# Conductance fluctuations and distribution at the metal-insulator transition induced by an electric field in a disordered chain

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A simple Kronig-Penney model for one-dimensional (1D) mesoscopic systems with $\delta$ peak potentials is used to study numerically the influence of a constant electric field on the conductance fluctuations and distribution at the metal-insulator transition. We use the Levy laws to investigate the statistical properties of the eigenstates. We found at this transition that the conductance probability distribution has a system-size independent shape with large fluctuations in good agreement with previous works in 2D and 3D systems.

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## I. INTRODUCTION

During the last two decades several works have been devoted to understanding transport properties in mesoscopic systems.[1–9] Experiments performed on such systems showed that the conductance $g$ is not a self-averaged quantity,[4] but that it fluctuates with the Fermi energy, chemical potential, and sample size. In the metallic regime both experimental[3–5] and theoretical[6–9] studies indicated that the conductance fluctuations are of order of $e^2/h$, and are universal (called universal conductance fluctuations).

In the localized (insulating) regime, the conductance exhibits strong fluctuations which tend to diverge for large system sizes.[10,11] At the critical point of the metal-insulator transition (MIT), large fluctuations were predicted analytically for $d=2+\varepsilon$ $(\varepsilon \ll 1)$,[12,13] but not found numerically by Markos and Kramer in two-dimensional (2D) and 3D systems.[14,15] They concluded that the large conductance fluctuations are not the general feature of the metal-insulator transition. It was also found that the variance of $\log(g)$ is of order of its mean value, and that the standard deviation of $g$ is also $\sim\langle g\rangle$.[16]

Since the conductance does not obey the central limit theorem,[2] it is necessary to investigate not only the first two moments but the whole probability distribution. Numerical results in 2D and 3D disordered systems showed that the conductance is normally distributed in a metallic regime, while for strongly localized systems (insulating regime) a log-normal distribution was found.[17] The correct form of the probability distribution at the transition is not well known. In such a regime, it was proven that the conductance distribution is independent of the microscopic details of the model (determined by the distribution of the disorder), of the system size, and of the position of the critical point which separates the metallic and localized regimes in the phase space of external parameters (energy and disorder). The universality of the conductance distribution was studied and confirmed for 2D and 3D models,[14,15] while the system-size invariance of $P(g)$ at the critical points of the MIT was confirmed for 3D and 4D systems.[18,19] It was also found that the critical distribution depends on the dimension[16] and the symmetry of the system,[16,18] as well as on the boundary conditions.[20]

The electric field was shown to delocalize the electronic states in 1D disordered systems (where all the states are localized).[21,22] In a previous work, we studied the transition from the localized to weakly localized states, and showed that the Anderson transition may occur for strong fields.[23] However, the position of this transition was not found for these systems.

The aim of this work is to find the Anderson transition region and test the size independence of the probability distribution of the conductance in disordered mesoscopic chains under an applied electric field. For this end we use Levy statistics.[24]

## II. MODEL DESCRIPTION

We consider a Kronig-Penney model applied to a 1D system of equally spaced potential $V(x)$ with random strengths under a constant electric field $F$. The corresponding Schrodinger equation can be read

$$\left\{-\frac{d^2}{dx^2}+\sum_n \beta_n\delta(x-n)-eFx\right\}\Psi(x)=E\Psi(x). \quad (1)$$

Here $\Psi(x)$ is the single-particle wave function at $x$, $\beta_n$ is the potential strength of the $n$th site, and $E$ is the single-particle energy in units of $\hbar^2/2m$, with $m$ the electronic effective mass. The electronic charge $e$ and the lattice parameter $a$ are taken here for simplicity to be unity. The chain length is identical to the number of scatterers $(L=N)$. The two ends of the system are assumed to be connected ohmically to ideal leads (where the electron moves freely) and maintained at a constant potential difference $V=FL$. The potential strength $\beta_n$ is uniformly distributed between $-W/2$ and $W/2$ ($W$ being the degree of disorder).

The exact solution of Eq. (1) is Airy-function-like. In order to reduce the computational time consumption, we use the so-called ladder approximation which is valid only for weak fields $Fa<E$.[21,22] For stronger fields, we use the multistep function approximation,[25] which is very accurate, and use plane waves instead of Airy functions. This approxima-

 

tion consists of subdividing each step into $m$ steps, so that the condition $Fa/m<E$ is satisfied.

