Fermi-liquid interactions and the superfluid density in *d*-wave superconductors

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We construct a phenomenological superfluid Fermi-liquid-theory for a two-dimensional *d*-wave superconductor on a square lattice, and study the effect of quasiparticle interactions on the superfluid density. Using simple models for the dispersion and the Landau interaction function, we illustrate the deviation of these results from those for the isotropic superfluid. This allows us to reconcile the value and doping dependence of the superfluid density slope at low temperature obtained from penetration depth measurements, with photoemission data on nodal quasiparticles.

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I. INTRODUCTION

The high-temperature superconductors appear to support well-defined quasiparticle (QP) excitations at low temperatures ($T \ll T_c$) as suggested by penetration depth,[1] transport,[2–4] and angle-resolved photoemission spectroscopy[5] (ARPES) experiments. Low-temperature superconducting (SC) state properties of the cuprates thus appear to be consistent with *d*-wave BCS theory with nodal QP excitations. However, the importance of correlations at low T is evident with underdoping: experiments[6] show that the superfluid stiffness $D_s(T=0) \sim x$, and the QP weight at $(\pi,0)$ diminishes on approaching the Mott insulator.[7] In this paper we address the question of interaction corrections to the temperature dependence of $D_s(T)$.

The in-plane superfluid stiffness $D_s(T) = (c^2 d/4\pi e^2 \lambda^2)$, with d the mean interlayer spacing along the c axis, can be directly obtained from measurements of the in-plane penetration depth, $\lambda(T)$. $D_s(T)$ is found to decrease linearly with temperature,[1] $D_s(T) = D_s(0) - AT$, for $T \ll T_c$, with a slope A which is nearly doping independent[8,9] (or weakly decreasing but nonsingular) as $x \to 0$.

Clearly the linear drop in $D_s(T)$ is due to thermally generated excitations which contribute to the normal-fluid density. BCS theory with noninteracting QP excitations around the four *d*-wave nodes leads to the result

$$D_s(T) = D_s(0) - \frac{2\ln 2}{\pi}\frac{v_F}{v_2}T, \tag{1}$$

where v_F is the Fermi velocity and v_2 is related to the slope of the SC gap via $v_2 = (1/k_F)\partial\Delta(\theta)/\partial\theta|_{\theta=\pi/4}$, at the nodal Fermi wave vector k_F. Mesot *et al.*[10] obtained the nodal QP dispersion parameters v_F and v_2 as a function of doping from ARPES data on $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212), and compared the D_s slope obtained from Eq. (1) with λ measurements. They found that the slope estimated in this manner is too large by more than a factor of 2 at optimal doping—the ARPES results[10] of $v_F = 2.5\times 10^7$ cm/sec and $v_2 = 1.25 \times 10^6$ cm/sec lead to an estimated slope $dD_s/dT = 0.77$ meV/K, while the slope obtained from penetration depth experiments[11–13] is approximately 0.33 meV/K. Furthermore, this discrepancy increases with underdoping since v_2 measured in ARPES decreases marginally leading to a slight *increase* in the estimated slope dD_s/dT on underdoping, while the slope obtained from penetration depth experiments in Bi2212 *decreases* somewhat with underdoping.[13] This is in contrast to the rather striking agreement between estimates from thermal transport measurements[4] and ARPES data[10] for the ratio $v_F/v_2 \approx 20$ at optimal doping in Bi2212.

Following Refs. 14 and 15, we attribute this discrepancy to residual QP interactions or Fermi-liquid corrections. We here use a phenomenological superfluid Fermi-liquid theory (SFLT) to explore the effects of lattice anisotropy on QP interactions in more detail than in earlier studies (see, however, Ref. 16). Some of the results obtained below were summarized without derivation in a conference report.[17] We note that thermal phase fluctuations[18] are ignored here, since we have shown elsewhere[19] that a proper treatment of the long-range Coulomb interaction results in their contribution to $D_s(T)$ being subdominant to that of the nodal QP's.

