Dynamics of electron tunneling in semiconductor nanostructures

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We have modeled the transmission of an electron wave packet through a resonant tunneling semiconductor nanostructure by solving the time-dependent Schrödinger equation using the finite-difference method. We have found in all cases that the passage of the electron wave packet through the tunneling barrier is accompanied by a propagation delay relative to the propagation of an undisturbed wave packet. Tunneling transport is shown to be causal, and no evidence of superluminal behavior is seen, either for resonant or for off-resonant tunneling. In the case of off-resonant tunneling, the peak of the transmitted wave packet is observed to exit a double resonant tunneling barrier before the peak of the incident wave packet enters the structure. However, these two peaks are not directly related, and the appearance of a well-formed peak in the transmitted intensity is shown to be a result of the transient behavior of the tunneling event.

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I. INTRODUCTION

Tunneling is a fundamental manifestation of the Heisenberg uncertainty principle and quantum mechanics. In the regime of quantum dimensions, the uncertainty principle teaches that the wave function of a wave particle may be nonzero on both sides of a potential barrier, the height of which exceeds the kinetic energy of the particle wave. Since a measurement must find the particle either on one side or the other, there is a finite probability of finding that the particle has tunneled through the barrier. The calculation of steady-state tunneling probabilities based on the solution of the time-independent Schrödinger equation probabilities is a standard problem in introductory quantum mechanics. However, this approach leaves a key question unanswered: How long does it take the particle to tunnel from one side to the other?

A closely related problem concerns that of photon propagation across thin regions where the wave is evanescent. These regions act like barriers for photons and are analogous to tunneling barriers for electrons. In 1982 time-resolved tunneling experiments were reported concerning the propagation of microwave photon pulses in a wave guide containing a segment with a lower cutoff frequency below the central frequency of the pulse.[1] Access to this region by the photon pulse is classically forbidden, but tunneling transmission is allowed. The measured transmission time of the pulse through this wave guide appears to be shorter than through a wave guide of the same length having no such obstacle. These experiments, while suggestive that tunneling acts to speed up the pulse, can be explained by the frequency dispersion of the microwave pulse and the filter effect of the tunneling region that preferentially transmits the faster components of the pulse. A better experiment using single photon transmission has also shown an apparent superluminal propagation of light in the tunneling regime.[2] In a recent publication it was argued by Wang *et al.* that photon transmission through a region of anomalous dispersion can lead to superluminal transmission velocities.[3]

The case of tunneling electrons is somewhat different but perhaps even more interesting than the case of tunneling photons. First of all the question of superluminal transport of electrons in solids is not evident. Typical drift velocities are well below the speed of light. Therefore it is perfectly legitimate to ask the following question: If an electron is to traverse a finite region of space, such as a section of conducting wire, can it do so even faster if we put a tunneling structure in its path? Through the research reported in this study we are able to address and answer this question.

Part of the phenomenal progress in the information revolution is based on the idea of creating smaller and smaller transistors that can respond in increasingly shorter times.[4] However, we are close to the end of this development road. Tunneling structures for electrons have been developed for electronic logic implementation as an alternative to transistor-based architectures. These devices have an associated transit time that depends fundamentally on the tunneling time.[5] The justification of using a tunneling device is that its much smaller size compared to a transistor implies a correspondingly higher speed. However, since the dynamics of tunneling remain unmeasured, it does not at all follow that device speed can be so simply scaled when the physics of transport is completely different. In this paper we develop a model of time-dependent tunneling using the finite-difference time-domain method (FDTD) to obtain an exact numerical solution of the Schrödinger equation. We use this model to study electron transport through a resonant tunneling diode (RTD), the fundamental element in tunneling-based logic circuits. Using the FDTD solutions, we can follow in considerable detail the time evolution of the wave packet as it moves through the tunneling structure, and through this approach we are able to answer a number of important issues of the physics.

