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Polarizability, correlation energy, and dielectric liquid phase of Bose-Einstein condensate of two-dimensional excitons in a strong perpendicular magnetic field

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The coherent pairing of electrons and holes occupying only the lowest Landau level in a two-dimensional (2D) system with a strong perpendicular magnetic field is studied using the Keldysh-Kozlov-Kopaev method and generalized random phase approximation. Bose-Einstein condensation of the correlated pairs takes place in a single particle state with an arbitrary wave vector **k** in a symmetric 2D model. We show that the ground state energy per exciton and the chemical potential are nonmonotonic functions of the filling factor, so that metastable dielectric liquid states with positive compressibility exist, consisting of Bose-Einstein condensate of magnetoexcitons. It is shown that this dielectric liquid phase of the Bose condensed excitons is more stable than the metallic electron-hole liquid phase. The polarizability of the Bose-condensed magnetoexcitons is calculated using Anderson-type wave functions of the coherent excited states, which correspond to the appearance of one out-of-condensate electron-hole (e-h) pair in the presence of the BCS-type ground state. The polarizability is characterized by a coherent factor which depends on **k** and vanishes when **k** tends to zero, as well as by a resonance frequency equal to the ionization potential of a magnetoexciton, and differs considerably from the polarizability of a noncondensed exciton gas. The condensate polarizability is used to determine the correlation energy of the system and the correction to the chemical potential beyond the Hartree-Fock-Bogoliubov approximation.

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I. INTRODUCTION

The observation of Bose-Einstein condensation (BEC) in alkali atomic gases using a laser trapping[1–3] has greatly expanded the related research in recent years. The transition temperature for BEC of alkali gases is extremely low, due to the heavy atomic mass and low gas density ($T_c \sim n^{2/3}/m$). The situation is quite different for a condensed phase of excitons in semiconductors due to much smaller mass of the excitons, which is often even smaller than the free electron mass. As is well known, under certain conditions excitons, i.e. bound states of electron-hole pairs in semiconductors have bosonic properties (see, e.g., Ref. 4). As neutral particles, excitons are weakly interacting, and therefore dephasing process are much slower than free electrons and holes. If the density is high enough, the Bose-Einstein statistics become important, and ideally one expects a Bose-Einstein condensation to occur, similar to the BEC observed in cold trapped atomic gases. Due to the macroscopic population of a single state which occurs in BEC, a robust macroscopic coherent quantum mechanical wave results, which should be directly accessible to experiments. Although theoretically recognized many years ago (for a review, see Ref. 4), experiments on BEC of excitons have made slow progress, because finite lifetime effects, strong interactions between excitons at high density, crystal imperfections and phonons in the crystal all act to complicate the system.

An essential requirement for BEC of excitons is supression of the electron-hole liquid (EHL) phase. In general, the Coulomb interaction between electrons and holes usually causes droplets of EHL to form at high carrier density instead of a Bose condensate. To avoid this, repulsive interactions between the excitons are necessary. This can occur in a bulk, three-dimensional (3D) semiconductor if the electron and hole masses are nearly equal, as is the case in the semiconductor Cu_2O (Refs. 5,6) (see also Ref. 4 for a review of experiments). Another way to have repulsive interactions is to engineer a quantum well structure with the right properties. In recent years, the system of coupled quantum wells in strong electric field has gained attention in both theoretical[7–11] and experimental[12–20] studies as a system with repulsive exciton-exciton interactions and long exciton lifetime, ideal for BEC of excitons. Another advantage of the 2D system is a possibility of much faster cooling of hot photoexcited excitons compared with their bulk counterparts.[21,22]

Another approach is to use strong magnetic field. It has been shown[23] that the properties of atoms and excitons are dramatically changed in a strong magnetic field such that the distance between Landau levels $e\hbar H/m_e c$ exceeds the Rydberg energy. In this case the binding energy of an exciton becomes independent of the masses of its constituent electron and hole, so that the exciton has universal properties and is known as a ''diamagnetic exciton'' in 3D samples and a ''magnetoexciton'' in 2D structures. The diamagnetic excitons in bulk crystals were revealed in (Refs. 24,25). In this case the diamagnetic exciton gas may form a Bose-Einstein condensate at low temperature due to the substantial decrease of the exciton-exciton interaction and increase of the exciton binding energy.[26–29] Even more attractive and worth investigating is the electron-hole (e-h) system in two dimensions (2D) in the presence of a strong perpendicular magnetic field. In this case, the energy spectrum of the e-h system is completely discrete, and one may expect that the properties of the system differ considerably from those of a 3D system in a strong magnetic field, where the electron and hole kinetic energies are functions of the momentum in one dimension. Unlike the 3D system, the kinetic energy in the 2D system is completely transformed into a discrete set of states by the magnetic field. The energy of electrons and holes in the 2D system in a strong magnetic field is simply characterized by the number of the Landau levels, which are N-fold degenerate, with $N=S/2\pi l^2$, where l is the magnetic length, $l^2=\hbar c/eH$, and S is the 2D sample dimension.

In the past two decades, a number of experimental[12–20] and theoretical[30–39] efforts have been dedicated to the study of 2D systems in a strong magnetic field. The problem of metallic electron-hole liquid formation using finite-temperature Green's function and diagram techniques, was studied[32,33] assuming that the electrons and holes are on the lowest Landau level (LLL) and taking into account the influence of the single-particle-excited states; the direct Coulomb interactions in the system are mutually compensated due to the electroneutrality condition. It was shown that the ground state of the e-h system can be found asymptotically exactly within the Hartree-Fock approximation for an infinitely high magnetic field H. In this case the exchange energy increases with the magnetic field strength as $\sqrt{H}$, whereas the correlation energy contains a small supplementary factor $l/a_{\mathrm{ex}}\sim 1/\sqrt{H}$, where a_{ex} is the exciton Bohr radius. In Refs. 34,35 the coherent pairing of electrons and holes resulting in the formation of the Bose-Einstein condensate of excitons in a single-particle state with wave vector $k=0$ was investigated. In the Hartree-Fock approximation, when the coupling to the higher Landau levels and the correlation energy are neglected, the magnetoexcitons with $k=0$ represent at $T=0$ an ideal excitonic gas, which has the same properties in the longwave approximation as an ideal 2D Bose gas. A surprising result was that the fermionic e-h droplets of metallic EHL, which corresponds to maximal local filling factor $\nu=1$ on the lowest Landau level (LLL), can be considered as an aggregate of excitons sticked together. Returning to the excited Landau levels and taking into account the corrections of the first order in l/a_{ex}, the authors[34,35] realized that the correlation energy is the same for both EHL and for excitonic phase. It is determined by the single-ring polarization operator and only the transitions from the LLL to the excited Landau level (ELL) give considerable contributions. The contribution of the all excited Landau levels is about four times greater than that of the only first Landau level. It is worth mentioning that the correlation energy calculated in this way does not contain any traces confirming existence of the Bose-Einstein condensate and is exclusively due to ELL. It is not surprising that it happened to be the same for both phases. Although the correlation energy is the same for both phases, it nevertheless makes the thermodynamic characteristics of the excitonic phase and of the EHL droplets slightly different. The inclusion of quantum transitions leads to weak nonideality; the low-energy spectrum becomes acoustic and the low-temperature behavior changes qualitatively.

Strictly speaking, it is known[40,41] that the BEC of an ideal Bose gas with a quadratic dispersion law in a 2D structure with infinite surface area S is possible only at $T=0$. Coupling to higher Landau levels, however, makes the system weakly nonideal,[34,35] which allows the Berezinskii-Kosterlitz-Thouless topological phase transition[42–44] at finite temperature.

The results of Refs. 34,35 were reproduced in Ref. 36 on the basis of a more simple and transparent approach using BCS-type wave functions of the BEC excitons and calculating the ground state energy as the average of the Hamiltonian of the Coulomb-interacting electrons and holes. The authors[36] used the Hartree-Fock-Bogoliubov approximation taking into account only the ground state wave functions, but considered the case of nonzero wave vectors and the possibility of coupling with higher Landau levels. They introduced the indirect interactions of the particles on the LLL due their virtual excitation to the ELL. The indirect interaction leads to corrections to the exciton binding energy and to the energy per one e-h pair in the BCS ground state, which are equivalent to the correlation effect discussed in Refs. 34,35. They also considered an asymmetric model, which takes into account the possibility that the electron and hole wave functions did not occupy the same region of space in a quantum well with finite thickness and finite barrier height. Interestingly, in this case, the Coulomb interaction of electrons and holes makes BEC of magnetoexcitons with $k=0$ unstable, but the possibility of the virtual transitions to the excited Landau level stabilizes the BEC (see also Ref. 45).

One of the aims of the present paper will be first to obtain the results of Refs. 33–36 from more simple arguments. The second aim is to investigate the properties of the system beyond the Hartree-Fock-Bogoliubov approximation, taking into account the possibility of coherent excited states, the corresponding polarizability of the Bose-Einstein condensate, the screening effects and the correlation energy due to just this channel of polarizability. Contrary to Refs. 34,35 our correlation energy is not related to excited Landau levels. We find that instead of coexistence of an excitonic condensate and electron-hole liquid, there is a single metastable dielectric phase formed by a Bose-Einstein condensate of magnetoexcitons with $k\neq 0$.

In the present paper we will consider only the case of a symmetric 2D model with arbitrary wave vectors of magnetoexcitons $k\neq 0$, which means that the excitons have nonzero dipole moments. An exciton with nonzero momentum must have a dipole moment because the electron and the hole are pulled in opposite directions by the magnetic field when they move together. As we will see, this effect leads to a number of interesting properties. It will turn out that the binding energy of the excitons decreases with increasing k, so that the magnetoexcitons have an energy term which increases as k^2, even though their kinetic energy is quantized in the Landau level spectrum.

Allowing a Bose-Einstein condensate of magnetoexcitons in a single particle state with $k \neq 0$, and therefore having a dipole moment, leads to novel and interesting properties of the system. We calculate the polarizability of the Bose-Einstein condensate, the screening effects, the correlation energy and the chemical potential of the metastable dielectric liquid phase in the range of filling factor far from the maximal value $\nu^2 = 1$.

