Comment on "Ultrasonic studies of the spin-triplet order parameter and the collective mode in Sr_2RuO_4"

J. D. Gavenda*
Department of Physics, The University of Texas at Austin, Austin, Texas 78712

(Received 12 March 2001; published 16 December 2002)

The "anomalies" in the temperature dependence of the ultrasonic attenuation in Sr_2RuO_4 reported by Matsui *et al.* [Phys. Rev. B **63**, 060505 (2001)] are well-understood consequences of the use of transverse waves. Their conclusion that the behavior at low temperatures is nonexponential is not justified because they have ignored the electric field contribution to the attenuation of transverse waves.

DOI: 10.1103/PhysRevB.66.216501 PACS number(s): 74.25.Ld, 74.70.−b

Matsui *et al.*[1] report very interesting ultrasonic measurements in the layered perovskite superconductor Sr_2RuO_4, in which the pairing may have *p* symmetry. Unfortunately, their interpretation of the ultrasonic measurements is marred by use of the Bardeen, Cooper, and Schrieffer (BCS) equation[2] for the ratio of the attenuation in the superconducting state to that in the normal state

$$\frac{\alpha_{\mathrm{BCS}}}{\alpha_n}=2f_0(\Delta(T))=\frac{2}{1+\exp(\Delta/kT)}, \tag{1}$$

which turns out to be valid for longitudinal sound but not for the transversely polarized waves used in their experiments.

Measurements of ultrasonic attenuation played an important role in determining the properties of the conduction electrons in conventional superconductors.[3] The first direct evidence for anisotropy of the energy gap took advantage of the directional sensitivity of the ultrasonic technique.[4] Most of the work was done in the decade centered about 1960, so it is not surprising that current workers in the field of *p*-wave superconductivity are not familiar with some of the published literature.

Since the electronic contribution to ultrasonic attenuation is appreciable only when the electron mean-free-path l is comparable with the sound wavelength λ, it is difficult to apply the ultrasonic technique to the truly high-temperature superconductors. However, thermal phonon scattering of the electrons in Sr_2RuO_4 is much reduced near the transition temperature since it lies below 1.5 K in this material.

Morse *et al.*[3,5] first reported the striking difference between the attenuation of transverse and longitudinal sound near the superconducting transition temperature. Figure 1 illustrates how the transverse wave attenuation $\alpha_{s,t}$ in a typical superconductor decreases very rapidly immediately below the transition temperature T_c until it reaches the temperature T_M. (The size of $\delta\equiv T_c-T_M$ has been exaggerated in the figure in order to clarify these points; in pure, conventional superconductors δ is typically less than $0.01T_c$.) Below this region it follows the BCS prediction in a fashion similar to that for longitudinal waves. The dashed curve shows the normal state attenuation $\alpha_{n,t}$ for a magnetic field greater than the critical field H_c. Note that the attenuation is lowered by an amount that depends on $\omega_c\tau$, where ω_c and τ are the electron cyclotron frequency and relaxation time, respectively.

The sharp peak in $d\alpha_s/dT$ just below T_c reported in Ref. 1 is what one would expect in the rapid-fall region, which can be explained by considering the two different mechanisms by which conduction electrons remove energy from the sound field. The most convenient way to describe them is in terms of Pippard's[6] theory based on a reference frame attached to the moving ions. In this frame there is a fictitious force $\mathbf{\Pi}$ that represents the departure of the electron from local equilibrium as a consequence of lattice strain. The electronic current caused by $\mathbf{\Pi}$ leads to an electric field $\mathbf{E}$ which, when combined with $\mathbf{\Pi}$, must result in essentiallly zero current in a good conductor.

The variations in ion density set up by a longitudinal sound wave lead to a longitudinal $\mathbf{E}$ that causes no dissipation of energy,[7] so the longitudinal attenuation in the normal state can be expressed as

$$\alpha_{n,l}=\alpha_{\mathbf{\Pi},l}. \tag{2}$$

For a transverse wave there is no density variation so $\mathbf{E}$ must be induced by the magnetic field associated with the

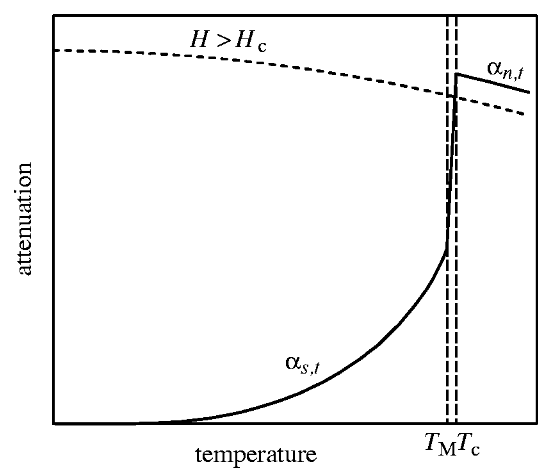

FIG. 1. Typical shear wave attenuation for a conventional superconductor with transition temperature T_c. The attenuation in the superconducting state $\alpha_{s,t}$ drops rapidly with temperature until the Meissner effect is complete at T_M, then shows BCS behavior. The dashed curve shows the effect on the normal state attenuation $\alpha_{n,t}$ of a magnetic field larger than the critical field.

