Superlattice with hot electron injection: An approach to a Bloch oscillator

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A semiconductor superlattice with *hot electron injection into the miniband* is considered. The injection changes the stationary distribution function and results in a qualitative change of the frequency behavior of the differential conductivity. In the regime with Bloch oscillating electrons and *injection into the upper part of the miniband* the region of negative differential conductivity is shifted from low frequencies to higher frequencies. We find that the dc differential conductivity can be made positive and thus the domain instability can be suppressed. At the same time the *high-frequency* differential conductivity is negative *above* the Bloch frequency. This opens another way to make a Bloch oscillator operating at THz frequencies.

Miniband electron transport in semiconductor superlattices is an important subject of experimental and theoretical investigations beginning with the pioneering paper of Esaki and Tsu.[1] If a static electric field E_s is applied along the superlattice axes, electrons begin to move in accordance with the semiclassical Newton's law (neglecting scattering),

$$\frac{dp}{dt} = eE_s, \tag{1}$$

where p is the momentum along the superlattice axes. If the energy gap Δ_G between the first and the second miniband is large enough, $\Delta_G \gg edE_s$ (d is the superlattice period) and the scattering rate ν is small, $h\nu < edE_s$, then electrons oscillate inside the first miniband with the so-called Bloch frequency $\Omega = edE_s/\hbar$. The quasiparticle energy $\epsilon_p = [(\Delta/2) \times (1 - \cos(dp/\hbar)]$ and the quasiparticle group velocity along the superlattice axes $v_p = \partial\epsilon_p/\partial p = (\Delta d/2\hbar)\sin(dp/\hbar)$ are periodic functions of time (Δ is the miniband width). We assume that the condition of semiclassical approximation $\Delta \gg edE_s$ is fulfilled.

In semiconductor superlattices the Bloch frequency can be varied by the electric field up to 1–10 THz, so it is a great challenge to make a tunable Bloch oscillator in the THz frequency range. However, in most cases the phases of Bloch oscillating electrons are random and high-frequency emission is noiselike. The amplification of an external high-frequency signal is possible due to a phase bunching of electrons. In 1972 Ktitorov *et al.*[2] showed that the linear differential conductivity $\sigma_E(\omega)$ of a biased superlattice is negative up to the Bloch frequency [see Fig. 2(a), broken line]. Later Ignatov *et al.*[3] calculated the finite-field ac response of a biased superlattice and showed that the amplification efficiency, i.e., the ratio of power absorbed by a THz field and the power delivered from a bias source, is larger at larger ac fields. That is what one needs to make an oscillator work at $\omega \sim \Omega$. However, $\sigma_E(\omega)$ is negative also at $\omega \to 0$, and this dc negative differential conductivity (NDC) leads to a low-frequency instability of space-charge waves and to the formation of domains.[4,5] This effect can be used itself for a

generation of microwaves by moving domains with frequencies above 100 GHz,[6] but the efficiency of a THz radiation at Bloch frequencies cannot be large in the inhomogeneous regime. *For a Bloch oscillator to operate the domain instability should be suppressed.* It was shown by Romanov *et al.*[7] and by H. Kroemer[8] that in the *nonlinear* regime with high enough high-frequency current the dc differential conductivity is *positive*, while the large-signal high-frequency differential conductivity still remains negative, consequently, the steady-state operation of a Bloch oscillator in large-signal regime can be achieved. However, the question of "device turn-on" is not answered. Note also that the nonlinear regime can be unstable.[9]

We present here a different approach to this long-standing problem. The key idea is to inject electrons into the upper part of the first miniband (note the discussion of other interesting effects in a superlattice with hot electrons in Ref. 10). We calculate the nonequilibrium stationary distribution function, and both *linear* and *nonlinear* (large ac current) differential conductivities with the help of a one-dimensional kinetic equation and three-dimensional (3D) Monte Carlo simulations. We show that the low-frequency domain instability is suppressed and the region of negative differential conductivity is shifted from low frequencies to higher frequencies. Thus the self-excitation of high-frequency oscillations becomes possible. At a finite level of high-frequency current this effect is supported by dynamic domain suppression found by Romanov and Kroemer.