The paper is organized as follows. In Sec. II the wave functions of electrons, holes, and magnetoexcitons and the Hamiltonian of the interacting quasiparticles are presented in a simple form. In Sec. III we use the Keldysh-Kozlov-Kopaev method to transform the initial Hamiltonian, introducing the Bose-Einstein condensate of magnetoexcitons. The chemical potential, the single particle energy spectrum and the ground state energy are calculated in the Hartree-Fock-Bogoliubov approximation. In this approximation the existence of the dipole moments results in an attractive interaction between excitons. In Sec. IV we introduce the coherent excited states and calculate the polarizability of the Bose-Einstein condensate. This polarizability is characterized by the coherence factor and resonance frequency, and we show the difference between polarizability of noncondensed and condensed magnetoexcitons. The screening effects, the correlation energy and the corrections to chemical potential are studied in Sec. V, using the generalized random phase approximation. We show that these corrections lead to the formation of metastable states of a Bose-Einstein condensed dielectric liquid phase with positive compressibility. We also calculate the energy per e-h pair of the EHL and compare it with that of this new dielectric liquid phase. Finally, the obtained results are summarized and discussed in Sec. VI.

II. HAMILTONIAN OF 2D ELECTRON-HOLE SYSTEM IN A STRONG MAGNETIC FIELD

We consider first a simple 2D model with a perpendicular magnetic field, assuming that the Zeeman splitting of the Landau levels is large enough that the electrons and holes are restricted to the lowest Landau level and the typical electrostatic energies are much smaller than the cyclotron energies. We choose the 2D layer plane as the (x,y) plane with the z axis directed along the direction of the magnetic field, with the Landau gauge for the vector potential $\mathbf{A} = (-Hy, 0, 0)$. In this case, the electron and exciton wave functions in a magnetic field are well known (see, e.g., Refs. 45–48). The states of an electron (hole) are characterized by two quantum numbers n and p_x (q_x), corresponding to its two degrees of freedom. For the lowest Landau level, $n=0$, which is considered below, the envelope Bloch wave functions of the electron and the hole are

$$\Psi^{e}_{n=0,p_x}(x,y) = \frac{1}{\sqrt{L_x l \sqrt{\pi}}} \exp(ip_x x) \exp[-(y - p_x l^2)^2/2l^2],$$

$$\Psi^{h}_{n=0,q_x}(x,y) = \frac{1}{\sqrt{L_x l \sqrt{\pi}}} \exp(iq_x x) \exp[-(y + q_x l^2)^2/2l^2], \tag{1}$$

where l is the magnetic length and $S = L_x L_y$ is the surface area of the layer, p_x and q_x are wave vectors of electron and hole, respectively.

In the solution given by Eq. (1), the electron moves in the x direction with translational symmetry and are confined by the oscillator potential $H^2 y^2$ caused by the magnetic potential in the y direction. This results in Landau quantization with cyclotron frequency $\omega^e_c = eH/m_e c$, where m_e is the mass of the conduction electron. The motion of the electron in the x direction with the wave vector p_x corresponds to a shift of the minimum of the oscillator potential from the point $y=0$ to the point $y_0 = p_x l^2$, due to the Lorentz force.

Since the values of the wave vectors p_x and q_x are restricted by the size of the layer surface in the y direction $-L_y/2 < p_x l^2, q_x l^2 \leq L_y/2$, the total number of possible states is

$$N = \frac{L_x L_y}{2\pi l^2} = \frac{S}{2\pi l^2}, \tag{2}$$

and the lowest Landau levels of the electrons and holes are N-fold degenerate. We define the filling factor $v^2 = N_e/N$, which is the ratio of the number of occupied electron states N_e to the total number N of electron states. The condition that the gaps between the Landau levels for electrons and holes are larger than the binding energy of the 2D exciton in the absence of the magnetic field, and that the magnetic length l is smaller than the Bohr radius of 2D exciton, leads to the following inequalities:

$$\hbar\omega^e_c \approx \hbar\omega^h_c > \frac{2\mu e^4}{\hbar^2 \varepsilon_0^2},$$

$$l < a^{2D}_{\rm ex} = \frac{\hbar^2 \varepsilon_0}{2\mu e^2}, \tag{3}$$

where $\mu = m_e m_h / m_{\rm ex}$ is the exciton reduced mass, $m_{\rm ex} = m_e + m_h$ is the translational exciton mass, and ε_0 is zero-frequency dielectric constant of the layer. The magnetic field which corresponds to the binding energy of the exciton equal to the cyclotron energy is

$$H_{\rm cr} = \frac{4ce^3\mu^2}{\hbar^3 \varepsilon_0^2}. \tag{4}$$

Typical values of $H_{\rm cr}$, e.g., for GaAs, are $\mu = 0.1 m_e$, $\varepsilon_0 = 11$, $l = a^{2D}_{\rm ex} = 100$ Å, and $H_{\rm cr} = 6.25$ T.

The condition that only the lowest Landau levels are occupied means that the first excited Landau level is situated much higher on the energy scale, at least of order of the ionization potential of the exciton in the 2D structure. Since, as we shall see, the ionization potential of the magnetoexcitons with large wave vectors is typically smaller than that with $k=0$, this condition is always satisfied for $H>H_{\rm cr}$.

We consider the wave function of magnetoexciton in the spatial coordinate representation, following Ref. 36. In a strong magnetic field, the 2D exciton wave function in the lowest Landau level $(n=0)$ can be written using the envelope functions (1)

$$\begin{aligned}\Psi_{\rm ex}(x_e,y_e;x_h,y_h)&\\ &=\sum_{p,q}\sum_{n_e,n_h}C_{n_e,n_h}(p,q)\frac{e^{ipx_e}}{\sqrt{L}}\frac{e^{iqx_h}}{\sqrt{L}}\phi_{n_e,p}(y_e)\phi_{n_h,q}(y_h)\\ &=\sum_{p}\sum_{q}C_{0,0}(p,q)\frac{e^{ipx_e}}{\sqrt{L}}\frac{e^{iqx_h}}{\sqrt{L}}\frac{1}{l\sqrt{\pi}}\\ &\quad\times\exp[-(y-pl^2)^2/2l^2]\\ &\quad\times\exp[-(y+ql^2)^2/2l^2].\end{aligned}\tag{5}$$

We consider the particular case $m_e=m_h=m_{\rm ex}/2$, and introduce the coordinates of the center of mass of the electron and hole and their relative motion with the corresponding wave vector k_x of the exciton translation motion, and wave vector t of the relative motion

$$\begin{gathered}\xi=\frac{1}{2}(x_e+x_h),\quad \eta=\frac{1}{2}(y_e+y_h),\\ x=x_e-x_h,\quad y=y_e-y_h,\\ k_x=p+q,\quad t=\frac{1}{2}(p-q).\end{gathered}\tag{6}$$

Then the coefficient in Eq. (5) takes the form

$$\begin{gathered}C_{0,0}(p,q)=\frac{1}{\sqrt{L}}\delta_{p+q,k_x}\varphi_{k_y}(t);\\ \varphi_{k_y}(t)=\frac{\sqrt{2\pi l^2}}{\sqrt{L}}\exp(ik_ytl^2).\end{gathered}\tag{7}$$

After summation over p and q, the magnetoexciton wave function in the coordinate representation takes the form

$$\begin{aligned}\Psi_{\rm ex}(\xi,\eta,x,y)=&\frac{e^{ik_x\xi}}{\sqrt{L}}\frac{e^{ik_y\eta}}{\sqrt{L}}e^{ix\eta/l^2}\\ &\times\exp\left[-\frac{(x+k_yl^2)^2}{4l^2}-\frac{(y-k_xl^2)^2}{4l^2}\right].\end{aligned}\tag{8}$$

The wave function (8) reflects the interaction between the translational and relative e-h motion expressed by the plane wave $\exp(ix\eta/l^2)$. The radius of the relative electron-hole motion is of the same order as the magnetic length l. We define the exciton dipole moment ρ_0, which arises perpendicular to the wave vector $\mathbf{k}$ when $\mathbf{k}\neq 0$, as

$$\rho_0=[\hat{\mathbf{z}}\times\mathbf{k}]\cdot l^2=(-\mathbf{i}k_y+\mathbf{j}k_x)\cdot l^2.\tag{9}$$

The Hamiltonian of the 2D electron-hole system in a strong perpendicular magnetic field was derived in Ref. 36 in the second quantization representation taking into account the electron-hole Coulomb interaction in the ideal symmetric 2D layer (corresponding to the electrons and holes confined to exactly the same region) and in a quantum well with finite thickness (with different electron and hole wave functions in the z direction), and also taking into account the electron-hole indirect interaction through virtually excited Landau levels. We consider here only the case of an ideal symmetric 2D layer and introduce another representation of the Coulomb matrix elements, which allows us to demonstrate their symmetry properties in a more simple way.

First, we introduce the chemical potentials for electrons and holes μ_e and μ_h, respectively, which allow us to take into account the constancy of their average numbers during phase transitions. The spin-oriented electrons and holes are assumed to be in the lowest Landau level, and therefore their kinetic energy does not appear in the Hamiltonian. The Hamiltonian describing the Coulomb interaction of electrons and holes is

$$\begin{aligned}\mathcal{H}=&\hat{H}-\mu_e\hat{N}_e-\mu_h\hat{N}_h\\ =&\frac{1}{2}\sum_{p,q,s}F_{e\text{-}e}(p,q;p-s,q+s)a_p^+a_q^+a_{q+s}a_{p\text{-}s}\\ &+\frac{1}{2}\sum_{p,q,s}F_{h\text{-}h}(p,q;p-s,q+s)b_p^+b_q^+b_{q+s}b_{p-s}\\ &-\sum_{p,q,s}F_{e\text{-}h}(p,q;p-s,q+s)a_p^+b_q^+b_{q+s}a_{p-s}\\ &-\mu_e\sum_p a_p^+a_p-\mu_h\sum_p b_p^+b_p,\end{aligned}\tag{10}$$

where a_p^+,b_p^+ and a_p,b_p are the electron and hole creation and annihilation operators, respectively. The operators are marked by the wave vectors p,q,s, which are oriented in the direction along the x axis. The translational symmetry and momentum conservation law are provided by the chosen Landau gauge. For example, the matrix elements of the electron-electron Coulomb interaction are

$$\begin{aligned}F_{e\text{-}e}(p,q;p-s,q+s)=&\int\int\Psi_p^{e\star}(\rho_1)\Psi_{p-s}^{e}(\rho_1)\frac{e^2}{\varepsilon_0\rho_{12}}\\ &\times\Psi_q^{e\star}(\rho_2)\Psi_{q+s}^{e}(\rho_2)d\rho_1d\rho_2,\end{aligned}\tag{11}$$

where only the envelope parts of the conduction electron Bloch function are used. The periodic parts of the Bloch functions can be excluded by integration over the volume of the unit cell. This part of the calculations can be omitted when we are not interested in such matters as exchange electron-hole interaction, band-to-band transition dipole moments, etc.

