Thermal conductivity and thermal Hall effect from vortex motion

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In the mixed state of type-II superconductors the motion of vortices in response to an applied temperature gradient ∇T leads to a longitudinal and a transverse heat current along ∇T and $\nabla T\times\mathbf{B}$, respectively, where $\mathbf{B}$ is the magnetic field. We calculate the corresponding contributions to the thermal conductivity and the thermal Hall effect in terms of other thermomagnetic transport coefficients and the electrical resistivity. The vortex contribution to the thermal Hall effect may be observable in superconductors with weak pinning and small quasiparticle thermal conductivity.

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The study of heat transport is a valuable tool for the investigation of quasiparticle (QP) dynamics in superconductors. For example, the order parameter symmetry as well as the QP relaxation time can be obtained by analyzing the thermal conductivity k_{xx} and the thermal Hall effect k_{xy}.[1–10] This latter quantity is the thermal analog of the electrical Hall effect. Whereas k_{xx} is often dominated by a phononic contribution to the heat current, k_{xy} is purely electronic. Important information regarding the origin of the scattering processes as well as the dynamics of QPs in the high-T_c superconductors was recently obtained from studies of k_{xy}.[3–10]

It is usually assumed that the electronic heat current in the superconducting state probes the QP response only, since the superfluid does not carry heat.[11] However, it is well known that in the mixed state of type-II superconductors vortices move in response to a temperature gradient, if they are not pinned, and that they carry heat.[12–15] Therefore, moving vortices contribute to the heat current; their contribution adds to that of the phonons and the QPs. To our knowledge this vortex heat current in response to a temperature gradient has not been considered up to now. However, an estimate of its magnitude is important for the interpretation of experimental data, in particular, for k_{xy}, which is small in the weak-field regime usually exploited experimentally.

In this paper we calculate the thermal conductivity and the thermal Hall effect resulting from vortex motion in terms of other, well-known thermomagnetic effects such as the Nernst or Ettingshausen effect, the thermopower, and the electrical resistivity. We find that the vortex contribution to the thermal Hall effect is small compared to the measured k_{xy} in the high-T_c superconductors. It may be observable in other superconductors with weak pinning and with a small QP contribution to the heat current.

We define transport coefficients according to[16]

$$\mathbf{E}=\rho_{xx}\mathbf{j}+\rho_{xy}\mathbf{j}\times\mathbf{z}+S\nabla T+QB\nabla T\times\mathbf{z}, \tag{1}$$

$$\mathbf{j}_h=\Pi\mathbf{j}+\epsilon k_{xx}B\mathbf{j}\times\mathbf{z}-k_{xx}\nabla T-k_{xy}\nabla T\times\mathbf{z}. \tag{2}$$

We consider isotropic materials for simplicity. We take the magnetic field $\mathbf{B}=B\mathbf{z}$ along the z direction ($\mathbf{z}$ is a unit vector). $\mathbf{E}$ denotes the gradient of the electrochemical potential, ∇T denotes the temperature gradient, and $\mathbf{j}$ and $\mathbf{j}_h$ denote the electrical and heat current density, respectively. We assume in the following that these quantities are within the (x,y)-plane, perpendicular to $\mathbf{B}$. ρ_{xx}, ρ_{xy}, k_{xx}, and k_{xy} are the electrical resistivity, the Hall resistivity, the thermal conductivity, and the thermal Hall conductivity, respectively. S is the thermopower (or Seebeck coefficient) and Q is the Nernst coefficient. S and Q are related to the Peltier coefficient Π and the Ettingshausen coefficient ϵ according to the Kelvin and Bridgeman relations,

$$\Pi=ST \quad \text{and} \quad QT=\epsilon k_{xx}, \tag{3}$$

respectively. If more than one type of excitation contributes to the electrical and thermal currents the corresponding conductivities have to be added, e.g. $k_{xx}=k_{xx}^{ph}+k_{xx}^{QP}+k_{xx}^{v}$, if phonons, QPs, and vortices contribute to the heat current, and correspondingly $k_{xy}=k_{xy}^{QP}+k_{xy}^{v}$, where we have assumed that phonons do not contribute to k_{xy}.

The transport coefficients may be divided into longitudinal effects (parallel to $\mathbf{j}$ or ∇T) and transverse effects (parallel to $\mathbf{j}\times\mathbf{B}$ or $\nabla T\times\mathbf{B}$). In the normal state the transverse effects are much smaller than the longitudinal ones in the weak-field regime $\omega_c\tau\ll 1$. Here $\omega_c=eB/m$ is the cyclotron frequency and τ is the relaxation time. In the mixed state of type-II superconductors, in the same magnetic-field regime, we still find $\rho_{xy}\ll\rho_{xx}$, but, in contrast to the normal state, the transverse Nernst and Ettingshausen effects are large, comparable in magnitude to the longitudinal thermopower and Peltier effect. These large transverse thermomagnetic effects are a hallmark of vortex motion in a temperature gradient.[12–15]

