Acoustic phonon-assisted tunneling in GaAs/AlAs superlattices

S. A. Cavill, L. J. Challis, A. J. Kent, F. F. Ouali,* A. V. Akimov, and M. Henini
School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, United Kingdom
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Measurements have been made of the transient changes in tunnel current ΔI through a biassed 50 period Si doped GaAs/AlAs superlattice produced by pulses of nonequilibrium acoustic phonons. The bias $V_{\max}$ at which ΔI is greatest varies linearly with the heater temperature T_h and hence with the dominant phonon frequency, the SL is acting as a phonon spectrometer with a linewidth (HWHM) $\lesssim k_B T_h / h \sim 200$ GHz (0.8 meV) for $T_h = 10$ K. The spectral response was confirmed using the phonons that are scattered and hence delayed in their passage across the substrate as a source of higher average frequency than those crossing ballistically. The measured values of $\Delta I(V)$ are compared with values calculated using a similar model to that of Glavin *et al.* [JETP Lett. **51**, 191 (2000)].

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I. INTRODUCTION

In a semiconductor superlattice (SL), the additional periodicity of the lattice potential splits the conduction band into a series of minibands of width Δ_m. If $\Delta_m \gtrsim \hbar/\tau$, where τ is the mean collision time, electrical conduction takes place through the minbands for small values of applied bias and hence electric field F.[1] However, at higher fields $eFNd > \Delta_m$, where d is the SL period and N is the number of periods, it is no longer possible for electrons to travel ballistically from one end of the SL to the other and the eigenstates become localized within a distance $\lambda \sim \Delta_m / eF$.[1,2] This localization splits each miniband into a ladder of levels separated by eFd which are well-defined if $eFd > \hbar/\tau$. These ladder states, each centered on a different quantum well (QW), are known as Wannier-Stark (WS) states[3] and transport takes place by tunneling (hopping) between them.[4] If F is large, so that $\lambda \sim d$, tunneling is essentially limited to states centered on neighboring wells but in lower fields it can also occur to wells further away. The tunneling can either occur by an elastic process involving impurity or disorder scattering with the subsequent emission of one or more phonons, or by inelastic phonon-assisted tunneling. If $\Delta_m \lesssim \hbar/\tau$, most of the states will be localized within the QWs, with transport taking place by tunneling between them for all values of F.

The electron density in the wells, due both to the doping and to charge movement under bias, can have the effect of replacing the single current peak in the $I(V)$ characteristic of undoped SLs by a series of peaks.[1] This multi-peak structure is attributed to the formation and expansion of electric field domains. At the first current peak, the system switches from a single domain of constant field throughout the SL to two (or more) domains of significantly different field strength separated by a domain boundary which contains space charge to provide the field gradient. Miniband transport occurs in the low field domain if the scattering is not too strong, while, in the high field domain, the potential energy drop between neighboring wells is $eFd \sim E_2 - E_1$ (E_1 and E_2 are the energies of the ground and first excited states in the well), close to the condition for resonant tunneling. As the bias is increased, a peak occurs in the differential conductance as the tunneling becomes resonant and this is followed by a nearly discontinuous fall as the high field domain suddenly expands by one period at the expense of the low field domain. The process is then repeated, leading to a series of stable conductance peaks in $I(V)$. The space charge arising from doping is not always enough for stable domains to form, although these can still occur if, under bias, the contact is able to supply enough additional electrons. If it is still insufficient, however, domains can form but their boundary is unstable. This results in oscillating currents so that the series of peaks in $I(V)$ is replaced by a plateau. The period of the oscillation is equal to the time the domain boundary takes to travel the length of the SL and the corresponding frequency can vary over a wide range depending on the domain velocity and the superlattice length.[5,6]

Phonon-assisted tunneling is a process in which the tunneling of an electron or hole through a barrier or barriers is accompanied by the absorption or emission of a phonon (for a recent review see Ref. 7). An electron with wavevector **k** and energy E_k tunnels into a quantum well and a phonon of wavevector **q** and energy $\hbar\omega_q$ is simultaneously absorbed or emitted. Phonon-assisted tunneling can occur both for optic modes, predominantly LO although much weaker TO phonon-assisted tunneling should also occur, and for acoustic modes, both LA and TA. The LO phonon emission process appears as a satellite peak in the $I(V)$ of resonant tunneling diodes (RTDs) which was first reported in 1987 in a double barrier resonant tunneling diode (DBRTD)[8] and shortly afterwards in a triple barrier resonant tunneling diode (TBRTD).[9] However, in a superlattice, although LO phonon-assisted tunneling contributes significantly to the transport current I, no satellite peak is seen in zero magnetic field but peaks appear when fields B are applied parallel to I. The first measurements of this type were by Higman *et al.*[10] A more detailed investigation was made by Müller *et al.*[11] who observed a number of resonant peaks in the photoconductivity, some of which could be assigned to LO phonon-assisted tunneling between Landau levels.

Phonon-assisted tunneling involving acoustic modes is not immediately apparent in the $I(V)$ characteristics of RTDs and SLs. However, the process can be seen using differential techniques; techniques in which a change in $I(V)$ is produced by a change in phonon occupation number at the device. This was the approach used in both the first observation

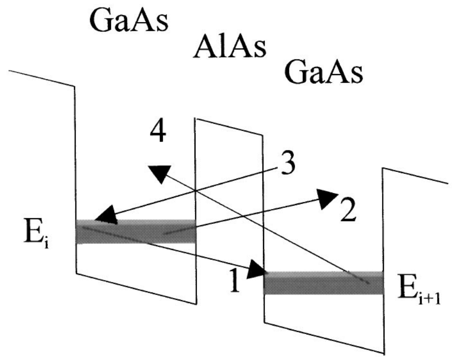

FIG. 1. Phonon-assisted tunneling processes.

of acoustic phonon-assisted tunneling which was in a DBRTD[12] and in subsequent work on a TBRTD.[13] The reason why the effects of acoustic phonon-assisted tunneling are less marked than those of the LO phonon process can be attributed to the weaker electron-phonon coupling of acoustic modes together with the broader frequency range of modes involved in the assisted processes.

The first measurements of acoustic phonon-assisted tunneling in a superlattice were reported briefly in 1999 by Cavill *et al.*[14] The devices used were GaAs/AlAs SLs that were uniformly doped with Si. The present article describes these experiments in more detail and includes later measurements made using a complementary technique which achieves greater time resolution. It also compares the experimental data with calculated values for the assisted current due to the four possible one-phonon-assisted tunneling processes shown in Fig. 1. Two of these correspond to stimulated phonon emission and two to absorption.

II. EXPERIMENTAL DETAILS

The approach used, which is similar to that used in the work on RTDs,[12] is to observe the effect on the tunnel current $I(V)$ of increasing the occupation number of phonons incident on the SL device. All the SL devices used were taken from NU1727 which is an $N=50$ period GaAs/AlAs SL grown by MBE on a 400 μm thick semi-insulating substrate. The wells were 5.9 nm wide and the thickness of the barriers, 3.9 nm, was chosen to provide weak coupling: estimates for the ground state miniband width from the Kronig-Penney and Bastard[15] models are, respectively, $\Delta_m \sim 1.0$ and 0.7 meV. The structure was uniformly doped with Si to 2×10^{16} cm^{-3} which leads to a sheet density per well, n_s, of 8×10^{9} cm^{-2} and hence to $E_F\sim0.3$ meV although these values should increase with bias as noted earlier. The superlattice was separated from the n^+ contact regions (2×10^{18} cm^{-3}) by 20 nm thick undoped spacer layers and etched down to a 50 μm diameter mesa.