Using the series expansion of 2D Coulomb interaction

$$\frac{e^2}{\varepsilon_0 \rho} = \sum_{\varkappa} V_{\varkappa} e^{-i\varkappa\rho},$$

$$V_{\varkappa} = V_{\varkappa_x,\varkappa_y} = \frac{2\pi e^2}{\varepsilon_0 S|\varkappa|},$$

$$|\varkappa| = \sqrt{\varkappa_x^2 + \varkappa_y^2}, \tag{12}$$

and the electron envelope function (1), one can reduce Eq. (11) to

$$\begin{aligned} F_{e\text{-}e}(p,q;p-s,q+s) &= \frac{1}{l^2\pi} \sum_{\varkappa} V_{s,\varkappa} \exp[-s^2 l^2 - (q-p)sl^2] \\ &\times \int_{-\infty}^{\infty} \exp\left[-\frac{(y_1 - pl^2)^2}{l^2} - (s - i\varkappa)y_1\right] dy_1 \\ &\times \int_{-\infty}^{\infty} \exp\left[-\frac{(y_2 - ql^2)^2}{l^2} + (s - i\varkappa)y_2\right] dy_2 . \end{aligned}$$

After integration, all other Coulomb matrix elements can be obtained in a similar way, and we have

$$\begin{aligned} F_{e\text{-}e}(p,q;p-s,q+s) &= \sum_{\varkappa} V_{s,\varkappa} \exp\left[-\frac{(s^2+\varkappa^2)l^2}{2} + i\varkappa(p-q-s)l^2\right], \\ F_{h\text{-}h}(p,q;p-s,q+s) &= \sum_{\varkappa} V_{s,\varkappa} \exp\left[-\frac{(s^2+\varkappa^2)l^2}{2} - i\varkappa(p-q-s)l^2\right], \\ F_{e\text{-}h}(p,q;p-s,q+s) &= \sum_{\varkappa} V_{s,\varkappa} \exp\left[-\frac{(s^2+\varkappa^2)l^2}{2} + i\varkappa(p+q)l^2\right] \end{aligned} \tag{13}$$

with the following symmetry properties:

$$\begin{aligned} F_{e\text{-}e}(p,q;p-s,q+s) &= F_{e\text{-}e}(q,p;q+s,p-s) \\ &= F_{h\text{-}h}(p,q;p-s,q+s) \\ &= F_{h\text{-}h}(-p,-q;-p+s,-q-s), \\ F_{e\text{-}e}(p,-q-s;p-s,-q) &= F_{h\text{-}h}(s-p,q;-p,q+s) \\ &= F_{e\text{-}h}(p,q;p-s,q+s). \end{aligned} \tag{14}$$

We introduce the exciton creation operator following[36]

$$d_{\mathbf{k}}^{+} = \frac{1}{\sqrt{N}} \sum_t e^{-ik_y t l^2} a_{k_x/2+t}^{+} b_{k_x/2-t}^{+}, \tag{15}$$

with the 2D vector $\mathbf{k} = (k_x, k_y)$, with k_x corresponding to the exciton translational motion, and k_y being the quantum number describing the internal e-h motion. The magnetoexciton wave function is obtained by acting on the vacuum state $|0\rangle$ with the operator $d_{\mathbf{k}}^{+}$,

$$|\Psi_{\text{ex},\mathbf{k}}\rangle = d_{\mathbf{k}}^{+}|0\rangle, \quad \langle\Psi_{\text{ex},\mathbf{k}'}|\Psi_{\text{ex},\mathbf{k}}\rangle = \delta_{\text{kr}}(\mathbf{k}',\mathbf{k}). \tag{16}$$

The energy of exciton formation can be obtained by acting on the exciton wave function with the Hamiltonian $\hat{H}$. Relative to the lowest Landau level it equals

$$\hat{H}|\Psi_{\text{ex},\mathbf{k}}\rangle = E_{\text{ex}}(k)|\Psi_{\text{ex},\mathbf{k}}\rangle, \quad E_{\text{ex}}(k) = -I_{\text{ex}}(k), \tag{17}$$

where the exciton ionization potential is

$$I_{\text{ex}}(k) = \sum_{\mathbf{Q}} V_Q \exp\left[-\frac{Q^2 l^2}{2} + i(k_x Q_y - k_y Q_x)l^2\right] \tag{18}$$

and $\mathbf{Q}$ stands for the wave vector of the exciton excited state. The ionization potential of the magnetoexciton is[36]

$$I_{\text{ex}}(\mathbf{k}) = \frac{e^2}{\varepsilon_0 l}\int_0^{\infty} e^{-x^2/2} J_0(xkl)\,dx = I_l e^{-k^2 l^2/4} I_0\left(\frac{k^2 l^2}{4}\right), \tag{19}$$

where

$$I_l = \frac{e^2}{\varepsilon_0}\int_0^{\infty} e^{-Q^2 l^2/2} dQ = \frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}} \tag{20}$$

and $I_0(k^2 l^2/4)$ is a modified Bessel function. The asymptotic behavior of the modified Bessel function means that the ionization potential $I_{\text{ex}}(k)$ tends to zero as $1/kl$ when the dipole moment $\rho_0 = kl^2 \sim k$ tends to infinity. Using the operator of the electron-hole density fluctuation

$$\hat{\rho}_{\mathbf{Q}} = \sum_t e^{iQ_y t l^2}\left(a_{t-Q_x/2}^{+} a_{t+Q_x/2} - b_{-t-Q_x/2}^{+} b_{-t+Q_x/2}\right), \tag{21}$$

we can rewrite the Hamiltonian of Coulomb interaction in the form

$$\hat{H} = \frac{1}{2}\sum_{\mathbf{Q}} W_{\mathbf{Q}}(\hat{\rho}_{\mathbf{Q}}\hat{\rho}_{-\mathbf{Q}} - \hat{N}_e - \hat{N}_h),$$

$$W_{\mathbf{Q}} = V_{\mathbf{Q}} e^{-Q^2 l^2/2}, \quad V_{\mathbf{Q}} = \frac{2\pi e^2}{\varepsilon_0 S|\mathbf{Q}|}, \quad \sum_{\mathbf{Q}} W_{\mathbf{Q}} = I_l, \tag{22}$$

and the full Hamiltonian $\hat{\mathcal{H}}$ will be

$$\begin{aligned} \hat{\mathcal{H}} = \frac{1}{2}\sum_{\mathbf{Q}} W_{\mathbf{Q}}\hat{\rho}_{\mathbf{Q}}\hat{\rho}_{-\mathbf{Q}} - \frac{1}{2}\sum_{\mathbf{Q}} W_{\mathbf{Q}}(\hat{N}_e + \hat{N}_h) - \mu_e \hat{N}_e \\ - \mu_h \hat{N}_h, \end{aligned} \tag{23}$$

which will be used below to calculate the polarizability of magnetoexcitons.

III. COHERENT PAIRING OF ELECTRONS AND HOLES IN A SINGLE-PARTICLE STATE WITH FINITE WAVE VECTOR: THE KELDYSH-KOZLOV-KOPAEV METHOD

The numbers of electrons and holes which occupy the lowest Landau levels are determined by the experimental conditions, e.g., the generation rate due to laser excitation and the exciton recombination rate. Assuming an quasipermanent population with an equal number of electrons and holes, their quasiequilibrium states are characterized by the chemical potentials μ_e and μ_h. The energy of electrons and holes as well as their chemical potentials are measured relative to their lowest Landau levels. The exciton formation reaction $e+h\leftrightarrow ex$ implies the relation between the chemical potentials

$$\mu_e+\mu_h=\mu_{\text{ex}}. \tag{24}$$

Below, we assume $\mu_e=\mu_h=\mu_{\text{ex}}/2$ and denote $\mu_{\text{ex}}\equiv\mu$. The coherent macroscopic state corresponding to Bose-Einstein condensation of correlated electron-hole pairs in a single-particle state with the wave vector **k** can be introduced following Keldysh-Kozlov-Kopaev method (KKK method)[49] by applying the unitary transformation

$$\begin{aligned}\mathcal{D}(\sqrt{N_{\text{ex}}})&=\exp[\sqrt{N_{\text{ex}}}(d_{\mathbf{k}}^{+}-d_{\mathbf{k}})]\\ &=\prod_t \exp[\sqrt{2\pi l^2 n_{\text{ex}}}(e^{-ik_y t l^2}a^{+}_{k_x/2+t}b^{+}_{k_x/2-t}\\ &\quad -e^{ik_y t l^2}b_{k_x/2-t}a_{k_x/2+t})],\end{aligned} \tag{25}$$

where

$$\frac{N_{\text{ex}}}{N}=2\pi l^2 n_{\text{ex}},\quad n_{\text{ex}}=\frac{N_{\text{ex}}}{S}. \tag{26}$$

We introduce the BCS-type wave function of the new coherent macroscopic state acting on the electron-hole vacuum state $|0\rangle$ by the unitary transformation operator $\mathcal{D}(\sqrt{N_{\text{ex}}})$

$$\begin{aligned}|\Psi_g(\mathbf{k})\rangle&=\mathcal{D}(\sqrt{N_{\text{ex}}})|0\rangle\\ &=\prod_t (u+v e^{-ik_y t l^2}a^{+}_{k_x/2+t}b^{+}_{k_x/2-t})|0\rangle,\end{aligned} \tag{27}$$

where the coefficients u and v are

$$\begin{aligned}u&=\cos g,\quad v=\sin g,\\ g&=\sqrt{2\pi l^2 n_{\text{ex}}},\\ u^2+v^2&=1.\end{aligned} \tag{28}$$

The transformed Hamiltonian is

$$\tilde{\mathcal{H}}=\mathcal{D}(\sqrt{N_{\text{ex}}})\mathcal{H}D^{+}(\sqrt{N_{\text{ex}}}). \tag{29}$$

For this Hamiltonian, the new ground state wave function (27) plays the same role as the initial vacuum state $|0\rangle$ for the Hamiltonian $\mathcal{H}$,

$$\tilde{\mathcal{H}}|\Psi_g(\mathbf{k})\rangle=\mathcal{D}HD^{+}\mathcal{D}|0\rangle=\mathcal{D}H|0\rangle=0.$$