We restrict our analytical consideration to the one-dimensional problem, where the dynamics of the system can be described by a semiclassical Boltzmann equation for the distribution function $f(p,x,t)$. The distribution function determines the particle density,

$$n(x,t) = \int_{-\pi\hbar/d}^{\pi\hbar/d} f(x,p,t)\,dp, \tag{2}$$

and the current density,

$$j(x,t)=e\int_{-\pi\hbar/d}^{\pi\hbar/d} v_p f(x,p,t)dp. \qquad (3)$$

In a one-miniband approximation with a Bhatnagar-Gross-Krook (BGK) scattering integral, which conserves the number of particles and makes it possible to describe space-charge effects,[11] we have

$$\frac{\partial f}{\partial t}+v_p\frac{\partial f}{\partial x}+eE\frac{\partial f}{\partial p}=-\nu\left(f-\frac{n}{n_0}f_0\right)+S(p), \qquad (4)$$

where

$$f_0=\frac{n_0 d}{2\pi\hbar I_0(\Delta/2T)}\exp\left(\frac{\Delta}{2T}\cos\frac{dp}{\hbar}\right) \qquad (5)$$

is the equilibrium distribution function of a nondegenerate electron gas and n_0 is the equilibrium particle density. $S(p)$ is the source of hot particles. For simplicity (and to conserve the total charge) we choose a ''narrow'' source of the simplest form,

$$S(p)=Q\delta(p-p')-\frac{Q}{n_0}f_s(p), \qquad (6)$$

where $f_s(p)$ is the stationary (static and homogeneous) solution of Eq. (4). Q is the injection rate of hot electrons and p' is their momentum.

In the homogeneous case (constant electric field E_s) and $S(p)=0$ the solution of Eq. (4) is

$$f_E(\varphi)=\frac{n_0 d}{2\pi\hbar I_0\left(\dfrac{\Delta}{2T}\right)}\sum_l I_l\left(\frac{\Delta}{2T}\right)\frac{\nu}{\nu+il\Omega}e^{il\varphi}, \qquad (7)$$

where $\varphi=dp/\hbar$ is the dimensionless momentum and I_l is the modified Bessel function of lth order. Note, that due to the one-miniband approximation $f(x,\varphi,t)$ is a periodic function of φ with a period of 2π.

In presence of the hot electron source (6) the stationary homogeneous distribution function $f_s(\varphi)$ can be represented as $f_s=f_E+f'$ and $f'(\varphi)$ can be obtained from the equation

$$\Omega\frac{\partial f'}{\partial\varphi}=-\left(\nu+\frac{Q}{n_0}\right)f'+\frac{dQ}{\hbar}\delta(\varphi-\varphi')-\frac{Q}{n_0}f_E(\varphi). \qquad (8)$$

Thus we find

$$f'(\varphi)=\eta\frac{n_0 d}{2\pi\hbar}\sum_l\frac{\Omega e^{il\varphi}}{\nu+\eta\Omega+il\Omega}$$
$$\times\left[e^{-il\varphi'}-\frac{I_l(\Delta/2T)}{I_0(\Delta/2T)}\frac{\nu}{\nu+il\Omega}\right], \qquad (9)$$

