Absence of singular superconducting fluctuation corrections to thermal conductivity

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We evaluate the superconducting fluctuation corrections to thermal conductivity in the normal state, which diverge as T approaches T_c. We find zero total contribution for one-, two- and three-dimensional superconductors for arbitrary impurity concentration. The method used is diagrammatic many-body theory, and all contributions—Aslamazov-Larkin (AL), Maki-Thompson (MT), and density of states (DOS)—are considered. The AL contribution is convergent, whilst the divergences of the DOS and MT diagrams exactly cancel.

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I. INTRODUCTION

The discovery of the high-T_c superconductors has led to a renewed interest[1] in superconducting fluctuation corrections to normal-state transport properties.[2] While much of the work has focused on the electrical resistivity, ρ, several experiments[3–6] have reported fluctuation corrections to the thermal conductivity, κ. Since there is some dispute between theorists[7–12] as to the predicted magnitude of the effect, we have performed a detailed microscopic calculation valid for all impurity concentrations. We find no divergent fluctuation contribution, and conclude that the experimental features seen near T_c must have some other physical origin.

Let us try to understand the reason for the lack of singular fluctuation contributions to thermal conductivity. There are several processes involved, and we will try to develop a physical picture[13] for each. The Aslamazov-Larkin (AL) process involves the transfer of heat by fluctuation Cooper pairs. The corresponding term for the electrical conductivity has a strong divergence,

$$\sigma_{AL} \sim (T-T_c)^{d/2-2}. \tag{1}$$

The size of the contribution to thermal conductivity can be estimated from Eq. (1) using the Wiedemann-Franz law, which has the general form

$$\kappa T \sim \left(\frac{k_B T_0}{Q_0}\right)^2 \sigma, \tag{2}$$

where $k_B T_0$ is the amount of heat and Q_0 is the electric charge, carried by the excitations in a given system. For fluctuation Cooper pairs, $T_0 \sim T - T_c$ and $Q_0 = 2e$ so that

$$\kappa_{AL} T \sim \left(\frac{k_B (T-T_c)}{2e}\right)^2 \sigma_{AL} \sim (T-T_c)^{d/2}, \tag{3}$$

which is clearly nonsingular as $T \to T_c$. The density-of-states (DOS) correction arises from the fact that when electrons form fluctuation Cooper pairs, they cannot simultaneously act as normal electrons; there is a corresponding decrease in the normal-state density of states and hence normal-state thermal conductivity,

$$\kappa_{DOS} \sim -\frac{n_{cp} k_B^2 T^2 \tau}{m} \sim -(T-T_c)^{d/2-1}, \tag{4}$$

where $n_{cp} \sim (T-T_c)^{d/2-1}$ is the number density of fluctuating Cooper pairs. This term is singular for $d \leqslant 2$, but is exactly canceled by Maki-Thompson (MT) terms. The latter terms are due to new heat transport channels opened up by Andreev-scattering processes. An electron can Andreev scatter into a hole, and since electrons and holes carry the same heat current, this leads to a net increase in thermal conductivity. The amplitude for the Andreev scattering is exactly the same as for an electron to scatter into a fluctuation Cooper pair, so the MT and DOS terms have the same magnitude but opposite sign, and hence cancel. These MT processes lead to a further suppression of electrical conductivity since holes carry electric charge opposite to electrons, i.e., the MT and DOS contributions cancel for thermal conductivity and reinforce for electrical conductivity.

Before we proceed to the details of our calculation, we present a short history of superconducting fluctuation corrections to thermal conductivity. They were first predicted[7] in 1970 by Abrahams *et al.* in the diffusive regime. These authors concluded that the AL terms were convergent, but that the DOS terms led to divergent contributions in two and one dimensions, of the form $\ln(T-T_c)$ and $(T-T_c)^{-1/2}$, respectively. They appear to have missed the cancellation between DOS and MT contributions. Shortly afterwards, fluctuation effects with the predicted power-law behavior were observed[14] in one-dimensional Pb-In wires. After this intial work, there was apparently no theoretical or experimental activity in this area for nearly two decades. Indeed, in Skocpol and Tinkham's 1975 review,[2] thermal conductivity is described as one of those quantities which ''have not yet benefited from sustained interaction between theory and experiment, perhaps because such effects are small, and hard to interpret.'' In 1990, Varlamov and Livanov[8] predicted AL contributions with the same strong divergence found in the electrical conductivity, $(T-T_c)^{d/2-2}$; this erroneous result appears to be due to an incorrect treatment of the heat-current operator. The same authors[9] also discussed the relative magnitudes of DOS, MT, and AL contributions in layered superconductors, and argued that the DOS and MT terms dominate in κ_c while AL terms dominate in κ_{ab}. The predicted fluctuation effects have since been seen experimentally in an $YBa_2Cu_3O_{7-\delta}$ single crystal,[3] and $Bi_2Sr_2CaCu_2O_8$ and $DyBa_2Cu_3O_{7-\delta}$ polycrystals.[4,5] Excellent quantitative agreement was found between theory and experiment; indeed, even the predicted two- to three-