The unitary transformation (29) of the Hamiltonian $\mathcal{H}$ means the unitary transformations of the operators a_p, b_p

$$\begin{aligned}\mathcal{D}a_p\mathcal{D}^{+}&=\alpha_p=ua_p-v\left(p-\frac{k_x}{2}\right)b^{+}_{k_x-p},\\ \mathcal{D}b_p\mathcal{D}^{+}&=\beta_p=ub_p+v\left(\frac{k_x}{2}-p\right)a^{+}_{k_x-p},\end{aligned} \tag{30}$$

where

$$\begin{aligned}v(t)&=ve^{-ik_y t l^2},\\ v(t)v(s)&=v\cdot v(t+s),\\ v^{\star}(t)&=v(-t),\end{aligned} \tag{31}$$

and the inverse transformation is

$$\begin{aligned}a_p&=u\alpha_p+v\left(p-\frac{k_x}{2}\right)\beta^{+}_{k_x-p},\\ b_p&=u\beta_p-v\left(\frac{k_x}{2}-p\right)\alpha^{+}_{k_x-p}.\end{aligned} \tag{32}$$

The ground state (27) plays the role of vacuum state for the new Fermi operators α_p and β_p,

$$\begin{aligned}\alpha_p|\Psi_g(\mathbf{k})\rangle&=0,\\ \beta_p|\Psi_g(\mathbf{k})\rangle&=0,\end{aligned} \tag{33}$$

which can be verified directly,

$$\alpha_p|\Psi_g(\mathbf{k})\rangle=\mathcal{D}a_p\mathcal{D}^{+}\mathcal{D}|0\rangle=\mathcal{D}a_p|0\rangle=0.$$

The average numbers of the electrons and holes in the new ground state at $T=0$ can be determined from equalities

$$\langle\Psi_g(\mathbf{k})|a_p^{+}a_p|\Psi_g(\mathbf{k})\rangle=\langle\Psi_g(\mathbf{k})|b_p^{+}b_p|\Psi_g(\mathbf{k})\rangle=v^2. \tag{34}$$

This expression means that the total average numbers of electrons, holes and excitons are

$$\begin{aligned}N_{\text{ex}}&=\sum_p \langle\Psi_g(\mathbf{k})|a_p^{+}a_p|\Psi_g(\mathbf{k})\rangle=Nv^2,\\ n_{\text{ex}}&=\frac{v^2}{2\pi l^2},\end{aligned} \tag{35}$$

with the following restriction that is extracted from Eqs. (35) and (28):

$$v^2=\sin^2(\sqrt{2\pi l^2 n_{\text{ex}}})=\sin^2 v. \tag{36}$$

The applicability of the theory is therefore restricted to values of the filling factor v^2 defined by $v^2 \approx \sin^2 v$. This restriction reflects the physical assumption that only the lowest Landau level is occupied. When the filling factor v^2 approaches unity, one must take into account the first excited Landau level. Below we shall assume $v^2 \leqslant \frac{1}{4}$.

Following the KKK method, the transformed Hamiltonian $\tilde{\mathcal{H}} = \mathcal{D}HD^+$ must be expressed in terms of the new operators α_p^+, α_p, β_p^+, and β_p using Bogoliubov's u,v transformations (32). In this way the Hamiltonian $\tilde{\mathcal{H}}$ can be represented in the form

$$\tilde{\mathcal{H}} = U + H_2 + H'. \tag{37}$$

The first term U does not contain operators α_p and β_p and plays the role of the new ground state energy. The second term H_2 is quadratic in the operators α_p and β_p and appears as a result of transpositions of the new operators and their normal ordering. In this transposition, the commutation relations of the Fermi operators α_p and β_p transform terms with four operators into quadratic terms. The term H' contains the remaining normal-ordered terms with four operators, which is treated as a perturbation. The term U can be represented as

$$\begin{aligned} U &= N_{\mathrm{ex}}[E_{\mathrm{ex}}(\mathbf{k}) - \mu] - N_{\mathrm{ex}} v^2 [I_l - I_{\mathrm{ex}}(\mathbf{k})] \\ &= -N_{\mathrm{ex}}[I_{\mathrm{ex}}(\mathbf{k}) + \mu] - N_{\mathrm{ex}} v^2 [I_l - I_{\mathrm{ex}}(\mathbf{k})], \end{aligned} \tag{38}$$

where

$$N_{\mathrm{ex}} = N v^2, \quad E_{\mathrm{ex}}(\mathbf{k}) = -I_{\mathrm{ex}}(\mathbf{k}),$$

$$I_{\mathrm{ex}}(\mathbf{k}) = I_l e^{-k^2 l^2/4} I_0\left(\frac{k^2 l^2}{4}\right).$$

The last term in Eq. (38), containing the correction proportional to filling factor v^2, is negative due to the inequality $I_{\mathrm{ex}}(\mathbf{k}) \leqslant I_l$. The term H_2 contains diagonal quadratic terms as well as the terms describing the creation and annihilation of the new e-h pairs from the new vacuum state $|\Psi_g(\mathbf{k})\rangle$. It has the form

$$\begin{aligned} H_2 = {} & \sum_p E(\mathbf{k},v^2,\mu)(\alpha_p^+ \alpha_p + \beta_p^+ \beta_p) \\ & - \sum_p \left[uv\left(\frac{k_x}{2} - p\right) \psi(\mathbf{k},v^2,\mu) \beta_{k_x - p} \alpha_p \right. \\ & \left. + uv\left(p - \frac{k_x}{2}\right) \psi(\mathbf{k},v^2,\mu) \alpha_p^+ \beta_{k_x - p}^+ \right], \end{aligned} \tag{39}$$

where

$$E(\mathbf{k},v^2,\mu) = 2u^2 v^2 I_{\mathrm{ex}}(\mathbf{k}) + I_l(v^4 - u^2 v^2) - \frac{\mu}{2}(u^2 - v^2) \tag{40}$$

and

$$\psi(\mathbf{k},v^2,\mu) = 2v^2 I_l + I_{\mathrm{ex}}(\mathbf{k})(u^2 - v^2) - \mu. \tag{41}$$

It is seen from the Hamiltonian H_2 that the new quasiparticles described by the operators α_p, β_p can appear spontaneously from the new vacuum state as a pair with total momentum k_x, which coincides with the translational wave vector of the Bose-Einstein condensate of magnetoexcitons. Such terms in the Hamiltonian and the corresponding diagrams are called dangerous ones and make the new vacuum state unstable. To avoid this instability, the condition of compensation of the dangerous diagrams is used. In the Hartree-Fock-Bogoliubov approximation, when only the dangerous diagrams in H_2 are taken into account, the condition of their compensation is

$$\psi(\mathbf{k},v^2,\mu) = 0$$

This condition determines the unknown parameter of the theory, namely the chemical potential μ of the system. In the Hartree-Fock-Bogoliubov approximation it is

$$\begin{aligned} \mu^{\mathrm{HFB}} &= E_{\mathrm{ex}}(\mathbf{k}) - 2v^2 [I_l - I_{\mathrm{ex}}(\mathbf{k})] \\ &= -I_{\mathrm{ex}}(\mathbf{k}) - 2v^2 [I_l - I_{\mathrm{ex}}(\mathbf{k})]. \end{aligned} \tag{42}$$

This condition introduces the breaking of the $u(1)$ gauge symmetry of the initial Hamiltonian $\mathcal{H}$ and makes nonequivalent its ground state and the new ground state U. With the help of μ^{HFB} we can determine self-consistently the ground state energy U and the energy of the single-particle elementary excitations, which in the given approximation are

$$U^{\mathrm{HFB}} = N_{\mathrm{ex}} v^2 [I_l - I_{\mathrm{ex}}(\mathbf{k})], \quad E(\mathbf{k},v^2,\mu) = \frac{1}{2} I_{\mathrm{ex}}(\mathbf{k}). \tag{43}$$

As one can see, the single-particle elementary excitation has an energy spectrum without dispersion. It does not depend on the wave vectors p or $k_x - p$ of the electron or of the hole appearing from the new vacuum state, and for each particle is equal exactly to one half of the ionization energy of the condensed excitons. To excite one electron-hole pair from the vacuum, the energy $I_{\mathrm{ex}}(\mathbf{k})$ is required, because it is equivalent to an unbound single exciton with the wave vector **k**. The absence of dispersion reflects the absence of the kinetic energy of the electrons and holes in the lowest Landau level. It was shown[36] that there are no plasma oscillations in the case $k=0$, whereas the dispersion relation of the collective excitations in this case is given by the exciton dispersion relation $E_{\mathrm{ex}}(\mathbf{k}) - E_{\mathrm{ex}}(0)$. These results can be generalized when condensation of excitons with dipole moments occurs. In this case, the ground state energy can be determined with BCS-type wave functions (27) by the expression

$$\begin{aligned} E_g(\mathbf{k}) &= \langle \Psi_g(\mathbf{k}) | \hat{H} | \Psi_g(\mathbf{k}) \rangle \\ &= -N v^2 I_{\mathrm{ex}}(k) - N v^4 [I_l - I_{\mathrm{ex}}(k)]. \end{aligned} \tag{44}$$

It was obtained in the scope of the Hartree-Fock approximation. Its derivative $dE_g(k)/dN_{\mathrm{ex}}$ determines the chemical potential μ^{HFB} in full accordance with Eq. (42), whereas the rate $E_g(k)/N_{\mathrm{ex}}$ characterizes the mean energy per one exciton

$$\frac{E_g(k)}{N_{\mathrm{ex}}}=E_{\mathrm{ex}}(k)-v^2[E_{\mathrm{ex}}(k)-E_{\mathrm{ex}}(0)]$$

$$=E_{\mathrm{ex}}(k)-v^2[I_l-I_{\mathrm{ex}}(k)],$$

$$E_g(k)=U+\mu^{\mathrm{HFB}}N_{\mathrm{ex}}. \tag{45}$$

In the next section we show that this result can be extended beyond the Hartree-Fock-Bogoliubov approximation, taking into consideration the polarizability of the Bose-Einstein condensed magnetoexcitons, at least for a symmetric 2D model.