II. SUPERFLUID FERMI-LIQUID THEORY

Fermi-liquid (FL) theory for a normal Fermi system is based on the existence of well-defined (coherent) QP excitations which are adiabatic continuations of the single-particle excitations of a free Fermi gas. While transport and ARPES experiments suggest that the normal state of optimal and underdoped high-T_c superconductivity is not a FL, nevertheless, sharp QP peaks do appear all over the Fermi surface (FS) deep in the SC state (for $T \ll T_c$). Naturally, one is then led to consider a description of the SC state and its low-lying QP excitations as an adiabatic continuation of a BCS state with Bogoliubov QP excitations.

The approach advocated in Refs. 14 and 15, and adopted below, assumes that such a SC state may be viewed as a correlated FL in which a pairing interaction has been turned on.[20] In this case, one can use the SFLT developed many years ago,[21–23] and generalize it to the anisotropic case.

For a normal Fermi system, the change in free energy due to a change in the QP momentum distribution $\delta n_{\mathbf{k}}$ takes the standard form

$$\delta F[\delta n_{\mathbf{k}}] = \sum_{\mathbf{k}} \xi^0_{\mathbf{k}} \delta n_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\,\delta n_{\mathbf{k}}\delta n_{\mathbf{k}'}, \tag{2}$$

where $\xi^0_{\mathbf{k}}$ is the dispersion for the QP of momentum $\mathbf{k}$ in the absence of other QP's, $f(\mathbf{k},\mathbf{k}')$ is the Landau interaction function, and $\delta n_{\mathbf{k}}=\Sigma_\sigma \delta n_{\mathbf{k},\sigma}$. We have ignored the spin-dependent part of $f(\mathbf{k},\mathbf{k}')$ in order to simplify the notation; the generalization with spin is straightforward, but not relevant for the present discussion. We will refer to the QP's, obtained by setting $f(\mathbf{k},\mathbf{k}')=0$ in the above equation, as noninteracting QP's. The dispersion for these QP's is $\xi^0_{\mathbf{k}}$ which does include the mass renormalization.

We now use the above functional to calculate the superfluid stiffness at low temperatures, in two steps: (i) we calculate the diamagnetic response to a vector potential and (ii) we calculate the renormalization of the current carried by the interacting QP's, relative to free QP's, and use this to compute the paramagnetic current correlator of the QP's. We next use the above quantities as inputs to a Kubo formula in the QP basis, which allows us to determine the superfluid stiffness $D_s(T)$.

A. Diamagnetic term

Let $n^0_{\mathbf{k}}$ be the unperturbed equilibrium QP distribution. In the presence of the vector potential, $n^0_{\mathbf{k}}\to n^0_{\mathbf{k}+e\mathbf{A}/c}$ leads to a shift of the momentum distribution $\delta n_{\mathbf{k}}=n^0_{\mathbf{k}+e\mathbf{A}/c}-n^0_{\mathbf{k}}$. We calculate the diamagnetic term[24] as the change δF to order A^2:

$$\delta F=\sum_{\mathbf{k}} \xi^0_{\mathbf{k}}\left(A^\mu \nabla_\mu n^0_{\mathbf{k}}+\frac{1}{2}A^\mu A^\nu \nabla_{\mu\nu} n^0_{\mathbf{k}}\right)+\frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')A^\mu A^\nu \nabla_\mu n^0_{\mathbf{k}}\nabla'_\nu n^0_{\mathbf{k}}. \tag{3}$$

Here we set $e=c=1$, and ∇_μ,∇'_μ denote derivatives with respect to $\mathbf{k}_\mu$ and $\mathbf{k}'_\mu$, respectively, where $\mu,\nu=x,y$ and the sum over μ,ν is implicit. The term linear in $\mathbf{A}$ vanishes, since the integrand is odd in $\mathbf{k}$, and we get

$$\delta F=\frac{1}{2}A^\mu A^\nu\left[\sum_{\mathbf{k}} \xi^0_{\mathbf{k}}\nabla_{\mu,\nu} n^0_{\mathbf{k}}+\sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\nabla_\mu n^0_{\mathbf{k}}\nabla'_\nu n^0_{\mathbf{k}}\right]\equiv\frac{1}{2}A^\mu A^\nu K_{\mu\nu}, \tag{4}$$

where $K_{\mu\nu}$ is the diamagnetic response.

