Crossing lattices in thin films of isotropic superconductors

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There has been considerable recent interest in the vortex state in highly anisotropic high-temperature superconductors in which the orthogonal pancake chain and Josephson vortices interpenetrate to form a crossed lattice. This paper reports magnetization measurements on thin films of conventional superconductors in which the field is applied at a small angle to the film plane. The experimental results demonstrate that the equilibrium vortex lattice structure under this condition consists of a similar interpenetrating lattice of orthogonal Abrikosov vortices and the crossed lattice is calculated to be the lowest-energy configuration.

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The discovery of high-temperature superconductivity (HTS) stimulated a resurgence of interest in the mutual interactions of quantized flux vortices in superconductors. The extreme anisotropy of HTS materials has been shown to lead to a variety of exotic vortex structures, including crossing lattices of interpenetrating orthogonal chain and Josephson vortices.[1] Even in conventional isotropic materials the response of a flux lattice in which vortices are not parallel remains of great interest, particularly in configurations in which the potential exists for mutually inclined Abrikosov vortices to cut or cross-join.[2,3] The most obvious occurrence of this is the so called force-free configuration in which a transport current flows in the direction of the applied field.[3–5] Flux flow to generate the measured electric field requires a movement of vortices inclined at the angle of the surface field toward the center of the conductor; vortex cutting and cross-joining is required to prevent the continuous accumulation of longitudinal flux within the material. The analysis of this geometry is complicated by the direct coupling between the measured parameter (the critical current) and the inclination angle of the vortices which is set by the self-field of the current; the geometry also restricts vortex interaction angles to comparatively small values. This paper focuses instead on magnetization measurements on thin films, and shows that even in conventional, isotropic materials, when the applied field is inclined at a small angle to the film plane, the equilibrium configuration consists of a crossed-lattice of orthogonal Abrikosov vortices.

Polycrystalline Nb and amorphous $Mo_{22}Si_{78}$ thin films with thicknesses between 50 and 500 nm were deposited by dc magnetron sputtering on Si(001) substrates. Magnetization measurements were performed using a vibrating sample magnetometer (VSM) in which the vibration direction and the axes of the pickup coils and the superconducting magnet are parallel. The 10×2 mm^2 samples were mounted so that the magnetic field was applied in-plane and parallel to the long axis. The misalignment angle between the film plane at the applied field was less than 5°.

Figure 1 shows a series of magnetization versus applied field (M vs H) curves for a 200-nm-thick film at a variety of measurement temperatures. It can been seen that superimposed on the hysteretic response is a series of peaks which appear at *identical*, temperature-independent fields on both the increasing and decreasing portions of the magnetization curves. These peaks are similar in form those measured by Brongersma *et al.*[6] and Pan *et al.*[7] using torque and vibrating reed magnetometry respectively and are associated with commensurability re-arrangements of the *in-plane* vortex lattice.

In analyzing VSM data, it is conventional to use the Bean model to give a simple relationship between the hysteretic magnetization and the critical current density of the form

$$J_c=(M^+-M^-)/\beta d \tag{1}$$

where $M^\pm$ is the magnetization for increasing/decreasing field, d is the sample thickness, and β is a demagnetization parameter.[8] Thus perpendicular magnetization measurements