IV. ANDERSON-TYPE WAVE FUNCTIONS OF THE COHERENT EXCITED STATES: POLARIZABILITY OF THE BOSE-EINSTEIN CONDENSED MAGNETOEXCITONS

The coherent excited states of the Bose-condensed magnetoexcitons can be constructed following the method proposed by Anderson in the theory of superconductivity.[50] The excited state can be obtained by acting with the electron part of the density fluctuation operator (21) on the ground state wave function (27),

$$\left|\Psi^e\left(q\pm\frac{Q_x}{2}\right)\right\rangle=a^{+}_{q+Q_x/2}a_{q-Q_x/2}|\Psi_g(k)\rangle. \tag{46}$$

Using the operators α_p,β_p, the u-v transformation (32) and taking into account

$$\alpha_p|\Psi_g(k)\rangle=\beta_p|\Psi_g(k)\rangle=0, \tag{47}$$

this function can be reduced to

$$\left|\Psi^e\left(q\pm\frac{Q_x}{2}\right)\right\rangle=\Bigg[v\cdot v(-Q_x)\beta_{k_x-q-Q_x/2}\beta^{+}_{k_x-q+Q_x/2}+uv\left(q-\frac{Q_x}{2}-\frac{k_x}{2}\right)\alpha^{+}_{q+Q_x/2}\beta^{+}_{k_x-q+Q_x/2}\Bigg]|\Psi_g(k)\rangle. \tag{48}$$

Taking into account that Q_x is different from zero, one can simplify Eq. (48) neglecting the first term in the right hand side. The set of functions (48) obeys the following orthogonality and normalization conditions

$$\left\langle\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\middle|\Psi^e\left(q\pm\frac{Q_x}{2}\right)\right\rangle=u^2v^2\,\delta_{kr}(\mathcal{P}_x,Q_x)\,\delta_{kr}(p,q). \tag{49}$$

The excitation energy in the Hartree-Fock-Bogoliubov approximation can be found using the $\hat{H}_2$ part of the Hamiltonian (37),

$$\hat{H}_2=\sum_p\frac{I_{\mathrm{ex}}(k)}{2}(\alpha^{+}_p\alpha_p+\beta^{+}_p\beta_p) \tag{50}$$

and equals

$$E\left(q\pm\frac{Q_x}{2}\right)=\frac{\langle\Psi^e(q\pm Q_x/2)|\hat{H}_2|\Psi^e(q\pm Q_x/2)\rangle}{\langle\Psi^e(q\pm Q_x/2)|\Psi^e(q\pm Q_x/2)\rangle}=I_{\mathrm{ex}}(k). \tag{51}$$

The excitation energy of this state equals the ionization potential of Bose-condensed magnetoexcitons with wave vector **k**. It does not depend on the wave vectors q and Q_x, which characterize the excited state. The excitation energy and the energy spectrum of single-particle elementary excitations are the same for the full set of the excited states (47) and have no dispersion.

We also introduce the excited states generated by the fluctuation of the hole density and by the action of the corresponding operator on the ground state wave function $|\Psi_g(k)\rangle$

$$\left|\Psi^h\left(-p\pm\frac{\mathcal{P}_x}{2}\right)\right\rangle=b^{+}_{-p+\mathcal{P}_x/2}b_{-p-\mathcal{P}_x/2}|\Psi_g(k)\rangle$$

$$=\Bigg[v\cdot v(\mathcal{P}_x)\alpha_{k_x+p-\mathcal{P}_x/2}\alpha^{+}_{k_x+p+\mathcal{P}_x/2}+uv\times\left(p+\frac{\mathcal{P}_x}{2}+\frac{k_x}{2}\right)\alpha^{+}_{k_x+p+\mathcal{P}_x/2}\beta^{+}_{-p+\mathcal{P}_x/2}\Bigg]\times|\Psi_g(k)\rangle. \tag{52}$$

They have the same properties of orthogonality and normalization as Eq. (49),

$$\left\langle\Psi^h\left(-p\pm\frac{\mathcal{P}_x}{2}\right)\middle|\Psi^h\left(-q\pm\frac{Q_x}{2}\right)\right\rangle=u^2v^2\,\delta_{kr}(\mathcal{P}_x,Q_x)\,\delta_{kr}(p,q), \tag{53}$$

as well as the same excitation energy $I_{\mathrm{ex}}(k)$. Because of the obvious relations between these two sets of wave functions

$$\left\langle\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\middle|\Psi^h\left(-q\pm\frac{Q_x}{2}\right)\right\rangle=u^2vv(Q_x)\,\delta_{kr}(\mathcal{P}_x,Q_x)\,\delta_{kr}(p-k_x,q),$$

$$\left\langle\Psi^h\left(-p\pm\frac{\mathcal{P}_x}{2}\right)\middle|\Psi^e\left(q\pm\frac{Q_x}{2}\right)\right\rangle=u^2vv(-Q_x)\,\delta_{kr}(\mathcal{P}_x,Q_x)\,\delta_{kr}(q,p+k_x), \tag{54}$$

they are not independent and there is a correspondence between each function from one set with some function from another set. This means that one should take into account only one of them.

In a similar way, the excited wave functions generated by the pair of electron and hole creation operators are introduced in the form

$$\left|\Psi^{e\text{-}h}\left(\pm p,\frac{\mathcal{P}_x+k_x}{2}\right)\right\rangle=a^{+}_{p+(\mathcal{P}_x+k_x)/2}b^{+}_{-p+(\mathcal{P}_x+k_x)/2}|\Psi_g(k)\rangle. \tag{55}$$

Their orthogonality and normalization properties are

$$\left\langle \Psi^{e\text{-}h}\left(\pm q,\frac{Q_x+k_x}{2}\right)\middle|\Psi^{e\text{-}h}\left(\pm p,\frac{\mathcal{P}_x+k_x}{2}\right)\right\rangle = u^4\delta_{kr}(\mathcal{P}_x,Q_x)\,\delta_{kr}(p,q),$$

$$\left\langle \Psi^{e\text{-}h}\left(\pm q,\frac{Q_x+k_x}{2}\right)\middle|\Psi^{e}\left(p\pm\frac{\mathcal{P}_x}{2}\right)\right\rangle = u^3v\left(p-\frac{\mathcal{P}_x+k_x}{2}\right)\delta_{kr}\left(q+\frac{k_x}{2},p\right), \tag{56}$$

$$\left\langle \Psi^{e\text{-}h}\left(\pm q,\frac{Q_x+k_x}{2}\right)\middle|\Psi^{h}\left(-p\pm\frac{\mathcal{P}_x}{2}\right)\right\rangle = u^3v\left(p+\frac{\mathcal{P}_x+k_x}{2}\right)\delta_{kr}\left(q,p+\frac{k_x}{2}\right).$$

The third set of excited wave functions in Eq. (55) is dependent on the other two. One can expect similar properties of the fourth set of excited wave functions generated by the pair of electron and hole annihilation operators. To verify the completeness of the first set of the excited wave functions in the framework of these four sets, we use the identity operator

$$\hat{I}=\sum_{p,\mathcal{P}_x}\frac{\left|\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\right\rangle\left\langle\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\right|}{\left\langle\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\middle|\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\right\rangle}. \tag{57}$$

The completeness of the wave functions (48) can be verified by direct substitution into the normalization and orthogonality relations (54) and (56). The matrix elements of the operator (21) can be calculated using these excited-state wave functions

$$|(\hat{\rho}_{\mathbf{Q}})_{n,0}|^2=\left|\frac{1}{uv}\left\langle\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\middle|\hat{\rho}_{\mathbf{Q}}|\Psi_g(\mathbf{k})\right\rangle\right|^2, \tag{58}$$

and they determine the polarizability of the system. In our case

$$\hat{\rho}_{\mathbf{Q}}|\Psi_g(\mathbf{k})\rangle=\sum_q e^{iQ_yql^2}\left(\left|\Psi^e\left(q\mp\frac{Q_x}{2}\right)\right\rangle-\left|\Psi^h\left(-q\mp\frac{Q_x}{2}\right)\right\rangle\right). \tag{59}$$

We denote the normalized excited state as

$$|n\rangle\equiv|e,\mathcal{P}_x,p\rangle=\frac{1}{uv}\left|\Psi^e\left(p\pm\frac{\mathcal{P}_x}{2}\right)\right\rangle. \tag{60}$$

It is characterized by two independent quantum numbers $\mathcal{P}_x$ and p or by their combinations.

The matrix elements of the density fluctuation operator $\hat{\rho}_{\mathbf{Q}}$ (21) are

$$(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}=(\hat{\rho}_{-\mathbf{Q}})_{n,0}=uv\,\delta_{kr}(\mathcal{P}_x,Q_x)e^{-iQ_ypl^2}[1-e^{-i(k_yQ_x-k_xQ_y)l^2}]. \tag{61}$$

It is seen that

$$|(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}|^2=|(\hat{\rho}_{-\mathbf{Q}})_{n,0}|^2=4u^2v^2\,\delta_{kr}(\mathcal{P}_x,Q_x)\sin^2(\{k_yQ_x-k_xQ_y\}l^2/2), \tag{62}$$

and these do not depend on the wave vector p and contain the coherence factor $\sin^2(\{k_yQ_x-k_xQ_y\}l^2/2)$. The corresponding excitation energy is determined by Eq. (51) and will be denoted below as

$$\hbar\omega_{n,0}=I_{\text{ex}}(k). \tag{63}$$

We shall calculate the polarizability in the approximation of a weak response to the external longitudinal perturbation.[51] In the case of a 2D structure with Hamiltonian of the form (22), the perturbation caused by an external probe charge can be written in the form

$$H_{\text{ext}}(t)=-\frac{ez}{e^2}W_Q[\rho_{\text{ext}}(\mathbf{Q},\omega)\rho^{+}_{\mathbf{Q}}e^{-i\omega t}+\rho^{\star}_{\text{ext}}(\mathbf{Q},\omega)\rho_{\mathbf{Q}}e^{i\omega t}]. \tag{64}$$

The specific properties of a 2D structure appear due to the Coulomb interaction coefficient W_Q, determined in Eq. (22), which differs essentially from the case of 3D structure. Substituting Eq. (62) into a general expression for the polarizability, we find

$$4\pi\alpha_0^{\text{HF}}(\mathbf{Q},\omega)=-\frac{W_{\mathbf{Q}}}{\hbar}\sum_n\left[\frac{|(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}|^2}{\omega-\omega_{n,0}+i\delta}-\frac{|(\hat{\rho}_{\mathbf{Q}})_{n,0}|^2}{\omega+\omega_{n,0}+i\delta}\right], \tag{65}$$

where $W_{\mathbf{Q}}$, $|(\hat{\rho}_{\mathbf{Q}})_{n,0}|^2$, and $\omega_{n,0}$ depend on the wave functions of the electrons, holes, and magnetoexcitons in the 2D structure. We obtain, after straightforward calculations,

$$4\pi\alpha_0^{\text{HF}}(\mathbf{Q},\omega)\underset{\delta\to+0}{=}-4u^2v^2W_QN\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\times\left[\frac{1}{\hbar\omega-I_{\text{ex}}(k)+i\delta}-\frac{1}{\hbar\omega+I_{\text{ex}}(k)+i\delta}\right]. \tag{66}$$

This expression can be rewritten as

$$4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega) \underset{\delta\to+0}{=} -4u^2v^2\frac{e^2}{\varepsilon_0 l^2|\mathbf{Q}|}e^{-Q^2l^2/2}\times\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\times\left[\frac{1}{\hbar\omega-I_{\mathrm{ex}}(k)+i\delta}-\frac{1}{\hbar\omega+I_{\mathrm{ex}}(k)+i\delta}\right]. \tag{67}$$

The same result for the polarizability can be obtained from another possible set of excitations. The polarizability has a resonance frequency equal to the ionization potential $I_{\mathrm{ex}}(k)$ of the magnetoexciton with dipole moment $\rho_0 = kl^2$. The polarizability vanishes when the wave vector k approaches zero, and the magnetoexcitons behave as an ideal noninteracting gas. The polarizability is an anisotropic function on the wave vector $\mathbf{Q}$ and decreases exponentially when Q goes to infinity.