A nonequilibrium phonon pulse is generated by heating a metal film deposited on the opposite surface of the substrate from the SL, Fig. 2, and produces a transient change in tunnel current $\Delta I(t)$ on arriving at the SL device. Two complementary techniques are used to heat the film. In the electrical technique, a constantan film of area $A=50\times500$ μm^2 evaporated opposite the device is heated by 50–100 ns electrical pulses. In the optical technique, an approximately 50 μm diameter area of an extended constantan film is heated by 12 ns focused NdYAG laser pulses. In both cases, the substrate is cooled to $T=1.8$ K by helium gas. The electrical technique has the advantage that the power P absorbed in the film is accurately known so that the temperature T_h the film reaches for the duration of the pulse can be determined from the rate at which the power is emitted into the GaAs substrate as acoustic phonons. This can be calculated[16] from the acoustic properties of constantan and GaAs and is given by the expression $P/A=\sigma(T_h^4-T^4)$ Wm$^{-2}\cong\sigma T_h^4$ Wm^{-2} since $T_h^4\gg T^4$, where $\sigma=524$ Wm^{-2} K^{-4} (this neglects the small amount of cooling by the gaseous helium). This technique does, however, have the disadvantage that electrical pick-up prevents the use of the shorter pulses needed to separate the contributions to $\Delta I(t)$ from LA and TA modes. The source position is also fixed so that several heaters are needed if measurements are to be made as a function of the mean incident angle θ of the phonons. In the optical technique, the intensity of the beam is varied by inserting neutral density filters. The intensities, and hence the *relative* powers absorbed in the constantan film, are measured using a Si photodiode with a 1 ns rise time. Absolute powers and hence approximate T_h values are obtained from these by calibrating the bias at the peak of the $\Delta I(V)$ plot for the TA mode at one optical intensity against those of plots obtained using electrically heated films of known T_h.[17] The advantages of the optical technique are that much narrower pulses can be used, since the optical pick-up is short-lived, and that θ can be varied by moving the laser spot across the film using computer-controlled mirrors.

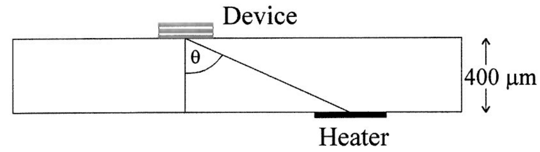

FIG. 2. Experimental arrangement.

Finally, we note that semi-insulating GaAs at these temperatures is very nearly transparent to phonons at frequencies less than ~1000 GHz although there is still some scattering by isotopes and traces of magnetic ions.[18] This increases rapidly with frequency so that the two ballistic peaks in $\Delta I(t)$ consist largely of phonons with $\nu\leqslant1000$ GHz. The scattered phonons travel diffusively and hence more slowly so that the ballistic peaks are followed by diffusive tails due to phonons whose average frequency is higher than that of those travelling ballistically.

III. EXPERIMENTAL RESULTS

A typical $I(V)$ characteristic is shown in Fig. 3. For both bias directions the conductance does not start until $V_0^{(1)}$ ~50 mV, presumably the bias needed to align the Fermi energies of the emitter and the nearest well. At higher biases, $I(V)$ becomes strongly asymmetric, suggesting it is affected by the differences in the emitter contacts: an extended sheet for $V<0$, and a 50 μm diameter disc for $V>0$. For $V<0$, the curve shows a peak at $V_c=-275$ mV followed by a series of

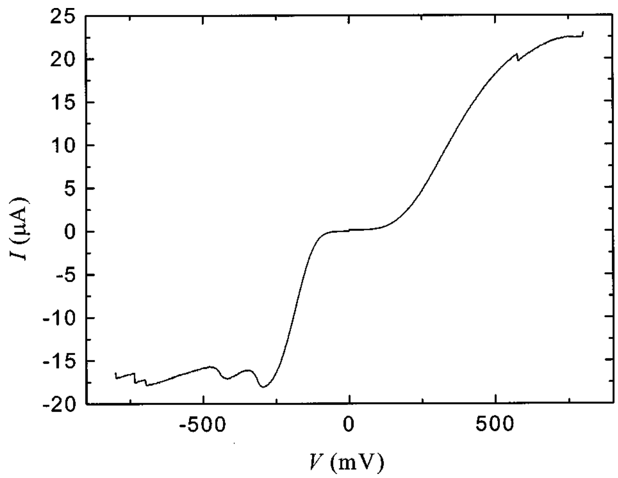

FIG. 3. $I(V)$ characteristic.

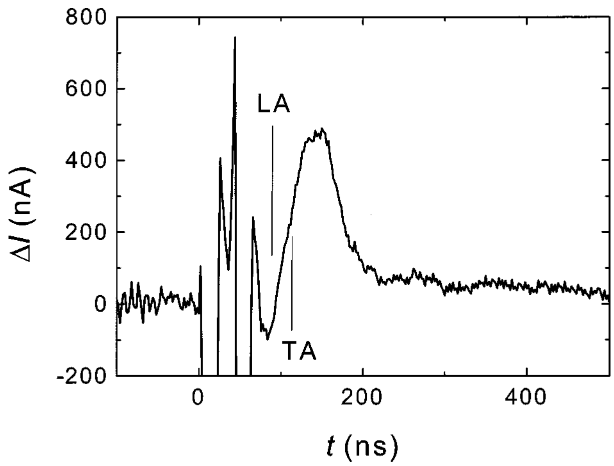

FIG. 4. Response $\Delta I(t)$ to a phonon pulse from an electrically heated film at $T_h = 13.3$ K for $V = +200$ mV.

time-independent peaks in $I(V)$ attributed to domain formation and expansion. For $V>0$, a single peak occurs at $V_c = +700$ mV followed by a plateau, the time-independent oscillations being replaced by oscillating currents. This suggests that the sheet densities in the wells are too small for stable domains to form when $V>0$[6] but not when $V<0$. Presumably, when $V<0$, enough electrons flow under bias from the extended emitter into the quantum wells to give rise to space charge large enough to stabilize the domain boundary. This is evidently not in general the case for $V>0$. However, the bigger value of V_c, 700 mV, that is found for $V>0$ does suggest, however, that a large fraction of the bias is dropping across part of the SL. The reason for this is not understood, but it might perhaps be due to the presence of a single stable high field domain next to the emitter. A value of $V_c = +275$ mV (equal in magnitude to that for $V<0$) would be obtained if the potential drop across the rest of the SL were $0.35(V - V_0^{(1)})$ where $0.35 = (275 - V_0^{(1)})/(700 - V_0^{(1)})$.

