Currents in a system of noisy mesoscopic rings

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A semiphenomenological model is proposed to study magnetic fluxes and currents in mesoscopic rings at nonzero temperature. The model is based on a Langevin equation for a flux subject to thermal equilibrium Nyquist noise. Quenched randomness, which mimics disorder, is included via the fluctuating parameter method. It is shown that self-sustaining and persistent currents survive in the presence of Nyquist noise and quenched disorder but the stability threshold can be shifted by noise.

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Quantum phenomena manifested at the mesoscopic level have attracted much experimental and theoretical attention. Phase coherence and persistent currents can be mentioned as examples. It is known that a small metallic ring threaded by a magnetic flux displays a persistent current,[1] signifying quantum coherence of electrons called coherent electrons. Moreover, it has been theoretically shown that in such a system self-sustaining currents can run even if the external flux is switched off.[2] At temperature $T=0$, the system is in the ground state and only coherent electrons exist.[3] Then the persistent current flows without dissipation. At temperature $T>0$ the amplitude of the persistent current run by coherent electrons decreases and some electrons become "normal" (i.e., noncoherent). The motion of normal electrons is random and their flow is dissipative. Under some conditions, coherent conduction and normal conduction coexist, resulting in dissipation of a total current. It was confirmed experimentally[4] in that mesoscopic rings connected to a current source presented a nonzero ohmic resistance.

Thermal motion of charge carriers in any conductor is a source of random fluctuations of current which is called Nyquist noise.[5] This thermal equilibrium noise is universal and exists in any conductor, irrespective of the type of conduction. Moreover, this noise increases with temperature. Therefore at relatively high temperatures and relatively large rings, universal conductance fluctuations and shot noise can be neglected,[6,7] and only Nyquist noise can play an important role. Nyquist noise generates the flux fluctuations which indirectly influence persistent currents run by coherent electrons. In the paper we analyze the steady states of magnetic fluxes and currents in a mesoscopic system subject to dissipation and fluctuations. Our main goal is to answer the question of whether persistent and self-sustaining currents survive in the presence of dissipation and fluctuations. We introduce a semiphenomenological model formulated as a Langevin equation with a noise term and with terms of a quantum origin. Our model is minimal in the sense that in the limiting cases it reduces to the well-established models of the quantum persistent current of coherent electrons and the classical Nyquist current of normal electrons. The approach used could be justified in a more elegant way applying the methods of thermofield dynamics.[8]

Now, let us formulate our model: The system is a collection of rings (individual current channels) stacked along a certain axis forming a cylinder. There are N_z channels in the direction of the cylinder axis and N_r in the direction of the cylinder radius. We assume that the thickness of the cylinder wall is small if compared with the radius. Because of the mutual inductance between rings, the current in one ring induces flux in other rings. In turn, the flux induces a current, and so on. We will analyze the effect of the mutual inductance among the rings. We assume that the rings are not contacted. So, there is no tunneling of electrons among the channels and the charge carriers moving in the different rings are independent. It has been shown[9] that the effective interaction between the ring currents, when taken in the self-consistent mean-field approximation, results in the magnetic flux $\phi=LI_{tot}$ felt by all electrons, where L is the cylinder inductance and I_{tot} is the total current in a cylinder. For a cylinder of radius r and height l_z the inductance[10] is $L=\mu_0\pi r^2/l_z$, where μ_0 is the permeability of the free space. At temperature $T>0$, the current $I_{coh}(\phi,T)$ of the coherent electrons (in a set of $N=N_r\times N_z$ current channels forming a cylinder) is an average (with a weight p) of the paramagnetic current I_{even} coming from the channels with an even number of coherent electrons and diamagnetic I_{odd} coming from the channels with an odd number of coherent electrons:

$$I_{coh}(\phi,T)=pI_{even}(\phi,T)+(1-p)I_{odd}(\phi,T), \tag{1}$$

where[3]

$$I_{even}(\phi,T)=NI_0\sum_{n=1}^{\infty} A_n(T)\sin(2n\pi\phi/\phi_0) \tag{2}$$

and $I_{odd}(\phi,T)=I_{even}(\phi+\phi_0/2,T)$. The flux quantum $\phi_0:=h/e$ and $I_0=heN_e/(2l_x^2m_e)$ where N_e is the number of coherent electrons in a single channel of circumference l_x and m_e is the electron mass. The amplitude is

$$A_n(T)=\frac{4T}{\pi T^*}\frac{\exp(-nT/T^*)}{1-\exp(-2nT/T^*)}\cos(nk_Fl_x). \tag{3}$$

