Nonreciprocal frequency doubler of electromagnetic waves based on a photonic crystal

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It is shown that an electromagnetic wave corresponding to the second harmonic transmitted through a dual periodic photonic crystal possessing $\chi^{(2)}$ nonlinearity exhibits a profound difference in output intensity depending on the direction of the propagation and, therefore, such a system appears to be suitable in the design of a frequency doubler of electromagnetic waves which allows propagation of the wave in one direction and forbids propagation in the opposite direction.

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The theory of electromagnetic wave propagation in periodic structures, also called photonic crystals, was developed on the basis of the close analogy with quantum particle dynamics in atomic crystals.[1] The close similarity between the two systems yields ideas for designing photonic crystals possessing new remarkable properties, which can be of great practical importance. In recent years the mechanisms that constitute the basis for nonreciprocal transmission have been numerically investigated in the one-dimensional (1D), nonlinear, anisotropic photonic band gap material with a spatial gradation in the linear refractive index[2] and it was found that such a structure can result in unidirectional pulse propagation. In the present paper we have employed yet another principle that constitutes the operational mechanism for a nonreciprocal frequency doubler (which also can be viewed as a basic element of a light diode), i.e., a device which for a given frequency of the electromagnetic radiation allows propagation of a wave in one direction but forbids propagation in the opposite direction. Such a device follows the principles known from the theory of a *p-n* transition and a semiconductor diode.[3]

To start with qualitative description of a nonreciprocal frequency doubler of electromagnetic wave we consider a TE electromagnetic wave incident from the left on a 1D multilayer stack which consists of two kinds of periodic media as is shown in Fig. 1(a).

The two stratified media are chosen in such a way that the relative positions of band gaps are as shown in Fig. 1(b). The band gaps of the subsystem on the left (below simply the left structure) can be arbitrarily small and even absent; i.e., it can be homogeneous in the limit of the vanishing filling fraction of one of its components. The choice of the periodic left structure is justified by the fact that it allows one to modify the dispersion relation, which is necessary in providing resonant conditions for second-harmonic generation (SHG).[6] Although the left part of the proposed device can be replaced by a homogeneous SHG material, the use of a photonic crystal seems to have several advantages, like the possibility to provide the matching condition in a wide range of wavelengths, the possibility of the realization giant SHG (see, e.g., Ref. 4), and the possibility of controlling the matching conditions by geometrical parameters (lattice constant of a periodic structure, filling fraction, etc.[6]). Simultaneously, we require for the left structure to possess $\chi^{(2)}$ nonlinearity, while for the right structure to be preferably linear or to have nonlinear properties that are weak enough to avoid strong SHG.

Let us choose the x axis orthogonal to the layers and directed from the left to the right (see Fig. 1) and suppose that TE, $\mathbf{E}=(0,0,E(x,t))$, the electromagnetic wave of the frequency ω, is incident on the stack from the left. We are interested in the case when (i) ω belongs to an allowed zone of the left structure and falls into the gap of the right structure while (ii) the second harmonic of the incident wave, with frequency $\omega_2=2\omega_1$ and wave vector $q_2=2q_1+Q$ (where Q is the vector of the reciprocal lattice of the left structure), which is generated in the left structure belongs to an allowed zone of the both left and right structures. The characteristic distance L_1 at which the most energy of the first harmonic is transformed into the second one is proportional to $v_{11}/A\chi^{(2)}$, where A is the amplitude of the incident wave in the left structure and we use the notation v_{ij} for the