II. CALCULATION OF THE TUNNELING TIME

An excellent review has been given by Hauge and Støvneng of the principal analytic approaches to this problem, who have found all methods wanting to some degree.[6] These approaches aim to define tunneling times. For example, a phase delay time due to Wigner[6,7] can be identified from the solution of the Schrödinger equation for a free-electron wave with a narrow distribution of k vectors:

$$\tau_{\phi}=\frac{m}{\hbar k}\left(w+\frac{d\phi}{dk}\right), \tag{1}$$

and a dwell time in the barrier:

$$\tau_{D}=\frac{2mk}{\hbar\kappa k_{o}^{2}}, \tag{2}$$

where w is the total width of the RTD and ϕ is the phase associated with the transmission coefficient, $T=|Ae^{i\phi}|^{2}$. The term $d\phi/dk$ has units of length and can be thought of as the extra "distance" the electron has to travel in crossing the barrier, and κ is the wave-vector component inside the barrier. The phase term can in principle be positive corresponding to a delay by the barrier, or negative, corresponding to an acceleration. The phase time model can be applied to a monochromatic electron wave with a single k vector at any point in space. Electrons are localized in materials like semiconductors to a few nanometers, and therefore the momentum distribution is not particularly narrow, as required by this simple model. In a real experiment involving electron passage through a resonant tunneling barrier, it is quite evident that the electron is localized both in space and time. Without this condition, the meaning of tunneling time is ambiguous. The same argument applies to the case of photon tunneling. It is possible to treat each k vector separately and imagine an ensemble average of times. This approach clearly illustrates, however, that different k components of the wave packet will tunnel at different times resulting in a reshaping of the wave packet.

In the discussion that follows, we model RTD structure by two barriers each with a width of 0.5 nm and a separation between them of 1 nm. The potential height of the barrier is 2 eV. The effective mass of the electrons is unity. This model is easily scaled to parameters representing specific RTD devices, without any change in the principles illustrated by this study. We consider two important cases: tunneling close to resonance, where the transmission probability is a maximum, and tunneling far from resonance, where the transmission probability is a minimum.

To introduce the problem, we show the transmission coefficient in Fig. 1(a) and the "tunneling time" for a free electron corresponding to the phase model in Fig. 1(b). There is a straightforward one-to-one relationship between the peaks in the transmission coefficient and the peaks in the transmission time. The most tightly bound resonances have the longest tunneling times. In Fig. 1(b) we also show the time required for an electron particle to traverse the same region of space containing no barrier. It can be seen that the time for an electron to transit an RTD at resonance, when the transmission probability is high, is about two orders of magnitude longer than for the time required to transit the same

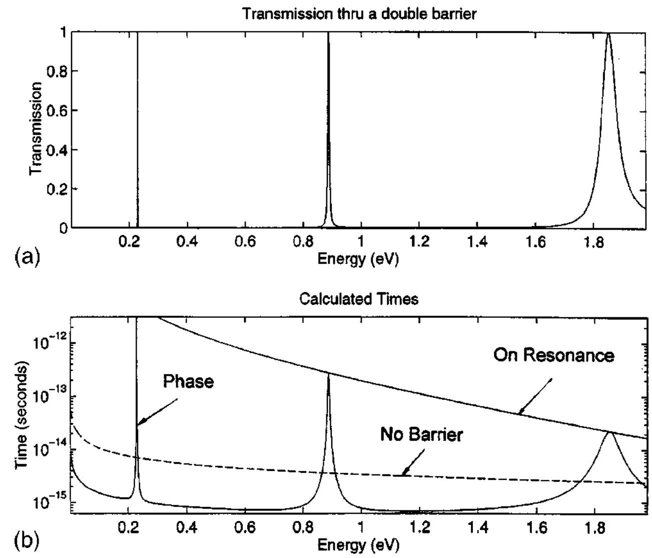

FIG. 1. (a) Transmission probability for an electron incident on a double potential barrier of height 2 eV and width equal to 150 Å. (b) The Wigner tunneling time can be calculated as a function of kinetic energy from Eq. (1) presuming a perfectly monochromatic electron. This result shows that a nonresonant electron will tunnel through the barrier in a time that is about three orders of magnitude shorter than an electron that tunnels on resonance. In between we show the time for an electron to traverse the same region of space when no barrier is present.

region by the classical physics of electron transport for an electron particle having the same kinetic energy. On the other hand, off-resonance tunneling results in a transmission time that is shorter than that of classical transport, although the transmission probability is much lower. This simple model shows some of the important features of resonant tunneling, but since the electron is modeled as a delocalized wave, the physical meaning of the Wigner phase tunneling time is not clear. A more sophisticated model that treats the particlelike behavior of the electron would be an improvement.