where we have introduced the parameter of nonequilibrium

$$\eta=\frac{Q}{\Omega n_0}. \qquad (10)$$

Distribution functions $f_s(\varphi)$ at different injection rates are shown in Fig. 1(a) for a superlattice[5] with $\Delta=16$ meV,
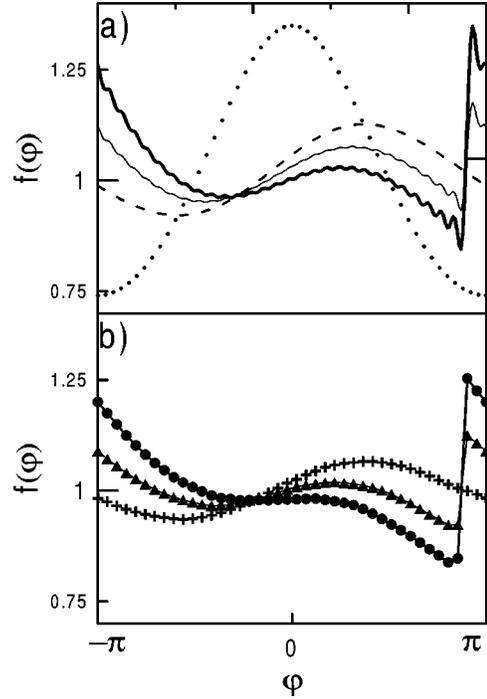
FIG. 1. Stationary distribution functions in the miniband: (a) analytical results for $f_0(\varphi)$ (dots), $f_E(\varphi)$ (broken line), $f_s(\varphi)$ at $\eta=0.036$ (thin line), and $\eta=0.072$ (thick line); (b) 3D Monte Carlo simulation without injection (crosses); at $\eta=0.036$ (triangles) and $\eta=0.072$ (circles); $\nu/\Omega=0.36$, $\Delta/2T=0.3$, $\varphi'=0.9\pi$.

$d=5.06$ nm, $\nu=0.5\times10^{13}$ sec^{-1}, $E_s=18$ kV/cm, $\Omega=1.38\times10^{13}$ sec^{-1}, at room temperature. We also calculated the distribution function by using a one-particle Monte Carlo method,[12] where the motion of an electron described by Eq. (1) is interrupted by three-dimensional scattering events (on acoustical and optical phonons), randomly distributed in time with the average frequency ν. The averaged frequency for each scattering process was calculated from quantum-mechanical perturbation theory taking into account the superlattice energy dependence and the material parameters of bulk GaAs. The injection is modeled in the same manner by the random change of the momentum to p' with the averaged frequency Q/n_0. The results of our Monte Carlo simulations are presented in Fig. 1(b). The comparison with the analytical results shows that in this parameter range the BGK scattering integral in the one-dimensional Boltzmann equation (4) adequately models the scattering in the 3D system.

The next step is to find the high-frequency conductivity in this nonequilibrium state. We consider perturbations with frequency ω and wave vector k of the form $E(x,t)=E_s+E_{\omega,k}e^{-i\omega t+ikx}$, $f(x,\varphi,t)=f_s(\varphi)+f_{\omega,k}(\varphi)e^{-i\omega t+ikx}$ and solve the linearized kinetic equation

$$\frac{\partial f_{\omega,k}}{\partial\varphi}+i(\alpha+\kappa\sin\varphi)f_{\omega,k}=-\frac{E_{\omega,k}}{E_s}\frac{\partial f_s}{\partial\varphi}+\frac{\nu n_{\omega,k}}{\Omega n_0}f_0, \qquad (11)$$

with

$$\alpha = -(\omega + i\nu)/\Omega, \quad \kappa = \frac{\Delta d}{2\hbar\Omega}k.$$

For the differential conductivity we find (following the method developed in Ref. 11)

$$\sigma(\omega,k) = \frac{j_{\omega,k}}{E_{\omega,k}} = \frac{2\pi\varepsilon_0 e^2 \Delta d}{\hbar\kappa} \sum_{m=-\infty}^{\infty} \frac{i^{m-1} m J_m(\kappa)}{\omega + i\nu - m\Omega}$$

$$\times \int_{-\pi}^{\pi} e^{-i\kappa\cos\varphi - im\varphi} \left[\frac{\partial f_s}{\partial\varphi} - i\frac{\nu\Omega\kappa}{\omega_p^2} f_0 \right] d\varphi,$$

$$(12)$$

where we introduced plasma frequency $\omega_p = \sqrt{e^2 \Delta n_0 d^2 / \hbar^2 \varepsilon' \varepsilon_0}$, ε' is the interlayer dielectric constant, ε_0 is the vacuum dielectric constant. This is the general expression for any nonequilibrium function $f_s(\varphi)$. With the injection source (6) and $f_s(\varphi)$ given by Eq. (9) we obtain