Consider vortices with (areal) density $n_v=B/\Phi_0$ (where Φ_0 is the flux quantum) moving with velocity $\mathbf{v}_L=(v_{L,x},v_{L,y},0)\perp\mathbf{B}=B\mathbf{z}$ in the (x,y) plane. Each vortex carries an entropy s_v (per unit length). The heat current associated with these moving vortices is[12,13]

$$\mathbf{j}_h^v=n_v s_v T\mathbf{v}_L=\frac{s_v T}{\Phi_0}\mathbf{E}\times\mathbf{z}. \tag{4}$$

Both the fact that vortices move in a temperature gradient, if they are not pinned and that they carry heat are well established empirically from experiments on conventional and high-T_c superconductors.[12–15] For an overview and for a discussion of the equation of motion of vortices in a tempera-

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ture gradient the reader is referred to Refs. 14 and 15. In the second step of Eq. (4) we have exploited that in the mixed state of a superconductor the vortex velocity $\mathbf{v}_L$ is unambiguously related to the electric field by the Josephson relation $\mathbf{E}=\mathbf{B}\times\mathbf{v}_L$. Using Eqs. (1) and (4) it is thus possible to relate the heat current in a temperature gradient to the thermopower and the Nernst coefficient.

We first consider the thermal Hall effect. In the presence of a magnetic field $\mathbf{B}\|\mathbf{z}$ the heat current arising from an applied temperature gradient $\nabla_x T$ in the x direction is deflected and gives rise to a transverse temperature gradient $\nabla_y T$ in the y direction under adiabatic conditions, i.e., for $j_{h,y}=0$. Here $\nabla_x T$ and $\nabla_y T$ are the x and y components of the temperature gradient, respectively. The thermal Hall effect is measured without electrical current flow, i.e., for $\mathbf{j}=0$. The thermal Hall conductivity k_{xy} is defined as

$$k_{xy}=k_{xx}\frac{\nabla_y T}{\nabla_x T}, \tag{5}$$

where k_{xx} is the *total* thermal conductivity. The vortex contribution k_{xy}^v to the thermal Hall effect arises from the vortex heat current $j_{h,y}^v=n_v s_v T v_{L,y}$ in the y direction, where $v_{L,y}=-E_x/B$ is determined by the electrical field E_x in the x direction. For $\nabla_x T\neq 0$ and $\mathbf{j}=0$ we find, from Eq. (1), that E_x is determined by the thermopower, i.e., $E_x=S\nabla_x T$. We therefore obtain

$$j_{h,y}^v=-n_v s_v T\frac{E_x}{B}=-\frac{s_v T}{\Phi_0}S\nabla_x T. \tag{6}$$

This heat current gives rise to a temperature gradient in the y direction, which in turn sets up a heat current $j_{h,y}^k=-k_{xx}\nabla_y T$ along the y direction by regular thermal conduction. Here k_{xx} is the *total* thermal conductivity, with contributions from vortices, QPs, and phonons. In the steady state, with $j_{h,y}=0$, we obtain

$$j_{h,y}=j_{h,y}^v+j_{h,y}^k=0=-\frac{s_v T}{\Phi_0}S\nabla_x T-k_{xx}\nabla_y T, \tag{7}$$

which yields

$$k_{xy}^v=k_{xx}\frac{\nabla_y T}{\nabla_x T}=-\frac{s_v}{\Phi_0}TS. \tag{8}$$

We may obtain the isothermal thermal conductivity (measured for $\nabla_x T\neq 0$, $\nabla_y T=0$) similarly be noting that the vortex heat current along the applied temperature gradient is related to the electrical field in the y direction. Using $v_{L,x}=E_y/B$ and $E_y=-QB\nabla_x T$ as obtained from Eq. (1), we find

$$j_{h,x}^v=n_v s_v T v_{l,x}=-\frac{s_v}{\Phi_0}TQB\nabla_x T, \tag{9}$$

which yields

$$k_{xx}^v=\frac{s_v}{\Phi_0}TQB. \tag{10}$$

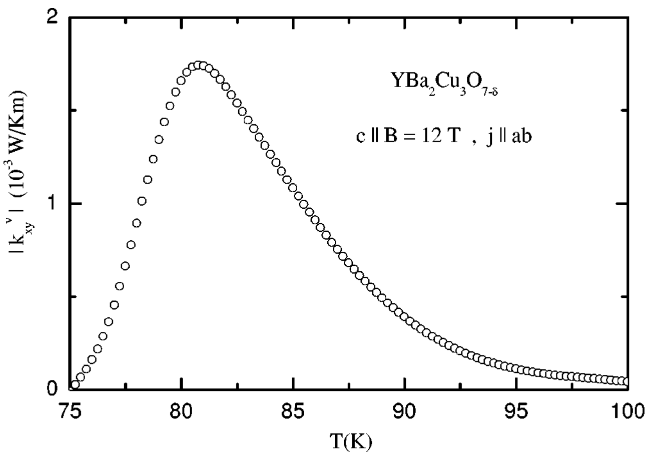

FIG. 1. Vortex contribution $|k_{xy}^v|$ to k_{xy} calculated for YBCO using the data of Ref. 20 (see the text).

