconventional Friedel oscillations should reappear.

Finally, we observe that the presence of the Friedel oscillations implies that the state of a critical system is in some sense a random charge-density wave, characterized by infinite ranged charge oscillations emerging from the various impurity sites. The resistivity and other properties of such a state are an interesting issue for future research.

VII. NONVANISHING IMPURITY DENSITY

The preceding treatment is valid for correlation lengths less than the mean-free path. We now consider what happens when parameters are tuned so that the correlation length ξ exceeds the mean-free-path l. We first note that the problem has two energy scales: the impurity scattering rate $\tau^{-1}=v_F/l$ and the characteristic quantum critical frequency $\omega^*=v\xi_0^2/\xi^3$ so that near criticality, when $\xi>\xi_0$ we have $\omega^*\tau<1$.

For length scales longer than the mean-free path the dynamic term in the action is modified to be[9]

$$S_{dyn,dirty}=\frac{a^2}{2E_0}\int\frac{d^2q}{(2\pi)^2}T\sum_{\nu_n}\frac{|\nu_n|}{D'q^2}|\phi(\mathbf{q},i\nu_n)|^2 \quad (16)$$

with D' of the order of the diffusion constant $D=v_F^2\tau/2$. We note $D'/D=v/v_F$ from the definition of g^2 in Eq. (8). Here $1/\tau=2\pi u^2N_0$ is the impurity scattering rate, u^2 is the renormalized squared impurity scattering strength (renormalized due to the effects discussed in Sec. IV), and N_0 is the density of states at the Fermi energy. Thus the dynamical critical exponent is $z=4$ in this case and the nonlinearity in the critical theory remains dangerously irrelevant.

The leading order correction to the conductivity is given by the sum of two diagrams in Fig. 5,[1] where the wavy line represents the metamagnetic fluctuations with the propagator $D(\mathbf{q},i\nu_n)=1/[(|\nu_n|/D'q^2)+q^2\xi_0^2+h^{2/3}]$. Notice that there exist three more diagrams at the same order, but they cancel

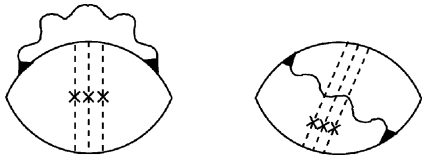

FIG. 5. Diagrams contributing to the leading order correction to conductivity. The black triangles and the dotted lines represent the vertex correction by impurity scattering and the Diffuson ladder.

each other.[1] Each interaction vertex that represents the coupling between the electrons and the bosonic mode is renormalized by the Diffuson correction, $(|\nu_n|+Dq^2)^{-1}\tau^{-1}$. We find that the leading correction to the dc conductivity at finite temperature is given by (we assume $T<1/\tau$)

$$\delta\sigma=-\frac{e^2}{\hbar}\frac{\lambda}{4\pi^2}\ln^2\left(\frac{\lambda D}{T\xi_0^2}\right) \quad \text{for} \quad T>T_h \tag{17}$$

$$=-\frac{e^2}{\hbar}\frac{\lambda}{3\pi^2}\ln\left(\frac{1}{h}\right)\ln\left(\frac{\lambda D}{T\xi_0^2}\right) \quad \text{for} \quad T<T_h, \tag{18}$$

where $\lambda=v_F/v$ and $T_h=h^{4/3}\lambda D/\xi_0^2$. Thus at a given temperature the conductivity will decrease as the critical point is approached $(h\to 0)$ but when $h<h_T$ with $h_T=(\xi_0^2 T/\lambda D)^{3/4}$ (or equivalently $T>T_h$) it levels off at a temperature-dependent constant value. The corresponding behavior of the resistivity as a function of h at a fixed temperature is sketched (somewhat exaggerated for clarity) in Fig. 6. Notice that these results are valid as far as we are in the perturbative regime, i.e., $\delta\sigma\ll\sigma_0=(e^2/2\pi\hbar)k_F l$.