Other models have been proposed to calculate the transit time of quantum-mechanical particle waves. The Larmor time was proposed to take advantage of the extra degree of freedom that exists because of the spin of the electron. The Buttiker-Landauer time follows a WKB-type derivation, and the end result ignores the complex nature of the particle wave vector in the barrier regions. The methods have been reviewed by Landauer and co-workers.[8,9] These calculational approaches rely on assumptions regarding the distribution of momentum vectors and the range of wave-function energies that enable semianalytic solutions of the Schrödinger equation. As a result it becomes possible to extract a characteristic time from the asymptotic behavior of the wave function. Chiao has identified five such approaches to calculating the transit time.[10,11] While each of these methods yields a number, each also suffers from an unrealistic model of the tunneling electron. Numerical simulation of the wave-function propagation represents an improvement over these methods, and illustrates some shortcomings of these approaches by showing the difficulty of assigning a simple number to the tunneling time.

Analysis of tunneling by numerical solution of the Schrödinger equation represents an alternate approach that does not require in principle any of the simplifying accommodations that are needed to derive an analytic model. A substantial body of research in this area has been carried out by Jauho and co-workers, examining both the spatial and momentum distributions of a wave packet.[12–15] Their calculations are based on the numerical integration of the Schrödinger equation, using an electron wave packet with a finite width in real space and momentum space. This procedure is described and is applied to a triple barrier tunneling structure in Ref. 12. This a structure more complicated than the one that we use here. Nonetheless, their results illustrate the filterlike effect of a tunneling barrier on a propagating wave packet where the resonant components are trapped in the tunneling barrier for a much longer time than the nonresonant components. Jauho and Jonson applied this numerical approach to the study of tunneling through barriers with a height that is modulated in time,[13] in order to compare results to those obtained by Buttiker and Landauer.[9] Significantly, they discovered that it is not possible to define a tunneling time by following the passage of a sharp wave front through the tunneling barrier. Their numerical method showed substantial pulse reshaping, due to the filter effect, obscuring the relationship between the incident and transmitted wave form. This work gives a much more detailed and complete picture of wave-packet evolution than those based on asymptotic solutions of the Schrödinger equation. We have been able to benefit from these results to confirm our numerical solutions based on the FDTD method.

We have looked at solutions of the time-dependent, one-dimensional Schrödinger equation for the propagation of an electron wave packet in an electric field, in the presence of a double barrier tunneling structure. We have developed a finite difference simulation of the time-dependent Schrödinger equation that gives the tunneling dynamics without recourse to a simplifying model,

$$i\hbar \frac{\partial}{\partial t}\Psi(x,t)=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right)\Psi(x,t). \tag{3}$$

The quantum-mechanical time-dependent Schrödinger equation gives the time evolution of a wave packet from which we can determine the tunneling time. We have calculated this parameter by discretization of Eq. (3) and solving the resulting equation by the finite-difference, time-domain method,

$$\begin{aligned} i\hbar \frac{\Psi(x,t+\Delta t)-\Psi(x,t)}{\Delta t} &= \frac{-\hbar^2}{2m}\left(\frac{\Psi(x,-\Delta x,t)-2\Psi(x,t)+\Psi(x+\Delta x,t)}{(\Delta x)^2}\right) \\ &\quad +V(x)\Psi(x,t). \end{aligned} \tag{4}$$

For $V(x)=0$, the Gaussian wave packet spreads and moves with a group velocity corresponding to the initial average momentum $\langle p_o\rangle=\sqrt{2mE}$.

The calculation proceeds by determining the wave-function intensity as a function of real space for each point in time. This solution method does not depend on simplifying assumptions regarding the nature of the incident wave packet, such as the distribution of momentum states. The results can be viewed as a movie of the transmission of the wave packet through the barrier.

In Fig. 2(a) we show the initial conditions of the simulation. A wave packet having spatial width of 40 Å is released with an initial velocity of 4.1×10^5 m/sec and is incident on a double barrier tunneling structure with a well that is 20 Å in width and 2 eV in height. Its energy width is less than 20% of the energy separation between resonant levels in the potential-well region between the two barriers. On the other hand, the energy width of the wave packet exceeds that of the resonant levels by about one order of magnitude. The incident wave packet has its peak initially at 500 Å and the left-hand edge of the double barrier structure is located at 580 Å. The resonant states of the barrier are diagrammed in Fig. 2(b). We can follow the time evolution of this wave packet for three different cases: (i) the wave packet is on resonance with the states in the barrier, (ii) the wave packet is off resonance with the states in the barrier, and (iii) the barrier is absent altogether. The FDTD simulation allows us observe the collision between the wave packet and the barrier. The ability to examine the evolution of the scattering event at various points in time is crucial to understanding the physics of the tunneling process.