It remains to determine the transport entropy s_v. It is related to the Nernst or Ettingshausen coefficient and to the electrical resistivity. To see this we consider experimental conditions appropriate for a measurement of the Ettingshausen effect. This effect consists in the generation of a temperature gradient $\nabla_y T$ in the y direction resulting from an applied electrical current j_x in the x direction in a magnetic field $\mathbf{B}=B\mathbf{z}$. It is measured with the conditions $j_y=j_{h,y}=\nabla_x T=0$. To calculate the vortex contribution to this effect we proceed similarly to the derivation of k_{xy}^v. The vortex heat current $j_{h,y}$ in the y direction is given by $j_{h,y}=-n_v s_v T E_x/B$. With the experimental conditions for a measurement of the Ettingshausen effect, Eq. (1) yields $E_x=\rho_{xx}j_x$, i.e., $v_{L,y}$ is determined by the electrical resistivity. We therefore obtain

$$j_{h,y}^v=-\frac{s_v}{\Phi_0}T\rho_{xx}j_x. \tag{11}$$

Requiring $j_{h,y}=j_{h,y}^v+j_{h,y}^k=0$, as above, we obtain the vortex contribution to the Ettingshausen coefficient as

$$\epsilon^v=-\frac{\nabla_y T}{Bj_x}=\frac{s_v}{\Phi_0}T\frac{1}{B}\frac{\rho_{xx}}{k_{xx}}. \tag{12}$$

In the mixed state of superconductors the total Ettingshausen coefficient is dominated by the vortex contribution, i.e. we may set $\epsilon^v\simeq\epsilon$. Using Eq. (3) we obtain

$$\frac{s_v}{\Phi_0}\simeq\frac{\epsilon k_{xx}B}{\rho_{xx}T}=\frac{QB}{\rho_{xx}}. \tag{13}$$

Equation (13) is well known and can also be derived from the equation of motion of vortices in a temperature gradient.[12–15] Using Eq. (13) both k_{xx}^v and k_{xy}^v can be expressed entirely using the thermomagnetic transport coefficients and the flux flow resistivity.

For an estimate of k_{xy}^v in high-T_c superconductors we take values typical for $YBa_2Cu_3O_{7-\delta}$ (YBCO) close to optimum doping.[17–20] Close to the superconducting transition temperature T_c we have $s_v \approx 5\times10^{-15}$ J/Km, $|S|\approx 2\times10^{-6}$ V/K for $B\approx 10$ T, and $T\approx 100$ K. Note that the sign of S (as well as its absolute value) depends sensitively on the oxygen (i.e., charge carrier) doping and may be positive or negative close to optimum doping.[21] Using these values we find $|k^v_{xy}|\approx 5\times10^{-4}$ W/Km close to T_c. The measured value for $|k_{xy}|$ in YBCO close to T_c is approximately $|k_{xy}|\approx 3\times10^{-2}$ W/Km, about two orders of magnitude larger than $|k^v_{xy}|$. Using experimental results[20] for $S(T)$, $Q(T)$, and $\rho_{xx}(T)$ of YBCO we calculate k^v_{xy} as a function of temperature (Fig. 1). The strong decrease of $|k^v_{xy}|$ below about 80 K is due to the onset of pinning. The increase of $|k^v_{xy}|$ below T_c is due to the temperature dependence of the transport entropy: s_v vanishes at $T=T_c$ [looking apart from superconducting fluctuations or vortex-like excitations above T_c (Ref. 23)] and for $T\rightarrow 0$, and has a maximum typically around $T_c/2$. In principle, k^v_{xy} may become a substantial contribution to k_{xy}, if pinning is weak and if the QP contribution to k_{xy} is small. In particular, this latter requirement is not fulfilled in the high-T_c superconductors, since the QP relaxation time[22] and thus k^{QP}_{xy} (Refs. 3–10) increase strongly below T_c. However, in conventional superconductors the QP heat current usually drops strongly below T_c. A conventional superconductor with weak pinning may therefore be a good candidate to observe k^v_{xy}.

The vortex contribution to the longitudinal heat current is of the same order of magnitude as k^v_{xy}, since $S\simeq QB$.[13,15,24] Since in the weak-field limit the total thermal conductivity k_{xx} is much larger than k_{xy} the contribution of the vortex motion to the longitudinal thermal conductivity is quantitatively unimportant.

In summary, we have calculated the contribution of moving vortices to the thermal conductivity and the thermal Hall effect in terms of other thermomagnetic transport coefficients, i.e., the Nernst coefficient and the thermopower, and the electrical resistivity. The vortex contribution to the thermal Hall effect is smaller by 1–2 orders of magnitude than the measured value of k_{xy} in $YBa_2CuO_3O_{7-\delta}$ close to optimum doping. In superconductors with weak pinning and a small QP heat current the vortex contribution to k_{xy} may be observable. The contribution of vortices to the longitudinal thermal conductivity is found to be quantitatively unimportant.

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