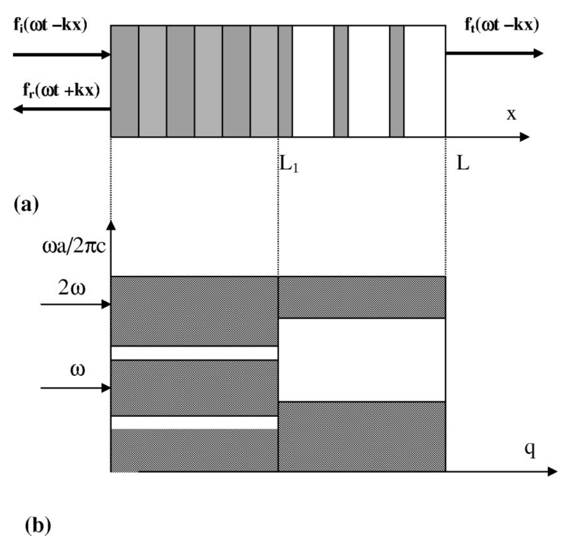

FIG. 1. (a) Multilayered 1D stack composed of two periodic substructures. (b) Schematic picture of the photonic band structure of the left and right parts of the dual 1D stack. Shadowed regions indicate the allowed bands while empty regions correspond to the stop bands.

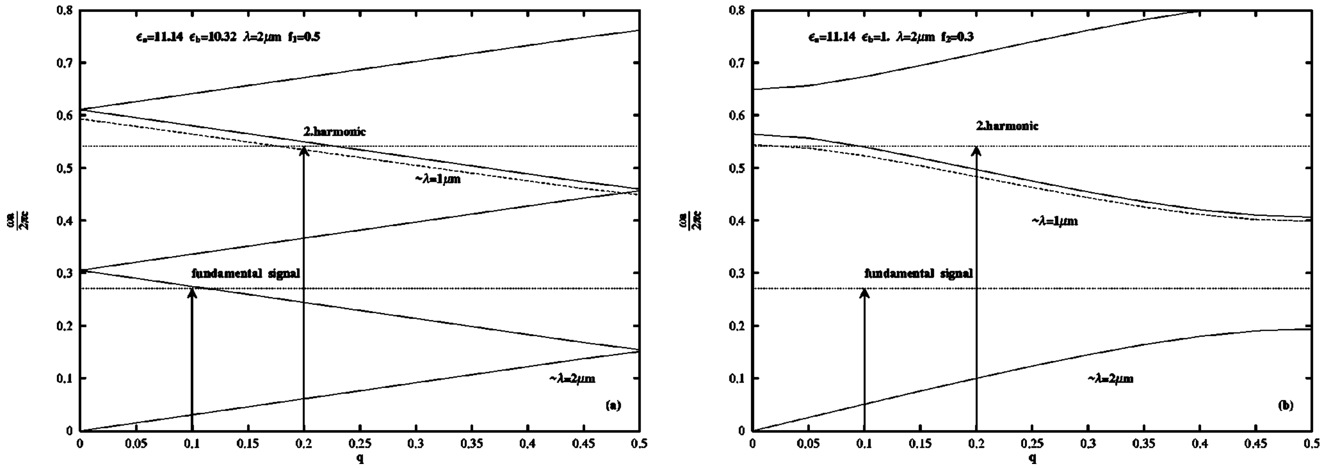

FIG. 2. (a) PBS's of the 1D GaAs/AlGaAs stack when the filling fraction is $f_1=0.5$. (b) PBS's of the 1D GaAs/vacuum stack when the filling fraction is $f_2=0.3$. In the both figures solid lines correspond to dispersion relation at $\lambda=2$ μm, while dashed curves refer to the fourth and second lowest bands at $\lambda=1$ μm, respectively, in the presence of material dispersion. The wave vector q is measured in units of $2\pi/a$. The frequencies of the fundamental and second harmonics are indicated by dotted lines. The vertical arrows indicate the matching conditions.

group velocity of the jth mode in the left $(i=1)$ and right $(i=2)$ structures. Then, if the length of the left structure is L_1, the wave incident from the left to the right structure has frequency ω_2 belonging to the allowed band and thus can propagate through the right structure. If the right structure is linear, the output signal will have frequency ω_2. On the other hand, if the wave with frequency ω_1 is incident on the stack from the right, it undergoes Bragg reflection and thus the transmission coefficient of such a wave is exponentially small.