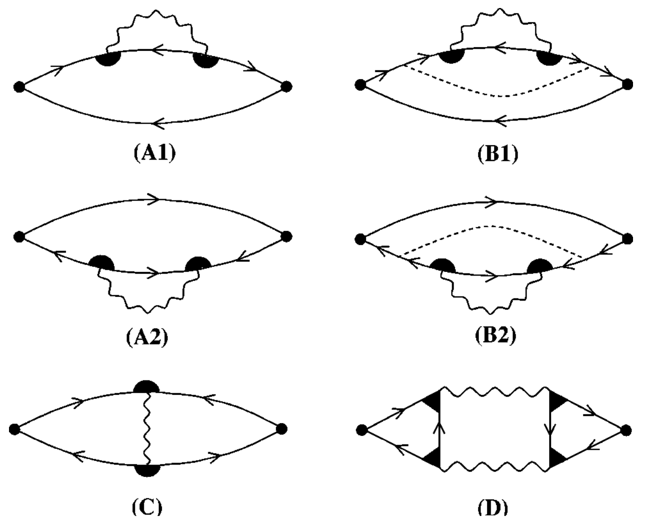

FIG. 1. Feynman diagrams which give singular contributions to the heat-current response function. Diagrams (a) and (b) are the density-of-states (DOS) correction diagrams; diagram (c) is the Maki-Thompson (MT) diagram; diagram (d) is the Aslamazov-Larkin (AL) diagram.

dimensional crossover is seen at roughly the predicted temperature. Fluctuation effects have also been seen in (Nd/Y)BCO intergrowth crystals,[6] although these have not been compared in detail with theory. However, there are problems with this apparent agreement between theory and experiment. The AL contributions have been reanalyzed in two works using phenomenological hydrodynamic[10] and Gaussian fluctuation[11] approaches, and are argued to be convergent. Very recently Savona *et al.*[12] have agreed that there is no divergent AL correction, but argue that there are still divergent DOS and MT terms; we believe that these authors have missed the cancellation between the DOS and MT terms.

II. THE CALCULATION

We now proceed to the details of our microscopic calculation. The thermal conductivity is obtained from the imaginary time heat response kernel, $Q_{hh}(i\Omega_n)$, by analytic continuation from positive Bose-Matsubara frequencies, $\Omega_n = 2\pi T n$,

$$\kappa = \lim_{\Omega\to 0} \frac{Q_{hh}(i\Omega_n \to \Omega + i0)}{i\Omega T}. \tag{5}$$

The diagrammatic contributions to the heat response kernel of the lowest order in perturbation theory are detailed in Fig. 1. The solid lines are disordered electron Green's functions,

$$G(k,i\varepsilon_l) = \frac{1}{i\varepsilon_l - \xi_k + \dfrac{i}{2\tau}\mathrm{sgn}(\varepsilon_l)}, \tag{6}$$

where $\varepsilon_l = 2\pi T(l+1/2)$ is a Fermi-Matsubara frequency, $\xi_k = k^2/2m - \mu$ is the electronic excitation spectrum, and τ is the elastic-scattering time. The black dots represent heat-current vertices, which are given by

$$\mathbf{j}_h(\mathbf{k},\varepsilon_l,\varepsilon_l+\Omega_n) = \frac{\mathbf{k}}{2m} i(2\varepsilon_l + \Omega_n). \tag{7}$$