The characteristic temperature T^* is given by the relation $k_BT^*=\Delta_F/2\pi^2$, where k_B is the Boltzmann constant, Δ_F is the energy gap at the Fermi surface, and k_F is the Fermi momentum. For temperatures $T<T^*$ the coherent current flows in such a system without dissipation but its amplitude (3) is reduced.[3] On the other hand, at temperature $T>0$, normal electrons occur, their flow is dissipative, and it generates random currents. The current coming from the normal electrons can be induced by, e.g., the change of the magnetic flux ϕ. According to Lenz's rule and Ohm's law one gets[11]

$$RI_{nor}(\phi)=-\frac{d\phi}{dt}, \tag{4}$$

where R is the effective resistance of the system.[12] In the absence of fluctuations, the magnetic flux is related to the total current via the expression

$$\phi=\phi_{ext}+L[I_{nor}(\phi,T)+I_{coh}(\phi,T)], \tag{5}$$

i.e., it is a sum of the external flux ϕ_{ext} and the flux coming from the currents.

Combining Eqs. (4) and (5) and adding the term describing Nyquist noise yields the equation

$$\frac{1}{R}\frac{d\phi}{dt}=-\frac{1}{L}(\phi-\phi_{ext})+I_{coh}(\phi,T)+\sqrt{\frac{2k_BT}{R}}\Gamma(t), \tag{6}$$

where $\Gamma(t)$ is Gaussian white noise. This equation takes the form of a classical Langevin equation and is our basic evolution equation.

Now, let us introduce dimensionless variables. The flux is scaled as $x=\phi/\phi_0$, and the time $\tilde{t}=t/\tau_0$ where $\tau_0:=L/R$ is the relaxation time of the averaged normal current. In this case, Eq. (6) can be transformed into its dimensionless form,

$$\dot{x}=-V'(x)+\sqrt{2D}\tilde{\Gamma}(\tilde{t}), \tag{7}$$

where the dot denotes a derivative with respect to the rescaled time $\tilde{t}$ and the prime denotes a derivative with respect to x. The generalized potential

$$V(x)=\frac{1}{2}x^2-\lambda x-i_0\int f(x,p,T)dx, \tag{8}$$

where $\lambda=\phi_{ext}/\phi_0$, is the rescaled external flux. The coupling constant $i_0=NLI_0/\phi_0$. The function $f(x,p,T)=pg(x,T)+(1-p)g(x+1/2,T)$ and

$$g(x,T)=\sum_{n=1}^{\infty}A_n(T)\sin(2n\pi x). \tag{9}$$

The dimensionless intensity D of rescaled Gaussian white noise $\tilde{\Gamma}(\tilde{t})\equiv\sqrt{\tau_0}\Gamma(\tau_0\tilde{t})$ is a ratio of thermal energy to the elementary energy stored up in the inductance, $D:=\frac{1}{2}k_BT/\epsilon_0$ with $\epsilon_0:=\phi_0^2/2L$. Let us notice that the resistance R does not occur explicitly in the rescaled Eq. (7).

Let us evaluate the order of magnitude of the parameters appearing in our equations. Consider the cylinder of the radius $r=3\times10^4$ Å and the height $l_z=100$ Å consisting of a set of $N\sim50$ current channels[13] in a wall of width much smaller than the radius. If the number of electrons in each channel is $N_e\sim2\times10^5$ then $i_0\sim1$. The energy gap at the Fermi surface $\Delta_F=\hbar^2N_e/(2m_er^2)$ gives the rescaled noise amplitude $D=\mu_0e^2/(16\pi^3m_e)(N_e/l_z)$. For the above values of parameters the diffusion coefficient $D\sim0.001T/T^*$.

