

Aging and fluctuation-dissipation ratio for the dilute Ising model

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We consider the out-of-equilibrium, purely relaxational dynamics of a weakly diluted Ising model in the aging regime at criticality. We derive at first order in a $\sqrt{\epsilon}$ expansion the two-time response and correlation functions for vanishing momenta. The long-time limit of the critical fluctuation-dissipation ratio is computed at the same order in perturbation theory.

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According to universality hypothesis, critical phenomena can be described in terms of quantities that do not depend on the microscopic details of the systems, but only on global properties such as symmetries, dimensionality, etc. A question of theoretical and experimental interest is whether and how this critical behavior is altered by introducing in the systems a small amount of uncorrelated nonmagnetic impurities leading to models with quenched disorder.

The static critical behavior of these systems is well understood thank to the Harris criterion.¹ It states that the addition of impurities to a system that undergoes a second-order phase transition does not change the critical behavior if the specific-heat critical exponent α_p of the pure system is negative. If α_p is positive, the transition is altered.

For the very important class of the three-dimensional $O(M)$ -vector models it is known that $\alpha_p < 0$ for $M \geq 2$,² and the critical behavior is unchanged in the presence of weak quenched disorder. Instead, the specific-heat exponent of the three-dimensional Ising model is positive.² Therefore the introduction of a small amount of nonmagnetic impurities (dilution) leads to a new universality class [as confirmed by renormalization group (RG) analyses, Monte Carlo simulations (MCs), and experiments, see Refs. 2 and 3 for a review], to which the weakly diluted random Ising model (RIM) belongs. The RIM is a lattice spin model with nearest-neighbor interaction Hamiltonian (see Ref. 2 and references therein) $\mathcal{H}_{RIM} = -\sum_{\langle ij \rangle} \rho_i \rho_j s_i s_j$, where s_i are the spins and ρ_i are uncorrelated quenched random variables such that $\rho_i = 1$ with probability c ($0 < c \leq 1$ being the spin concentration in the lattice), $\rho_i = 0$ with probability $1 - c$. The coarse-grained version of RIM is given by the random temperature Landau-Ginzburg Hamiltonian (see Ref. 2 and the discussion below). The purely relaxational equilibrium dynamics (model A of Ref. 4) of this new universality class is under intensive investigation.⁵⁻⁹ The dynamic critical exponent z differs from the mean-field value already in the one-loop approximation,⁵ at variance with the pure model. This exponent is known up to three-loop level in an $\sqrt{\epsilon}$ (Ref. 7) and in fixed ($d=2,3$) dimension⁸ expansion, and has a value in good agreement with several MCs.⁹

The out-of-equilibrium dynamics is less studied. The initial slip exponent θ of the response function was determined up to two-loop order¹⁰ and the response function only up to one-loop order, both for conservative and nonconservative dynamics.¹¹

In this work we take advantage of the studied short-time

scaling behavior of a critical relaxation process^{10,12} to investigate long-time dynamics in the aging regime. The relaxation of the system from an out-of-equilibrium initial state is characterized by two different regimes: a transient one with off-equilibrium evolution, for $t < t_R$, and a stationary one with equilibrium evolution of fluctuations for $t > t_R$, where t_R is the relaxation time. In the former a dependence of the behavior of the system on the initial condition is expected, while in the latter homogeneity of time and time reversal symmetry (at least in the absence of external fields) are recovered. Consider the system in a disordered state for the initial time $t=0$, and quench it to a given temperature $T \geq T_c$, where T_c is the critical temperature. Calling $\phi_{\mathbf{x}}(t)$ the order parameter of the model, its response to an external field h applied at a time $s > 0$ and in $\mathbf{x}=0$ is given by the response function $R_{\mathbf{x}}(t,s) = \delta \langle \phi_{\mathbf{x}}(t) \rangle / \delta h(s)$, where $\langle \dots \rangle$ stands for the mean over stochastic dynamics. The two-time correlation function $C_{\mathbf{x}}(t,s) = \langle \phi_{\mathbf{x}}(t) \phi_{\mathbf{0}}(s) \rangle$ is useful to describe the dynamics of order-parameter fluctuations. When the system does not reach the equilibrium (i.e., $t_R = \infty$) all the previous functions will depend both on s (the ‘‘age’’ of the system) and t . This behavior is usually referred to as aging and was first noted in glassy systems.^{13,14} To characterize the distance from equilibrium of an aging system, evolving at a fixed temperature T , the fluctuation-dissipation ratio (FDR) is usually introduced,¹⁵

$$X_{\mathbf{x}}(t,s) = \frac{T R_{\mathbf{x}}(t,s)}{\partial_s C_{\mathbf{x}}(t,s)}. \quad (1)$$

When $t, s \gg t_R$ the fluctuation-dissipation theorem holds and thus $X_{\mathbf{x}}(t,s) = 1$.