The shaded regions are impurity vertex renormalizations which, at zero momentum, take the form

$$C(q=0,\varepsilon_1,\varepsilon_2) = \Theta(+\varepsilon_1\varepsilon_2) + \frac{\Theta(-\varepsilon_1\varepsilon_2)}{(|\varepsilon_1|+|\varepsilon_2|)\tau}, \tag{8}$$

whilst the dashed lines are single impurity renormalizations. The wavy lines are superconducting fluctuation propagators, $L(q,i\omega_m)$, which for small q are given by

$$L(q,i\omega_m)^{-1} = N(0)\left[\ln\left(\frac{T}{T_c}\right) + \psi\left(\frac{1}{2} + \frac{|\omega_m|}{4\pi T}\right) - \psi\left(\frac{1}{2}\right) + A(\omega_m)Dq^2\right], \tag{9}$$

where $N(0)$ is the electronic density of states per spin at the Fermi surface, $\omega_m = 2\pi T m$ is a Bose-Matsubara frequency, $\psi(x)$ is the digamma function, $D = v_F^2\tau/d$ is the diffusion constant, and $A(\omega_m)$ is given by

$$A(\omega_m) = \frac{1}{4\pi T}\psi'\left(\frac{1}{2} + \frac{|\omega_m|}{4\pi T}\right) - \tau\left[\psi\left(\frac{1}{2} + \frac{|\omega_m|}{4\pi T} + \frac{1}{4\pi T\tau}\right) - \psi\left(\frac{1}{2} + \frac{|\omega_m|}{4\pi T}\right)\right]. \tag{10}$$

The zero-frequency fluctuation propagator, $L(q,0)$, has a $1/q^2$ divergence as T approaches T_c,

$$L(q,0)^{-1} = N(0)\left[\frac{T-T_c}{T_c} + A(0)Dq^2\right]. \tag{11}$$

It is this feature that leads to divergent contributions to various physical properties as T approaches T_c.

Diagrams (a) and (b) of Fig. 1, in which a fluctuation propagator affects only one electron line yield the DOS contributions; diagram (c), in which a fluctuation propagator leads to interference between electron lines, yields the MT contribution; diagram (d), which possesses two fluctuation propagators, yields the AL contributions. Note that since the object of this paper is merely to show that there are no divergent contributions to κ at $T = T_c$, we have omitted all diagrams that cannot have such divergences. In particular, we have ignored all DOS and MT diagrams that have an impurity line or ladder between the two heat-current vertices. Such diagrams possess an extra factor of q^2, which removes the low-momentum singularity of the fluctuation propagator, $L(q,0)$. We also need to consider only the lowest power of q in any diagram since this will have the most divergent behavior—we, therefore, set $q=0$ in all terms except the fluctuation propagators. Finally, since all DOS and MT diagrams have only one superconducting fluctuation propagator, we can take the static limit and consider only terms $L(q,i\omega_m)$ with zero Cooper pair frequency, $\omega_m=0$. The AL term has two fluctuation propagators, and here we have to be more careful and keep all ω_m terms as there is an anomalous region of frequencies, where one propagator can have positive frequency and the other negative frequency.

The regular parts of the DOS and MT diagrams, which come from diagrams (a) and (c), give the total contribution,

$$Q_{hh}^{reg}(i\Omega_n)=-\pi N(0)DT^2\sum_{\varepsilon_l>0}\frac{(2\varepsilon_l+\Omega_n)^2}{[1+(2\varepsilon_l+\Omega_n)\tau]}\left\{\frac{1}{\varepsilon_l^2}+\frac{1}{(\varepsilon_l+\Omega_n)^2}-\frac{2}{\varepsilon_l(\varepsilon_l+\Omega_n)}\right\}\sum_q L(q,0). \tag{12}$$

The sum of the three terms in the curly brackets is easily seen to be proportional to Ω_n^2, so upon analytical continuation, division by Ω, and setting Ω to zero, we get zero contribution. The two DOS and one MT term have exactly canceled each other. Note that the same terms in the electromagnetic response function reinforce rather than cancel each other because the electric current vertex has the opposite electron-hole parity to the heat-current vertex (i.e., holes carry opposite charge but the same excitation energy to electrons).