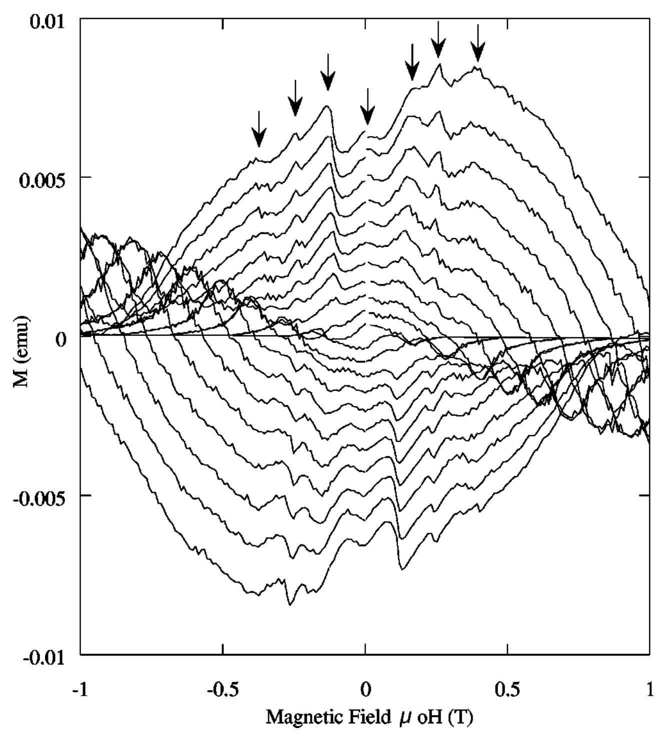

FIG. 1. Magnetization vs applied magnetic field for a 150-nm-thick Nb film. Measurements at 0.4 K intervals between 3.9 K (outer loop) and 8.1 K (inner loop).

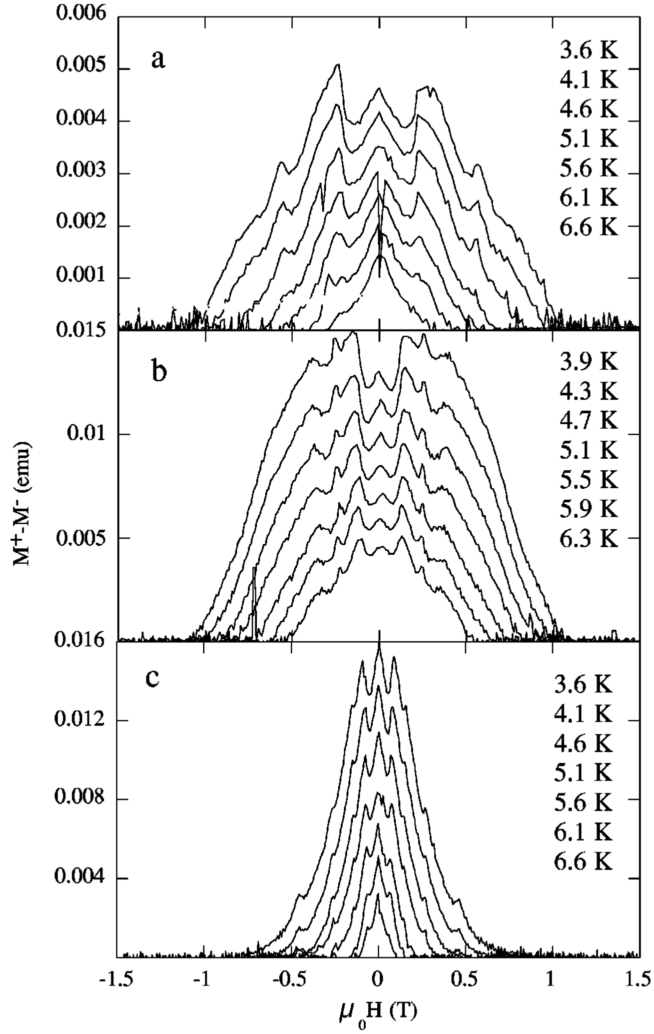

FIG. 2. Magnetization hysteresis vs applied magnetic field for Nb films: (a) 140 nm, (b) 220 nm, and (c) 290 nm. Measurements at temperatures indicated.

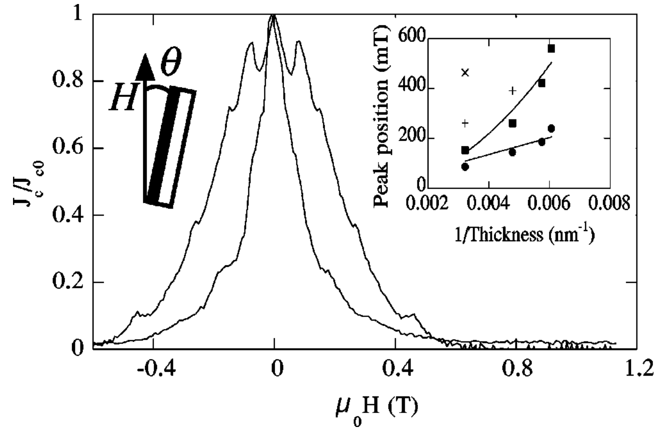

FIG. 3. Normalized critical current density at 4.1 K vs applied magnetic field for Nb films: nominally aligned (upper curve) and $\theta=15°$ offset (lower curve). Inset: peak positions vs inverse film thickness for a series of Nb films at $\theta=0$. The inset diagram shows the film orientation.