This result should be compared with the well-known behavior of the quantum correction to the conductivity, $\delta\sigma=-(e^2/2\pi^2\hbar)\ln(1/T\tau)$, in weakly disordered interacting two-dimensional electron systems.[1] The notable differences are: (i) $\delta\sigma$ in the present case has more singular temperature dependence when $T>T_h$ and at the critical point $(h=0)$. (ii) If $T<T_h$, the coefficient of the logarithmic term explicitly depends on h and it increases as the critical point is approached until it is cutoff by temperature.

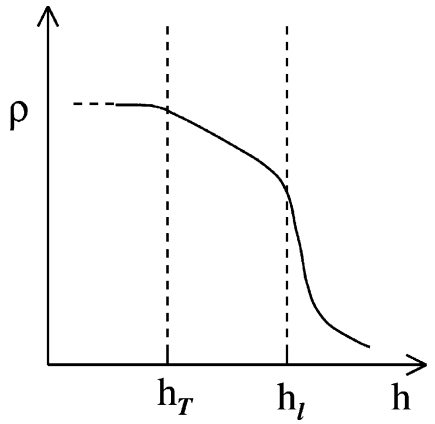

FIG. 6. The schematic behavior of the resistivity as a function of h at a fixed temperature. Here $h_T=(\xi_0^2 T/\lambda D)^{3/4}$ and $h_l=(\xi_0/l)^3$. When $h>h_l$, $\xi=\xi_0 h^{-1/3}<l$ so that the considerations of Secs. IV and V apply while $h<h_l$ we have $\xi<l$. If $h_T<h<h_l$, Eq. (18) applies and the resistivity increases logarithmically as h is decreased. When $h<h_T$, Eq. (17) applies and the resistivity becomes a temperature-dependent constant.

Now let us estimate the size of the perturbative quantum correction in $Sr_3Ru_2O_7$. We assume that $D'\approx D$ and ξ_0 is of the order of the in-plane lattice constant ~ 4 Å. Using $v_F\sim 1$ eV–Å and $l\sim 300$–400 Å, we get $1/\tau\sim 30$ K and $D\sim 45$–60 cm^2 sec. Let us consider $h\sim 10^{-6}$, then $T_h\sim 10^{-3}$ K. In this case, $T>T_h$ for experimentally relevant temperature range and Eq. (17) should be used. Using the leading-order result, $\sigma_0=(e^2/2\pi\hbar)k_F l$, in two dimensions the relative size of the correction can be estimated as $\delta\sigma/\sigma_0\sim 0.07$ at $T=2$ K and $\delta\sigma/\sigma_0\sim 0.1$ at 0.1 K. Thus the relative correction is only 7–10%. Given that the residual resistivity is about 5 $\mu\Omega$–cm, it will be hard to see the effect of these corrections. If the material were more dirty, say $k_F l\sim 50$, we would get $1/\tau\sim 150$–200 K and $T_h=10^{-4}$ K. Similar estimation would predict that the relative correction is 30–50% at $T=0.1$–2 K. It would be interesting to test these predictions in more dirty samples.

To conclude this section we note that the scattering time τ is the impurity scattering rate, *as renormalized by critical fluctuations at scales less than the (renormalized) l*. We also observe that our result is perturbative in both the disorder strength and the interaction. Presumably when the resistivity becomes of the order of the Mott value a crossover to insulating behavior occurs. The insulating state should presumably be interpreted in terms of the random charge-density wave state discussed at the end of Sec. VI, but the issue deserves more careful investigation.

VIII. CONCLUSION

In summary, the effect of critical fluctuations on the residual resistivity is studied near a two-dimensional metamagnetic quantum critical point. When the correlation length is smaller than the mean-free path, the critical fluctuations induce a singular renormalization of the amplitude of the backscattering off an impurity of an electron. This leads to a pronounced cusp in the residual resistivity near the metamagnetic critical point in accordance with the experimental results. When the correlation length becomes larger than the mean-free path, the critical fluctuations convert the ''Aronov-Altshuler'' logarithmic correction to the conductivity to the more singular squared-logarithmic behavior near the critical point. Our results imply a divergent resistivity at criticality. We argued that the state which gives rise to this divergence is some sort of randomly phased charge-density wave, but a detailed investigation of its properties has not been performed.