The second-order differential equation (1) can be mapped by means the Poincaré map representation[21]

$$\Psi_{n+1}=\left[\cos(k_{n+1})+\frac{k_n\sin(k_{n+1})}{k_{n+1}\sin(k_n)}\cos(k_n)+\beta_n\frac{\sin(k_{n+1})}{k_{n+1}}\right]\Psi_n-\frac{k_n\sin(k_{n+1})}{k_{n+1}\sin(k_n)}\Psi_{n-1}, \quad (2)$$

where $\Psi_n$ is the value of the wave function at site $n$ and $k_n=\sqrt{E+Fn}$ is the electron wave number at the site $n$. The solution of Eq. (2) is carried out iteratively by taking the two initial wave functions at sites 1 and 2: $\Psi_1=\exp(-ik)$ and $\Psi_2=\exp(-2ik)$. Here we consider an electron having a wave number $k$ incident at site $N+3$ from the right (by taking the chain length $L=N$, i.e., $N+1$ scatterers). The transmission coefficient ($T$) reads

$$T=\frac{k_0}{k_L}\frac{|1-\exp(-2ik_L)|^2}{|\Psi_{N+2}-\Psi_{N+3}\exp(-ik_L)|^2} \quad (3)$$

where $k_0=\sqrt{E}$ and $k_L=\sqrt{E+FN}$.

The dimensionless four-probe conductance ($g=G/e^2/h$) can be obtained from the transmision coefficient $T$ via the Landauer formula for 1D systems,[26]

$$g=\frac{2T}{1-T}, \quad (4)$$

where the factor 2 arises from the two possible states of the electron spin.

### III. RESULTS AND DISCUSSION

In this section we discuss numerical results of the conductance fluctuations and distribution at the transition regime. In a previous work,[23] we showed that the true metal-insulator transition does not occur at $E\approx V$ as believed from the results of Mato and Caro,[27] but this regime can be obtained for stronger fields (this point corresponds to a transition to power-law localized states). In order to obtain the probabilty distribution of the conductance, we build a statistical ensemble of $10^4$ samples which differ only in the realization of the disorder. The electric field has been shown to delocalize the electronic states in 1D disordered systems where the wave function becomes power-law decaying,[21,22] while for sufficiently large field strengths the eigenstates become extended.[23] The metal-insulator transition regime in this system is unknown. In the absence of a field, the conductance exhibits large fluctuations signature of strong localization. When the field increases, these fluctuations decrease, indicating a metallic regime.

In Fig. 1, we show the variance of Ln($g$) as a function of the applied voltage (for large field strengths $F\geqslant 1$). The conductance fluctuations have peaks due to the Stark ladder localization.[28] Beyond $F=3$ the fluctuations decrease leading to the metallic regime. To understand the behavior of the conductance fluctuations for different values of the applied

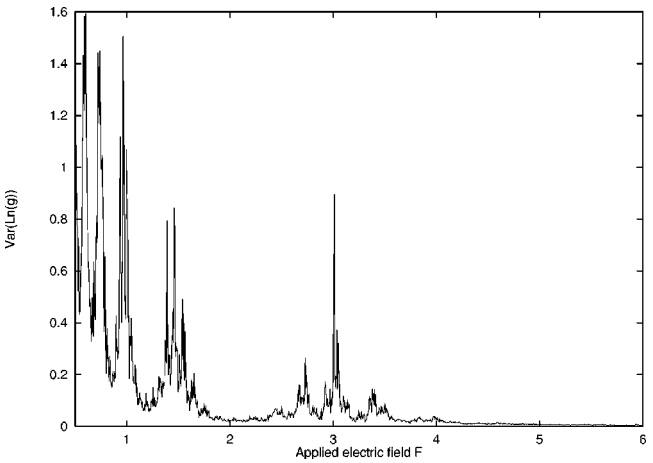


FIG. 1. Variance of Ln($g$) vs applied electric field for $L=500$, $E=0.4$, and $W=0.25$.