Given the jump discontinuity in the ''normal'' state QP distribution at the FS, we use $\nabla_\mu n^0_{\mathbf{k}}=-2v^0_{\mathbf{k}\mu}\delta(\xi^0_{\mathbf{k}})$, where the factor of 2 arises from summing over both spins. Using the definition $\mathbf{v}^0_{\mathbf{k}}=\nabla\xi^0_{\mathbf{k}}$ leads to

$$K_{\mu\nu}=2\sum_{\mathbf{k}} v^0_{\mathbf{k}\mu}v^0_{\mathbf{k}\nu}\delta(\xi^0_{\mathbf{k}})+4\sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')v^0_{\mathbf{k}\mu}v^0_{\mathbf{k}\nu}\delta(\xi^0_{\mathbf{k}})\delta(\xi^0_{\mathbf{k}'})\equiv\alpha_{_F}K^0_{\mu\nu}, \tag{5}$$

where $K^0_{\mu\nu}\equiv 2\Sigma_{\mathbf{k}}v^0_{\mathbf{k}\mu}v^0_{\mathbf{k}\nu}\delta(\xi^0_{\mathbf{k}})$ is the diamagnetic term for noninteracting QP's.

B. Quasiparticle current renormalization

The QP energy $\xi_{\mathbf{k}}=\xi^0_{\mathbf{k}}+\Sigma_{\mathbf{k}'}f(\mathbf{k},\mathbf{k}')\delta n_{\mathbf{k}'}$ leads to the QP velocity $\mathbf{v}_{\mathbf{k}}=\mathbf{v}^0_{\mathbf{k}}+\Sigma_{\mathbf{k}'}\nabla f(\mathbf{k},\mathbf{k}')\delta n_{\mathbf{k}'}$. The total QP current $\mathbf{J}$ is then $\Sigma_{\mathbf{k}}\mathbf{v}_{\mathbf{k}}n_{\mathbf{k}}$, which reduces to

$$\mathbf{J}=\sum_{\mathbf{k}} \mathbf{v}^0_{\mathbf{k}}\delta n_{\mathbf{k}}-\sum_{\mathbf{k},\mathbf{k}'} f(\mathbf{k},\mathbf{k}')\delta n_{\mathbf{k}'}\nabla n^0_{\mathbf{k}}, \tag{6}$$

where we have used $\Sigma_{\mathbf{k}}\mathbf{v}^0_{\mathbf{k}}n^0_{\mathbf{k}}=0$ in the first term, since the equilibrium QP population does not carry any current. In the second term, we have transferred the $\mathbf{k}$ derivative from $f(\mathbf{k},\mathbf{k}')$ to $n_{\mathbf{k}}$, with $n_{\mathbf{k}}\approx n^0_{\mathbf{k}}$ at this order. This relates the current carried by the interacting QP, to that carried by a noninteracting QP which only has a mass renormalization.

To make further progress in the specific case of a d-wave superconductor, we note that the dominant excitations in the low-temperature state are those near the gap nodes. We therefore restrict our attention to the renormalization of the current carried by the QP's at the four nodal points located at $\mathbf{k}^M_{_F}$, with $M=1\cdots 4$. Setting $\nabla_\mu n^0_{\mathbf{k}}\simeq -2v^0_{\mathbf{k}\mu}\delta(\xi^0_{\mathbf{k}})$ as before, we find that the contribution to the current at the Mth node is

$$J_\mu(M)=J^0_\mu(M)\left[1+\frac{2}{v_{_F\mu}(M)}\sum_{\mathbf{k}'} f(\mathbf{k}^M_{_F},\mathbf{k}')v^0_{\mathbf{k}'\mu}\delta(\xi^0_{\mathbf{k}'})\right]\equiv J^0_\mu(M)\beta_{_F}, \tag{7}$$

where $J^0_\mu(M)\equiv v_{_F\mu}(M)\delta n_{\mathbf{k}}(M)$ is the noninteracting QP current. In arriving at the above result, we have interchanged the $\mathbf{k},\mathbf{k}'$ labels in the second term, used the symmetry $f(\mathbf{k},\mathbf{k}')=f(\mathbf{k}',\mathbf{k})$. There is no implicit sum over μ in Eq. (7).