are routinely used for critical current density characterization of thin films. In the almost parallel geometry used in this work, the analysis is considerably more complex, but the measured signal remains dependent on J_c.

In Fig. 2 we plot M^+-M^- for three sample thicknesses; it can be seen that the fields corresponding to each peak reduce with increasing film thickness; they are not apparent in a 430-nm-thick film. Figure 3 shows the how the normalized J_c depends on field at two different sample angles: it can be seen that at more oblique angles (larger θ) J_c falls off more rapidly with applied field, but that the peak structure is preserved.

The field positions of the various peaks are plotted as a function of Nb film thickness (d) in the inset to Fig. 3. The solid lines are fits to the first and second peaks which are respectively linear and quadratic with inverse field. The linearity of the first peak is consistent with this being the lower critical field or the superheating field of Matawari and Yamafuji.[9] Higher-order peaks in systems which are limited by surface barriers to flux entry are expected to remain linear in inverse thickness, but purely geometrical matching effects will follow a d^{-2} dependence.[5]

The peaks or steps are therefore associated with the packing of longitudinal (B_x) vortices within the plane of the film and their redistribution into successively more rows as the longitudinal field (H_x) is increased. However, the accurate coincidence of the peaks during increasing and decreasing field, shows that B_x is reversible. Han *et al.*[10] used polarized neutron reflectometry (which is sensitive only to B_x) to show directly that longitudinal flux enters and exits Nb films reversibly. During VSM measurements of thin films the reversibility of B_x is assured by the penetration of the effective ac field caused by vibration within the imperfectly homogeneous field of practical magnets.[11]

In Fig. 1 it can be seen that the maxima in the positive moment branches of the data align with minima in the negative branch; the peaks are therefore not due simply to stepwise changes in B_x with applied field. The increase in M^+-M^- which occurs at the nucleation of an additional row of longitudinal vortices must instead be due to a modulation of the out-of-plane moment M_z, a component of which is measured because of sample misalignment and the finite acceptance angle of the pick-up coils. Again this was confirmed by Han *et al.*[10] who, while not seeing the peak structure, demonstrated that a superconducting quantum interference device magnetometer records an irreversible B_z in the geometry in which B_x is reversible.

Changes in M_z therefore occur at fields which correspond to structural rearrangements of the in-plane flux lattice. This confirms the suggestion of Pan *et al.*[7] that in this geometry out-of-plane vortices must coexist with in-plane vortices. However, the important observation is that since B_z is increased and decreased during the measurement of a hysteresis loop, this coexistence must persist under conditions of changing magnetic field which necessitate the mutual cutting of orthogonal Abrikosov vortices.[12] Therefore, in the process of establishing an equilibrium flux lattice, vortex cutting will enable the Abrikosov vortices to form an interpenetrating

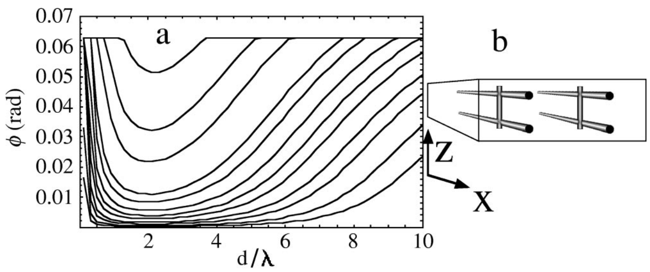

FIG. 4. Inclination angle vs film thickness for the model described in the text and a field angle of 0.06 rad. The curves represent decreasing values of the vortex line-tension (top to bottom).

crossing lattice [see Fig. 4(b)]. The experiments show that step increases in J_c vs B with increasing field correspond to the nucleation of additional longitudinal vortex rows; thus the pinning force on a perpendicular vortex is dependent on the number of longitudinal vortex rows. This indicates that there is a significant interaction energy between orthogonal vortices. In the remainder of this paper we show that the minimum-energy configuration for orthogonal vortices is achieved through the formation of an interpenetrating crossing lattice and that the pinning force on a perpendicular vortex which threads the longitudinal rows is proportional to the number of these rows.