In the previous section it was noted that transient changes in $I(V)$ are produced by phonons emitted from a heated film located on the other side of the substrate and that two different techniques were used to heat the film. We first describe the results obtained using an electrically heated film directly opposite the device. Measurements of $\Delta I(t)$ were made as a function of bias for a range of values of T_h and an example is given in Fig. 4 for $V = +200$ mV and $T_h = 13.3$ K. The vertical arrows show the calculated arrival times for LA and TA phonons traveling ballistically across the substrate. The LA arrow coincides with the onset of the rise in $\Delta I(t)$ and, while there may be a point of inflection at the arrival time for TA phonons, it is not possible to resolve the effects of the two modes. The average value of $\Delta I(t)$ over the interval between the TA arrow and the peak in $\Delta I(t)$ was used to obtain the values of $\Delta I(V)$ shown in Fig. 5(a) for $V>0$ and a range of values of T_h. Figure 6 shows that the magnitude of the bias V_{max} at the maximum value of $\Delta I(V)$ increases linearly with T_h. This can be written $\Delta_{max} = 0.7k_BT_h + 0.25$ meV where $\Delta = \gamma e(V - V_0^{(1)})/50$ is the energy drop between neighboring wells (this assumes the fraction, $\gamma = 0.35$, of the applied voltage across the SL is the same at these lower positive biasses as at $V = +700$ mV) and Δ_{max} is the value of Δ at $V = V_{max}$. The form of $\Delta I(V)$ for $V<0$ is similar to that for $V>0$. Electrical measurements were only taken for three values of T_h but, if a linear dependence at low bias is assumed, $\Delta_{max} \sim 0.6k_BT_h + 1.0$ meV ($\gamma = 1$ for $V<0$).

We next discuss the results obtained using the optical technique with the heated area opposite the device. Figure 7 shows $\Delta I(t)$ for $V = -130$ mV and $T_h = 7.6$ K. The phonon pulses using this technique are much shorter, 12 ns, than the difference in the arrival times of the LA and TA ballistic phonons so that the LA contribution to $\Delta I(t)$ has time to decay significantly before the TA pulse arrives. The two contributions are now resolved so that values of $\Delta I(V)$ can be determined for each and the TA peak is seen to be roughly twice the size of the LA peak. The relative sizes will be affected by the fact that the [001] direction of the normal is a focusing direction for TA phonons but not for LA phonons. Peaks in $\Delta I(t)$ can also be seen from phonons that have made three transits of the substrate before interacting with the SL demonstrating that a substantial fraction of the phonons have mean free paths greater than 1.2 mm (three times the substrate thickness). $\Delta I(V)$ plots for $V<0$ and for TA phonons are shown in Fig. 5(b) for various values of T_h and are seen to be similar to those obtained using the electrically heated source although appreciably smaller. A few $\Delta I(V)$ plots were also taken for LA phonons and found to be very similar in form to those in Fig. 5(b). $\Delta I(V)$ was approximately half the size but the peaks coincided with those for TA phonons. Figure 6 shows that the dependence of V_{max} on T_h obtained from the plots of Fig. 5(b) corresponds to $\Delta_{max} = 0.7k_BT_h + 1.1$ meV which is in good agreement with the dependence obtained using the electrical technique for $V<0$. In summary then, the slope of $\Delta_{max}(T_h)$ is the same for both bias directions but the intercept is appreciably bigger, 1.1 meV, for $V<0$ than for $V>0$, 0.25 meV, and we tentatively attribute this to the larger sheet density and so Fermi energy for $V<0$ that is indicated by the presence of stable domains as discussed earlier.

The size of the $\Delta I(V)$ signal within the gated time interval for the electrical heater is approximately five times larger than that of $\Delta I(V)$ for the TA phonons from an optically

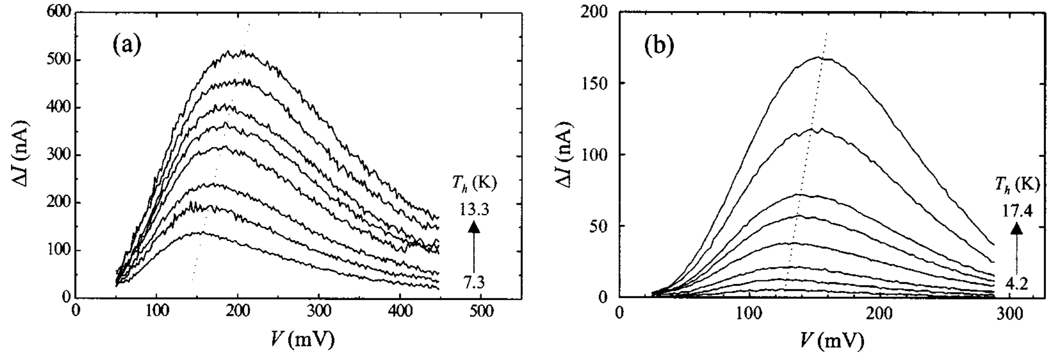

FIG. 5. Bias dependence of the current change due to TA phonons, $\Delta I(V)$ for a range of heater temperatures T_h. (a) $V>0$, electrical technique; (b) $V<0$, optical technique.

heated area at the same temperature and we attribute this to its appreciably larger area. It also seems likely to contain a significant contribution from LA phonons emitted towards the end of the electrical pulse, particularly from the furthermost parts of the electrical heater whose distance to the SL is nearly 20% greater than the substrate thickness. The relative size of this LA contribution compared with that from the TA phonons will be enhanced by the fact that the incident angles of the TA phonons emitted from the furthermost parts of the electrical heater are well outside the focusing directions. Allowing for these factors suggests that the LA contribution to the signal from the electrical heater might be comparable to that of the TA phonons. The use of $V_{\max}$ values from these data to calibrate T_h for the TA data from the optical heater is justified, however, by the fact that $V_{\max}$ is found to be essentially the same for both modes.

The predominant electron-phonon interaction at the frequencies of most of the phonons emitted in these experiments is expected to be by deformation coupling. This is evidently not the case for TA modes in GaAs with k parallel or close to the [001] direction normal to the plane of the substrate but has been shown to be[19] for off-axis TA modes. So if any q-dependent factors associated with the discrete nature of the structure were neglected, the transition probability for phonons incident normally should vary approximately as $q^3 n_q$ and we should expect $\Delta I(V) \propto \omega_q^3 n_q$ where $\hbar\omega_q \sim \Delta$. This would lead to a maximum at $\Delta_{\max} \approx 2.8 k_B T_h$ and hence to a slope of $\Delta_{\max}(k_B T_h)$ which is four times greater than that found experimentally. This difference is largely attributed to the fact that, in the experiments, most of the phonons are not incident close to the normal but at angles up to some maximum $\theta_{\max}$ so that they have appreciable in-plane momentum. The significance of this can be seen by considering, for example, process 1 in Fig. 1 which involves forward transitions assisted by phonon emission. To conserve in-plane momentum, the energy difference between the initial and final electron states must be Δ for phonons incident normally and $<\Delta$ for those incident obliquely. Hence the average frequency and so wave number of the phonons involved falls with $\theta_{\max}$ and this has the effect of displacing the slope of $\Delta_{\max}(k_B T_h)$ to values <2.8. The q-dependent factors associated with the layered structure of the SL will also weigh the frequency dependence of the phonon contribution to the assisted tunneling in some way which in turn will affect the value of the slope. These aspects will be considered in a later section together with the apparent correlation of the intercept with the Fermi energy.