C. The superfluid stiffness

From the Kubo formula, we find $D^{\mu\nu}_s=K_{\mu\nu}-\Lambda_{\mu\nu}(\mathbf{q}\to 0,i\omega_n=0)$ where $\Lambda_{\mu\nu}(\mathbf{q},i\omega_n)\equiv\langle j_\mu(\mathbf{q},i\omega_n)j_\nu(-\mathbf{q},-i\omega_n)\rangle$ is the current correlator and we take the transverse limit of $\mathbf{q}\to 0$. In the QP basis, there are no excitations at $T=0$ and $D^{\mu\nu}_s(T=0)=K_{\mu\nu}$. At low temperatures, there are nodal QP excitations and the current operator in $\Lambda(\mathbf{q},i\omega_n)$ has matrix elements between the ground state and these excited states. The current carried by the QP's is, however, renormalized by the factor $\beta_{_F}$ which leads to $\Lambda=\beta^2_{_F}\Lambda^0$, with Λ^0 being the correlator for the noninteracting QP's. The correlator Λ^0 is easily evaluated within BCS theory using the dispersion $\xi^0_{\mathbf{k}}$, and is linear in T at low temperature in a d-wave superconductor. Further, there are polarization effects by which the flowing QP's lead to an internal (fictitious) vector potential arising from the $f(\mathbf{k},\mathbf{k}')$, in addition to the applied vector potential.[23] This effect is important close to T_c when there are a large number of QP's, but it is unimportant at low temperature when there are very few thermally excited QP's.[22,23] The superfluid stiffness in a d-wave superconductor at low T is thus given by

$$D_s(T)=\alpha_F K^0-\beta_F^2\left(\frac{2\ln 2}{\pi}\frac{v_F}{v_2}\right)T, \tag{8}$$

where $K^0=(1/d)\ \mathrm{Tr}\,K^0_{\mu\nu}$ in d dimensions, assuming cubic symmetry. We now proceed to discuss the FL corrections α_F,β_F in more detail.

III. ISOTROPIC LIMIT

For an isotropic system v_F and k_F are independent of the location on the FS and $m^*\equiv k_F/v_F$ is the effective mass. The Landau interaction $f(\mathbf{k},\mathbf{k}')\equiv f(\mathbf{k}\cdot\mathbf{k}')$ and depends only on the angle between the two momenta on the FS. Retaining only the *single* Landau parameter relevant for this discussion, $f(\mathbf{k}\cdot\mathbf{k}')=(dn/d\epsilon)^{-1}F_1\cos\theta$, where $\cos\theta=\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'$ and $(dn/d\epsilon)=m^*/\pi$ is the total "normal"-state QP density of states for both spins. It is then easy to see that in two dimensions, (2D)

$$K_{\mu\nu}=\delta_{\mu\nu}\frac{n}{m^*}(1+F_1/2), \tag{9}$$

where the two-dimensional electron density $n=k_F^2/2\pi$. From Eq. (5), we thus find $\alpha_F=(1+F_1/2)$. [For the special case of a Galilean-invariant system, using the Landau relation $(1+F_1/2)=m^*/m$ in 2D, we find $K_{\mu\nu}=\delta_{\mu\nu}(n/m)$.] It is also easy to find that the renormalization of the current in the isotropic case is given by $J_\mu=J^0_\mu\ (1+F_1/2)$, and the current correlator is then $\Lambda=\beta_F^2\Lambda^0$ with $\beta_F=(1+F_1/2)$. These results for α_F,β_F are in agreement with the earlier work of Larkin and Migdal[21] and Leggett.[22]

We now discuss the shortcomings of isotropic SFLT as applied to the high-T_c superconductors following Ref. 14. Low-temperature penetration depth experiments[6] suggest that $D_s(x,T=0)\sim x$. At the same time, ARPES experiments, as well as theoretical studies of superconductivity in doped Mott insulators,[25] suggest that m^* does not diverge on underdoping. Within the isotropic SFLT framework, these two together imply $(1+F_1/2)\sim x$ which in turn means the slope of $D_s(x,T)$ is proportional to $(1+F_1/2)^2\sim x^2$. This scaling of the slope,[26] however, is in strong disagreement with penetration depth measurements. Following the suggestion[14] that this problem may be resolved by including anisotropy of the Landau interaction function over the FS, we next try to understand FL corrections in the anisotropic case.