The real and imaginary parts of the polarizability are

$$\begin{aligned}
4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega) &= 4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega)+i4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega),\\
4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega) &= -4u^2v^2(W_QN)\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\times\left[\frac{Pf}{\hbar\omega-I_{\mathrm{ex}}(k)}-\frac{Pf}{\hbar\omega+I_{\mathrm{ex}}(k)}\right],\\
4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega) &= 4\pi u^2v^2(W_QN)\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\times\{\delta[\hbar\omega-I_{\mathrm{ex}}(k)]-\delta[\hbar\omega+I_{\mathrm{ex}}(k)]\}.
\end{aligned} \tag{68}$$

The symbol (Pf) in Eq. (68) denotes that the singular term which may appear in the expression $4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega)$ at the point $\hbar\omega = I_{\mathrm{ex}}(k)$ must be removed. The polarizability $4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega)$ determines the dielectric constant $\varepsilon(\mathbf{Q},\omega)$, which in the Hartree-Fock (HF) approximation is

$$\frac{1}{\varepsilon^{\mathrm{HF}}(\mathbf{Q},\omega)} = 1-4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega), \tag{69}$$

contrary to the random phase approximation (RPA), where the expression for the dielectric constant is different. This difference appears due to the summation of the diagrams with polarization loops, which are related to the screening of the effective Coulomb interaction[4,51] and give the following expression:

$$\begin{aligned}
W_{Q\ \mathrm{eff}} &= W_Q+W_Q\Pi(\mathbf{Q},\omega)W_Q+W_Q[\Pi(\mathbf{Q},\omega)W_Q]^2+\cdots\\
&= \frac{W_Q}{1-\Pi(\mathbf{Q},\omega)W_Q} = \frac{W_Q}{\varepsilon^{\mathrm{RPA}}(\mathbf{Q},\omega)}.
\end{aligned} \tag{70}$$

The dielectric constant $\varepsilon(\mathbf{Q},\omega)$ obtained by this method is denoted $\varepsilon^{\mathrm{RPA}}(\mathbf{Q},\omega)$:

$$\varepsilon^{\mathrm{RPA}}(\mathbf{Q},\omega) = 1+4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega). \tag{71}$$

The relation between the polarizability $4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega)$ and the contribution of the polarization loop $\Pi(\mathbf{Q},\omega)$ is

$$\begin{aligned}
\varepsilon^{\mathrm{RPA}}(\mathbf{Q},\omega) &= 1-\Pi(\mathbf{Q},\omega)W_Q = 1+4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega),\\
4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega) &= -\Pi(\mathbf{Q},\omega)W_Q.
\end{aligned} \tag{72}$$

One can see that the expression for the dielectric constant $\varepsilon^{\mathrm{HF}}(\mathbf{Q},\omega)$ appears as the first two terms in the series expansion on the polarizability $4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega)$ of the inverse value of $\varepsilon^{\mathrm{RPA}}(\mathbf{Q},\omega)$. This result can be easily obtained if only two diagrams are taken into account when the effective Coulomb interaction is calculated.

To compare the obtained results with the polarizability of noncondensed magnetoexcitons, we consider the wave function (16) of the magnetoexciton with wave vector $\mathbf{k}$. The excited state of the magnetoexciton

$$|\Psi_{\mathrm{ex}},k'\rangle = \frac{1}{\sqrt{N}}\sum_p a^{+}_{p+k'_x/2}b^{+}_{-p+k'_x/2}|0\rangle, \tag{73}$$

is characterized by the wave vector $\mathbf{k}'$. Taking into account that the square matrix element of the density fluctuation operator is

$$\begin{aligned}
|(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}|^2 &= |(\hat{\rho}_{-\mathbf{Q}})_{n,0}|^2\\
&= 2\delta_{kr}(\mathbf{Q},\mathbf{k}-\mathbf{k}')[1-\cos(Q_yk_x-Q_xk_y)l^2].
\end{aligned} \tag{74}$$

We calculate the polarizability of the noncondensed magnetoexcitons

$$\begin{aligned}
4\pi\alpha_0^{\mathrm{HF}}(\mathbf{k},\mathbf{Q},\omega) &= 4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{k},\mathbf{Q},\omega)+i4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{k},\mathbf{Q},\omega),\\
4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{k},\mathbf{Q},\omega) &= -4W_Q\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\times\left[\frac{Pf}{\hbar\omega-[I_{\mathrm{ex}}(k)-I_{\mathrm{ex}}(k-Q)]}-\frac{Pf}{\hbar\omega+[I_{\mathrm{ex}}(k)-I_{\mathrm{ex}}(k+Q)]}\right],\\
4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{k},\mathbf{Q},\omega) &= 4\pi W_Q\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)(\delta\{\hbar\omega-[I_{\mathrm{ex}}(k)-I_{\mathrm{ex}}(k-Q)]\}-\delta\{\hbar\omega+[I_{\mathrm{ex}}(k)-I_{\mathrm{ex}}(k+Q)]\}).
\end{aligned} \tag{75}$$

The resultant polarizability can be obtained by summation over all the wave vectors $\mathbf{k}$ of noncondensed magnetoexcitons with Bose distribution function

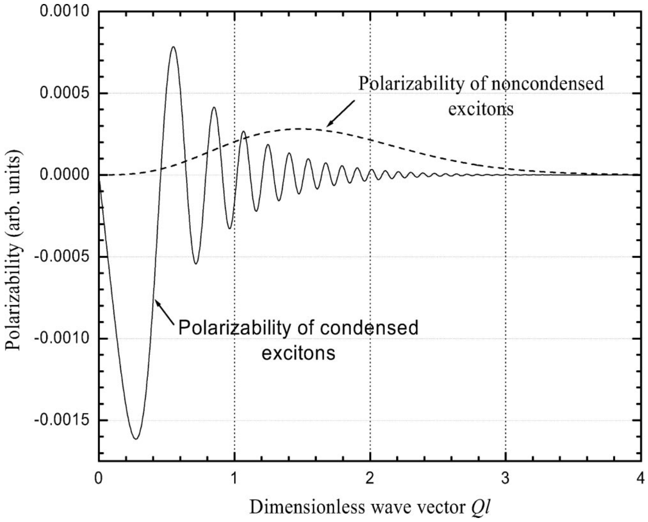

FIG. 1. Real part of the polarizability versus dimensionless wave vector. Solid curve: polarizability of condensed magnetoexcitons at $kl=4.6$. Dashed curve: polarizability of noncondensed magnetoexcitons.

$$4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega) \underset{\delta\to+0}{=} \sum_{\mathbf{k}} \frac{1}{\exp\left[\dfrac{E(k)-\mu}{k_BT}\right]-1} 4\pi\alpha_0^{\mathrm{HF}}(\mathbf{k},\mathbf{Q},\omega). \tag{76}$$

To simplify the expression (76) the following approximations were introduced:

$$E_{\mathrm{ex}}(k)=-I_{\mathrm{ex}}(k)=-I_l+\frac{k^2\hbar^2}{2m_H},$$

$$k\leqslant k_{\max},\quad I_{\mathrm{ex}}(k_{\max})=0,\quad \mu=-I_l,$$

$$\sin^2\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\approx\frac{Q^2k^2l^4}{8}\Bigg/\left(1+\frac{Q^2k^2l^4}{4}\right). \tag{77}$$

They allows to perform the exact integration over the polar angles, which appear in the denominators (75) and the numerical calculations of the integrals on the modulus k in the interval $0\leqslant k\leqslant k_{\max}$.

The real part of the polarizability for the condensed and noncondensed magnetoexcitons is shown in Fig. 1 versus the dimensionless vector $\tilde{Q}\equiv Ql$. One can see that the polarizability vanishes for both the condensed and noncondensed magnetoexcitons for large wave vectors Q. If the direction of the polarization is perpendicular to the single particle wave vector of the condensate, the polarizability of the condensed magnetoexcitons is zero. One can see that polarizability is essentially different for the condensed and noncondensed excitons. The polarizability of the condensed excitons is strongly oscillating function of Q, and this fact opens up the possibility for an experimentally observable signature of the appearance of Bose-Einstein condensation.

V. SCREENING EFFECTS AND CORRELATION ENERGY: NOZIÈRES AND COMTE APPROACH

In this section, we calculate the correlation energy of the condensed excitons, taking into account the screening effects. For this purpose it is convenient to use the method proposed in Ref. 52, which allows us consider simultaneously the binding processes and the screening effects in a system of Bose condensed e-h pairs, which can be used both in the cases of dense and dilute limits of exciton concentrations.

The formulation of this method, called the generalized random-phase approximation (GRPA), is based on the Pauli-Feynman theorem[4,51,52] for the ground state energy. In the case of interacting e-h pairs, their ground state energy E_0 can be expressed in the form

$$E_0=E_{\mathrm{kin}}+\int_0^{e^2}E_{\mathrm{int}}(\lambda)\frac{d\lambda}{\lambda}. \tag{78}$$

Here E_{kin} is the kinetic energy of the Bose condensed ideal e-h pairs without Coulomb interaction between them, and $E_{\mathrm{int}}(\lambda)$ is the mean value of the Coulomb interaction, with λ being the square electric charge, which changes from zero to the real value e^2.