Our experimental results imply that orthogonal vortices exist within the films and that the cutting and cross-joining of these vortices does not create any other orientation. However, the various models of the cutting process which have been proposed[2,13,14] all predict the decomposition of two cutting vortices into two parallel vortices which bisect the mutual angle of incidence. Generally, this process is energetically favorable since the total vortex length in the superconductor is reduced by the process. For our crossed lattice model to be physically reasonable we have to show that in thin films this process is modified by the anisotropy imposed by the film geometry.

To start our analysis we consider the stability of vortices whose angle with respect to the film plane (ϕ) is not too small. This is a problem treated by Brandt,[15] who found that for a field inclination θ which is not too close to zero (see the inset diagram to Fig. 3 for the definition of θ),

$$\cos\phi \approx \frac{4d^2}{a_0^2 \pi \ln(d/\pi\xi)} \cot\theta. \tag{2}$$

for films in which the thickness d is comparable to the London penetration depth λ (in our Nb films of the order of 100 nm). Thus at low fields vortices will penetrate the superconductor at angles ϕ close to the film normal (a_0 is the vortex spacing and ξ is the coherence length). With this vortex arrangement the longitudinal flux density within the superconductor is very small (i.e., in general $B_x \approx 0$); consequently for small θ the analysis becomes invalid because of the large demagnetizing field induced.

To model the low-θ configuration we assume straight vortices initially aligned such that $\phi=\theta$. We allow the angle of a vortex to vary between θ and zero, with additional sections of vortex which are normal to the film plane added to maintain constant exit and entry points for the vortex; in this way the external field remains unperturbed. We use a simple functional form to represent the vortex interaction energy per unit length E_0 with the film surface $E_0(z)=\exp(-d/\lambda)\cosh(z/\lambda)$, where z is the distance from the center of the film. Therefore the total energy of a vortex can be expressed as

$$E = kd(1+\cot\theta - \tan\phi) + \int_{-d\cot(\theta)/2}^{d\cot(\theta)/2} E_0(x\tan\phi)\,dx, \tag{3}$$

where k is the vortex line tension energy (largely the condensation energy associated with the core). We minimize this energy with respect to ϕ to determine the equilibrium angle of the vortex lattice. Figure 4 shows a plot of the equilibrium angle ϕ as a function of d and k for an experimentally reasonable value of θ. The plot demonstrates that for large film thicknesses $\phi \approx \theta$, but for thinner films there is a substantial field range over which ϕ is close to zero. The range of d over which this is true is increased by decreasing k.

Combining the two analyses confirms the experimental result that vortices are energetically stable when they are either parallel or perpendicular to the film plane. The presence of both vortex directions leads to the orthogonal lattice which minimizes both demagnetizing effects and vortex-surface interactions. As described above, changing the external field will cause the lattices to displace with respect to one another; however since the decomposition of a crossed orthogonal vortex pair into cross-joined vortices which bisect the incidence angle is unfavorable compared to separation into two orthogonal vortices it follows that a crossed-lattice structure persists even when the field is swept.