Let us now demonstrate that the resonant conditions formulated above can be achieved in a one-dimensional dual structure which consists of substructures composed of alternating GaAs/AlGaAs layers on the left with $\epsilon_{GaAs}=11.14$, $\epsilon_{AlGaAs}=10.32$ at $\lambda=2$ μm, and GaAs/vacuum stack on the right. To scale our simulations into the near-infrared region we choose the lattice constant $a=1$ μm which is common to both substructures while the filling fractions are $f_1=0.5$ on the left and $f_2=0.3$ on the right. In Fig. 2 we present the photonic band structures (PBS's) of both GaAs/AlGaAs and GaAs/vacuum substructures determined by means of the standard plane-wave method[7] in terms of normalized frequency $\Omega=\omega a/2\pi c$ as a function of the wave vector q.

The fundamental signal which can be generated by a laser source operating at 2 μm corresponds to the frequency $\Omega_1\approx 0.2705$ which in the PBS's of the left part is within the second lowest pass band while it falls within the lowest gap in the PBS's of the GaAs/vacuum stack. The second-harmonic signal with wavelength 1 μm corresponds to the frequency $\Omega_2\approx 0.541$ which is within the pass band of the PBS's of both substructures. In evaluating the PBS's for the second harmonic we take into account the material dispersion; namely, we use the tabulated values of the dielectric constants[8] at $\lambda=2$ μm mentioned above and $\epsilon_{GaAs}=12.19$ and $\epsilon_{AlGaAs}=10.53$ at $\lambda=1$ μm. We underline an important feature of the obtained matching condition: the group velocities of all modes have the same direction (otherwise the energy transfer from one mode to another would be accompanied by the effective wave reflection). In the case at hand the absolute values of the group velocities are given as $v_{11}\approx 0.31$, $v_{21}\approx 0.3$, and $v_{22}\approx 0.48$.

As has been mentioned above the right structure is preferably linear, since harmonic generation in it is not desirable for our purposes. The generation of higher harmonics can also be prevented by the proper choice of periodic structure (such that, for example, $4\omega_1$ belongs to a stop gap, in the case of quadratic nonlinearity) or choosing a shorter length L_2 of the right structure, which, however, must be large enough to provide an effective Bragg reflection of the wave incident from the right. To this end one can use a rough estimate $L_2\sim v_{22}/\Delta\omega$, where $\Delta\omega$ is the frequency detuning into the stop gap when $L_2\ll L_1$.

In the present statement of the problem the wave equation for the amplitude E can be written in the form

$$-\frac{\partial^2 E(x,t)}{\partial x^2}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty}\epsilon(x,t-t')E(x,t')dt' = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\chi^{(2)}(x)[E(x,t)]^2, \quad (1)$$

where

$$\epsilon(x,t)=\begin{cases}\epsilon_1(x,t), & 0<x<L_1,\\ \epsilon_2(x,t), & L_1<x<L_2,\end{cases}$$

and the functions $\epsilon_j(x,t)(j=1,2)$ are periodic: $\epsilon_j(x+a,t)=\epsilon_j(x,t)$. A similar requirement is imposed on $\chi^{(2)}$.

We are interested in transmission properties of the slab, in the case when an incident wave is monochromatic with the frequency ω. Then the solution of Eq. (1) can be written in a general form as

$$E(x,t)=\sum_{n=0}^{\infty}\left[A_n(x)\cos(n\omega t)+B_n(x)\sin(n\omega t)\right], \quad (2)$$

which is supported by the chosen type of nonlinearity. The Fourier coefficients $A_n(x)$ and $B_n(x)$ satisfy the following system of equations:

$$\frac{d^2A_n}{d\xi^2}=-(n\omega)^2[A_n\epsilon'(\xi,n\omega)-B_n\epsilon''(\xi,n\omega)]$$
$$-(n\omega)^2\chi(\xi)\sum_{n_1,n_2}[\alpha_{nn_1n_2}A_{n_1}A_{n_2}+\beta_{nn_1n_2}B_{n_1}B_{n_2}], \quad (3)$$

$$\frac{d^2B_n}{d\xi^2}=-(n\omega)^2[A_n\epsilon''(\xi,n\omega)+B_n\epsilon'(\xi,n\omega)]$$
$$-(n\omega)^2\chi(\xi)\sum_{n_1,n_2}\beta_{n_2n_1n}A_{n_1}B_{n_2}. \quad (4)$$