Recently¹⁶⁻²³ attention has been paid to the FDR, for non-equilibrium, nondisordered, and unfrustrated systems quenched at their critical temperature T_c from an initial disordered state.²⁴ The scaling form for $R_{\mathbf{x}=\mathbf{0}}$ can be obtained by using general RG arguments,¹²

$$T_c R_{\mathbf{x}=\mathbf{0}}(t,s) = A_R (t-s)^a (t/s)^{\theta} F_R(s/t), \quad (2)$$

where $a = (2 - \eta - z)/z$. Moreover, local scale invariance predicts $F_R(v) = 1$.^{23,25} For the correlation function the RG analysis gives¹²

$$C_{\mathbf{x}=\mathbf{0}}(t,s) = A_C s (t-s)^a (t/s)^{\theta} F_C(s/t), \quad (3)$$

$$\partial_s C_{\mathbf{x}=\mathbf{0}}(t,s) = A_{\partial C} (t-s)^a (t/s)^{\theta} F_{\partial C}(s/t), \quad (4)$$

with the same θ and a as in Eq. (2). The functions $F_C(v)$, $F_{\partial C}(v)$, and $F_R(v)$ are universal, provided one fixes the nonuniversal, normalization constants A_R , A_C , and $A_{\partial C}$ to have $F_i(0)=0$. Obviously, $A_{\partial C}$ and $F_{\partial C}(v)$ may be derived from A_C and $F_C(v)$.

It has been argued that the limit

$$X_{\mathbf{x}=0}^{\infty} = \lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} X_{\mathbf{x}=0}(t, s) = \frac{A_R}{A_{\partial C}} = \frac{A_R}{A_C(1-\theta)} \quad (5)$$

is a novel universal quantity of nonequilibrium critical dynamics.^{18,20}

In a recent work²² we evaluated $X_{\mathbf{x}=0}^{\infty}$ at criticality for $O(M)$ -vector models up to the second order in the ϵ expansion, finding values in very good agreement with two- and three-dimensional numerical simulations for the Ising model.^{18,20} We also confirmed the validity of the scaling laws (2) and (4) up to the same order in perturbation theory. A different model of relaxation (model C of Ref. 4) has also been studied.²⁶

The extension of this kind of investigation to disordered systems is very interesting because, besides giving a check of the expected scaling laws, it predicts a new universal dynamical quantity (the long-time limit of the FDR) that could be measured in MCs and could be used to identify a universality class, as in the case of other universal quantities.

The time evolution of a scalar field $\varphi(\mathbf{x}, t)$ under a purely dissipative relaxation dynamics (model A of Ref. 4) is described by the stochastic Langevin equation,

$$\partial_t \varphi(\mathbf{x}, t) = -\Omega \frac{\delta \mathcal{H}_{\psi}[\varphi]}{\delta \varphi(\mathbf{x}, t)} + \xi(\mathbf{x}, t), \quad (6)$$

where Ω is the kinetic coefficient, $\xi(\mathbf{x}, t)$ a zero-mean stochastic Gaussian noise with correlations

$$\langle \xi(\mathbf{x}, t) \xi(\mathbf{x}', t') \rangle = 2\Omega \delta(\mathbf{x} - \mathbf{x}') \delta(t - t'), \quad (7)$$

and $\mathcal{H}_{\psi}[\varphi]$ the static Landau-Ginzburg Hamiltonian with random temperature,²

$$\mathcal{H}_{\psi}[\varphi] = \int d^d x \left[\frac{1}{2} (\partial \varphi)^2 + \frac{1}{2} [r_0 + \psi(\mathbf{x})] \varphi^2 + \frac{1}{4!} g_0 \varphi^4 \right]. \quad (8)$$

Here $\psi(\mathbf{x})$ is a spatially uncorrelated random field with Gaussian distribution,

$$P(\psi) = \frac{1}{\sqrt{4\pi w}} \exp\left[-\frac{\psi^2}{4w}\right]. \quad (9)$$

Dynamical correlation functions, generated by Langevin equation (6) and averaged over the noise ξ , can be obtained by the field-theoretical action,²⁷

$$S_{\psi}[\varphi, \tilde{\varphi}] = \int dt d^d x \left[\tilde{\varphi} \frac{\partial \varphi}{\partial t} + \Omega \tilde{\varphi} \frac{\delta \mathcal{H}_{\psi}[\varphi]}{\delta \varphi} - \tilde{\varphi} \Omega \tilde{\varphi} \right], \quad (10)$$

where $\tilde{\varphi}(\mathbf{x}, t)$ is the response field.

