Analytical results for coupled collective modes in a periodic superlattice of quantum wires embedded in a local bulk medium

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We have analyzed the coupled-collective mode spectrum of a periodic superlattice (SL) of one-dimensional (1D) quantum wires in interaction with the background local plasma. Based on reasonable approximations we have given analytical results for the modes by showing explicitly how, in the extreme strong coupling limit, the 1D SL tends to change character to become a single 2D electron layer, and how, in the extreme weak coupling limit, the SL wire system reduces to an isolated single wire. We have also analyzed the mode coupling of the bulk (3D) plasmon and the lower-dimensional (2D/1D) plasmons in the appropriate limits.

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I. INTRODUCTION

There has been a great deal of interest in the collective electrostatic modes of the one-dimensional quantum wire (1DQW) systems. The mode spectra of such systems are described by either numerical or experimental techniques. In this manner, plasmons and magnetoplasmons in multiwire systems have been studied theoretically[1–8] and experimentally.[9–11] Our considerations here are focused on obtaining analytical results for the coupled-mode spectrum of a periodic superlattice (SL) of 1DQWs in interaction with the plasmalike local host medium in which the SL is lodged. To carry out our calculation, we perform a closed form determination of the inverse dielectric function $K(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)$ of the combined system (the SL and host medium) in position representation within the framework of the random phase approximation (RPA). The frequency poles of $K(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2)$ define the coupled-collective modes resulting from the coupling of the SL plasmon with the bulk plasmon of the host medium. Also, the residues at the pole positions provide the spectral weights (oscillator strengths) of the coupled-collective modes.

II. JOINT POLARIZABILITY AND INVERSE DIELECTRIC FUNCTION

In our analysis, we assume that the wires constituting the SL are identical, and are equally spaced (spacing a) along the z direction at $z=na$, $(n=-\infty,\ldots,0,\ldots,\infty)$, and that the electrons in each wire are free to move along the x direction, but are confined in deep potential wells with only the lowest populated subband states in the y and z directions so that we neglect both overlap and intersubband transitions.

Considering translational invariance along the x axis parallel to the quantum wires and time translational invariance, we Fourier transform into the single one-dimensional (1D) wave vector q_x and frequency ω representation, writing

$$K(\mathbf{r}_1,\mathbf{r}_2;t_1-t_2)\rightarrow K(y_1,z_1,y_2,z_2;q_x,\omega)\rightarrow K(y_1,z_1;y_2,z_2).$$

(Henceforth we suppress the q_x, ω dependence in our discussion, exhibiting only the transverse positional dependence explicitly, unless otherwise stated.) The determination of the inverse dielectric function $K(y_1,z_1;y_2,z_2)$ follows from the inversion relation

$$\int dy_3\int dz_3\varepsilon(y_1,z_1;y_3,z_3)K(y_3,z_3;y_2,z_2)=\delta(y_1-y_2)\,\delta(z_1-z_2), \tag{1}$$

where $\varepsilon(y_1,z_1;y_3,z_3)$ is the corresponding direct dielectric function of the joint system. In terms of the polarizability α of the composite system, the inversion relation may be rewritten in the form of the RPA integral equation as

$$K(y_1,z_1;y_2,z_2)=\delta(y_1-y_2)\,\delta(z_1-z_2)-\int dy_3\int dz_3\alpha(y_1,z_1;y_3,z_3)\times K(y_3,z_3;y_2,z_2). \tag{2}$$

The structure of the free-electron polarizability of a single quantum wire may be expressed as[12]

$$\alpha_{1D}(y_1,z_1;y_3,z_3)=\alpha_{1D}\delta(y_3)\,\delta(z_3)K_0(|q_x|\sqrt{y_1{}^2+z_1{}^2}), \tag{3}$$

where $\alpha_{1D}\equiv\alpha_{1D}(q_x,\omega)$ describes the wave vector and frequency-dependent 1D polarizability function of a single wire and $K_0(x)$ represents the zeroth order modified Bessel function which stems from the Fourier transform of the 1D Coulomb potential. In writing Eq. (3), we assumed that the wire is very thin and has its lowest subband functions $\chi(y)$ and $\xi(z)$ wholly confined in narrow potential wells, so we took $|\chi(y)|^2\rightarrow\delta(y)$ and $|\xi(z)|^2\rightarrow\delta(z)$. In the local limit, α_{1D} is given by