The Langevin Eq. (7) defines a Markov diffusion process. Its probability density $p(x,\tilde{t})$ obeys a Fokker-Planck equation.[14] Its stationary solution $p_s(x)$ reads

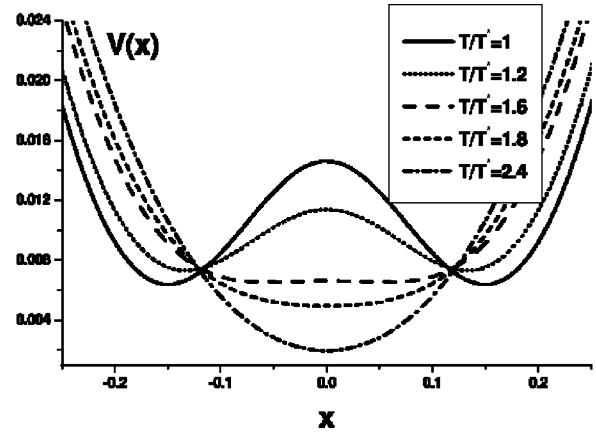

FIG. 1. The dimensionless generalized potential $V(x)$ is shown as a function of the dimensionless magnetic flux x for several values of the scaled temperature T/T^*. The scaled amplitude $i_0=1$ and scaled external magnetic flux $\lambda=0$.

$$p_s(x)=N_0e^{-V(x)/D} \tag{10}$$

with a normalization constant N_0. The probability density (10) looks like a Boltzmann distribution. However, it is not strictly a Boltzmann distribution because temperature T enters into it in two ways: into the intensity $D=k_BT/\epsilon_0$ of Nyquist noise and into the generalized potential (8). Nevertheless, it describes an equilibrium state. There are examples of equilibrium distributions with temperature-dependent effective (generalized) potentials in thermofield dynamics[8] or for quantum Smoluchowski systems [cf. Eq. (11) in Ref. 15)].

Let us consider the case of absence of external flux, $\lambda=0$ and $p=1/2$. The properties of $p_s(x)$ are determined by the properties of the potential $V(x)$. In high temperature, where no coherent electrons are present, the potential (8) is monostable. If temperature decreases, a bifurcation occurs—the potential becomes bistable and two nonzero symmetric minima appear at $x_s=\pm x_m$. Physically, it means that below some critical temperature T_c the *spontaneous flux*[16,17] appears and nonzero stationary current flows in the system. This critical temperature T_c is defined by the condition $V''(x_s=0)=0$. The formation of a bistability is shown in Fig. 1. The phenomenon is analogous to the *continuous phase transition* in macroscopic systems, and appears here as a result of the interaction of ring currents. Because the potential is reflection symmetric, $V(x)=V(-x)$, the mean values of both the flux x and the current are zero. From this point of view, properties of stationary states are trivial and nonzero fluxes and currents are impossible. However, it is possible to define the phase transition in the following way:[18] the phase transition point is a value of the relevant parameter γ of the system at which the profile of the stationary distribution function changes drastically (e.g., if the number of maxima of the distribution function changes) or if a certain most probable point x_0 begins to change to an unstable state. In the case considered here, for sufficiently low temperatures, thermal fluctuations are small and one expects the experimental results to be accumulated around the *most prob-*

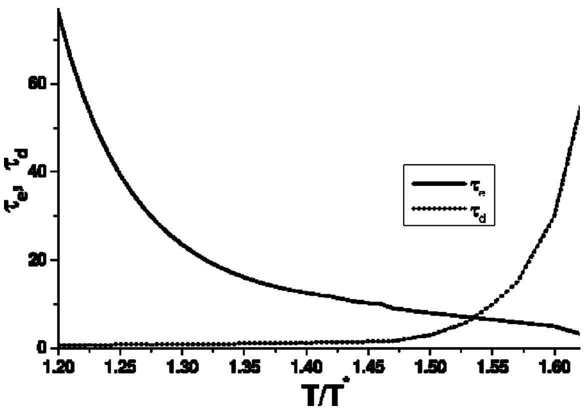

FIG. 2. Plot of the characteristic escape time τ_e and decay time τ_d as a function of the temperature T/T^* for $i_0=1$ and the strength of Nyquist noise $D=0.001T/T^*$.