electric field, we investigate the conductance distribution $P(g)$ for different values of the field. All these distributions show long power-law tails decreasing for large values of the conductance $g$. Therefore, we use the Levy statistics $L_\mu(Z)$ of index $\mu$ which decrease as $Z^{-(1+\mu)}$ for large values of $Z$.[24] We found that the conductance distribution $P(g)$ behaves as $g^{-(1+\mu)}$ for large values of $g$. The exponent $\mu$ is then calculated from the log-log plot of $P(g)$ for large values of $g$, which is linear with a slope equal to $-(1+\mu)$. In Fig. 2, we show the index $\mu$ as a function of the applied field strength. The exponent $\mu$ starts to increase when the field strength increases with a minimum for $F=3$. For $F\simeq 1$ (corresponding to a peak in Fig. 1), the exponent $\mu$ is still less than 2, corresponding to the localized regime. When the field increases above 1, the exponent $\mu$ becomes greater than 2, indicating the normal law of the distribution. The surprising decrease of the exponent $\mu$ near $F=3$ is expected to be a Stark localization[28] with a competing metallic behavior.

Now let us check the nature of the regime at the minimum of Fig. 1 ($F=3$). In Fig. 3, we show the probabilty distribution of Ln($g$) [Fig. 3(a)] and g [Fig. 3(b)] for different sizes of the system ($L=500$, 700, 800, and 900) and for $F=3$.

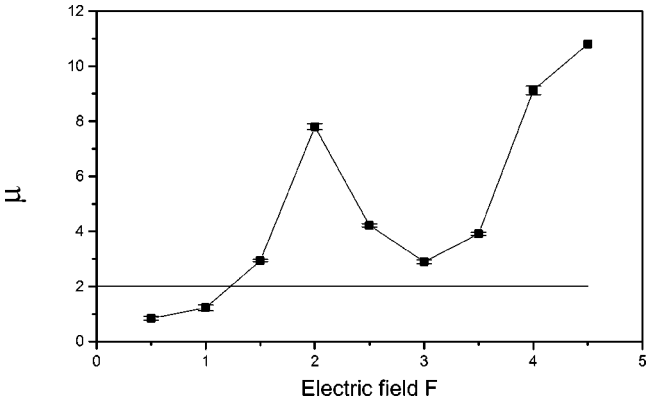


FIG. 2. The index $\mu$ as a function of the applied electric field for the same parameters as in Fig. 1.

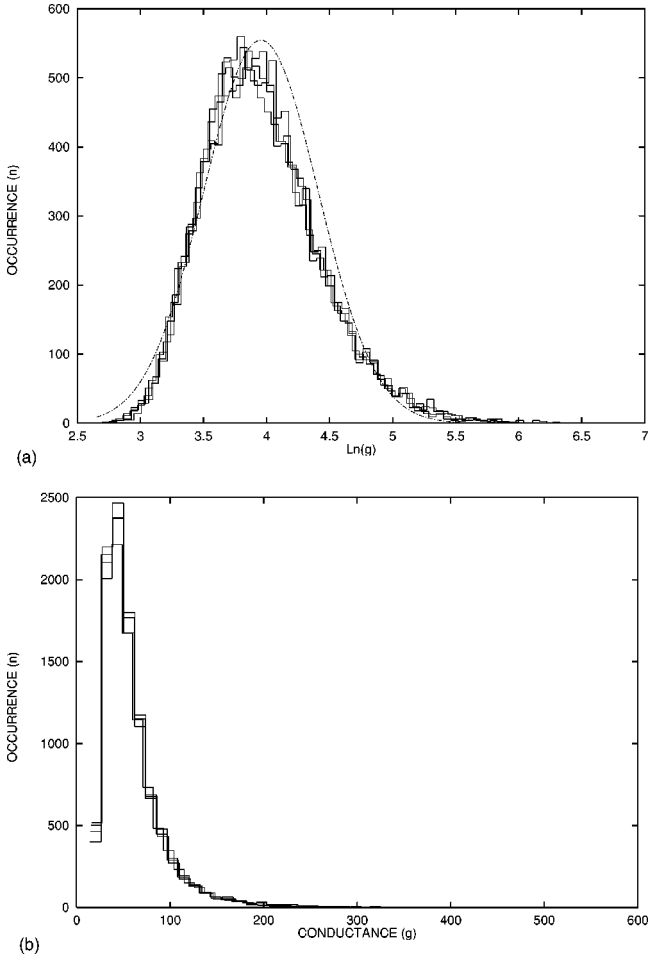


FIG. 3. (a) Probability distribution of $\mathrm{Ln}(g)$ for $F=3$, $E=0.4$, and $W=0.25$ and different system sizes ($L=500$, 700, 800, and 900) compared with a Gaussian with the same mean and variance (dashed curve). (b) Distribution of $g$ for the same parameters as in (a).