$$\alpha_{1D}=-2e^2\left(\frac{n_{1D}}{m_{1D}}\right)\left(\frac{q_x^2}{\omega^2}\right),\qquad \hbar\omega>\hbar^2q_xq_F/m_{1D},\hbar^2q_x^2/2m_{1D}, \tag{4}$$

where $q_F=\pi n_{1D}/2$ is the 1D Fermi wave number, and n_{1D} and m_{1D} represent 1D electron density and electron effective mass, respectively. Since the electrons and the wave functions in the wires are wholly confined in their own wires (no tunneling), the 1D SL polarizability $\alpha_{SL}(y_1,z_1;y_3,z_3)$ vanishes everywhere except at the wire sites, $z=na$. Therefore, it may be expressed by the sum of the polarizabilities $\alpha_{1D}(y_1,z_1;y_3,z_3)$ of the individual wires as

$$\alpha_{SL}(y_1,z_1;y_3,z_3)=\alpha_{1D}\sum_{n=-\infty}^{\infty}\delta(y_3)\,\delta(z_3-na)\times K_0(|q_x|\sqrt{y_1{}^2+(z_1-na)^2}). \quad (5)$$

Within the framework of the RPA, the joint polarizability $\alpha(y_1,z_1;y_3,z_3)$ of the compound system is determined by summing the polarizability of the 1D SL with the polarizability $\alpha_{3D}(y_1,z_1;y_3,z_3)$ of the local host medium,

$$\alpha_{3D}(y_1,z_1;y_3,z_3)=\alpha_{3D}\delta(y_1-y_3)\,\delta(z_1-z_3), \quad (6)$$

from which

$$\alpha(y_1,z_1;y_3,z_3)=\alpha_{1D}\sum_{n=-\infty}^{\infty}\delta(y_3)\,\delta(z_3-na)\times K_0(|q_x|\sqrt{y_1{}^2+(z_1-na)^2})+\alpha_{3D}\delta(y_1-y_3)\,\delta(z_1-z_3). \quad (7)$$

Employing the joint polarizability in Eq. (2), the RPA integral equation for $K(y_1,z_1;y_2,z_2)$ takes the form

$$K(y_1,z_1;y_2,z_2)=\frac{1}{\varepsilon_{3D}}\delta(y_1-y_2)\,\delta(z_1-z_2)-\frac{\alpha_{1D}}{\varepsilon_{3D}}\sum_{n=-\infty}^{\infty}K_0(|q_x|\sqrt{y_1{}^2+(z_1-na)^2})\times K(0,na;y_2,z_2), \quad (8)$$

where $\varepsilon_{3D}=1+\alpha_{3D}$ is the dielectric function of the local host medium. To determine $K(0,na;y_2,z_2)$ in Eq. (8), we set $y_1=0$ and $z_1=n'a$, $(n'=-\infty,\ldots,0,\ldots,\infty)$, and regard y_2 and z_2 as fixed parameters, writing

$$K(0,n'a;y_2,z_2)=\frac{1}{\varepsilon_{3D}}\delta(y_2)\,\delta(z_2-n'a)-\frac{\alpha_{1D}}{\varepsilon_{3D}}\sum_{n=-\infty}^{\infty}K_0(|q_x||n'-n|a)\times K(0,na;y_2,z_2), \quad (9)$$

which is a matrix equation of infinite order for $K(0,na;y_2,z_2)$. Recalling the periodicity of the SL (period a),[13] we solve Eq. (9) by means of a Fourier series:

$$G(na)=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}dk\,e^{-ikna}\tilde{G}(k),$$

$$\tilde{G}(k)=\sum_{n=-\infty}^{\infty}G(na)e^{ikna}. \quad (10)$$

In this view, we denote the Fourier series representation of $K(0,na;y_2,z_2)$ by $\tilde{K}(0,k;y_2,z_2)$ and that of $K_0(|q_x||n'-n|a)$ by $\tilde{K}_0(|q_x|,k)$. Then application of $\Sigma_{n'=-\infty}^{\infty}e^{ikn'a}$ to Eq. (9) yields

$$\tilde{K}(0,k;y_2,z_2)=\frac{1}{\varepsilon_{3D}}\delta(y_2)\sum_{n'=-\infty}^{\infty}\delta(z_2-n'a)e^{ikn'a}-\frac{\alpha_{1D}}{\varepsilon_{3D}}\left(\frac{a}{2\pi}\right)\int_{-\pi/a}^{\pi/a}dp\left(\sum_{n'=-\infty}^{\infty}e^{i(k-p)n'a}\right)\times\tilde{K}_0(|q_x|,p)\tilde{K}(0,p;y_2,z_2). \quad (11)$$