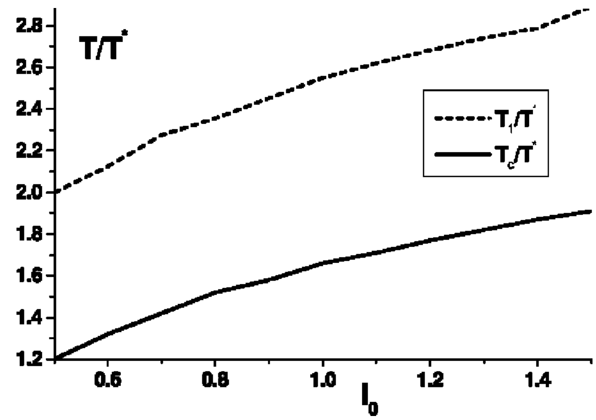

FIG. 3. Plot of the critical temperature T_c and the temperature T_1 corresponding to minimal current fluctuations or minimal energy $\langle E\rangle$ versus coupling constant i_0 for $D=0.001T/T^*$.

able values of the stationary probability distribution. It follows from Eq. (10) that the most probable values of the flux correspond exactly to the minima of the generalized potential $V(x)$. One can introduce characteristic time scales of the system. The first characteristic time $\tau_d=1/V''(x_s)$ describes decay within the attractor $x_s=\pm x_m$ of the potential $V(x)$. The second characteristic time is the escape time τ_e from the well around $\pm x_m$. This time is related to the mean first passage time from the minimum of the potential to the maximum.[14] If these time scales are well separated, i.e., if $\tau_e \gg \tau_d$, then the description based on the most probable value is correct. Otherwise, this description fails and we should characterize the system by averaged values of relevant variables. We have calculated τ_e for the transition from the left minimum of the potential (8) assuming that the left boundary at $-\infty$ is reflecting and the right boundary at the maximum x_M of the potential is absorbing. In Fig. 2 we show the dependence of two characteristic times τ_e and τ_d upon the rescaled temperature T/T^*. From Eq. (8) we estimated the critical temperature $T_c-1.66T^*$ for the parameters chosen as in Fig. 1. One can observe that roughly for temperatures $T<14T^*$, the characteristic time τ_d is more than one order-of-magnitude less than τ_e. Both time scales are then well separated and self-sustaining currents are long-living states. In this sense, they are not destroyed by Nyquist noise. In the opposite case, self-sustaining currents are short lived and "jump" between various states.

Fluctuations of the magnetic flux and currents exhibit nonlinear dependence on temperature of the system and are minimal at a certain temperature $T=T_1$ which is always larger than T_c. This has been confirmed by numerical analysis (Fig. 3). Because $\langle x\rangle=0$, the temperature dependence of these fluctuations is exactly the same as for the average energy stored in the magnetic field, i.e., $\langle E\rangle=\langle\phi^2\rangle/2L=\epsilon_0\langle x^2\rangle$. For low temperatures, Nyquist fluctuations are small and the main contribution to the energy comes from the deterministic part $\phi^2/2L$. Because the magnetic flux ϕ decreases as temperature increases, hence $\langle E\rangle$ decreases as well. On the other hand, for high temperature, the stationary state approaches the Gaussian state and as a consequence the main contribution to the average magnetic energy comes from thermal energy, $\langle E\rangle\propto kT$, which obviously increases when T grows. The competition between these two mechanisms leads to the minimal value of $\langle E\rangle$ and *minimal current fluctuations* at $T_1>T_c$.

The influence of the external field on the critical points of the stationary density (10) was studied in Ref. 19. An interesting feature is the occurrence of the hysteresis loop. It is a hallmark of the *first-order phase transition*. The transition can occur only below the critical temperature T_c. Due to the "structural periodicity" the hysteresis loop is repeated with the period $\lambda=1/2$ which results in the formation of a family of loops.

The last problem we want to consider is a system with disorder. There are several sources of disorder. The first one is caused by impurities distributed randomly in the system. The presence of impurities leads to modification of the coherent current amplitude.[13] There is also a geometrical source of disorder: The radius of rings is not exactly of the same value, and may change randomly from one ring to the other. Also the number N_r of current channels in the direction of the cylinder radius can be, in general, different at every point of the vertical axis of the cylinder. These sources of disorder can influence the flux and current. Consequently, they should be included at the microscopic level of description. Their caricature can be modeled phenomenologically assuming that the coupling parameter i_0 defined below Eq. (8) is a random variable, i.e., $i_0=j_0+\epsilon\xi$, where j_0 is an averaged value of the amplitude i_0, ϵ characterizes magnitude of disorder, and ξ is a zero-mean random variable of values in the interval $[-1,1]$ and of the probability density $P_\xi(z)$. It means that disorder is described by quenched fluctuations. The stationary probability density $p_s(x)$ of the flux is now expressed as