III. RESULTS

In Fig. 3 we show the results of all three cases. The unscattered curve represents the propagation of the wave packet in the absence of any potential barrier. The ''on'' curve corresponds to the case in which the peak of the wave packet is centered in energy on a resonant level of the potential well. We refer to this as the on-resonance wave packet. The ''off'' curve corresponds to the case where the same wave packet, with the same initial energy, is incident on a RTD barrier so that the wave-packet peak energy lies halfway between two resonant levels. In our simulation the value of the resonance energy is selected by tuning the energy of the bottom of the well, while keeping all other parameters, such as the well width and the wave-packet properties, fixed. After an elapsed time of 5×10^{-14} sec, the peak of the unscattered wave packet has advanced to 710 Å, as expected given its initial velocity. Both the resonant and nonresonant wave packets experience strong reflection when incident on the barrier. In the case of on-resonance transmission, most of the wave-packet intensity has been reflected toward the left, because the packet contains substantial nonresonant components due to its width. There is a component that continues to resonate in the barrier, and there is a transmission corresponding to about 10% of the packet amplitude toward the right. Almost

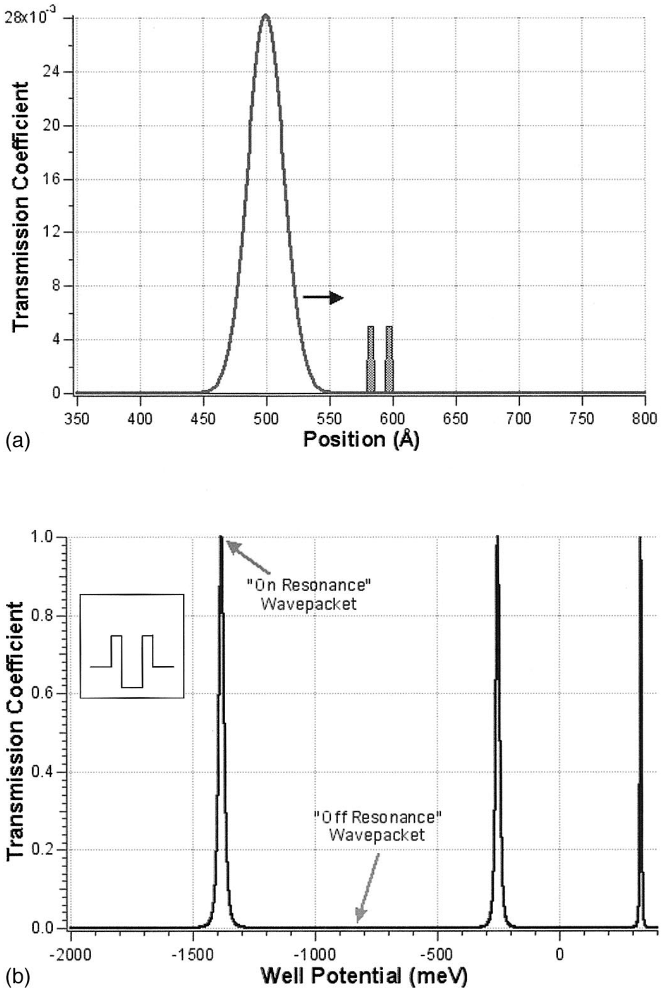

FIG. 2. (a) Diagram showing the initial position and spatial width of the pulse used to interrogate the resonant double barrier. (b) Resonant tunneling energies for the 2-eV quantum well. The horizontal axis gives the depth of the well relative to the zero of energy for free-space propagation. Thus for the on-resonance case, the bottom of the well lies 1480 meV below the zero energy level outside the well.