The effect of a macroscopic initial condition $\varphi_0(\mathbf{x}) = \varphi(\mathbf{x}, t=0)$ may be taken into account by averaging over the initial configuration with a weight $e^{-H_0[\varphi_0]}$,^{12,10} where

$$H_0[\varphi_0] = \int d^d x \frac{\tau_0}{2} [\varphi_0(\mathbf{x}) - a(\mathbf{x})]^2. \quad (11)$$

This specifies an initial state $a(\mathbf{x})$ with short-range correlations proportional to τ_0^{-1} . In this way, all response and correlation functions may be obtained as averages on the functional weight $\exp\{-S_{\psi}[\varphi, \tilde{\varphi}] - H_0[\varphi_0]\}$. In the analysis of static critical behavior, the average over the quenched disorder ψ is usually performed by means of the replica trick.² If instead we are interested in dynamic processes it is simpler to perform directly the average at the beginning of the calculation,²⁸

$$\int [d\psi] P(\psi) \exp(-S_{\psi}[\varphi, \tilde{\varphi}]) = \exp(-S[\varphi, \tilde{\varphi}]) \quad (12)$$

with the ψ -independent action¹⁰ (with $v_0 \propto w$),

$$S[\varphi, \tilde{\varphi}] = \int d^d x \left\{ \int_0^{\infty} dt \tilde{\varphi} [\partial_t \varphi + \Omega(r_0 - \Delta)\varphi - \Omega \tilde{\varphi}] + \frac{\Omega g_0}{3} \int_0^{\infty} dt \tilde{\varphi} \varphi^3 - \frac{\Omega^2 v_0}{2} \left(\int_0^{\infty} dt \tilde{\varphi} \varphi \right)^2 \right\}. \quad (13)$$

The perturbative expansion is performed in terms of the two fourth-order couplings g_0 and v_0 and using the propagators of the free theory with an initial condition at $t=0$, $\langle \tilde{\varphi}(\mathbf{q}, s) \varphi(-\mathbf{q}, t) \rangle_0 = R_q^0(t, s)$ and $\langle \varphi(\mathbf{q}, s) \varphi(-\mathbf{q}, t) \rangle_0 = C_q^0(t, s)$.¹² It has also been shown that τ_0^{-1} is irrelevant (in RG sense) for large times behavior.¹² From the technical point, the breaking of time homogeneity gives rise to some problems in the renormalization procedure in terms of one-particle irreducible correlation functions (see Ref. 12) so all the computations are done in terms of connected functions.

In this field-theoretical approach the calculation are simpler in momentum space. The response and correlation functions for $\mathbf{q}=0$ are expected to scale as their analog in real space, Eqs. (2), (3), and (4). As a consequence, the ratio²¹

$$\mathcal{X}_{\mathbf{q}=0}(t, s) = \frac{\Omega R_{\mathbf{q}=0}(t, s)}{\partial_s C_{\mathbf{q}=0}(t, s)} \quad (14)$$

is also an universal quantity. In Ref. 21 a heuristic argument is given to show that $X_{\mathbf{x}=0}^{\infty} = \mathcal{X}_{\mathbf{q}=0}^{\infty}$ [where $\mathcal{X}_{\mathbf{q}=0}^{\infty} = \lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} \mathcal{X}_{\mathbf{q}=0}(t, s)$].

To compute the one-loop contributions to the response function, we have to evaluate the two Feynman diagrams depicted in Fig. 1. In the following we report the expressions of all the diagrams only at criticality ($r_0=0$ in dimensional regularization) and for vanishing external momentum, since we are only interested in that limit, and since expressions for nonzero \mathbf{q} are long and not very illuminating. The bare response function is thus given by (we assume $t > s$)

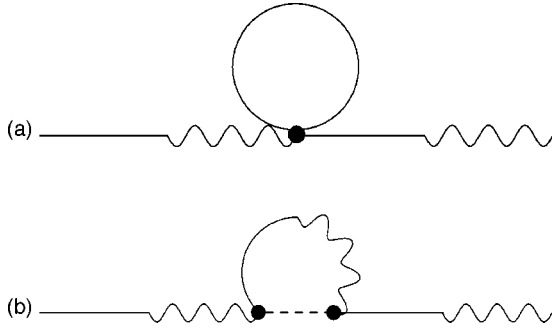


FIG. 1. Feynman diagrams contributing to the one-loop response function. Wavy-normal lines represent R_q^0 , whereas C_q^0 is drawn as a normal line. The dotted line is a non-local v -like vertex.

$$R_B(t,s) = 1 - \frac{1}{2}g_0(a) + v_0(b) + O(g_0^2, v_0^2, g_0v_0), \quad (15)$$

where we set $\Omega = 1$ to lighten the notation.