The linear dependence of $V_{\max}$ on the dominant phonon frequency indicates that the SL device is acting as a tunable

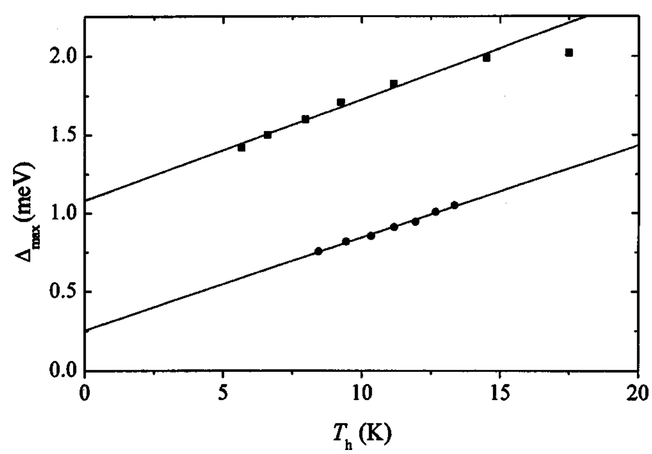

FIG. 6. The energy separation of the wells at the maximum of $\Delta I(V)$, $\Delta_{\max}$, as a function of heater temperature T_h for both bias directions. The data are obtained from Fig. 5(a) (lower values) and (b) (upper values).

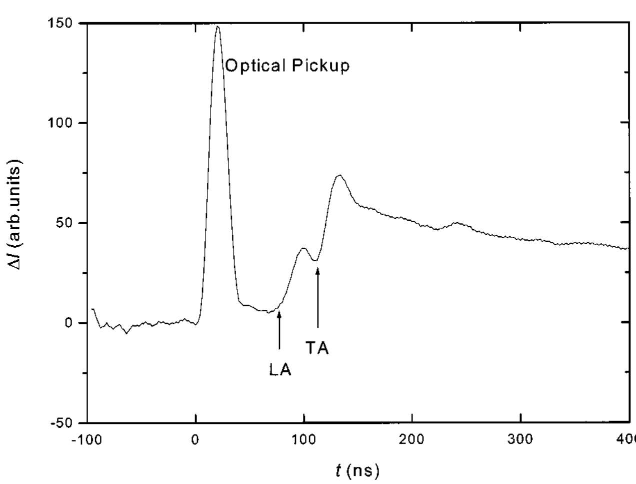

FIG. 7. Response $\Delta I(t)$ to a phonon pulse from an optically heated film at $T_h = 7.6$ K for $V = -130$ mV.

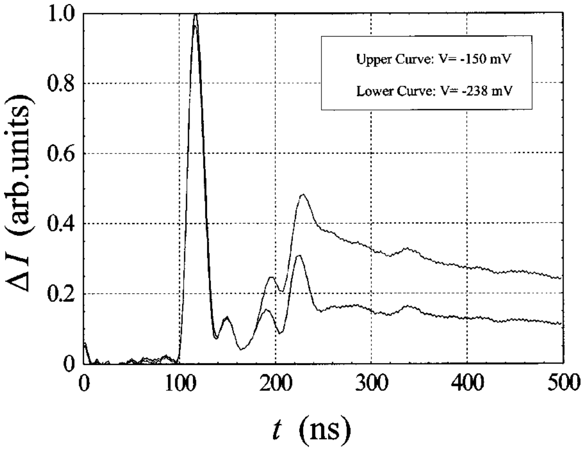

FIG. 8. Response $\Delta I(t)$ to a phonon pulse from an optically heated film at $T_h = 14.5$ K for two bias values. The ratio ΔI(tail)/ΔI(peak) decreases with bias from 0.8 (-150 mV) to 0.5 (-238 mV).

frequency sensitive detector with a small bandwidth, i.e., a spectrometer. A further demonstration of this was obtained using the fact that the probability of phonon scattering in the substrate increases with frequency. Because of this, the unscattered phonons that produce the ballistic peaks in $\Delta I(t)$ have a lower average frequency than the scattered and hence delayed phonons in the diffusive tails. The measurements were made using the optical technique and with $T_h = 14.5$ K so that somewhat over half the phonons emitted have frequencies above the peak in the distribution at $\nu \sim 2.8 k_B T_h / h \sim 900$ GHz. These would mostly propagate diffusely and contribute to the tail of $\Delta I(t)$. The rest would have $\nu < 900$ GHz and would make the major contribution to the LA and TA ballistic peaks. Measurements were made for two bias values, -150 and -238 mV, and hence energy splittings $\Delta = h\nu = e(V - V_0^{(1)})/50$ corresponding to $\nu = 480$ and 910 GHz, respectively. For the idealized case in which the phonons are incident normally and the effect of the SL of the q-dependence of the phonon interaction can be neglected, these are also the phonon frequencies that would be expected to contribute most to ΔI. In practice, for the reasons discussed above, the actual frequencies detected are expected to be lower than both of these values but would still be larger for a bias of -238 mV than for a bias of -150 mV. When biased at the higher value the SL should detect higher frequency phonons compared to when biased at the lower value and so the proportion of scattered to unscattered phonons detected should be appreciably greater at this bias. So if indeed the response of the SL is frequency dependent, we should expect ΔI (diffusive)/ΔI(ballistic) to be greater at -238 mV than at -150 mV. Figure 8 shows $\Delta I(t)$ obtained for the two bias values. It can be seen that ΔI(diffusive)/ΔI(ballistic) for TA phonons is about 30% larger at $V = -238$ mV than at -150 mV [ΔI (diffusive) is obtained from the value of $\Delta I(V)$ 20 ns afterthe ballistic peak] confirming that the response of the SL device is frequency dependent.

Measurements of $\Delta I(V)$ made as a function of θ for both TA and LA phonons using the optical technique with $T_h = 7.6$ K show that the signal size falls steadily as θ increases, Fig. 9. However, $V_{\max}$ remains approximately constant for LA phonons, and, for TA phonons, it only decreases slightly, by less than 10%, when θ is increased from 0° to 30°. The phonon distribution has a peak frequency of about 450 GHz so that most of the phonons should travel ballistically to the SL. The fall in signal with θ is mainly attributed therefore to the decrease in the proportion of the phonons leaving the heater that fall in the solid angle subtended by the device as the distance between them is increased (for TA phonons there will be the additional effect of moving away from a focusing direction). The fact that $V_{\max}$ depends so weakly on angle is somewhat surprising, however. The frequency distribution should not be affected by the change in angle but the angular distribution of the q vectors would be and the arguments presented earlier would suggest that this should lead to a decrease in $V_{\max}$ as θ is increased.

The form of $\Delta I(V)$ in a magnetic field $B = 6.5$ T ($B \| I$) for $T_h = 7.6$ K is similar to that in zero field[17] and it would be of interest to extend this work to see if $V_{\max}$ remains linearly dependent on T_h. If it did, the SL device should still act as a spectrometer in the presence of magnetic fields.