transverse ion current, which leads to energy dissipation, thus the transverse attenuation in the normal state is described by

$$\alpha_{n,t}=\alpha_{\mathbf{\Pi},t}+\alpha_{\mathbf{E},t}\,. \tag{3}$$

Claiborne and Morse[8] pointed out that the rapid decrease in shear wave attenuation just below the transition temperature could be attributed to the rapid decrease in the London penetration depth λ_L in this temperature range. Below T_M, $\lambda_L \ll \lambda$ and the Meissner effect suppresses the self-consistent field generated by the sound wave, so that the attenuation in the superconducting state depends only on $\alpha_{\mathbf{\Pi},t}$. They and Leibowitz[9] found good agreement between calculations based on this model and experimental results.

Kadanoff and Pippard[7] rederived the attenuation expressions for a real metal using the model from Ref. 6 but in a more general fashion. They included the BCS energy expression at the outset and found, for longitudinal waves,

$$\alpha_{s,l}=2f(\Delta)\alpha_{\mathbf{\Pi},l}\,, \tag{4}$$

in agreement with Eq. (1), and

$$\alpha_{s,t}=2f(\Delta)\alpha_{\mathbf{\Pi},t} \tag{5}$$

below T_M for transverse waves. Since $\alpha_{n,t}$ has the additional term $\alpha_{\mathbf{E},t}$, Eq. (1) is not valid for this case.

When l is impurity-limited so that α_n is independent of temperature, one can get around this difficulty for transverse waves by finding the electric field contribution,[8,9]

$$\alpha_{\mathbf{E},t}=\alpha_{n,t}(T_c)-\alpha_{s,t}(T_c)\approx\alpha_{n,t}(T_c)-\alpha_{s,t}(T_M), \tag{6}$$

and subtracting it from $\alpha_{n,t}$ before forming the ratio in Eq. (1), i.e.,

$$\frac{\alpha_{s,t}}{\alpha_{n,t}-\alpha_{\mathbf{E},t}}=\frac{2f(\Delta)\alpha_{\mathbf{\Pi},t}}{\alpha_{\mathbf{\Pi},t}}=2f(\Delta). \tag{7}$$

However, l appears to be temperature-dependent in Ref. 1, since their Fig. 1(c) shows the normal state attenuation increasing with decreasing temperature. Thus, $\alpha_{\mathbf{E},t}$ will have a temperature dependence (which may be different from that of $\alpha_{\mathbf{\Pi},t}$) and Eq. (6) cannot be used. It might be possible to separate the temperature dependence of l from that of $\Delta(T)$ by making meaurements at a number of different frequencies, since q and l affect $\alpha_{n,t}$ somewhat differently.[6]

It is surprising that the normal state attenuation below T_c shown in Fig. 1(c) of Ref. 1 is not displaced below its value just above T_c in the absence of an external magnetic field (see the typical behavior in Fig. 1 above). This might be because τ is very small at T_c; measurements of $\alpha_{n,t}$ as a function of magnetic field H could help clarify this point, since it should go to zero as $1/H^2$.[10,11]

In summary, an alternative explanation for the apparently nonexponential behavior of ultrasonic attenuation in the superconducting phase reported by Matsui *et al.*[1] is their failure to take into account the electric field contribution to shear wave attenuation.

---

*Electronic address: gavenda@physics.utexas.edu

[1] H. Matsui, Y. Yoshida, A. Mukai, R. Settai, Y. Ōnuki, H. Takei, N. Kimura, H. Aoki, and N. Toyota, Phys. Rev. B **63**, 060505 (2001).

[2] This equation was derived in a note added in proof on p. 1191 of J. Bardeen, L.N. Cooper, and J.R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[3] A good early review of the field was presented by R. W. Morse, in *Progress in Cryogenics*, edited by K. Mendelssohn (Heywood and Company, London, 1959), pp. 219–259.

[4] R.W. Morse, T. Olsen, and J.D. Gavenda, Phys. Rev. Lett. **3**, 15 (1959); **3**, 193 (1959).

[5] R.W. Morse and H.V. Bohm, J. Acoust. Soc. Am. **31**, 1523 (1959).

[6] A.B. Pippard, Philos. Mag. **46**, 1104 (1955).

[7] L.P. Kadanoff and A.B. Pippard, Proc. R. Soc. London, Ser. A **292**, 299 (1966).

[8] L.T. Claiborne, Jr. and R.W. Morse, Phys. Rev. **136**, A893 (1964).

[9] J.R. Leibowitz, Phys. Rev. **136**, A22 (1964).

[10] A.B. Pippard, Proc. R. Soc. London, Ser. A **257**, 165 (1960).

[11] M.H. Cohen, M.J. Harrison, and W.A. Harrison, Phys. Rev. **117**, 937 (1960).