Here $\xi=\sqrt{2}x/c$, $\chi(\xi)\equiv 8\pi\chi^{(2)}(x)$,

$$\epsilon'(\xi,n\omega)=\frac{1}{2}[\epsilon(\xi,n\omega)+\epsilon(\xi,-n\omega)],$$

$$\epsilon''(\xi,n\omega)=\frac{1}{2i}[\epsilon(\xi,n\omega)-\epsilon(\xi,-n\omega)],$$

$$\alpha_{nn_1n_2}=\langle\cos(n\omega t)\cos(n_1\omega t)\cos(n_2\omega t)\rangle,$$

$$\beta_{nn_1n_2}=\langle\cos(n\omega t)\sin(n_1\omega t)\sin(n_2\omega t)\rangle,$$

and the angular brackets stand for the average over one period $T=2\pi/\omega$.

General boundary conditions to the above system, which take into account the absence of the reflected wave to the right from the slab, read as

$$B_n'(0)+k_nA_n(0)=2\alpha_nk_n, \quad k_nB_n(0)-A_n'(0)=2k_n\beta_n,$$

$$k_nA_n(L)-B_n'(L)=0, \quad A_n'(L)+k_nB_n(L)=0,$$

where α_n and β_n are constants characterizing amplitudes of the Fourier harmonics of the incident wave, while k_n are their wave vectors.

To study transmission properties of the structure shown in Fig. 1(a) with the dispersion relation depicted in Fig. 2, we consider a finite 1D stack of the length L, the left part of which consists of four alternating layers of GaAs and $Al_{0.5}Ga_{0.5}As$ with filling fraction $f_1=0.5$, while the right part consists of four alternating layers of GaAs and vacuum with filling fraction $f_2=0.3$. Both GaAs and $Al_{0.5}Ga_{0.5}As$ possess second-order nonlinearity with $\chi^{(2)}=1.68\times10^{-10}$ m/V and $\chi^{(2)}=1.73\times10^{-10}$ m/V, respectively.

The energy of the transmitted wave is characterized by the total transmittance, which for the wave incident from the left side can be defined as $T=\Sigma_{n=1}^{\infty}I_n(L)/\Sigma_{n=1}^{\infty}I_n(0)$ where $I_n=A_n^2+B_n^2$ (for the wave incident from the right, one has to exchange L and 0 in this formula). It is evident that, when an incident light contains only the fundamental harmonic, the first and second harmonics are dominant and thus $T\approx T_1+T_2$ [where $T_1=I_1(L)/I_1(0)$ is the transmittance of the first harmonic and $T_2=I_2(L)/I_1(0)$ is the transformation coefficient describing the energy transfer from the fundamental to the second harmonic]. In the case considered, we seek a solution for the monochromatic cosine incident wave; i.e., we set the following values of the free parameters: $\alpha_1=1$ V/m and $\alpha_n=0$ V/m for $n\neq1$ and $\beta_n=0$ V/m for all n. By taking into account the finite length of the structure and relatively weak nonlinear interactions the system of equations (3) and (4) for the coefficients A_n and B_n can be restricted only to the two lowest modes; i.e., effectively it can be rewritten in a form of four coupled ordinary differential equations (ODE's) of the first order, which are solved by using the shooting method.[9] It is to be emphasized here that although the generation of harmonics higher than the second one is neglected, the reduced system of equations does take into account an exhaustion of the intensity of the incident fundamental radiation (i.e., the energy decay of the first harmonic due to transfer into the second one). For evaluation of integrals $A_n'(L)$, $A_n(L)$, $B_n'(L)$, and $B_n(L)$ the Runge-Kutta method is exploited. We note that the method outlined above represents an independent approach that allows us to treat the problem of the propagation of the electromagnetic wave through a one-dimensional nonlinear stratified structure with arbitrary geometrical and material parameters. In contrast to the nonlinear transfer-matrix formalism[5] employed in Ref. 4 within the effective medium approximation the accuracy of our method is limited only by the precision of the spatial discretization of the tabulated values of the dielectric constants.