IV. THEORETICAL ANALYSIS AND DISCUSSION

The potential energy drop over a SL lattice period is greater than Δ_m over most of the range at which phonon experiments were made. This implies that transport is by hopping rather than miniband transport. The first theoretical treatment of acoustic phonon-assisted tunneling between the WS states of SLs was by Tsu and Döhler[4] who calculated its contribution to electrical transport. At low fields and for narrow barriers, the WS states may extend over several periods so that assisted tunneling can take place between wells some distance apart. However, at higher fields, or at all fields if the barriers are sufficiently thick, the transport is dominated by nearest neighbor transitions leading to a "two well model." This model was extended by Glavin *et al.*[20] to the situation of the present experiments where the incident phonons have a temperature T_h higher than that of the electrons T and they also calculated the contribution to $\Delta I(V)$ from electron heating. The present treatment is similar to that of Glavin *et al.* although the parameters used were chosen to model more closely the conditions of the experimental work.

The transient change in current ΔI is calculated by including the four processes shown in Fig. 1 although omitting spontaneous emission since this is not affected by the incident phonons. ΔI can be written

$$\Delta I = eA \sum_{\mathbf{q}} W(\mathbf{q}) (n_q^*(T_h) - n_q^*(T)),$$

where A is the area of the device and the transition probability, $W(\mathbf{q}) = W_+^+ + W_+^- - W_-^+ - W_-^-$ is the sum of the probabilities of the four possible stimulated emission and absorption processes shown in Fig. 1; the superscript $+/-$ indicates emission/absorption and the subscript indicate the direction of the electron current with respect to the bias. Since only a proportion of the phonons from the heater are incident on the

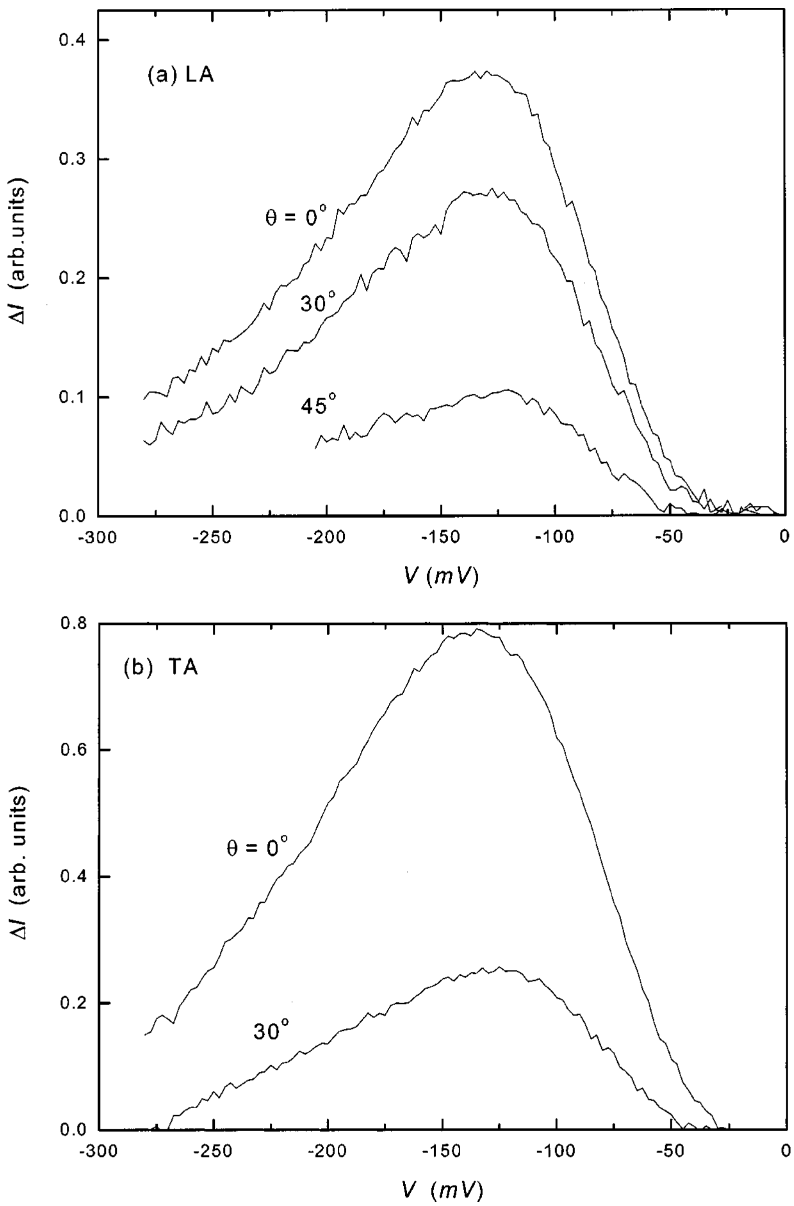

FIG. 9. $\Delta I(V)$ from an optically heated film. θ is the angle to the normal of the line joining the center of the heated area to the center of the SL device. (a) LA phonons and (b) TA phonons.

device, their occupation number is written $n_q^* = a n_q$ where $a<1$. For simplicity, a is assumed to be constant, so its introduction only reduces the size of ΔI from that given by Glavin *et al.* and not its form. The term $-n_q^*(T)$ is included here so that $\Delta I=0$ when $T_h=T$ but omitted by Glavin *et al.* since they assume $T=0$. This should have little effect on ΔI and is omitted from now on:

$$W(\mathbf{q}) = \frac{2\pi}{\hbar} C^2(q)|M(q_z)|^2 \sum_{\mathbf{k},\mathbf{k}'} \delta_{\mathbf{k},\mathbf{k}'\mp\mathbf{q}_\parallel} \times \delta(E_k - E_{k'} \mp \hbar\omega_q) \times f_{\mathbf{k}}(1-f_{\mathbf{k}'}),$$

$C(q) = iD(\hbar q/2\rho\Omega s)^{1/2}$ where ρ is the density, Ω is the volume of the device and s is the velocity of sound. ψ,ψ' and $\mathbf{k},\mathbf{k}'$ are respectively the electron wave-functions and in-plane wavevector components of the initial and final states in the two neighboring wells involved in the tunneling. The phonon matrix element between the two states is given[4] by

$$|M(q_z)|^2 = |\langle \psi'|\exp(iq_z z)|\psi'\rangle|^2 \sim \left[\left(\frac{\alpha}{\Delta}\right) \sin\left(\frac{q_z d}{2}\right) \frac{\sin\left(\frac{1}{2}q_z w\right)}{\frac{1}{2}q_z w} \times \frac{1}{1-(q_z w/2\pi)^2} \right]^2 \quad (1)$$

where α is the coupling between the wells, d is the period of the SL and w is the width of the QWs.