The diagram (a) in Fig. 1 contributes also the response function of nondisordered models, and it has been computed in Ref. 21, obtaining

$$(a) = -N_d \frac{1}{4} \ln \frac{t}{s} + O(\epsilon). \quad (16)$$

where $N_d = 2/[\Gamma(d/2)(4\pi)^{d/2}]$. For diagram (b) we find

$$\begin{aligned} (b) &= \int_0^\infty dt' dt'' \int \frac{d^d p}{(2\pi)^d} R_p^0(t, t') R_p^0(t', t'') R_p^0(t'', s) \\ &= \frac{1}{(4\pi)^{d/2}} \frac{1}{1-d/2} \frac{1}{2-d/2} (t-s)^{2-d/2}. \end{aligned} \quad (17)$$

Inserting the expression for (a) and (b) in Eq. (15) and expanding (b) up to the first order in ϵ , one obtains

$$\begin{aligned} R_B(t,s) &= 1 + \tilde{g}_0 \frac{1}{8} \ln \frac{t}{s} - \frac{\tilde{v}_0}{2} \left[\frac{2}{\epsilon} + \ln(t-s) + \gamma_E \right] \\ &+ O(\epsilon^2, \epsilon \tilde{g}_0, \epsilon \tilde{v}_0, \tilde{g}_0^2, \tilde{v}_0^2, \tilde{g}_0 \tilde{v}_0), \end{aligned} \quad (18)$$

where $\tilde{g}_0 = N_d g_0$ and $\tilde{v}_0 = N_d v_0$. The dimensional pole of this expression (i.e., its singular part for $\epsilon \rightarrow 0$) can be canceled out by the multiplicative renormalizations $\varphi \mapsto Z^{1/2} \varphi$, $\tilde{\varphi} \mapsto \tilde{Z}^{1/2} \tilde{\varphi}$ [thus $R(t,s) = Z^{-1/2} \tilde{Z}^{-1/2} R_B(t,s)$ and $C(t,s) = Z^{-1} C_B(t,s)$], $\Omega \mapsto (Z/\tilde{Z})^{1/2} \Omega$, $Z = 1 + O(\tilde{g}_0^2, \tilde{v}_0^2, \tilde{g}_0 \tilde{v}_0)$, with^{10,7} $\tilde{Z} = 1 + 2\tilde{v}_0/\epsilon + O(\tilde{g}_0^2, \tilde{v}_0^2, \tilde{g}_0 \tilde{v}_0)$.

The critical response function is then obtained by setting the renormalized couplings at their fixed point values. We remind that the stable fixed point of the RIM is of order $\sqrt{\epsilon}$ and not ϵ (see, e.g., Ref. 2), due to the degeneracy of the one-loop β functions. The nontrivial fixed point values at the first nonvanishing order (i.e., two loops) are (see, e.g., Refs. 10 and 7)

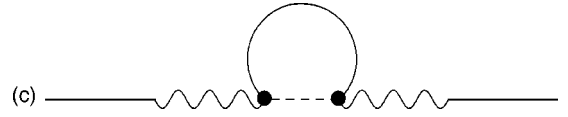


FIG. 2. Feynman diagram contributing to the one-loop correlation function that does not have an analog in the response.

$$\tilde{g}_0 = \tilde{g}^* = 4 \sqrt{\frac{6\epsilon}{53}} + O(\epsilon), \quad \tilde{v}_0 = \tilde{v}^* = \sqrt{\frac{6\epsilon}{53}} + O(\epsilon). \quad (19)$$

Finally we get

$$R(t,s) = 1 + \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} \left[\ln \frac{t}{s} - \ln(t-s) - \gamma_E \right] + O(\epsilon), \quad (20)$$

which is fully compatible with the expected scaling form (2) with the well-known exponents^{7,11}

$$a = -\frac{1}{2} \sqrt{\frac{6\epsilon}{53}} + O(\epsilon), \quad \theta = \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} + O(\epsilon), \quad (21)$$

and $F_R(x) = 1 + O(\epsilon)$, while

$$A_R = 1 - \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} \gamma_E + O(\epsilon). \quad (22)$$