Our results may have broader implications. The apparently successful comparison of our calculation to data suggests that the singular $2p_F$ renormalization discovered in the gauge theory context in Ref. 17 is more than a theoretical curiosity, and therefore, motivates examination of other systems where $2p_F$ effects might be important; for example in the transresistance of bilayer $\nu=1/2$ quantum Hall systems. Work in this direction is in progress. The relevance of the

results to other systems is an interesting issue. The essential ingredients in our calculation were two dimensionality, a susceptibility diverging at $q=0$ and a leading momentum dependence involving a power of q greater than unity. A material near a two dimensional *ferromagnetic* transition might fulfil these criteria, but known materials have (weakly) first-order transitions. Also theoretical arguments have been advanced suggesting that the asymptotic momentum dependence might be $|q|$[12,15] (while for larger q the conventional q^2 dependence might be dominant). Thus the relevance of our results to ferromagnetic transitions depends on whether the first-order transition is sufficiently weak, and the region of $|q|$ is sufficiently narrow, to allow an intermediate asymptotic regime. Conversely, we suggest that experimental study of the residual resisitivity might clarify this issue if direct (e.g., neutron) study of the momentum dependence of the susceptibility is impractical.

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[1] B.L. Altshuler, A.G. Aronov, and P.A. Lee, Phys. Rev. Lett. **44**, 1288 (1980); B.L. Altshuler, D. Khmel'nitzkii, A. Larkin, and P.A. Lee, Phys. Rev. B **22**, 5142 (1980).
[2] H.H. Weitering *et al.*, Science **285**, 2107 (1999).
[3] B.L. Altshuler, L.B. Ioffe, A.I. Larkin, and A.J. Millis, Phys. Rev. B **52**, 4607 (1995).
[4] R.S. Perry, L.M. Galvin, S.A. Grigera, L. Capogna, A.J. Schofield, A.P. Mackenzie, M. Chiao, S.R. Julian, S.I. Ikeda, S. Nakatsuji, Y. Maeno, and C. Pfeiderer, Phys. Rev. Lett. **86**, 2661 (2001).
[5] S.A. Grigera *et al.*, Science **294**, 329 (2001).
[6] M. Chiao *et al.*, cond-mat/0207697 (unpublished).
[7] S. Ikeda *et al.*, Phys. Rev. B **62**, 6089 (2000).
[8] A.J. Millis, A.J. Schofield, G.G. Lonzarich, and S. Grigera, Phys. Rev. Lett. **88**, 217204 (2002).
[9] J.A. Hertz, Phys. Rev. B **14**, 1165 (1976).
[10] A.J. Millis, Phys. Rev. B **48**, 7183 (1993).
[11] P. Coleman *et al.*, J. Phys.: Condens. Matter **13**, R723 (2001).
[12] D. Belitz, T.R. Kirkpatrick, and T. Vojta, Phys. Rev. B **55**, 9452 (1997).
[13] G.Y. Chitov and A.J. Millis, Phys. Rev. Lett. **86**, 5337 (2001); Phys. Rev. B **64**, 054414 (2001).
[14] Ar. Abanov, A.V. Chubukov, and J. Schmalian, cond-mat/0107421, Adv. Phys. (to be published).
[15] D. Belitz, T.R. Kirkpatrick, and T. Vojta, Phys. Rev. B **65**, 165112 (2002).
[16] L. Capogna, A.P. Mackenzie, R.S. Perry, S.A. Grigera, L.M. Galvin, P. Raychaudhuri, A.J. Schofield, C.S. Alexander, G. Cao, and S.R. Julian, Phys. Rev. Lett. **88**, 076602 (2002).
[17] B.L. Altshuler, L.B. Ioffe, and A.J. Millis, Phys. Rev. B **50**, 14 048 (1994).
[18] L.B. Ioffe and A.J. Millis, Usp. Fiz. Nauk **168**, 672 (1998) [Phys. Usp. **41**, 595 (1998)].
[19] Y.B. Kim, A. Furusaki, X.G. Wen, and P.A. Lee, Phys. Rev. B **50**, 17 917 (1994).