For phonons incident at angles $\theta \leqslant \theta_{\max}$, the change in current is the sum of the changes due to the two emission and two absorption terms and can be written

$$\Delta I(\Delta)=KA\int_0^{\theta_{\max}}\int_0^{\infty}\int_{E_m}^{\infty}\frac{|M(q_z)|^2q^2n_q(T_h)f(E)(1-f(E\mp\hbar\omega_q\pm\Delta))}{(E-E_m)^{1/2}}d\theta dqdE, \tag{2}$$

where the signs before $\hbar\omega_q$ and Δ are respectively the reverse of and equal to those of the superscript and subscript of the particular term in $W(q)$,

$$K=\frac{eaD^2m^{*3/2}}{8\pi^4 2^{1/2}\hbar^3\rho s},$$

and E_m is the minimum value of E for which tunneling is possible for phonons q incident at angle θ. For a particular value of Δ, conservation of energy and in-plane momentum determine the two values of $k\cos\beta$ that can interact with a phonon $\mathbf{q}$ incident at angle θ:

$$k\cos\beta=\frac{m^*}{\hbar^2q_\parallel}\left(\frac{\hbar^2q_\parallel^2}{2m^*}\pm\hbar\omega_q\mp\Delta\right), \tag{3}$$

where β is the angle between $\mathbf{k}$ and $\mathbf{q}_\parallel$. In this case, the signs before $\hbar\omega_q$ and Δ are respectively equal to and the reverse of those of the superscript and subscript of the particular term in $W(q)$. The minimum electron energy E_m at which interaction is possible with phonons of wavenumber q incident at angle θ occurs when $\beta=0$ and is given by

$$E_m=\frac{m^*}{2\hbar^2q_\parallel^2}\left(\frac{\hbar^2q_\parallel^2}{2m^*}\pm\hbar\omega_q\mp\Delta\right)^2,$$

where $q_\parallel=q\sin\theta$. The integral over E is integrated by parts to avoid the singularity at $E=E_m$.

$\Delta I(\Delta)$ has been calculated for both TA and LA phonons for heater temperatures T_h in the range 4 to 10 K assuming $T=1.5$ K, $w=6$ nm, $d=10$ nm, $m^*=0.067m_e$, $\theta_{\max}=5°$, 10° and 15° and $E_F=0.3$, 0.6 and 1.0 meV. An example of the data is given in Fig. 10 for TA phonons assuming $E_F=0.6$ meV and $\theta_{\max}=10°$. The curves are seen to be qualitatively similar to the experimental plots in Fig. 5; since the values of a and α are not well known, $\Delta I(\Delta)$ is expressed in arbitrary units in Fig. 10. For $\theta_{\max}=5°$, however, the calculated curves become negative at low bias. Absorption processes dominate in this limit and the total current becomes negative when the negative assisted current ΔI_- due to absorption becomes larger than the corresponding positive current ΔI_+. This can also be seen in Fig. 11 which shows calculated values of $\Delta I(\Delta)$ for LA phonons by Glavin *et al.*[20] for $E_F=0.8$ meV and $\theta_{\max}=6°$. The values were calculated for a superlattice of similar period, $d=10.5$ nm, but an appreciably smaller well width, $w=4.5$ nm, than that used in the present work.

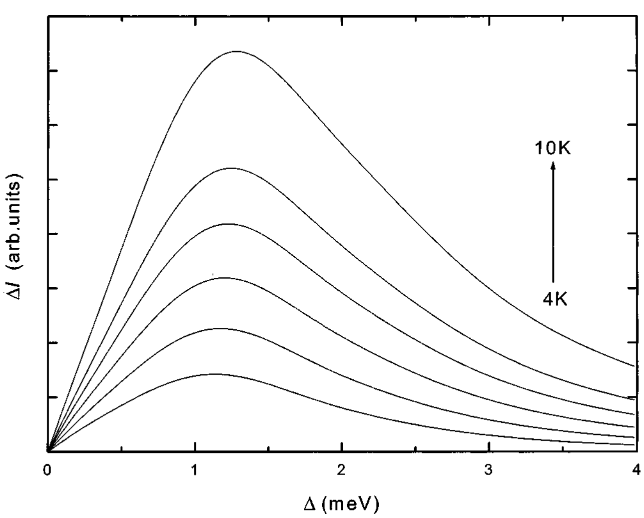

FIG. 10. Calculated values of $\Delta I(\Delta)$ due to TA phonons for a range of heater temperatures T_h for $\theta_{\max}=10°$ and $E_F=0.6$ meV.

The values of the bias at the peaks in Fig. 10 can be expressed approximately as $\Delta_{\max}\sim0.25k_BT_h+1.1$ meV. So for $E_F=0.6$ meV and $\theta_{\max}=10°$, the slope is appreciably smaller than the measured value of 0.7 while the intercept is similar to the measured value for $V<0$ but appreciably larger than that found for $V>0$. We recall that the difference in the measured values was attributed to the different values of E_F expected for these two bias directions but the calculated intercept values are in fact essentially independent of E_F. In contrast, the measured slopes are the same for both bias directions, while the calculated values are the same for $E_F=0.3$ and 0.6 meV but rise for $E_F=1.0$ meV to become approximately equal to the measured value. Qualitatively similar results are found for LA phonons, the main difference being that the calculated slopes and intercepts are somewhat bigger than those for TA phonons.

The small slope of $\Delta_{\max}(k_BT_h)$ given by the model is attributable partly to the angular distribution of the phonons and partly to the form of the phonon matrix element M. Figure 12 shows plots of $M^2(\hbar\omega)$ and $M^2(\hbar\omega)q^3n_q$ for a

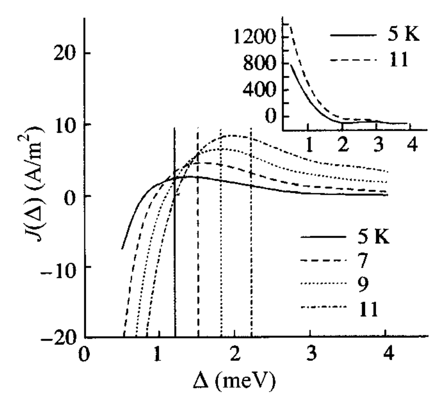

FIG. 11. Calculated values of $\Delta J(\Delta)$ for LA phonons from Glavin *et al.*[20] The inset shows the contribution due to electron heating.