IV. ANISOTROPIC CASE

In order to set up a phenomenological SFLT on a two-dimensional square lattice, we first rewrite all our functions in terms of an angle variable θ which sweeps over the large hole-barrel FS centered around (π,π). Then, the Fermi momentum $k_F\equiv k_F(\theta)$, the Fermi velocity $v_F\equiv v_F(\theta)$, and the Landau interaction function $f(\mathbf{k},\mathbf{k}')\equiv f(\theta,\theta')$. We expand these in an orthogonal basis,

$$v_{FX}(\theta)=\sum_{\ell=0}^{\infty}V_X^{(\ell)}\cos[(2\ell+1)\theta], \tag{10}$$

$$v_{FY}(\theta)=\sum_{\ell=0}^{\infty}V_Y^{(\ell)}\sin[(2\ell+1)\theta], \tag{11}$$

$$k_F(\theta)=k_{F0}+\sum_{\ell=1}^{\infty}k_F^{(\ell)}\cos(4\ell\theta), \tag{12}$$

where we have used the symmetries of the square lattice to restrict the form of the expansion, and also used the vector (scalar) character of the v_F (k_F). We may also generally expand the interaction, $f(\theta,\theta')=\Sigma_{\ell,m}F_{\ell,m}e^{i\ell\theta}e^{im\theta'}$. We restrict the form of $f(\theta,\theta')$ using the following symmetries: (i) $f(\theta,\theta')=f(\theta',\theta)$, (ii) $f(\theta,\theta')=f(-\theta,-\theta')$, and (iii) $f(\pi/2-\theta,\pi/2-\theta')=f(\theta,\theta')$. While (i) is generally valid, (ii) and (iii) are valid for a square lattice. This finally leads to

$$f(\theta,\theta')=\sum_{\ell\geqslant m}F_{\ell,m}[\cos(\ell\theta+m\theta')+\cos(\ell\theta'+m\theta)], \tag{13}$$

where $\ell,m:-\infty\rightarrow\infty$ with $(\ell+m)=4p$ and $p=0,\pm1,\pm2,\ldots$. We have set $\ell\geqslant m$ to avoid overcounting. We note that (i) the interaction function depends on θ and θ' separately in general and not only on $(\theta-\theta')$ as in the isotropic case, and (ii) there are many more Landau parameters on the lattice, labeled by two integers (ℓ,m). As we shall see, this considerably complicates our problem since many Landau parameters may contribute to a given response function, which prevents their unique determination.[27] This is unlike the isotropic case (say, in He^3) where usually a single Landau parameter renormalizes a particular correlation function.

We now write the results for α_F,β_F in these new coordinates. The diamagnetic term is given by

$$\begin{aligned}K_{xx}=&2\int_0^{2\pi}\frac{d\theta}{(2\pi)^2}\frac{k_F(\theta)}{|v_F(\theta)|}v_{FX}^2(\theta)\\&+4\int_0^{2\pi}\frac{d\theta d\theta'}{(2\pi)^4}\frac{k_F(\theta)}{|v_F(\theta)|}\frac{k_F(\theta')}{|v_F(\theta')|}\\&\times v_{FX}(\theta)v_{FX}(\theta')f(\theta,\theta')\end{aligned} \tag{14}$$

and the current renormalization for node M is

$$\frac{J_x(M)}{J_x^0(M)}=1+2\int_0^{2\pi}\frac{d\theta'}{(2\pi)^2}\frac{k_F(\theta')}{|v_F(\theta')|}\frac{v_{FX}(\theta')}{v_{FX}(\theta_M)}f(\theta_M,\theta'), \tag{15}$$

where θ_M is the angular position of node M. We can express this in a more compact form by defining $\langle O\rangle_\theta\equiv\int_0^{2\pi}d\theta k_F(\theta)O(\theta)/[2\pi|v_F(\theta)|]$. This yields

$$\alpha_F = 1 + \frac{\langle\langle v_{FX}(\theta) v_{FX}(\theta') f(\theta,\theta')\rangle\rangle_{\theta\theta'}}{\pi \langle v_{FX}^2\rangle_\theta}, \tag{16}$$

$$\beta_F = 1 + \frac{\langle v_{FX}(\theta') f(\theta_M,\theta')\rangle_{\theta'}}{\pi v_{FX}(\theta_M)}. \tag{17}$$

For $f(\theta,\theta') = (\pi/m^*) F_1 \cos(\theta-\theta')$ and k_F, v_F independent of θ, we easily recover the isotropic limit.