The conductance distribution seems to be neither normal nor log-normal. The size independence of the conductance distribution is in agreement with the observed one for 2D and 3D systems.[14–16,19] This behavior is not observed in the metallic and insulating regime (see Fig. 4) where the conductance distribution depends on the size of the system. The long tail of the distribution in Fig. 3(a) is representative for large fluctuations in good agreement with the results of Shapiro[12] and Shapiro and Cohen[13] for the metal-insulator transition. The distribution of $g$ in Fig. 3(b) has a similar shape, as found recently in the crossover region between metallic and insulating regime in disordered quasi-one-dimensional wires.[29] For an electric-field strength corresponding to the peak in Fig. 1 at $F\simeq 1$, the conductance distribution has a size-dependent form (not shown). All these arguments lead us to conclude that a MIT regime occurs at $F=3$ for $E=0.4$ and $W=0.25$. We can also observe that this regime is characterized by very large conductances $\langle g\rangle \simeq 58$ [Fig. 3(b)]. This was not observed in the metallic regime for large fields where the mean conductance is much smaller $\langle g\rangle \simeq 0.4$.[23] This is probably due to the competition between

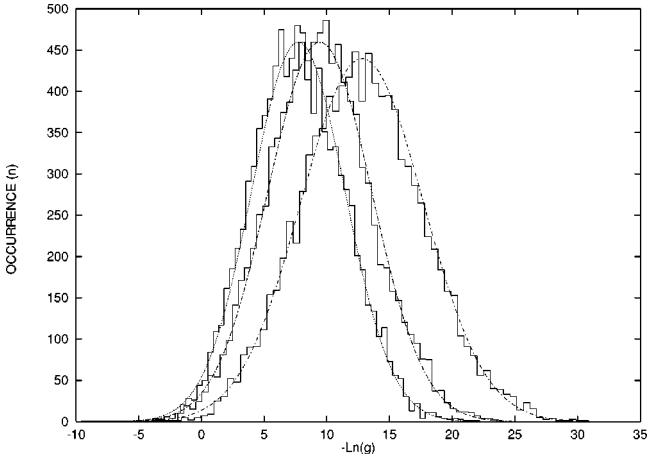


FIG. 4. Distribution of $-\mathrm{Ln}(g)$ in the insulting regime for $F=0$, $E=5$, and $W=2$, and different system sizes ($L=500$, 600, and 800) from left to right compared with a Gaussian with the same mean and variance (dashed curves).

the ''accelerating'' effect of the field and the destructive interferences within Stark localization.[28] In Table I we present a mean value of $\log g$ and its variance. We note clearly that $\mathrm{Var}[\ln(g)]$ is not of the order of $\ln(g)$ which is typical for the localized states as found for 3D and 4D systems.[16] This shows that in our system the extended states dominates due to an electric field. We can also see that $P(g)$ has a hole at small $g$, in agreement with the analytical result in the $\varepsilon$ expansion[12,13] and with the numerical results for 2D (Ref. 30) and 3D systems.[19,31]

## IV. CONCLUSION

We have used the Kronig-Penney model in a simple 1D disordered system in the presence of an electric field to determine the metal-insulator transition and examine the conductance fluctuations and the size independence of its distribution at this transition. To find this transition, we have used stable Levy laws.[24] We found this transition to be the result of the competition between the field-induced localization (Stark) and the field-induced delocalization. These effects make the transition not easy to determine. However, the point at $F=3$ seems to correspond to this transition. Indeed, the results for the conductance distribution are in good agreement with the previous works in 2D and 3D systems for other models[14,15] for metal-insulator transition (the field increases the effective dimension of the wave function). It is important to study the universality of the conductance distri-

TABLE I. Mean conductance $\langle \mathrm{Ln}(g)\rangle$ and its variance for different system sizes $L$ and for the same parameters as in Fig. 3(a).

| $L$ | $\langle \mathrm{Ln}(g)\rangle$ | $\mathrm{Var}[\mathrm{Ln}(g)]$ |
|---|---|---|
| 500 | 3.9557 | 0.20548 |
| 600 | 3.9314 | 0.20474 |
| 700 | 3.9596 | 0.21149 |
| 800 | 3.9632 | 0.21717 |

bution at the transition in a system of finite width potentials where the conductance fluctuations are less important in comparison to the present model,[23] and to find the critical points of the transition in the (energy-disorder electric field) phase space and for different kinds of disorder. These problems will be the subject of a forthcoming paper.