For the gas of electrons and holes occupying the lowest Landau levels only the Coulomb interaction term (22) enters the Hamiltonian. Applying the GRPA method, we introduce the hypothetical gas with the bare Coulomb interaction $W_Q(\lambda)=W_Q(\lambda/e^2)$ and wave functions $|n(\lambda)\rangle$. Then the interaction part of the ground state energy is

$$W_Q=\int_0^{e^2}\frac{W_Q(\lambda)}{\lambda}d\lambda. \tag{79}$$

The expression $E_{\mathrm{int}}(\lambda)$ takes the form

$$E_{\mathrm{int}}(\lambda)=-N_{\mathrm{ex}}\sum_{\mathbf{Q}}W_Q(\lambda)+\frac{1}{2}\sum_{\mathbf{Q}}W_Q(\lambda)\sum_{n(\lambda)}|(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}|^2, \tag{80}$$

which can be expressed through the imaginary part of the polarizability $4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega)$ obtained in the Hartree-Fock approximation.

Taking into account the definitions (65) and (69), we obtain the relations

$$\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\mathrm{Im}\,4\pi\alpha_0^{\mathrm{HF}}(\mathbf{Q},\omega)=\frac{1}{2}W_Q\sum_n|(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}|^2, \tag{81}$$

$$\frac{1}{2}W_Q(\lambda)\sum_{n(\lambda)}|(\hat{\rho}^{+}_{\mathbf{Q}})_{n,0}|^2=-\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\mathrm{Im}\frac{1}{\varepsilon^{\mathrm{HF}}(Q,\omega,\lambda)}, \tag{82}$$

and for the ground state energy we find

$$E_0=-N_{\mathrm{ex}}\sum_{\mathbf{Q}}W_Q-\sum_{\mathbf{Q}}\int_0^{\infty}\frac{\hbar d\omega}{2\pi}\int_0^{e^2}\frac{d\lambda}{\lambda}\mathrm{Im}\frac{1}{\varepsilon(Q,\omega,\lambda)}. \tag{83}$$

Taking into account the chemical potential, the Hamiltonian is $\hat{\mathcal{H}}=\hat{H}-\mu_e\hat{N}_e-\mu_h\hat{N}_h$, and the ground state energy becomes

$$\mathcal{E}_0=E_0-\mu N_{\mathrm{ex}}, \tag{84}$$

where $\mu=\mu_e+\mu_h$. As mentioned in Ref. 52, the choice of the approximation for $\varepsilon(Q,\omega,\lambda)$ determines the accuracy of the energy E_0 given by Eq. (83). The best results were obtained using the RPA together with the Habbard correction[4] for the one-component plasma and the electron-hole liquid. When $\varepsilon(Q,\omega,\lambda)$ is taken in the form $\varepsilon^{\mathrm{HF}}(Q,\omega,\lambda)$, we obtain the previous expressions (42),(43) written in the Hartree-Fock-Bogoliubov approximation. Representing $\varepsilon(Q,\omega,\lambda)$ in the form

$$\begin{aligned}\varepsilon(Q,\omega,\lambda)&=\varepsilon_1(Q,\omega,\lambda)+i\varepsilon_2(Q,\omega,\lambda),\\ \varepsilon_1(Q,\omega,\lambda)&=1+4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega,\lambda);\\ \varepsilon_2(Q,\omega,\lambda)&=4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega,\lambda),\end{aligned} \tag{85}$$

and supposing $\varepsilon_2(Q,\omega,\lambda)\approx\lambda$ and $\varepsilon_1(Q,\omega,\lambda)\approx 1$, we obtain

$$\begin{aligned}\int_0^{e^2}\frac{d\lambda}{\lambda}\,\mathrm{Im}\frac{1}{\varepsilon(Q,\omega,\lambda)}&=-\frac{1}{\varepsilon_1(Q,\omega)}\arctan\frac{\varepsilon_2(Q,\omega)}{\varepsilon_1(Q,\omega)}\\ &\approx-\frac{\varepsilon_2(Q,\omega)}{\varepsilon_1^2(Q,\omega)}.\end{aligned}$$

Substituting Eq. (85) into this expression and expanding it in series up to the first order in $4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega)$, inclusively we obtain

$$\begin{aligned}\frac{4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega)}{[1+4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega)]^2}&\approx 4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega)-2\\ &\quad\times 4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega)4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega).\end{aligned} \tag{86}$$

The first term in the series expansion (86) corresponds to Hartree-Fock-Bogoliubov approximation, whereas the second term determines the correlation energy

$$E_{\mathrm{corr}}=-2\sum_{\mathbf{Q}}\int_0^{\infty}\frac{\hbar\,d\omega}{2\pi}\,4\pi\alpha_{0,2}^{\mathrm{HF}}(\mathbf{Q},\omega)4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega). \tag{87}$$

As explained above, only the real part of the polarizability $4\pi\alpha_{0,1}^{\mathrm{HF}}(\mathbf{Q},\omega)$, which contains the denominator $[\hbar\omega+I_{\mathrm{ex}}(k)]$, gives a nonvanishing contribution to the correlation energy at the point $\hbar\omega=I_{\mathrm{ex}}(k)$. At this point the denominator equals $2I_{\mathrm{ex}}(k)$, which means that the correlation energy is due to the virtual excitation of two quasiparticle pairs out of the mean-field ground state $|\Psi_g(\mathbf{k})\rangle$. The correlation energy is negative due to the screening effects and therefore lowers the energy of the interacting system. However, this reduction is not monotonic and the local minimum of the correlation energy depends on the value of the filling factor v^2.

Substituting Eq. (75) in Eq. (87), we obtain the correlation energy in the form

$$\begin{aligned}E_{\mathrm{corr}}&=-\frac{8(u^2v^2)^2}{I_{\mathrm{ex}}(k)}\sum_{\mathbf{Q}}(W_QN)^2\sin^4\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)\\ &=-\frac{N(u^2v^2)^2}{\sqrt{\pi}}\frac{I_l^2}{I_{\mathrm{ex}}(k)}F(kl),\end{aligned} \tag{88}$$

where the sum over $\mathbf{Q}$ was substituted by the integral

$$\int\frac{d^2\mathbf{Q}}{Q^2}e^{-Q^2l^2}\sin^4\left(\frac{k_yQ_x-k_xQ_y}{2}l^2\right)=\frac{\pi\sqrt{\pi}}{8}F(kl). \tag{89}$$

These expressions can be presented through the modified Bessel function $I_0(z)$, as

$$\begin{aligned}F(kl)&=3+e^{-k^2l^2/2}I_0\left(\frac{k^2l^2}{2}\right)-4e^{-k^2l^2/8}I_0\left(\frac{k^2l^2}{8}\right),\\ I_{\mathrm{ex}}(k)&=I_lG(kl),\qquad G(kl)=e^{-k^2l^2/4}I_0\left(\frac{k^2l^2}{4}\right).\end{aligned} \tag{90}$$

The total mean energy per particle consists of two parts. One of them coincides with $E_g(k)/N_{\mathrm{ex}}$ (45) and the other is $E_{\mathrm{corr}}/N_{\mathrm{ex}}$,

$$\begin{aligned}\mathcal{E}=\mathcal{E}^{\mathrm{HF}}+\mathcal{E}_{\mathrm{corr}}&=\frac{E_g(k)+E_{\mathrm{corr}}}{N_{\mathrm{ex}}}\\ &=-I_{\mathrm{ex}}(k)-v^2[I_l-I_{\mathrm{ex}}(k)]-\frac{1}{\sqrt{\pi}}\frac{I_l^2}{I_{\mathrm{ex}}(k)}F(kl)u^4v^2.\end{aligned} \tag{91}$$

The corresponding correction to the chemical potential is

$$\begin{aligned}\mu_{\mathrm{corr}}&=\frac{dE_{\mathrm{corr}}}{dN_{\mathrm{ex}}}=\frac{d}{dv^2}\frac{E_{\mathrm{corr}}}{N},\\ N_{\mathrm{ex}}&=Nv^2\end{aligned} \tag{92}$$

and the total value of the chemical potential is

$$\begin{aligned}\mu&=\mu^{\mathrm{HFB}}+\mu_{\mathrm{corr}}\\ &=-I_{\mathrm{ex}}(k)-2v^2[I_l-I_{\mathrm{ex}}(k)]\\ &\quad-\frac{2}{\sqrt{\pi}}\frac{I_l^2F(kl)}{I_{\mathrm{ex}}(k)}v^2(1-v^2)(1-2v^2).\end{aligned} \tag{93}$$

It is convenient to express the total energy per particle and the chemical potential in terms of the exciton ionization potential at the point $k=0, I_l=I_{\mathrm{ex}}(0)$,

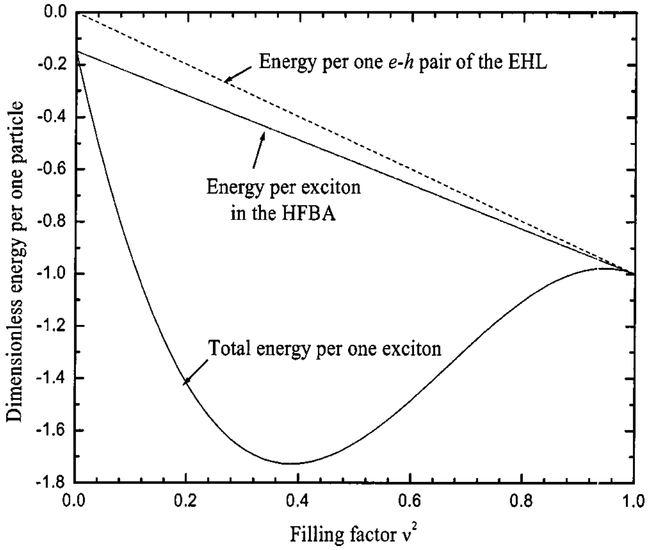

FIG. 2. Energy per particle versus filling factor v^2. Solid line: energy per exciton at $kl=4.6$. Dashed line: energy per one exciton in the Hartree-Fock-Bogoliubov approximation. Dash-dotted line: energy per e-h pair of the metallic EHL.

$$\frac{\mathcal{E}}{I_l}=-G(kl)-[1-G(kl)]v^2-\frac{1}{\sqrt{\pi}}\frac{F(kl)}{G(kl)}v^2(1-v^2)^2, \tag{94}$$

$$\frac{\mu}{I_l}=-G(kl)-2v^2[1-G(kl)]-\frac{2}{\sqrt{\pi}}\frac{F(kl)}{G(kl)}\times v^2(1-v^2)(1-2v^2). \tag{95}$$

The mean energy per particle and the chemical potential versus the filling factor v^2 are shown in Figs. 2 and 3. The first term in Eq. (95) gives the energy per exciton in the Hartree-Fock-Bogoliubov approximation. For small values of kl, the total energy and chemical potential, with the correlation corrections, are monotonic functions of v^2, and almost coincide with those found in the scope of the HFBA. For larger kl, the total energy and chemical potential deviate considerably their values in the HFBA and become nonmonotonic functions of v^2 with a well-pronounced local minimum. This minimum becomes deeper and more pronounced with the increase of the dipole moment kl^2, due to the increase of the coherent factor and the decrease of the ionization potential $I_{\rm ex}(k)$.