The anomalous parts of the DOS and MT diagrams give total contribution

$$\begin{aligned}Q_{hh}^{anom}(i\Omega_n)&=\frac{\pi N(0)D}{(1+\Omega_n\tau)^2}T^2\sum_{0<\varepsilon_l<\Omega_n}(2\varepsilon_l-\Omega_n)^2\left\{\frac{1+2\varepsilon_l\tau}{\varepsilon_l^2}+\frac{1+\Omega_n\tau}{\varepsilon_l^2}-\frac{1}{\varepsilon_l^2}+\frac{1+\Omega_n\tau}{\varepsilon_l(\Omega_n-\varepsilon_l)}\right\}\sum_q L(q,0)\\&=\frac{N(0)DT}{(1+\Omega_n\tau)^2}\left\{-2\Omega_n^2\tau-\Omega_n\left[\psi\left(\frac{1}{2}+\frac{|\Omega_n|}{2\pi T}\right)-\psi\left(\frac{1}{2}\right)\right]+\frac{(1+\Omega_n\tau)\Omega_n^2}{4\pi T}\left[\psi'\left(\frac{1}{2}\right)-\psi'\left(\frac{1}{2}+\frac{|\Omega_n|}{2\pi T}\right)\right]\right\}\sum_q L(q,0),\end{aligned} \tag{13}$$

where we have explicitly carried out the ε_l sum. Upon analytically continuing $i\Omega_n\to\Omega$, dividing by Ω, and taking the limit $\Omega\to0$, the above expression gives zero result. The net result is thus that the anomalous part of the DOS+MT diagrams do not yield a divergent contribution.

Finally, it only remains to show that there is no divergent contribution from the AL terms. Paradoxically, although this result does not appear to be in dispute, it is the trickiest to prove. The method used is simple power counting, applied to the analytical continuation of the complete Matsubara frequency sum. We need the complete sum because there is an anomalous region of Bose frequency, ω_m, for which the two superconducting propagators, $L(q,i\omega_m+i\Omega_n)$ and $L(q,i\omega_m)$, have opposite signs of Matsubara frequency. We cannot, therefore, simply take the static approximation, where one or the other superconducting propagator has zero Matsubara frequency. Instead, we must evaluate the two triangle blocks for general ω_m, and distinguish between the three summation regions: (i) $\omega_m+\Omega_n>0$, $\omega_m>0$; (ii) $\omega_m+\Omega_n>0$, $\omega_m<0$; (iii) $\omega_m+\Omega_n<0$, $\omega_m<0$. Note that the two summation terms, $\omega_m=0$ and $\omega_m=-\Omega_n$, which possess one divergent fluctuation propagator, $L(q,0)$, are both zero after analytic continuation $i\Omega_n\to\Omega+i0$, division by Ω, and taking the limit $\Omega\to0$. It follows that when we analytically continue using the contours shown in Fig. 2, we need not worry about contours passing through the poles.

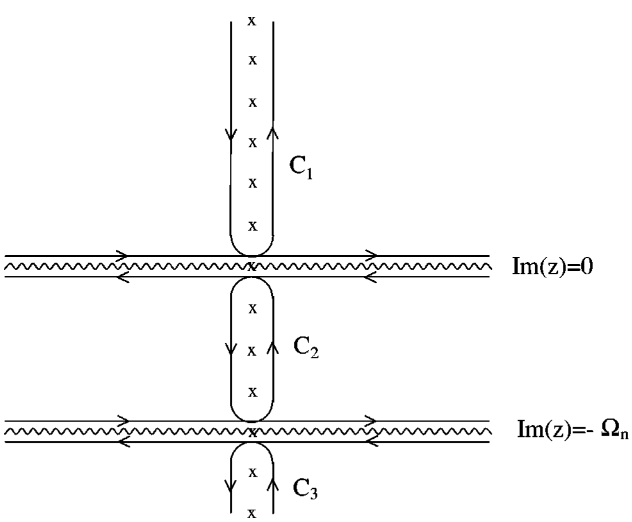

FIG. 2. Contour required to perform sum over Matsubara frequencies ω_m in the AL diagram. The branch cuts at $Im(\omega)=0$ and $Im(\omega)=-\Omega_n$ come from the fluctuation propagators $L(q,i\omega_m)$ and $L(q,i\omega_m+i\Omega_n)$. The poles summed over fall into three regions separated by the two branch cuts: (i) $\omega_m>0$; (ii) $0>\omega_m>-\Omega_n$; (iii) $-\Omega_n>\omega_m$. These contours can be deformed to contours parallel to the real axis as shown in the figure. Note that the poles, which lie on the branch cuts yield no singular contribution and can be ignored.