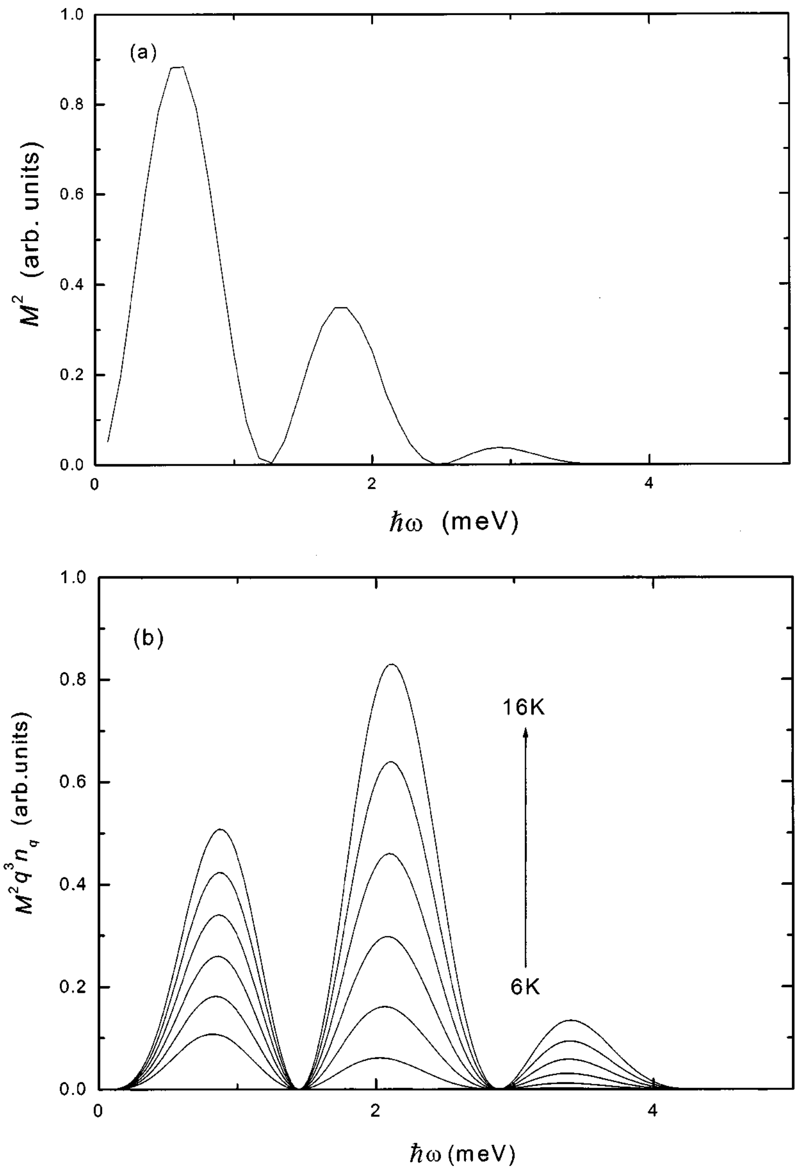

FIG. 12. (a) $M^2(\hbar\omega)$ and (b) $M^2(\hbar\omega)q^3n_q$ for $T=1.5$ K for TA phonons incident normally ($q_z=q=\omega/s$) on a SL with $d=10$ nm and $w=6$ nm.

SL with $w=6$ nm and $d=10$ nm for TA phonons incident normally so that $q_z=q=\omega/s$. The predominant features in $M^2(\hbar\omega)$ are the zeros (antiresonances) which occur when $q_z=0$ and $2n\pi/d$ ($n=1,2,35\ldots$) and the overall decrease as ω^{-4} when $q_z>2\pi/w$. The plot of $M^2(\hbar\omega)q^3n_q$ shows that the first two peaks are dominant in the temperature range of interest and that both move slightly to higher values with increasing T_h by amounts which correspond to slopes of $\Delta_{\max}(k_BT_h)$ ~0.2 and 0.3, respectively, close to the values found using Eq. (2), which of course includes the effects of energy and momentum conservation and the Fermi functions and is also integrated over a range of incident angles which should make the antiresonant effects less pronounced.

Figure 11 also shows calculations[20] of the change in tunnel current $\Delta I_s(\Delta)$ associated with transitions due to elastic scattering. The change occurs because of the increase in electron temperature resulting from the absorption of nonequilibrium phonons (electron heating). ΔI_s falls steadily with increasing bias and eventually becomes negative, and, since this behavior is very different from that observed for $I(V)$, we conclude that the contribution from electron heating is small compared with that by phonon-assisted tunneling.

A. Monchromatic phonons

In view of the potential of the SL for phonon spectroscopy, we have also calculated the response $\Delta I(\hbar\omega_0)$ to a quasi-monochromatic beam of TA phonons of frequency ω_0 by replacing the thermal frequency distribution by a Gaussian, $\exp(-\gamma(\omega-\omega_0)^2)$ with $\gamma=1000$ s^2. Figure 13 shows how, for $\Delta=3$ meV, $E_F=0.7$ meV and $T=1.5$ K, $\Delta I(\hbar\omega_0)$ varies as $\theta_{\max}$ is increased from 5° to 30°. For these angles, there is a small peak in ΔI at $\hbar\omega_0\sim\Delta$ which is barely dis-

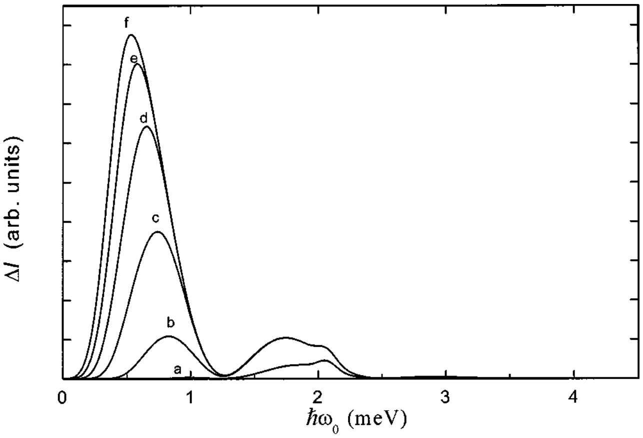

FIG. 13. Calculated values of $\Delta I(\hbar\omega_0)$ for TA phonons with a Gaussian frequency distribution $\exp(-\gamma(\omega-\omega_0)^2)$ with $\gamma = 1000\ \mathrm{s}^2$. The values shown are for $\Delta=3$ meV, $E_F=0.7$ meV and $T=1.5$ K. Curves a, b, c, d, e, and f correspond to $\theta_{\max}=5°,10°,15°,20°,25°$ and 30°, respectively.

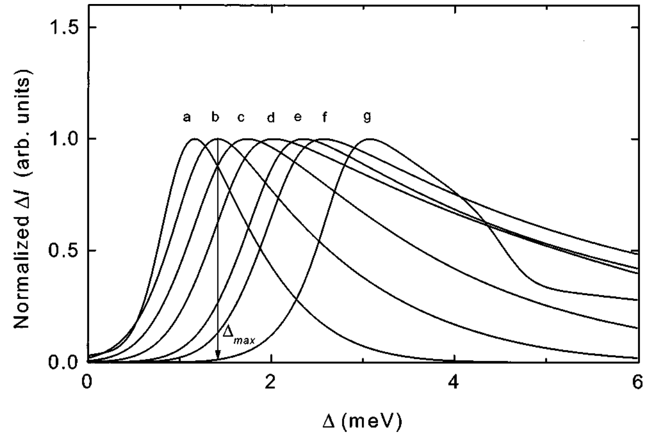

FIG. 14. Calculated values of the dominant positive emission contribution to $\Delta I(\Delta)$ for TA phonons with the Gaussian distribution of Fig. 13 for a range of values of $\hbar\omega_0$ assuming $E_F=0.7$ meV, $T=1.5$ K and $\theta_{\max}=15°$. Curves a, b, c, d, e, f and g correspond to $\hbar\omega_0=0.26$, 0.52, 0.77, 1.0, 1.3, 1.55 and 2.1 meV, respectively.