$$\sigma(\omega,\kappa) = \sigma_E(\omega,\kappa) + \sigma'(\omega,\kappa),$$

$$\sigma_E(\omega,\kappa) = -\frac{\varepsilon'\varepsilon_0\nu\Omega}{I_0(\Delta/2T)} \sum_{m,l=-\infty}^{\infty} \left(1 - \frac{l\omega_p^2}{\kappa\Omega(\nu+il\Omega)} \right)$$

$$\times \frac{m i^l I_l(\Delta/2T) J_m(\kappa) J_{m-l}(\kappa)}{\omega + i\nu - m\Omega},$$

$$(13)$$

$$\sigma'(\omega,\kappa) = \frac{\varepsilon'\varepsilon_0\Omega\,\eta\omega_p^2}{\kappa}$$

$$\times \sum_{m,l=-\infty}^{\infty} \frac{l m i^l J_m(\kappa) J_{m-l}(\kappa)}{(\nu+\eta\Omega+il\Omega)(\omega+i\nu-m\Omega)}$$

$$\times \left[e^{-il\varphi'} - \frac{I_l(\Delta/2T)}{I_0(\Delta/2T)} \frac{\nu}{\nu+il\Omega} \right],$$

$$(14)$$

$\sigma_E(\omega,\kappa)$ was obtained previously by Ignatov and Shashkin.[11]

Now let us consider the conductivity in long-wavelength limit $\sigma(\omega) = \sigma(\omega,\kappa\to0)$. Taking into account that $J_n(x) \to x^n/2^n n$ for $x\to0$ and $n>0$, we obtain

$$\sigma_E(\omega) = \sigma_0 \frac{I_1}{I_0} \frac{\nu^2(\Omega^2-\nu^2+i\nu\omega)}{(\nu^2+\Omega^2)[(\omega+i\nu)^2-\Omega^2]},$$

$$(15)$$

which was first obtained by Ktitorov et al., and

$$\sigma'(\omega) = \frac{\eta\sigma_0\nu\Omega}{(\tilde\nu^2+\Omega^2)[(\omega+i\nu)^2-\Omega^2]}$$

$$\times \left\{ (\Omega^2-\nu\tilde\nu+i\tilde\nu\omega)\cos\varphi' - \Omega(\omega+i\nu+i\tilde\nu)\sin\varphi' \right.$$

$$\left. - \frac{I_1}{I_0} \frac{\nu\Omega^2(\tilde\nu+\nu)+\nu(\tilde\nu\nu-\Omega^2)(i\omega-\nu)}{(\nu^2+\Omega^2)} \right\},$$

$$(16)$$

where $\sigma_0 = \varepsilon'\varepsilon_0\omega_p^2/\nu$, $\tilde\nu = \nu + \eta\Omega$, the arguments $\Delta/2T$ of Bessel functions are omitted.
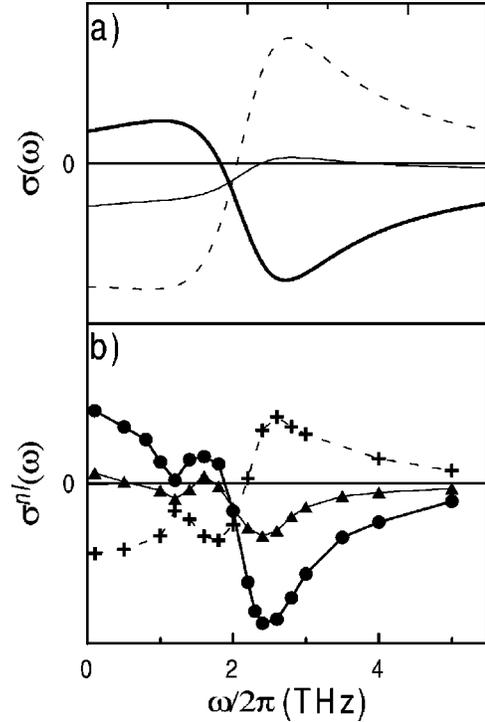
FIG. 2. Differential conductivities (a) analytical results: $\sigma_E(\omega)$ (broken line), $\sigma(\omega)$ at $\eta=0.036$ (thin line), and $\eta=0.072$ (thick line); (b) 3D Monte Carlo simulation: $\sigma_E^{nl}(\omega)$ (crosses), $\sigma^{nl}(\omega)$ at $\eta=0.036$ (triangles), and $\eta=0.072$ (circles); $\nu/\Omega=0.36$, $\Delta/2T=0.3$, $\varphi'=0.9\pi$.