Further, using the Poisson sum formula, we write the sums over n' in Eq. (11) in terms of the delta function, i.e.,

$$\sum_{n'=-\infty}^{\infty}e^{i(k-p)n'a}=\frac{2\pi}{a}\sum_{n=-\infty}^{\infty}\delta(k-p-2\pi n/a)\equiv\frac{2\pi}{a}\delta(k-p),$$

where, in the last step, we have used the fact that k and p are restricted to the first Brillouin zone, $(-\pi/a,\pi/a)$, so $n=0$. Carrying out the integration, we have

$$\tilde{K}(0,k;y_2,z_2)=\delta(y_2)\frac{\Sigma_{n'=-\infty}^{\infty}\delta(z_2-n'a)e^{ikn'a}}{[\varepsilon_{3D}+\alpha_{1D}\tilde{K}_0(|q_x|,k)]}. \quad (12)$$

Finally, inverting the Fourier series $\tilde{K}(0,k;y_2,z_2)$ back into Fourier series coefficient $K(0,na;y_2,z_2)$ and substituting it into Eq. (8), we obtain the inverse dielectric function of a periodic SL of 1DQWs embedded in a local bulk medium as

$$K(y_1,z_1;y_2,z_2)=\frac{1}{\varepsilon_{3D}}\delta(y_1-y_2)\,\delta(z_1-z_2)-\frac{\alpha_{1D}}{\varepsilon_{3D}}\sum_{n=-\infty}^{\infty}K_0(|q_x|\sqrt{y_1{}^2+(z_1-na)^2})\,\delta(y_2)\times\sum_{n'=-\infty}^{\infty}\delta(z_2-n'a)\left(\frac{a}{2\pi}\right)\int_{-\pi/a}^{\pi/a}dk\left(\frac{e^{ik(n'-n)a}}{\varepsilon_{3D}+\alpha_{1D}\Sigma_{m=-\infty}^{\infty}K_0(|mq_xa|)e^{ikma}}\right). \quad (13)$$

III. COUPLED-COLLECTIVE MODES

The coupled-mode dispersion relation for the plasmons of the periodic SL of 1DQWs in interaction with the bulk plasmons of the host medium is determined by the frequency poles of the inverse dielectric function, namely,

$$1+\left(\frac{\alpha_{1D}}{\varepsilon_{3D}}\right)\sum_{m=-\infty}^{\infty} K_0(|mq_xa|)e^{ikma}=0. \tag{14}$$

For the local limit the polarizability of a quantum wire [Eq. (4)] is $\alpha_{1D}=-2e^2(n_{1D}/m_{1D})(q_x^2/\omega^2)$. Also, in the local limit $\varepsilon_{3D}=\varepsilon_0-\omega_p^2/\omega^2$, where ε_0 is the background dielectric constant, and $\omega_p=(4\pi e^2 n_{3D}/m_{3D})^{1/2}$ is the 3D classical plasma frequency of the host plasma.

Because of the continuous variable k explicit in Eq. (14), we note that the dispersion relation introduces an uncountable set of collective modes into the spectrum. We also note that the series in the dispersion relation is dominated by the $m=0$ term due to the singular behavior of K_0 function at zero argument. This occurs for the wire at the origin. (The remaining terms in the series due to the wires labeled by $m\neq 0$ give a correction to the mode spectrum.) However, the singular nature of the Bessel functions at zero argument introduces a lack of definition in the summation of the series. To have a physically well-defined problem, we invoke a small finite thickness b ($b\ll a$) for the wires in the z direction, and replace the $K_0(|mq_xa|)$ term for $m=0$ by $K_0(|q_x|b)$ in the series above, rewriting the dispersion relation as

$$1+\left(\frac{\alpha_{1D}}{\varepsilon_{3D}}\right)K_0(|q_x|b)+2\left(\frac{\alpha_{1D}}{\varepsilon_{3D}}\right)S(q_x,k)=0, \tag{15a}$$

where $S(q_x,k)$ is defined by

$$S(q_x,k)\equiv\sum_{m=1}^{\infty} K_0(m|q_x|a)\cos(mka). \tag{15b}$$

An equation of this type has also been discussed by Das Sarma and Lai;[1] however they examined it using numerical techniques. In contrast to their analysis, our considerations here are concerned about obtaining analytical results for the mode spectrum. Since there seems to be no closed form expression for the sum in Eq. (15b), we examine the dispersion relation analytically in two limiting cases:

A. Strong coupling case ($|q_x|a\ll\pi$)

This is the case in which the wires are closely spaced. For very small values of $(|q_x|a)$, many terms in the series of Eq. (15b) are necessary to give an accurate approximation. The number of terms required to obtain an accurate value for the series is of the order of $m\gtrsim E(1/|q_x|a)$, where E signifies the integral part. For larger values of m, the corresponding terms become less significant since K_0 function decreases exponentially for increasing argument. Our examination of the series in the strong coupling case is restricted by the condition $(|q_x|a)\ll\pi$. Referring to Ref. 14 (p. 978, no. 8.526: set $x=|q_x|a,\ xt=ka$), we rewrite the sum of the series in Eq. (15b) in the form

$$\begin{aligned}S(q_x,k)=&\frac{1}{2}\left(C+\ln\frac{|q_x|a}{4\pi}\right)+\frac{\pi}{2\sqrt{(q_xa)^2+(ka)^2}}\\&+\frac{\pi}{2}\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{(q_xa)^2+(2\pi n-ka)^2}}-\frac{1}{2\pi n}\right)\\&+\frac{\pi}{2}\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{(q_xa)^2+(2\pi n+ka)^2}}-\frac{1}{2\pi n}\right).\end{aligned} \tag{16}$$

Since $(2\pi n\pm ka)\gg(|q_x|a)$ for any value of ka [recall that ka is restricted to the interval $(-\pi,\pi)$], we expand the square root functions under the n sums in power series up to the second order in $(|q_x|a)$, and further utilize the identity

$$\frac{\pi}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2\pi n\pm ka}-\frac{1}{2\pi n}\right)=-\frac{1}{4}\left[C+\psi\left(1\pm\frac{ka}{2\pi}\right)\right], \tag{17}$$

from which we obtain

$$\begin{aligned}S(q_x,k)\cong&\left[\frac{1}{2}\ln\left(\frac{|q_x|a}{4\pi}\right)+\frac{\pi}{2}[(q_xa)^2+(ka)^2]^{-1/2}\right.\\&\left.-\frac{1}{4}\psi\left(1-\frac{ka}{2\pi}\right)-\frac{1}{4}\psi\left(1+\frac{ka}{2\pi}\right)\right]+O([q_xa]^2),\\&(|q_x|a)\ll\pi,\end{aligned} \tag{18}$$

where $C\approx 0.577\,2$ is Euler's constant. The function $\psi(x)$ is called the psi function and is defined as the logarithmic derivative of the gamma function $\Gamma(x)$, i.e., $\psi(x)=(d/dx)\ln\Gamma(x)$.[14] Higher order corrections to the sum are possible in terms of the Riemann's zeta function $\zeta(n,ka)$, where $n=3,5,\ldots$.

Considering first the case where the 1D SL is embedded in a constant dielectric background ($\varepsilon_{3D}=\varepsilon_0$), Eq. (15) jointly with Eq. (18) yield the SL plasmon mode in the local limit accurate to the order of $(q_xa)^2$ as

$$\omega^2=\frac{\omega_{1D}^2}{\varepsilon_0}\left\{1+\frac{2S(q_x,k)}{K_0(|q_x|b)}\right\}, \tag{19}$$

where ω_{1D} represents the 1D intrasubband plasma frequency,

$$\omega_{1D}^2=(2e^2n_{1D}/m_{1D})q_x^2K_0(|q_x|b), \tag{20}$$

which, for small wave vectors, is of logarithmic character, $\omega_{1D}\propto|q_x|[-\ln(|q_x|b/2)]^{1/2}$, since $K_0(x)\to-[C+\ln(x/2)]$ for $x\ll 1$. Note that the first term in Eq. (19) represents the 1D plasmon corresponding to the wire at the origin and the second term accounts for the SL contribution (from other wires) to the spectrum. For $k\to\pm(\pi/a)$, we ignore $(|q_x|a)^2$ compared to $(ka)^2$ under the square root function of $S(q_x,k)$ [see Eq. (18)] since $(|q_x|a)\ll\pi$, thus rewriting Eq. (19) to the second order in $(|q_x|a)$ in the form

$$\omega^2 \approx \frac{\omega_{1D}^2}{\varepsilon_0}\left\{1+\frac{[C+\ln(|q_x|a/\pi)]}{K_0(|q_x|b)}\right\}$$
$$\cong \frac{\omega_{1D}^2}{\varepsilon_0}\left\{1-\frac{K_0(2|q_x|a/\pi)}{K_0(|q_x|b)}\right\}, \tag{21}$$

where, in the last step, we have used the representation of the Bessel function for small argument since $(|q_x|a)/\pi \ll 1$.