The contributions from regions (i) and (iii) give identical results, and their sum is

$$Q_1^{AL}(i\Omega_n)=-T\sum_{\omega_m>0}\sum_q\frac{q^2}{d}B_1(i\omega_m,i\Omega_n)^2\times L(q,i\omega_m)L(q,i\omega_m+i\Omega_n), \tag{14}$$

where the $B_1(i\omega_m,i\Omega_n)$ are from the triangle blocks. Upon replacing summation over ω_m by integration over ω, and analytically continuing $i\Omega_n\to\Omega+i0$, we get

$$Q_1^{AL}(\Omega)=-\frac{1}{4\pi i}\int_{-\infty}^{+\infty}d\omega\coth(\omega/2T)\sum_q\frac{q^2}{d}B_1(\omega,\Omega)^2\times L(q,\omega)L(q,\omega+\Omega). \tag{15}$$

For small ω, Ω, we can show that $B_1(\omega,\Omega)\approx\alpha\omega+\beta\Omega$, where α and β are constants, so that for power-counting purposes Eq. (15) at $T=T_c$ becomes, ignoring all irrelevant coefficients,

$$Q_1^{AL}(\Omega)\sim\int_{-\infty}^{+\infty}d\omega\,\coth(\omega/2T)\int d^dq q^2 \times\frac{(\omega+\Omega)^2}{(q^2-i\omega)(q^2-i\omega-i\Omega)}. \tag{16}$$

The $O(\Omega)$ piece can be found by expanding either the numerator or denominator. In both cases, the behavior as $\omega\sim q^2\sim 0$ is $O(q^d)$, and hence there is no infrared singularity for $d>0$.

The contribution from region (ii) has the form,

$$Q_2^{AL}(i\Omega_n)=-T\sum_{0>\omega_m>-\Omega_n}\sum_q\frac{q^2}{d}B_2(i\omega_m,i\Omega_n)^2 \times L(q,i\omega_m)L(q,i\omega_m+i\Omega_n), \tag{17}$$

which upon replacing summation over ω_m by integration over ω, gives

$$Q_2^{AL}(i\Omega_n)=-\frac{1}{4\pi i}\left[\int_{-\infty}^{+\infty}-\int_{-\infty-i\Omega_n}^{+\infty-i\Omega_n}\right]d\omega\,\coth(\omega/2T) \times\sum_q\frac{q^2}{d}B_2(\omega,i\Omega_n)^2L^A(q,\omega)L^R(q,\omega+i\Omega_n). \tag{18}$$

Shifting the variable in the second integral, $\omega\to\omega-i\Omega_n$, analytically continuing $i\Omega_n\to\Omega+i0$, shifting the variable back, $\omega\to\omega+\Omega$, dividing throughout by Ω, and letting $\Omega\to 0$ gives

$$\lim_{\Omega\to 0}\frac{Q_2^{AL}(\Omega+i0)}{\Omega}=\frac{1}{8\pi iT}\int_{-\infty}^{+\infty}\frac{d\omega}{\sinh^2(\omega/2T)} \times B_2(\omega,0)^2\sum_q\frac{q^2}{d}L^A(q,\omega)L^R(q,\omega). \tag{19}$$

For small ω, we can show that $B_2(\omega,0)=\gamma\omega$, where γ is a constant, so that for power-counting purposes Eq. (19) at $T=T_c$ becomes

$$\lim_{\Omega\to 0}\frac{Q_2^{AL}(\Omega+i0)}{\Omega} \sim\int_{-\infty}^{\infty}\frac{d\omega}{\sinh(\omega/2T)^2}\int d^dq q^2\frac{\omega^2}{(q^2-i\omega)(q^2+i\omega)}. \tag{20}$$

The behavior as $\omega\sim q^2\sim 0$ is $O(q^d)$, and hence there is no infrared singularity for $d>0$. We have, therefore, shown that there is no singular contribution from the AL diagrams.

III. CONCLUSIONS

We have shown that there are no superconducting fluctuation corrections to the thermal conductivity above the transition temperature, which are singular as T approaches T_c. The experimental features seen near T_c must, therefore have some other physical explanation, such as reduced phonon scattering from normal-state electrons. We hope that there will be continued experimental interest in thermal conductivity near T_c in one- and two-dimensional superconductors, of both the high-T_c and low-T_c variety. In future work, we also intend to evaluate the nonsingular fluctuation contributions to the thermal conductivity to see whether this can explain any of the experimental features (although, given their power-law behavior, this seems unlikely).

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