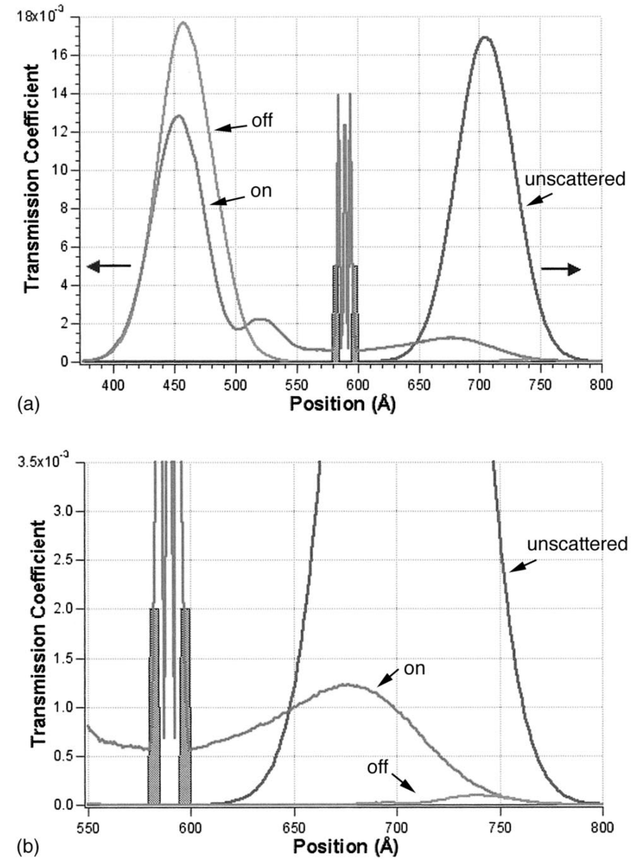

FIG. 3. (a) Transmission amplitudes after a propagation time of 5×10^{-14} sec. The unscattered curve shows the diffusion of the wave packet in the absence of any scattering. The off curve shows the propagation of the wave packet off-resonance, while the on wave packet is centered in energy on a resonant level in the tunneling barrier. The on-resonance packet shows transmission, reflection, and a substantial intensity in the well itself, due the resonance condition. (b) A blowup of the transmission plot shown in Fig. 3(b). Although it cannot be resolved in this view, the off-resonance wave packet is transmitted through the resonant tunneling barrier ahead of the on-resonance wave packet. Note that the peak of the off-resonance wave packet is found to lie ahead of the peak of the wave packet that encountered no barrier. However, the positions of these two peaks cannot be directly compared because the peaks do not have the same origin.

all of the off-resonance packet is reflected, but a small component, representing less than 1% of the incident intensity, has tunneled through the barrier. Figure 3(b) shows an expanded portion of these three events.

Looking at Fig. 3(b), we record the following conclusions: Although the occurrence rate is quite low, the peak of the off-resonance wave packet arrives before the peak of the on-resonance packet. The peak of the on-resonance packet arrives after that of the unscattered packet. These conclusions are in general similar to those drawn from the Wigner model shown in Fig. 1.

In an experiment, the situation is different. One does not measure the peak of the wave function, but rather the presence or absence of an electron in a given time period. To accomplish this measurement, the threshold for detection is set a certain level and observations are carried out. If the threshold is set so that all three events diagrammed in Fig. 3(b) can be detected: unscattered electrons, on-resonance tunneling electrons, and off-resonance tunneling electrons, then the conclusion would be that the unscattered electrons are detected in the shortest time. The off-resonance electrons arrive next and the on-resonance electrons arrive last. In this example there is no superluminal transport, even though the peak of the wave function for the unscattered electron lags in time compared to that for the transmitted part of the wave function corresponding to off-resonance tunneling.

We can use these data to calculate an effective group velocity of the wave packets in the on-resonance and off-resonance cases. In the on-resonance case, the peak of the transmitted wave packet travels 180 Å in 5×10^{-14} sec. The effective group velocity outside the well is given by the incident group velocity, and the effective velocity while crossing the well is 1.7×10^{5} m/sec, showing that the wave packet is retarded by the resonance, as expected. When we apply the same procedure to the peak of the off-resonance packet, we discover that the elapsed time in the well region is negative, suggesting that the off-resonance packet exits the well before the peak of the incident packet has entered it. Landauer and Martin[8] have concluded that this event demonstrates a lack of causality. However, we will show here that this interpretation is not correct, and we will thus remove the objection that these authors have given to the use of wave-packet transmission to the study of tunneling time. This type of event has been observed in the optical transmission experiments of Wang *et al.*,[3] and attributed to superluminal transmission of photons. Haché and Poirier have published quite recently microwave transmission experiments showing similar effects, claiming the observation of group velocities greater than $3c$.[16] As we will show presently, such estimates of time or velocity are naïve.