On the other hand, for $k=0$, Eq. (19) reduces to

$$\omega^2 = \frac{\omega_{1D}^2}{\varepsilon_0}\left\{1+\left[\frac{C+\ln(|q_x|a/4\pi)+\pi/(|q_x|a)}{K_0(|q_x|b)}\right]\right\} + O([q_x a]^2). \tag{22}$$

Further, in the limit $a\to 0$ [or $|q_x|a\to 0$] (the extreme strong coupling limit), we neglect $[C+\ln(|q_x|a/4\pi)]$, which is relatively small, compared to $[\pi/(|q_x|a)]$ in the second term which is much larger in comparison with the first term (unity), and rewrite Eq. (22) as

$$\omega^2 \approx \frac{1}{\varepsilon_0}\left(\frac{2\pi e^2(n_{1D}/a)|q_x|}{m_{1D}}\right) \to \frac{\omega_{2D}^2}{\varepsilon_0}. \tag{23}$$

Equations (21) and (23) are of great importance for they tell a great deal of information about the nature of the SL system in the strong coupling case. [In fact, it is $S(q_x,k)$, the sum of series in Eq. (15b), which determines the nature of the SL plasmon in appropriate limits.] On the one hand, close to the boundaries of the first Brillouin zone ($k\to \pm\pi/a$), we see from Eq. (21) that the SL plasmon is clearly 1D in nature. On the other hand, our discussion resulting in Eq. (23) explicitly shows how, in the extreme strong coupling limit for $k=0$, the 1D SL tends to change character to become a single 2D electron layer; consequently, at the center of the first Brillouin zone, the SL plasmon is strictly 2D in nature. This is understandable on an intuitive basis as well because, in the limit $a\to 0$, the wires gather gradually to form a 2D electron layer with plasma frequency $\omega_{2D}^2 = (2\pi e^2 n_{2D}|q_x|/m_{2D})$ if (n_{1D}/a) in Eq. (23) is interpreted as a 2D sheet-electron density n_{2D} and m_{1D} is replaced by m_{2D}. Thus, our analytically obtained results explicitly confirm the conclusions which were drawn by means of numerical techniques in the literature (see, for example, Refs. 1 and 6). In passing, we remark that we could have obtained the same result in Eq. (23) more easily had we written the sum of the series in Eq. (15b) in the limit $k=0$, $a\to 0$ and then replaced the sum by the corresponding integral by setting the lower limit to zero with negligible error,

$$S(q_x,0)=\sum_{m=1}^{\infty} K_0(m|q_x|a) \to \int_0^{\infty} K_0(x|q_x|a)dx = \frac{\pi}{2(|q_x|a)},$$

which, when employed in Eq. (15a), would immediately result in the mode given by Eq. (23).

Next, considering the case in which the 1D SL is embedded in a background host plasma, the dispersion relation [Eq. (15)] gives the general form of the SL plasmon mode in the local limit correct to the order of $(|q_x|a)^2$ as

$$\omega^2 = \frac{\omega_p^2}{\varepsilon_0} + \frac{\omega_{1D}^2}{\varepsilon_0}\left\{1+\frac{2S(q_x,k)}{K_0(|q_x|b)}\right\}, \tag{24}$$

which, for $k=\pm(\pi/a)$, becomes

$$\omega^2 = \frac{\omega_p^2}{\varepsilon_0} + \frac{\omega_{1D}^2}{\varepsilon_0}\left\{1-\frac{K_0(2|q_x|a/\pi)}{K_0(|q_x|b)}\right\}, \tag{25}$$

and describes the hybridization of the 1D SL plasmon with the 3D bulk plasmon. On the other hand, by the same discussion resulting in Eq. (23), the plasmon mode of Eq. (24) in the extreme strong coupling limit for $k=0$ reduces to

$$\omega^2 = \frac{\omega_p^2}{\varepsilon_0} + \frac{\omega_{2D}^2}{\varepsilon_0}, \tag{26}$$

and represents the coupling of the 2D plasmon with the 3D bulk plasmon.[15]