$$p_s(x)=\int_{-1}^{1}p(x|z)P_\xi(z)dz, \tag{11}$$

where the conditional probability distribution $p(x|z)=C_0(z)\exp\{-[V(x)-\epsilon zF(x)]/D\}$ has the same form as Eq. (10) with the replacement $i_0=j_0+\epsilon z$ in the potential (8). Now, the normalization constant depends on z, $C_0^{-1}(z)$

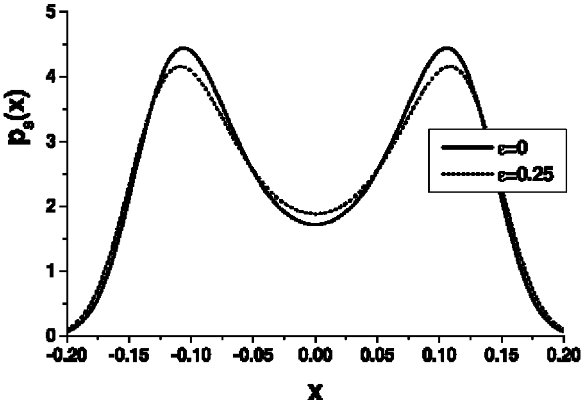

FIG. 4. Probability density for the system with ($\varepsilon = 0.25$) and without ($\varepsilon = 0$) quenched disorder for $T/T^* = 1.4$, $j_0 = 1$, $\lambda = 0$, and $D = 0.001T/T^*$.

$= \int_{-\infty}^{\infty} \exp\{-[V(x) - \epsilon z F(x)]/D\} dx$. In Fig. 4 we show the distribution (11) for $p = 1/2$, $\lambda = 0$ and the uniform probability distribution $P_\xi(z)$. The observed shift of the most probable value of the flux is rather small and can be interpreted as a small increase of the magnitude of the self-sustaining current (Fig. 5). The magnitude of peaks does not change significantly as well. One observes that both the depth of the minimum and the height of the maxima decrease. One concludes that the quenched disorder does not drastically change the properties of the flux with only one exception. The probability density in or near the critical temperature T_c may change qualitatively from one peak to two peaks. In the critical region the quenched disorder lowers the critical temperature T_c.

In summary, we have investigated the influence of Nyquist noise and quenched disorder on the persistent and self-sustaining currents in a set of mesoscopic rings having a cylindrical symmetry. Our discussion is limited to stationary states of the magnetic flux and current although the proposed model of the flux dynamics can be, in principle, applied to study time-dependent problems. The presented approach can be applied also to superconducting rings and carbon nanotubes.[20] The general conclusion is that persistent and self-sustaining currents survive in the presence of the above-mentioned fluctuations. However, if the intensity of Nyquist noise or quenched disorder is sufficiently strong, fluctuations lead to the lowering of the temperature below which the system is in the ordered state characterized by long-living self-sustaining currents. It is very favorable for the experimental observations.

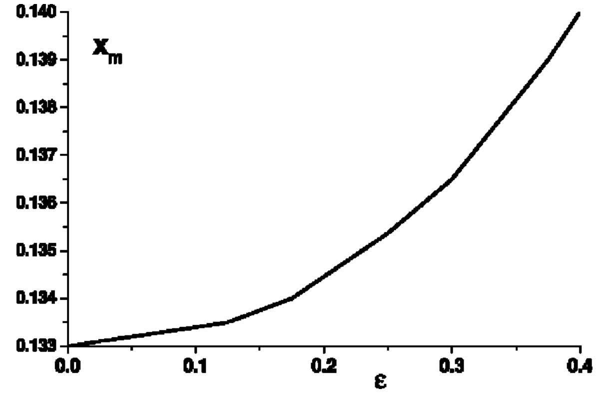

FIG. 5. The most probable value of x as a function of the intensity of disorder ε for $j_0 = 1$ and $D = 0.001T/T^*$.

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