To examine this unusual result for the case of off-resonance tunneling, we used the FDTD simulation to track the peak of the off-resonance packet. We first determined its velocity following its exit from the barrier. To our initial surprise, the group velocity is not 4.1×10^5 m/sec, as assumed above, but 4.8×10^5 m/sec. This increased propagation velocity of the peak of the off-resonance packet is an important piece of the puzzle. The Schrödinger equation is diffusive in time. As a result, the leading edge of a wave packet moves faster than the peak, causing the packet to broaden in time. By tracking the off-resonance packet with the leading edge of the unscattered packet, we confirm that the leading edge of the unscattered packet moves with the same velocity as that of the peak of the transmitted off-resonance packet once it has exited the double barrier structure. Further analysis confirms that the peak of the off-resonance transmitted pulse is not related in any simple way to the peak of the incident pulse. Above all, it is not an attenuated replica of the incident packet. Comparing their positions is meaningless. This result shows that the tunneling barrier acts like a time dependent gate that allows only the leading edge of the incident packet to be transmitted. The apparent increase in peak velocity due to diffusion has its analog in light propagation through a dispersive medium.[8] If optical pulse broadening occurs with time, the leading edge of the pulse may appear to be moving faster than the speed of light in the dispersive medium. Of course, this effect for photons would disappear in a vacuum, which is nondispersive.

To complete the study of the off-resonance tunneling, we examined the impact of the incident packet on the barrier. At the first barrier more than 95% of the packet is reflected setting up strong interference. This interference is nearly total and is instrumental in diminishing the additional packet intensity that enters the double barrier structure. The transmitted packet enters the well, but encounters no interference during this transient phase because there is not yet any counter propagating part of the wave function. At the second barrier, the wave packet is transmitted with about 5% efficiency, and starts to exit the well, forming the transmitted packet. The remaining 95% reflected intensity at the second barrier creates a destructive interference inside the well. By the time the backward traveling wave front reaches the left-hand barrier, the interference is nearly complete, and the peak of transmitted packet has exited the well. The packet is formed in space and time by the transient associated with the time to build up a nearly complete destructive interference, which could be thought of approximately as the time needed to complete two round trips in the well. At the time that the transmitted pulse has exited the well on the right, the peak of the incident packet has not yet reached the left-hand barrier. However, it is clear that the transmitted packet travels through the double barrier structure in a manner that is wholly compatible with ideas of causality.

Japha and Kurizki have studied the propagation of photons in the evanescent regime in order to investigate two features of photon tunneling that we have also noted in the previous discussion in the case of electrons: the appearance of a transmitted peak in the tunneling wave function that is (i) narrower in width and (ii) advanced in time compared to the peak of the wave function for an unscattered photon. Their calculation highlights the role of multiple reflections or interferences of the photon wave function in the barrier region, and confirms that the peak in the wave function of the tunneling photon is the result of a transient in the buildup of the interference that represents the reflectivity of the tunneling barrier. Their study emphasizes that the multiple reflections are interfering components of the same single-particle wave function.

On-resonance tunneling is less complicated. The leading edge of the packet is transmitted through the double barrier with approximately the same time delay and intensity as that of the off-resonance packet. However, the interference is replaced by resonance which leads to a continuous increase in the transmitted packet intensity until after the peak of the incident packet is reflected from the left-hand barrier. The spatial width of the resulting wave packet is enlarged compared to the unscattered packet. This is the direct result of the filtering effect of the resonant barrier structure, which transmits efficiently only the resonant wave-vector components of the wave packet.

There are obvious analogies between the behavior of photons and that of electrons because of their wavelike properties. It is equally important to remember that experiments with photons also involve wave packets, limited in time and space. When the wave packet contains wave-vector components, the energy of which is resonant with the double barrier, the double barrier tunneling structure acts like a *Fourier* filter, transmitting principally the resonant wave-vector components, and by consequence, enlarging the spatial/temporal width of the transmitted packet. The energy width of the resonance will also determine the delay in the transmission of the packet. It follows that transport of a wave packet via tunneling on-resonance is slower than transport in the absence of a barrier. These are features that are all well known from the wavelike behavior of resonant cavities of any type.