cernable on this scale but becomes the dominant feature at even lower angles when $\Delta \gg E_F,\ k_BT$ as is in the present calculations. The dominant features from 5° to 30° are seen to be two intermediate frequency peaks between 1.2 and 2.2 meV and a low frequency peak below 1.2 meV. The height of the low frequency peak increases steadily with $\theta_{\max}$, reflecting the increase in the number of allowed transitions, and then saturates showing that transitions can only occur for $\theta \lesssim 25°$. The intermediate frequency peaks also rise with increasing $\theta_{\max}$ but saturate at a lower angle showing that the peak is due to transitions occurring at $\theta \lesssim 10°$ The contribution to $\Delta I(\hbar\omega_0)$ from the positive emission process is dominant at most angles.[21] An approximate value of the spectral line width $\Delta\nu$ (HWHM) is given by that of the low frequency peak in Fig. 13. For $\theta_{\max}=15°$ which should correspond approximately to the geometry of the experiments for TA phonons if some allowance is made for focusing, $\Delta\nu \sim 100$ GHz. This value could be increased by interface roughness and scattering which are not included in the model although recent estimates suggest the scattering rates from these are $\leqslant 100$ GHz.[22]

The potential of the SL as a spectrometer can also be seen in the plots in Fig. 14 of $\Delta I(\Delta)$ for fixed values of $\hbar\omega_0$ and for $\theta_{\max}=15°$; the plots have been normalized to the same height. The values of $\Delta_{\max}$ at the peaks in ΔI increase approximately linearly with $\hbar\omega_0$ with a slope of 1.1. [The $\Delta I(\Delta)$ values are for the positive emission term which dominates at this angle.] It would also be of interest to analyze the effects of quantizing magnetic fields since the requirement $q_{\parallel} \lesssim 1/l_B$, where $l_B=(\hbar/eB)^{1/2}$ is the magnetic length, should reduce the range of incident angles at which interaction can occur and hence also reduce the spectral linewidth.

B. Discusssion

The theoretical model of Glavin *et al.* provides a qualitative explanation of the experimental data but there are differences in the dependence of the maximum of $\Delta I(V)$ on electron density and on T_h. This might be the result of a number of simplifications made in the model and its application to the experimental situation for TA phonons including for example, the neglect of anisotropic effects in both the phonon propagation (phonon focusing) and the electron-phonon interaction, the omission of piezoelectric coupling and the neglect of screening. More detailed calculations including these effects would clearly be of interest. We have also not considered the possibility that, as a result of stimulated phonon emission, amplification might be taking place when the phonons pass through 100 wells, 50 before reflection at the top surface and 50 after. How significant this would be evidently depends on the strength of processes resulting in losses, but Glavin *et al.*[20,23,24] concluded that these should be sufficiently weak at low temperatures for amplification to be present at least for modes traveling close to the normal. Phonon amplification has been observed in a number of other systems, most recently in optically pumped magnetic ions in dielectrics for phonon frequencies $\nu < 100$ GHz[25] and in two-level systems in glasses for $\nu < 1$ GHz.[26] However, it has not apparently previously been seen above 100 GHz.

As already noted, uncertainties in the values of a and α make it difficult to compare the measured and calculated values of ΔI and an alternative approach is to compare values of $\Delta I/I$. We consider the change in current due to TA modes. Before the phonon pulse arrives at the SL, the current component I_{sp} due to phonon-assisted tunneling is predominantly the result of spontaneous emission since $n_q \ll 1$ at the SL temperature T while the component due to stimulated emission is proportional to $n_q(T_h)$. Now since the stimulated and spontaneous emission transitions follow the same selection rules, the earlier discussion based on the results shown in Fig. 13 shows that they are both only possible for emission angles $\theta \lesssim 25°$. This is in fact larger than the range of angles $\theta \lesssim 5°$ ($\tan\theta \lesssim 25/400$) at which phonons from the optically heated film would arrive if they were emitted isotropically and traveled ballistically from a point on the heater.

However, an estimate of the effect on the phonon intensity at the device produced by the strong focusing of the TA phonons into directions close to the [001] indicate that it is increased by ∼20. If this occurred uniformly for all directions around the [001], this would correspond to an increase in the range of θ to a value close to 25°, showing that approximately all allowed transitions can be stimulated by phonons from the heater. So from this we conclude that $\Delta I/I_{sp} \sim n_q(T_h)/1 = 0.06$ for a heater at 13 K. However, this does not allow for the fact some of the phonons from the heater will be scattered as they cross the substrate and so will not contribute to the ballistic peak. For a heater at 13 K, somewhat more than half the phonons have frequencies >800 GHz so it seems likely that less than half of the TA phonons emitted travel ballistically across the substrate [this is supported by the ratio of the areas under the ballistic and diffusive parts of $\Delta I(t)$]. So a better estimate for the change in current is $\Delta I/I_{sp} \sim n_q(T_h)/2 = 0.03$. Now the total current I contains contributions from spontaneous emission due to TA and LA modes plus that from elastic scattering. It is usually assumed that the elastic contribution is dominant and hence that $I_{sp}(\mathrm{TA}) \ll I$ and, if this is the case, we conclude that $\Delta I/I \ll 0.03$.

The peak value of the TA phonon peak found experimentally for an optically heated film at 13 K corresponds to $\Delta I/I = 0.03$ ($\Delta I/I$ is approximately five times larger for the electrical heater), so the increase in current is greater than would be expected. This might therefore suggest that amplification is taking place. We recall, however, that this argument does not allow for anisotropy of the focusing around the [001] direction.

V. SUMMARY

Measurements of the transient change in tunnel current $\Delta I(t)$ through a biased 50 period Si doped GaAs/AlAs superlattice produced by pulses of nonequilibrium acoustic phonons show peaks at the ballistic arrival times of LA and TA phonons. The sizes of the peaks vary with the bias V. Investigations of the TA peak show that the bias $V_{\max}$ at which $\Delta I(V)$ is greatest varies linearly with the heater temperature T_h and hence with the frequency of the dominant phonons $\sim 60 T_h$ GHz incident on the device: the SL is acting as a phonon spectrometer. This was confirmed using the fact that the average frequency of the phonons that are scattered crossing the substrate is significantly greater than that of those traveling ballistically.

The form of $\Delta I(V)$ is qualitatively similar to that predicted by the model of Glavin *et al.*[20] developed from the two well model of Tsu and Döhler.[4] There are, however, quantitative differences particularly in the dependence of the position of the maximum of $\Delta I(V)$ on electron density and heater temperature. This might be the result of a number of simplifications made in the model and its application to the experimental situation for TA phonons. The relative change in current $\Delta I/I$ that was measured is similar to that estimated using a simple physical model which assumes that I is entirely due to spontaneous emission. Since, however, the component due to spontaneous emission is usually assumed to be small compared to that from elastic scattering, this suggests the measured value could be appreciably greater than that calculated. We cannot therefore rule out the possibility that phonon amplification is taking place, but further investigation both experimental and theoretical would be needed to determine whether or not this is the case.

The theoretical model suggests that the spectral linewidth $\Delta\nu$ (HWHM) of the SL detector is around 100 GHz, which is consistent with experimental data which indicate that $\Delta\nu \lesssim k_B T_h/h \sim 200$ GHz ($h\Delta\nu \lesssim 0.8$ meV). However, the linewidth could be less than these values if phonon amplification is taking place. It would clearly be of interest to measure the linewidth using a narrow band phonon source. The linewidth should also be reduced by the application of magnetic fields ($B \parallel I$).

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*Now at Bookham Technology Plc. Caswell, Towcester, Northamptonshire, NN12 8EQ, United Kingdom.

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