The differential conductivity with hot electron injection is shown in Fig. 2(a) (solid lines) together with the conductivity without injection (broken line). At strong enough injection (thick solid line) the hot differential conductivity is significantly different from the cold one: now NDC takes place above the Bloch frequency, and in the entire low-frequency part it is positive. The results of the corresponding Monte Carlo simulations at large ac signal ($E_\omega=12$ kV/cm) are shown in Fig. 2(b). They confirm our main conclusion about the "inversion" of the differential conductivity in a nonequilibrium state and demonstrate that at high fields the effect can be even stronger.

Note that in the regime with Bloch oscillations ($\Omega>\nu$) the important parameter η is determined by the competition between the pumping frequency $\nu_t=Q/n_0$ and Bloch frequency Ω. The desirable effect takes place when η is larger then some critical value η_c. Large η can be achieved at low temperatures with a low equilibrium density n_0. On the other hand, at high (room) temperatures, when $T>\Delta$, η_c becomes smaller, because the equilibrium distribution function is flat in the miniband and therefore the effect of pumping is more pronounced. That gives hope to observe the effect at low as well as at room temperatures.

An important question is the choice of the injection momentum p'. Our calculations show that the effect takes place if particles are injected at high enough energies, $\epsilon_p>\Delta/2$. The injection with p' and $-p'$ leads qualitatively to the same result, but at positive p' the effect is stronger and exists in a larger frequency range. It can be easily seen from Eq.

(16) that $\mathrm{Re}\,\sigma'(\omega\to 0)\propto-\cos\varphi'$ [at $\Omega^2>\nu(\nu+\eta\Omega)$] and $\mathrm{Re}\,\sigma'(\omega\gg\Omega)\propto-\sin\varphi'$, such that the low-frequency instability is suppressed equally by p' and $-p'$ injection, but the high-frequency instability with $\omega\gg\Omega$ exists only at $\varphi'>0$.

In conclusion, we have shown that hot particle injection into the upper part of the miniband leads to a qualitative change of the frequency behavior of the differential conductivity. The region of NDC is shifted from low frequencies (below the Bloch frequency) to higher frequencies. Consequently, the domain instability is suppressed. To further support this result we considered the instability of space-charge waves with finite wave vectors, the spectrum $\omega(k)$ of which is given by the solution of the dispersion equation

$$\varepsilon(\omega,k)=\varepsilon'+i\frac{\sigma(\omega,k)}{\varepsilon_0\omega}=0. \qquad (17)$$

In the high-field limit ($\Omega\gg\nu,\omega_p$) we found that a low-frequency drift-diffusion mode with $\omega(k)\ll\Omega$, which is unstable without injection, now becomes damped. This means that the homogeneous state becomes stable, and a superlattice in that state can be used as high-frequency oscillator if we couple it to some external circuit (resonator or wave guide).

The narrow hot particle source (6) may be realized by tunneling from a metal or doped semiconductor,[13,14] or by tunneling from a narrow miniband superlattice. In the case of tunneling from the end of a superlattice "from layer to layer" our approach is valid for an active region with the length of the order of the Bloch length $l_B=\Delta/2eE_s\gg d$. The consideration of an inhomogeneous distribution function in a real superlattice with leads requires more sophisticated approaches,[15,16] which link Bloch oscillator and quantum cascade laser physics.

Qualitatively the same results are obtained for the optical excitation when $S(p)\propto[\delta(p-p')+\delta(p+p')]$. The consideration of optical pumping is the subject of a future publication. We think that the prospective way to the practical realization of the considered effect is the combination of injection (or optical excitation) of particles into the second miniband with relaxation from the second miniband into the upper part of the first active miniband.

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