Off-resonance tunneling is another matter. Our simulations show that the barrier acts like a *temporal* filter, transmitting the off-resonance wave packet, albeit highly attenuated, until sufficient interference can build up in the tunneling structure to cut off further transmission. The peak of the transmitted wave packet is directly related to a reshaping of the leading edge of the incident wave packet. This is seen by comparing the propagation velocity of these two features. As a result it is observed that the peak of the transmitted packet leaves the tunneling structure before the peak of the incident packet has entered. However, this transmitted peak is not an attenuated replica of the incident wave packet. Since these two peaks are not directly related to each other, this result is not a paradox.

In Fig. 4 we show a snapshot taken just as the off-resonance packet exits the barrier. Inside the well, the wave-function intensity, which is nonzero, creates a nearly completely reflecting boundary to the wave incident from the left. On the right-hand side, we have confirmed that the leading edge of the off-resonance packet (red) is slightly ahead of the leading edge of the on-resonance packet (green). However, it is not ahead of the leading edge of the position of the unscattered packet (blue). The implications of these results for experimental measurements are discussed below.

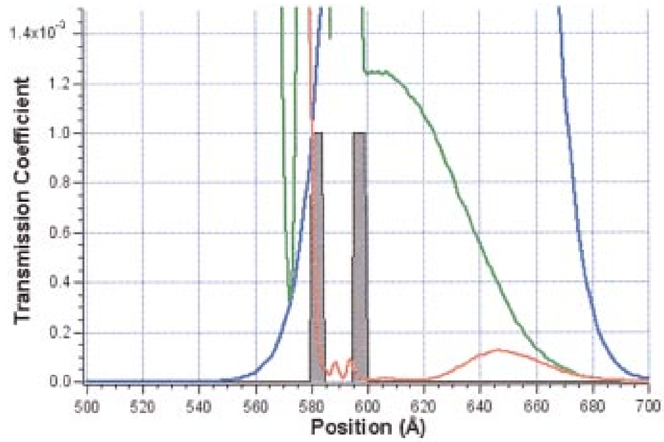

FIG. 4. (Color) A snapshot of the of the wave-function intensities shortly after the off-resonance wave packet (red) exits the tunneling structure. Note the presence of a small "echo" in the off-resonance transmission emphasizing that the transmission of this part of the wave packet is a transient effect. The leading edge of the off-resonance peak is slightly ahead of the on-resonance peak (green), but is lagging the leading edge of the unscattered freely propagating, wave packet (blue).

IV. DISCUSSION AND CONCLUSIONS

An electron is a quantum-mechanical entity. In an experiment one measures the presence or absence of an electron and not a wave-packet intensity. If we measure the transmission of a stream of electrons, these calculations tell us that for the off-resonance case we will detect the arrival of a quantum only occasionally. These quanta will arrive with a distribution of times. In an experiment, one sets the threshold of the electron detector. All of our results show that the threshold for detection of the unscattered wave packet is always reached before the threshold of the electron passing through the tunneling barrier. Thus our first important result is to show that an electron wave packet is retarded by passage through a tunneling barrier, both for on-resonance and above all for off-resonance packets. This result should be contrasted with the results of the phase-time model shown in Fig. 1 which predicts that off resonance tunneling through a barrier occurs in less time than an unimpeded passage. Figures 3 and 4 show that for any arrival time there is always a smaller probability of detecting a quantum passing through a tunneling structure compared to the case for propagation in the absence of tunneling. Thus our model shows that information is not transmitted by tunneling faster than in the absence of tunneling. Our results do not support the assertion by Landauer and Martin[8] that tunneling can act to speed up transmission.

These results show that the dynamics of propagation of an electron through a resonant tunneling structure will be very different depending on the relationship between the electron wave packet and the resonant energies of the barrier. For example, if the electron wave packet is only slightly off resonance then multiple peaks appear in the time-dependent transmitted wave packet due the combined participation of both resonant and nonresonant transmission channels. We illustrate such an event in Fig. 5, where we have chosen a larger well width in order to better resolve the transmitted wave-packet shape. The transmitted wave packet has a series of peaks. If a tunneling time were to be determined by the centroid of one of these peaks, then the tunneling time would be a multiple-valued function. This example shows that the dynamics of tunneling cannot be described meaningfully by a simple number designated as the tunneling time. As mentioned earlier, the interference that causes this temporal modulation of the electron wave function is a single-particle phenomenon, and not the result of the interaction of several electrons. In fact this effect is the temporal analog of the well-known two-slit interference experiment. Just as the passage of a single electron through a double slit produces a