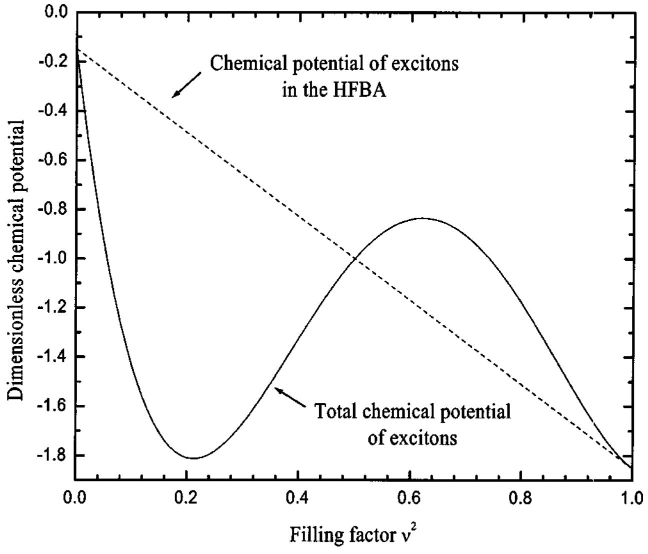

FIG. 3. Chemical potential versus filling factor v^2. Solid line: total chemical potential of the condensed magnetoexcitons at $kl=4.6$. Dashed line: chemical potential of the condensed magnetoexcitons in HFBA at $kl=4.6$.

At first, the local minimum of chemical potential appears for $kl=2.2$; it becomes deeper with further increase of kl. The relative minimum of the chemical potential of the Bose-Einstein condensed magnetoexcitons implies the formation of a metastable dielectric liquid phase with positive compressibility in this range of filling factor v^2. At the values $kl=4.6$, $I_{\rm ex}(k)=0.18I_l$, $\rho_0=4.6l$, and $v^2=0.25$, the minimum on the plot of the chemical potential achieves the same value as at the limiting point $v^2=1$. In spite of the fact that the curves in Figs. 2 and 3 are extrapolated up to the point $v^2=1$, it should be noticed that the applicability of the theory is limited by the upper boundary given by $v^2\approx\sin^2 v$, which means that we neglect the transitions to excited Landau levels.

Whether the dielectric liquid phase of the Bose-Einstein condensed magnetoexcitons or the electron-hole liquid (EHL) state is preferable depends on the energy of the system in these states. In the HFA, the ground state energy of an EHL can be found from the Hamiltonian of the Coulomb interaction of electrons and holes,

$$\begin{aligned}\hat{H}=&\frac{1}{2}\sum_{p,q,s}F_{e\text{-}e}(p,q;p-s,q+s)a_p^+a_q^+a_{q+s}a_{p-s}\\&+\frac{1}{2}\sum_{p,q,s}F_{h\text{-}h}(p,q;p-s,q+s)b_p^+b_q^+b_{q+s}b_{p-s}\\&-\sum_{p,q,s}F_{e\text{-}h}(p,q;p-s,q+s)a_p^+b_q^+b_{q+s}a_{p-s}\end{aligned} \tag{96}$$

averaging over the ground state. Using Wick's theorem and taking into account that all electrons and holes are in their lowest Landau levels, so that $\langle a^+a\rangle=\langle b^+b\rangle=\nu^2$ we obtain

$$\begin{aligned}E_{\rm EHL}=&\frac{\nu^4}{2}\left(\sum_{p,q}F_{e\text{-}e}(p,q;p,q)+\sum\,_{p,q}F_{h\text{-}h}(p,q;p,q)\right.\\&\left.-2\sum_{p,q}F_{e\text{-}h}(p,q;p,q)\right)-\frac{\nu^4}{2}\left(\sum_{p,s}F_{e\text{-}e}(p,p-s;p\right.\\&\left.-s,p)+\sum\,_{p,s}F_{h\text{-}h}(p,p-s;p-s,p)\right).\end{aligned} \tag{97}$$

From Eqs. (13) and (14) one can see that the direct Coulomb terms cancel each other, and only the exchange terms remain, so that after integration, we obtain

$$E_{\rm EHL}=-\nu^4 I_l N. \tag{98}$$

The energy per electron-hole pair is

$$\mathcal{E}=\frac{E_{\mathrm{EHL}}}{N_{e\text{-}h}}=-\nu^2 I_l \tag{99}$$

and has a global minimum at $v^2=1$, when the drop is formed of the completely filled Landau level. In Fig. 2, the dashed-dotted line corresponds to the energy per electron-hole pair of the metallic EHL. The energy of the metallic EHL is always above the energy of the dielectric liquid phase and has no local minimum.

VI. CONCLUSIONS

We have studied the coherent pairing of electrons and holes in an ideal 2D structure in a strong transverse magnetic field. The coherent pairing results in the Bose-Einstein condensation of 2D magnetoexcitons on the single-particle state with wave vector k. We have shown that within the model of the carriers confined to the lowest Landau level and low densities of electrons and holes, the correlation energy and the corrections to the chemical potential beyond the HFA result in the existence of a metastable dielectric liquid phase formed by the Bose-condensed magnetoexcitons. We have seen that this dielectric liquid phase of Bose-condensed magnetoexcitons with nonzero values of the dipole moments and comparatively low ionization potentials is more stable than the EHL phase. Although we have considered arbitrary wave vectors $\mathbf{k}$, the range of the main interest is $kl\geqslant 1$. The Keldysh-Kozlov-Kopaev method supplemented by the generalized random phase approximation was applied for the analysis. It was shown that the attractive interaction between excitons in the system prevails due to the existence of the dipole moments at $k\neq 0$ and the ground state of the Bose-Einstein condensed excitons is unstable within the HFB approximation. This fact leads to the tendency of the system to transform into another more stable phase. This new phase is a droplet of a metastable liquid dielectric phase formed by the magnetoexcitons with considerably large dipole moments $kl\leqslant 4.6$ and sufficiently low ionization potentials $I_l > I_{\mathrm{ex}}(k) \geqslant 0.18 I_l$. Each droplet has a well defined local direction of the inplane wave vector $\mathbf{k}$ as well as the inplane dipole moment $\rho_0=[\mathbf{k}\times\mathbf{z}]\ l^2$ perpendicular to $\mathbf{k}$. These values can slowly change in space and in time, for example, due to hydrodynamic evolution. A set of such droplets can form, for instance, a ring so that the wave vector $\mathbf{k}$ of the particular droplet coincides with the tangent to the ring, whereas the dipole moment is directed along the radius of the ring. Different structures of such type may appear if the energy per one e-h pair is the lowest one. This scenario differs from the case considered in Ref. 53. We have shown that this new phase may occur for the coherent excited states of the Bose-Einstein condensed magnetoexcitons even within the framework of the lowest Landau level approximation. Contrary to this approximation the correlation energy calculated in Refs. 34,35 and the indirect interaction in Ref. 37 are due to account of the excited Landau levels and they are proportional to a small parameter $l/a_{\mathrm{ex}}\approx I_l/\hbar\omega_c$. In our analysis we used the Anderson-type wave functions of the coherent excited states of the correlated pairs taking into account only the lowest Landau level. These excited states means the excitation and the unbinding of one of many e-h pairs bounded into Bose-Einstein condensate and its transformation into a free e-h pair. The energy of such unbinding equals to the ionization potential $I_{\mathrm{ex}}(k)$ of the magnetoexcitons with $k\neq 0$. The polarizability of the Bose-Einstein condensate with $k\neq 0$ is characterized by the coherent factor which equals to zero when $k=0$ as well as by the resonance frequency equal to the ionization potential $I_{\mathrm{ex}}(k)$. The correlation energy is the same for all condensed coherent excitons and therefore is proportional to the square of the coherence factor and inverse proportional to the ionization potential $I_{\mathrm{ex}}(k)$. The calculated correlation energy and the correction to the chemical potential do not contain any small parameters and have the same dependencies as energy and chemical potential in the HFB approximation. Comparing the polarizability of the Bose-Einstein condensed and noncondensed magnetoexcitons one can observe that the coherence factor which appears even if a single exciton is considered remained a common factor in the case of coherent macroscopic state and determines the existence of a superlattice with the period $2\pi/k$. Due to this fact the polarizability depends periodically on the wave vector $\mathbf{Q}$ of the longitudinal weak perturbation. In the case of noncondensed excitons the coherence factors of different excitons have different wave vectors and the average of these polarizabilities over the distribution function of the noncondensed excitons leads to a smooth dependence on $\mathbf{Q}$. This is the main difference of the polarizabilities of the condensed and noncondensed magnetoexcitons. They also have different resonance frequencies. The perturbative method have been used to simplify the calculation of the correlation energy. By this reason the correction to the chemical potential μ_{corr} cannot exceed the value μ^{HFB}. This means that we can consider only the limiting values $kl\leqslant 4.6$ and $I_l > I_{\mathrm{ex}}(k) \geqslant 0.18 I_l$. Another restriction of the theory is $\nu^2\approx\sin^2\nu$, which means that we can consider the phenomenon of the Bose-Einstein condensation of magnetoexcitons in the LLL approximation only in the limit of small filling factor $\nu^2<1$. This requirement does not concern the theory of EHL. The metastable states appear at the value of the filling factor $\nu^2\approx 0.25$ and this value satisfies the above restriction. To develop more general theory extended to the case of $\nu^2\approx 1$ it is necessary to generalize the definition of the exciton creation operator involving into its construction the creation operators of electrons and holes on the excited Landau levels.

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