There are five diagrams contributing to the correlation function. Four of them are obtained from the ones in Fig. 1, changing one of the two external response propagators with a correlation line (see Ref. 22 for a detailed explanation of this correspondence). We call these four diagrams (a_1) , (a_2) , (b_1) , and (b_2) . The sum $(a_1) + (a_2)$ was computed in Ref. 21 leading to

$$(a_1) + (a_2) = -\frac{N_d}{2} s \left(\ln \frac{t}{s} + 2 \right) + O(\epsilon). \quad (23)$$

The sum $(b_1) + (b_2)$ is instead

$$\begin{aligned} (b_1) + (b_2) &= \frac{N_d \Gamma(d/2)}{(1-d/2)(2-d/2)(3-d/2)} \\ &\times [t^{3-d/2} + s^{3-d/2} - (t-s)^{3-d/2}]. \end{aligned} \quad (24)$$

The octopus diagram in Fig 2 does not have a corresponding one contributing to the response function. It has the value

$$\begin{aligned} (c) &= \frac{N_d \Gamma(d/2)}{(1-d/2)(2-d/2)(3-d/2)} \\ &\times \left[\frac{(t-s)^{3-d/2} + (t+s)^{3-d/2}}{2} - t^{3-d/2} - s^{3-d/2} \right]. \end{aligned} \quad (25)$$

Collecting together these contributions and expanding in powers of ϵ we find, at $O(\epsilon^2, \epsilon \tilde{g}_0, \epsilon \tilde{v}_0, \tilde{g}_0^2, \tilde{v}_0^2, \tilde{g}_0 \tilde{v}_0)$,

$$\begin{aligned}
C_B(t,s) &= 2s - \frac{g_0}{2} [(a_1) + (a_2)] + v_0 [(b_1) + (b_2) + (c)] \\
&= 2s + \frac{\tilde{g}_0}{4} s \left(\ln \frac{t}{s} + 2 \right) + \tilde{v}_0 \left[-\frac{2s}{\epsilon} - \gamma_E s + s \right. \\
&\quad \left. + \frac{(t-s)\ln(t-s)}{2} - \frac{(t+s)\ln(t+s)}{2} \right]. \quad (26)
\end{aligned}$$

This expression is renormalized as already done for $R_B(t,s)$. At the nontrivial fixed point for couplings we have

$$\begin{aligned}
\frac{C(t,s)}{2} &= s \left\{ 1 + \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} \left[\ln \frac{t}{s} + 3 - \gamma_E - \ln(t-s) \right. \right. \\
&\quad \left. \left. + \frac{t+s}{2s} \ln \frac{t-s}{t+s} \right] \right\} + O(\epsilon), \quad (27)
\end{aligned}$$

which is again compatible with the expected scaling form [cf. Eq. (3)] with the same exponents of Eq. (21), the non-universal amplitude

$$\frac{A_C}{2} = 1 + \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} (2 - \gamma_E) + O(\epsilon), \quad (28)$$

and the universal regular scaling function

$$F_C(x) = 1 + \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} \left[1 + \frac{1}{2} \left(1 + \frac{1}{x} \right) \ln \frac{1-x}{1+x} \right] + O(\epsilon). \quad (29)$$

Note that at variance with the pure model,^{21,22} the function $F_C(x)$ receives a contribution already at one-loop order which should be observable in MCs.

Using Eq. (5) the FDR is

$$\chi_{q=0}^\infty = \frac{1}{2} - \frac{1}{4} \sqrt{\frac{6\epsilon}{53}} + O(\epsilon), \quad (30)$$

which, for $\epsilon=1$ leads to $\chi_{q=0}^\infty \sim 0.416$, and $\chi_{q=0}^\infty \sim 0.381$ for $\epsilon=2$. It would be interesting to see if this one-loop result is in as good agreement with MCs as in the case of the pure model (cf. Refs. 21 and 22). To this order it is not even clear whether or not randomness really changes in a sensible way the limit of the FDR. Two-loop computations and MCs could clarify this point.

For completeness, we report also the FDR for finite times,

$$\chi_{q=0}^{-1}(t,s) = 1 + \frac{1}{2} \sqrt{\frac{6\epsilon}{53}} \left[1 + \frac{1}{2} \ln \frac{t-s}{t+s} \right] + O(\epsilon). \quad (31)$$

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