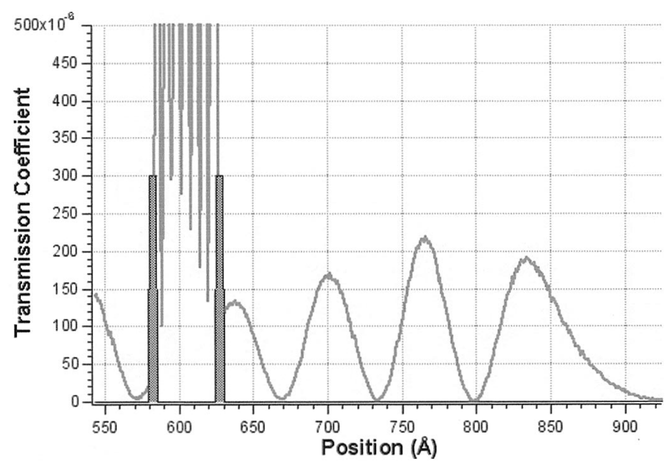

FIG. 5. A snapshot of the wave-function intensity of a off-resonance wave packet after 7.5×10^{-14} sec of propagation. The well in this figure is 40 Å wide in order to better resolve the series of multiple peaks in the transmitted wave function. The peak energy of the wave packet is about 0.15 eV above the resonant level in the quantum well. The presence of multiple peaks in the reshaped wave packet shows that a tunneling time consisting of a simple number cannot be meaningfully defined.

spatial modulation of the electron wave function, passage through a double barrier produces a modulation of the electron wave function in time.

The situation shown in Fig. 5 could be realized in real resonant tunneling circuits because of structural differences that occur in fabrication between individual tunneling barriers. In an experiment where a regular stream of electrons is incident on such tunneling barriers, measurement will detect a distribution of arrival times. Furthermore, the distribution is not a broadened Gaussian-type curve, but a series of peaks whose spacing is approximately related to the dwell time in the tunneling barrier. This ''dwell time'' depends on the momentum distribution of the packet in the well, which is changing with time, and is therefore also not constant. We identify the time jitter introduced by such a resonant tunneling diode as a different kind of *quantum telegraph noise* that has its origin in the quantum-mechanical dynamics of the tunneling process.

Our implementation of the FDTD method to model wave-packet propagation has allowed us to study the propagation of the wave-packet transmitted in off-resonant tunneling.[17] The transmitted peak is not an attenuated replica of the incident peak. Comparing their peak positions in time is meaningless. Under conditions of off-resonance propagation, the tunneling barrier acts like a temporal filter. It is a convincing demonstration of causality, as the opaqueness of the barrier is built up in time due to sequential reflection and interference in the quantum well. Only the leading edge of the wave packet is significantly transmitted because the interference is absent by causality during this initial time period. This is a feature of wave-packet propagation and will apply to the propagation of electrons or photons.

The relative importance of tunneling current increases exponentially with the decrease in device dimensions. For device structures such as RTD's or quantum dots, tunneling can be expected to dominate the transport behavior. In this study we have examined the dynamics of tunneling electrons. The results we have obtained show that the presence of a tunneling structure retards the electron wave packet in time. The amount of retardation is larger for on-resonance tunneling than for off-resonance tunneling. The differences between on-resonance and off-resonance tunneling will depend on the details of the tunneling barrier. These results, obtained by FDTD solution of the time-dependent Schrödinger equation, recall the results from using Eq. (1), with the significant difference that Eq. (1) predicts that off-resonance tunneling causes the electron to be transported through the tunneling region faster than would be the case in the absence of a tunneling barrier. None of the present work has shown this case to occur.

It may be asked if the situation would be changed for the case of transmission through a single tunneling barrier, instead of the double resonant barrier presented here. The interference at the barrier that causes the wave packet to be reflected will be present as in the case of the double barrier. As shown above, this interference takes time to build up. During this time, there will be transmission of the incident wave. A small peak in the transmitted wave-function intensity will appear at the output due to the reshaping of the leading edge of the wave function by the barrier, similar to the case of off-resonance tunneling described above. Thus tunneling through a single barrier does not represent a different case as far as the physics is concerned.

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[17] Our computer-generated tunneling ''movies'' are available at no charge. The interested reader should contact Steven Konsek at: skonsek@socrates.Berkeley.edu