V. SIMPLE MODELS FOR THE DISPERSION AND $f(\mathbf{k},\mathbf{k}')$

We now consider special cases of the general result which serve to illustrate the deviation from the isotropic limit.

A. Case I

Consider an isotropic dispersion, with v_F and k_F independent of θ, but we retain all allowed Landau parameters on the lattice. In this case, with $m^* \equiv k_F/v_F$, we find

$$\alpha_F = 1 + \frac{m^*}{\pi}(F_{1,1} + F_{1,-1}),$$

$$\beta_F = 1 + \frac{m^*}{2\pi}\Bigg[\sum_{p\leqslant 0} (-1)^p F_{1,4p-1} + \sum_{p\geqslant 0} (-1)^p F_{4p+1,-1} + \sum_{p>0} (-1)^p F_{4p-1,1} + \sum_{p<0} (-1)^p F_{-1,4p+1}\Bigg]. \tag{18}$$

Thus, many Landau parameters contribute to the renormalization in this anisotropic case unlike in the isotropic limit. Furthermore, different Landau parameters contribute to α_F and β_F. It is then easily possible that $\alpha_F \neq \beta_F$ and they could then also behave very differently with doping if several Landau parameters are nonzero.

B. Case II

We next consider the case where we keep a single Landau parameter $F_{1,-1} \neq 0$, and set all other $F_{\ell,m} = 0$. However, we retain the full anisotropy of the dispersion, as measured in ARPES. We take the tight-binding fit to the (normal-state) ARPES dispersion,[28] and numerically compute the above integrals to determine α_F, β_F. In order to study the doping dependence of α_F, β_F, we assume a doping dependence $F_{1,-1}(x) = B + Cx$, such that $\alpha_F(x) \sim x$ in agreement with the Uemura plot,[6] with reasonable values $\alpha_F(x=0.2) \approx 0.3-0.5$. This fixes B, C and we use this to determine the doping dependence of $\beta_F(x)$. The result of this calculation is plotted in Figs. 1(a) and 1(b), where we see a marked deviation from the isotropic result ($\beta_F^2 = \alpha_F^2$) in the anisotropic case, and β_F^2 is nonsingular as $x \to 0$, in qualitative agreement with penetration depth results.

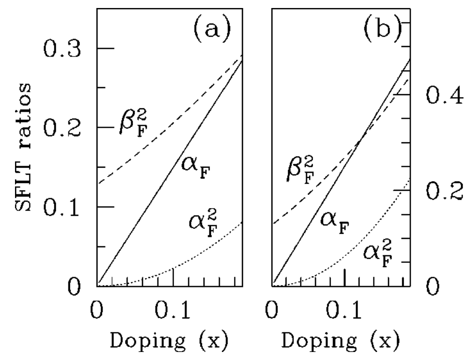

FIG. 1. Doping dependence of the SFLT renormalization (β_F^2) of the slope of $D_s(T)$ for a model with anisotropic QP dispersion and a single Landau parameter chosen such that (a) $\alpha_F(x) = 1.5x$ and (b) $\alpha_F(x) = 2.5x$ (see case II, Sec. V B for details). In the isotropic limit, $\beta_F^2(x) = \alpha_F^2(x)$, but there is marked deviation from this in the anisotropic case—most strikingly $\beta_F^2(x) \neq 0$ as $x \to 0$, as in the experiments. For this simple model and choice of dispersion, a larger renormalization of $D_s(0)$ [smaller $\alpha(x)$, as in (a)] appears to correlate with a weaker doping dependence of $\beta_F^2(x)$.

VI. CONCLUSIONS

We have used a phenomenological SFLT for a d-wave superconductor to determine the renormalization of $D_s(T=0)$ and dD_s/dT due to FL factors. Within simple models for the dispersion and the Landau interaction function, we find that anisotropy can cause strong deviations from the isotropic result. This allows us to understand the discrepancy between penetration depth and photoemission experiments for the temperature and doping dependence of the superfluid density in terms of SFLT corrections. While we discussed the case of a d-wave order parameter as appropriate for the high-T_c superconductors, our results are easily generalized to any unconventional superconductor with point nodes and well-defined QP's.

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