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[1] For a review, see Y. Imry, Rev. Mod. Phys. **71**, S306 (1999); C. W. J. Beenakker, *ibid.* **69**, 731 (1997); D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, New York, 1997); D. Reguera, G. Platero, L. L. Bonilla, and J. M. Rubi, *Statistical and Dynamical Aspects of Mesoscopic Systems* (Springer-Verlag, Berlin, 2000); A. M. Jayannavar, H. R. Krishnamurthy, and A. K. Raychaudhuri, *Mesoscopic and Disordered Systems* (Indian Academy of Sciences, Bangalore, 2002).
[2] B. L. Alt'shuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Éksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].
[3] R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985).
[4] S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).
[5] J. L. Pichard and M. Sanquer, Physica A **167**, 67 (1990).
[6] P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985); P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).
[7] H. A. Weidenmuller, Physica A **167**, 28 (1990).
[8] B. L. Alt'shuler, Pis'ma Zh. Éksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].
[9] I. V. Lerner, Phys. Lett. A **133**, 253 (1988).
[10] K. S. Chase and A. McKinnon, J. Phys. C **20**, 6189 (1987).
[11] B. Kramer, A. Kawabata, and M. Schreiber, Philos. Mag. B **65**, 596 (1992).
[12] B. Shapiro, Philos. Mag. B **56**, 1032 (1987); Phys. Rev. Lett. **65**, 1510 (1990).
[13] B. Shapiro and A. Cohen, Int. J. Mod. Phys. B **6**, 1243 (1992).
[14] P. Markos, Europhys. Lett. **26**, 431 (1994); J. Phys. I **4**, 551 (1994).
[15] P. Markos and B. Kramer, Philos. Mag. B **68**, 357 (1993).
[16] P. Markos, Phys. Rev. Lett. **83**, 588 (1999); M. Rühländer, P. Markos, and C. M. Soukoulis, Phys. Rev. B **64**, 212202 (2001).
[17] P. Sheng and Z. Zhang, J. Phys.: Condens. Matter **3**, 4257 (1991).
[18] K. Slevin and T. Ohtsuki, Phys. Rev. Lett. **78**, 4083 (1997).
[19] K. Slevin and T. Ohtsuki, Phys. Rev. Lett. **82**, 669 (1999).
[20] D. Braun, G. Montanmbaux, and M. Pascaud, Phys. Rev. Lett. **81**, 1062 (1998).
[21] C. M. Soukoulis, J. V. José, E. N. Economou, and P. Sheng, Phys. Rev. Lett. **50**, 764 (1983).
[22] E. Cota, J. V. Jose, and M. Ya. Azbel, Phys. Rev. B **32**, 6157 (1985).
[23] K. Senouci, N. Zekri, and R. Ouasti, Physica A **234**, 23 (1996).
[24] P. Lévy, *Théorie de l'Addition des Variables Aléatoires* (Gautier-Villars, Paris, 1937); M. Shlesinger, G. M. Zaslavsky, and U. Fish, *Lévy Flights and Related Topics in Physics* (Springer, Berlin, 1995); J. P. Bouchaud and A. Georges, Phys. Rep. **195**, 12 (1990).
[25] Y. Ando and T. Itoh, J. Appl. Phys. **61**, 1497 (1987).
[26] R. Landauer, Philos. Mag. **21**, 263 (1970).
[27] C. Mato and A. Caro, J. Phys.: Condens. Matter **1**, 901 (1989).
[28] N. Zekri, M. Schreiber, R. Ouasti, R. Bouamrane, and A. Brezini, Z. Phys. B: Condens. Matter **99**, 381 (1994).
[29] P. Wolfle and K. A. Muttalib, Ann. Phys. (Leipzig) **8**, 753 (1999).
[30] X. Wang, Q. Li, and C. M. Soukoulis, Phys. Rev. B **58**, 3576 (1998).
[31] C. M. Soukoulis, X. Wang, Q. Li, and M. M. Sigalas, Phys. Rev. Lett. **82**, 668 (1999).