In Fig. 3(a) we present the transmittance of the fundamental signal T_1 as a function of the normalized frequency Ω separately for the left and right structures. One can see that transmission spectrum through the AlGaAs/GaAs stack (solid line) with low index contrast does not resemble a PBS of the infinite system shown in Fig. 2(a) (which is due the small index contrast that gives rise to very small widths of stop gaps) and rather corresponds to a typical interference pattern. On the other hand, the transmissivity through the GaAs/vacuum stack (indicated by the dotted line) reveals large band gaps which correspond well to those found in the PBS's of the infinite GaAs/vacuum stack shown in Fig. 2(b) in the frequency ranges $0.19<\Omega<0.4$ and $0.54<\Omega<0.65$. In Fig. 3(b) we show the transmission spectra versus normalized frequency Ω for the whole stack which exhibits a mixed nature; i.e., it possesses large gaps found in the PBS's of the right part with the transmission modulated by the interference pattern resulting from the left AlGaAs substructure.

We found that the transmittance for the fundamental signal shown in Fig. 3(b) does not depend on the direction of propagation which is consistent with the well-known left-and-right incidence theorem.[10] The crucial property of the dual structure shown in Fig. 1(a), however, stems from the fact that the transmission spectra for the second-harmonic signal depend on the direction of propagation and on the intensity of the incident wave and that the intensity of the

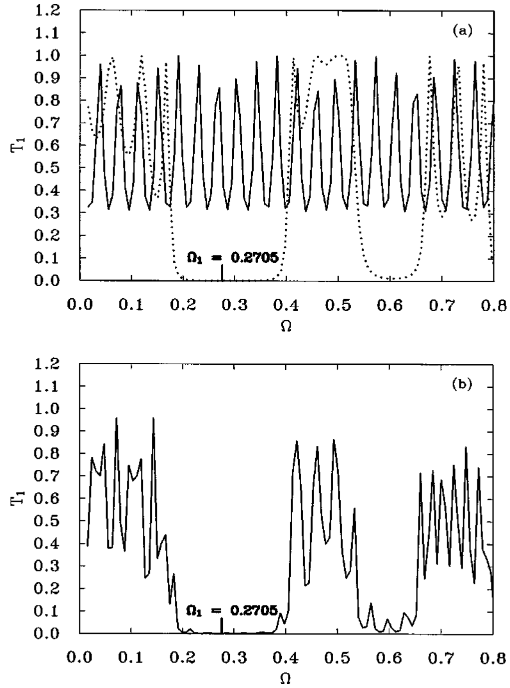

FIG. 3. Transmittance for the fundamental signal T_1 vs normalized frequency Ω (a) for the left structure consisting of four alternating layers of GaAs and $Al_{0.5}Ga_{0.5}As$ for $f_1=0.5$ (solid line) and for the right structure consisting of four alternating layers of GaAs and vacuum for $f_2=0.3$ (dotted line) and (b) for the whole one-dimensional dual structure consisting of GaAs/AlGaAs and GaAs/vacuum substructures.

second harmonic is substantially larger than that of the fundamental signal. This effect is demonstrated in Fig. 4 by a profound difference between the intensity of the second harmonic I_2 for the positive and negative incidence of the electromagnetic wave.

The obtained results corroborate with the qualitative theory of the frequency doubler presented above. Namely, considering positive incidence we have shown that for even for small amplitude of the wave transmitted through the left structure that is reflected from the right structure—note that the respective transmittance is extremely small; see Fig. 3(a)—the left medium width is large enough to provide transfer of a sufficiently large part of the energy to the second harmonic, which can propagate through the whole stratified medium.