B. Weak coupling case ($|q_x|a \gg \pi$)

In this case, the wires are sufficiently far apart and the Coulomb interaction between the wires becomes small. The evaluation of the series for $(|q_x|a)\gg\pi$ by Eq. (16) is not suitable, as $(|q_x|a)$ may become comparable in magnitude to $(2\pi n \pm ka)$. Therefore, for large values of $(m|q_x|a)$, we replace the Bessel functions by their asymptotic form[14]

$$K_0(x) \sim \sqrt{\frac{\pi}{2x}}e^{-x}\left[1+O\left(\frac{1}{x}\right)\right], \quad x\gg 1, \tag{27}$$

in the series in Eq. (15b), from which

$$S(q_x,k) \sim \sqrt{\frac{\pi}{2|q_x|a}}\sum_{m=1}^{\infty}\frac{e^{-m|q_x|a}}{\sqrt{m}} \times\left[1+O\left(\frac{1}{m|q_x|a}\right)\right]\cos(mka). \tag{28}$$

It is evident that the series rapidly approaches its asymptotic form given by the first term as the higher terms in the series decrease exponentially and become less significant. Therefore, to a very good approximation, we have

$$S(q_x,k) \cong \sqrt{\frac{\pi}{2|q_x|a}}e^{-|q_x|a}\cos(ka), \quad (|q_x|a)\gg\pi. \tag{29}$$

For a constant 3D bulk background, the dispersion relation in the local limit yields the SL plasmon mode as

$$\omega^2 = \frac{\omega_{1D}^2}{\varepsilon_0}\left\{1+\sqrt{\frac{2\pi}{|q_x|a}}\frac{e^{-|q_x|a}\cos(ka)}{K_0(|q_x|b)}\right\}. \tag{30}$$

Note that the first term is the 1D plasmon of the wire at the origin, while the second term, whose sole purpose is to exhibit the k dependence, is a very small contribution from other wires to the mode spectrum. Thus, in the weak coupling case, the mode spectrum is determined dominantly by a single wire plasmon, an experimentally well-known fact.[10,11] Also, note that the modes at the zone boundaries ($k=$

$\pm\pi/a$) are smaller than the mode at the zone center ($k=0$). For $k=\pm(\pi/2a)$, the contribution from other wires vanishes, hence

$$\omega^2=\frac{\omega_{1D}^2}{\varepsilon_0}, \tag{31}$$

which is the plasmon of the wire at the origin. Considering further the extreme weak coupling case in which $a\to\infty$ [or $(|q_x|a)\to\infty$] for all values of k, the system reduces to an isolated single wire whose plasma frequency is given by Eq. (31). On the other hand, if a background bulk plasma is considered, in the local limit, the dispersion relation takes the form

$$\omega^2=\frac{\omega_p^2}{\varepsilon_0}+\frac{\omega_{1D}^2}{\varepsilon_0}\left\{1+\sqrt{\frac{2\pi}{|q_x|a}}\frac{e^{-|q_x|a}\cos(ka)}{K_0(|q_x|b)}\right\}, \tag{32}$$

which, in the limit $a\to\infty$, reduces to

$$\omega^2=\frac{\omega_p^2}{\varepsilon_0}+\frac{\omega_{1D}^2}{\varepsilon_0}, \tag{33}$$

and describes the coupling of an isolated single wire plasmon to the bulk host plasmon.[12] Moreover, by a denominator factor $\varepsilon_{3D}=\varepsilon_0-\omega_p^2/\omega^2$ in the structure of the inverse dielectric function [Eq. (13)], the bulk plasmon ($\omega^2=\omega_p^2/\varepsilon_0$) is always present in the spectrum.

IV. SUMMARY

In this paper, we have presented analytical results for the coupled-collective modes of a periodic SL of 1DQWs in interaction with the plasmalike bulk host medium. We have considered first the case in which the SL is embedded in a constant background, and discussed the nature of the SL plasmon in the strong and weak coupling limits. Our results, based on an analytical calculation, show clearly how in the strong coupling case the 1D SL becomes a 2D electron system and how in the weak coupling case the 1D SL reduces to an isolated single wire, thus confirming the conclusions which were drawn earlier by means of numerical and experimental techniques in the literature. We then have considered the case in which the SL is embedded in a background bulk plasma and discussed the nature of the mode coupling of the bulk (3D) plasmon and the lower-dimensional (2D/1D) plasmons in the appropriate limits depending on the type of the interaction between the wires constituting the SL.

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