It now remains to determine the energy of orthogonal vortices and to show that a crossed lattice of intersecting vortices is actually the minimum energy state. Brandt *et al.*[14] calculate the interaction force between two straight vortices separated by a distance s and inclined at a mutual angle alpha to be

$$f_p = A\left(\frac{\phi_0^2}{2\lambda^2 \sin\alpha}\right)\left(\frac{s}{S}\right)\{(\cos\alpha)e^{-S/\lambda} - e^{-S\sqrt{2}/\xi}\} \tag{4}$$

where $S \approx (s^2+\xi^2)^{0.5}$ and A is close to unity for $\lambda/\xi \geq 2$; ϕ_0 is the flux quantum. As α tends to 90° the repulsive magnetic interactions tend to zero, leaving an attractive condensation energy interaction term in which the two vortices effectively act as mutual core pinning sites. In their analysis, the core interaction is approximated rather severely; however, to within a factor of 2 this force can be obtained by dividing the energy difference between the crossed and separated vortices (the condensation energy within a sphere of radius ξ,) by the interaction length (of order ξ). The critical current density due to the pinning by N_x longitudinal vortex rows is therefore given by

$$J_c \approx \frac{N_x \mu_0 H_c^2 \xi^2}{\phi_0 d}, \tag{5}$$

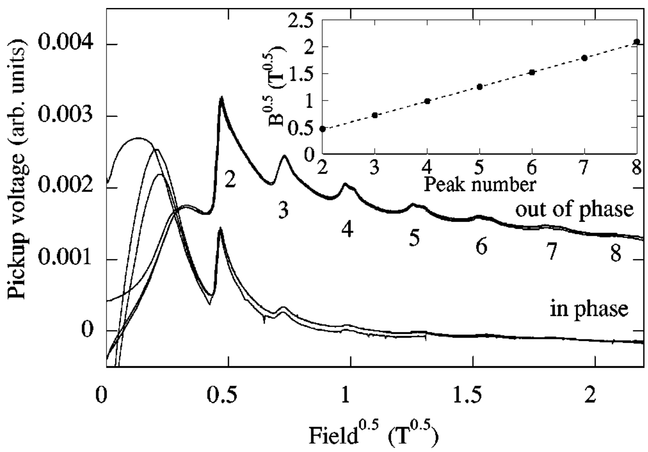

FIG. 5. In- and out-of phase VSM signals for a 140-nm MoSi thin film measured at 2 K. The inset shows the peak position plotted vs the square-root of the applied field; the dashed line is a least-squares linear fit to the data.

where H_c is the critical field. Typical values of H_c and ξ for thin-film Nb are 10^5 A m^{-1} and 35 nm respectively, which implies a value for the J_c increase at each longitudinal vortex rearrangement of the order of 10^{10} A m^{-2}. This estimate neglects a number of factors such as the true shape of the effective pinning potential and other vortex-vortex interactions; however, even if one allows these factors to reduce the effective pinning strength of the crossed structure, the value is easily sufficient to account for the peak structure observed in the experiments.

The peak structure associated with a crossed-lattice should become more pronounced as the extrinsic pinning of the material is reduced; under these circumstances the energy minima associated with the crossed vortex structure are better defined and the background J_c is lower. We therefore repeated the study using *a*-MoSi thin films in which $J_{c\perp}$ is several orders of magnitude lower than for polycrystalline Nb. Figure 5 shows the measured VSM pickup signal from a 140-nm *a*-MoSi film. A consequence of the lower J_c is that B_z becomes reversible at low field, and consequently there is no hysteresis in the measured moment.[11] However, the signal amplitude, particularly the out of phase loss signal, remains dependent on J_c.[11] In this data, the regularity of the peak spacing with $H^{0.5}$ is clearly evident up to high fields, partly as a consequence of the large λ in *a*-MoSi. As predicted, the relative peak height is significantly greater than for the Nb data.

In conclusion, in a superconducting film, whose plane is close to parallel to the applied magnetic field, the in-plane vortex density is proportional to the in-plane component of the applied field, with the out of plane flux forming a critical state characteristic of the perpendicular field component. There is a clear interaction between these vortex systems which is reflected in the modulation of J_c by H_x. We have shown that the vortex system in these films consists of two interpenetrating orthogonal vortex lattices whose minimum-energy configuration is a crossed lattice. Thin films of low-pinning conventional superconductors therefore provide an ideal model system for the study of crossed-lattices, and similar vortex structures are likely to be observed in thin films of the less anisotropic HTS materials such as $YBa_2Cu_3O_{7-\delta}$.

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