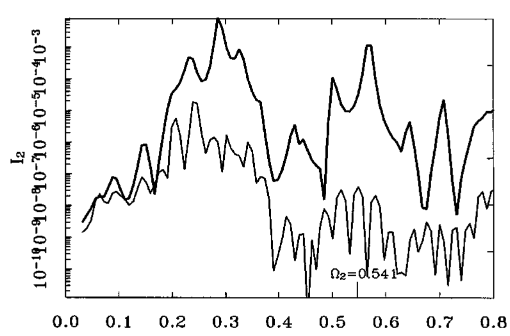

FIG. 4. Output energy of the second-harmonic signal I_2 vs normalized frequency Ω evaluated for the whole 1D dual structure for the positive (solid line) and negative (thin line) incidence of the electromagnetic wave when $A_1(0)=1.0^3$ V/m.

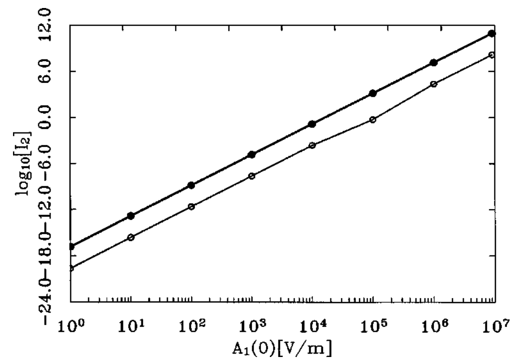

FIG. 5. Output energy of the second-harmonic signal I_2 at $\Omega_2=0.541$ as a function of the intensity of the amplitude of the input wave $A_1(0)$, measured in V/m, for the positive (solid line with full circles) and negative (thin line with open circles) incidence.

To inspect qualitatively the diodelike properties of the stratified nonlinear structure considered we first choose a fundamental signal with a frequency $\Omega_1=0.2705$ within the gap with the intensity of the second harmonic I_2 with the frequency $\Omega_2=0.541$ as a function of the intensity of the input wave for both positive and negative incidence in the range of the intensities of the amplitude $A_1(0)$ of the incident wave in the range up to 1.0^7 V/m. To do so we change the initial condition for α_1 in the range from 1 to 1.0^7 in V/m units, while $\alpha_2=0$ V/m and $\beta_i=0$ V/m, when $i=1,2$. In Fig. 5 we present the dependence of the output intensity I_2 of the second harmonic when the electromagnetic wave is incident in the opposite directions. The results obtained reveal the difference in the output intensities of the wave propagating in opposite directions of about four orders of magnitude as is shown in Fig. 5.

To conclude, based on the nonlinear 1D photonic crystal we proposed a new concept of a frequency doubler (a diodelike device) which exhibits nonreciprocal properties with respect to the direction of propagation of the incident wave. The nonreciprocity is achieved due to SHG in the left part of the dual periodic structure that possesses $\chi^{(2)}$ nonlinearity and Bragg reflection of the fundamental harmonic in the right component of a dual structure. We have shown that for an appropriate frequency of the fundamental signal which lies within the band gap, such a device can act as a light diode that operates at the frequency of the second harmonic with the amplitude that is predicted to be significantly stronger than the fundamental signal for sufficiently large intensity of the incident wave. To demonstrate the possibility of such a device we used a simple model structure fabricated from experimentally accessible materials. The effect can be enhanced by varying the geometrical and material parameters or by employing more sophisticated geometry. For example, an active layer, enhancing the second harmonic, can be included between the two structures (in this case the requirement of very efficient SHG in the left structure can be relaxed).

In the present paper we have neglected the effect of dissipation and thus we solved the independent set of second-order differential equations, which discard a generation of higher (than second) harmonics, but take into account an exhaustion of the intensity of the incident wave. Besides the effect of the dissipation the transmission spectra one also can explore nonreciprocal properties of the transmission spectra of one-dimensional nonlinear stratified structures with metallic or semiconductor components which are characterized by the dielectric function that are negative in certain frequency range. By using our technique we also can study characteristics of the electromagnetic wave propagation in 1D disordered systems that are periodic on average such as a localization length. Finally, to illustrate the physics of the phenomenon we have chosen photonic crystals, the characteristics of which are well known and which are experimentally available. This, of course, leaves unexplored the possibilities of increasing of the efficiency of the device, using other materials (like, for example, recently reported porous silicon photonic crystals[4]).

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