Electrons in a twisted quantum wire

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We study electronic states in a straight twisted quantum wire with nonround identical crosssections turned relative to each other along the wire. The wire internal torsion is supposed to be small compared to the inverse wire width. This assumption allows us to transform the Hamiltonian to the single-subband one-dimensional form. Both uniform and nonuniform torsions are considered. The effective Hamiltonian (including spin-orbit interaction) for one-dimensional motion along the wire has been constructed.

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Recent technological progress makes it possible to fabricate different low-dimensional systems with complicated geometric shape,[1–4] such as rolls, rings, spirals, or other structures. It is expected that the complicated geometry will provide new physical features and new functionality for electronic devices.

The electron states in curved low-dimensional systems were the subject of many recent publications, both theoretical and experimental (see, e.g, Refs. 5–12). Electrons in such systems are described in terms of the adiabatic effective Hamiltonian for the longitudinal motion by means of averaging over the states of transversal quantization.[13–15] The curvature of these systems leads to the appearance of effective potential. This curvature-induced potential complements large energies of transversal quantization, caused by the confinement alone. While the energy of transversal quantization grows as the inverse squared size d of transversal confinement, the geometric potential remains finite. In addition to the geometric potential, the longitudinal kinetic energy in a curvilinear low-dimensional system obtains the contributions $\propto 1/d$ (see Ref. 17), which exist if the confining potential has no inversion symmetry. What is important for further, the effective Hamiltonian turns out to depend on internal structure of confinement.

Along with external geometric parameters, the curvature, in particular, one-dimensional systems are also characterized by internal parameters, namely, the shapes of the system cross sections. When the wire cross-sections or their orientation vary along the wire, it effects the electron longitudinal motion.

The purpose of the present paper is to study electron states in a straight twisted quantum wire with identical cross sections. Electron motion will be described in terms of the effective 1D Hamiltonian. The cross sections are supposed to have a noncircular shape. Their relative orientation is defined by the rotation angle $\phi(z)$, depending on the coordinate z along the wire. Figure 1 exemplifies the systems under consideration. Such systems, twisted strips, in particular, can be fabricated by means of the technique of self-scrolling.[3]

We shall consider both the nonrelativistic case and spin-orbit (SO) corrections. Our interest to the SO interaction is stimulated by the hope that twisted wires are perspective for spin control. In the recent paper[17] we have considered the spin-orbit interaction caused by the surface curvature. The wire internal torsion induces the spin-orbit interaction similar to the curvature-induced SO interaction.

I. EFFECTIVE HAMILTONIAN FOR TWISTED WIRE

The potential, confining electrons in the twisted wire, can be written as

$$V(\boldsymbol{\rho},z)=V[\rho,\varphi-\phi(z)], \tag{1}$$

where $\boldsymbol{\rho}=(x,y)$, φ is the azimuthal angle. The starting point is the single-band effective-mass Hamiltonian of the system

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}^2+V(\boldsymbol{\rho},z)+\mathcal{H}_{\mathrm{SO}}, \tag{2}$$

where the Hamiltonian of SO interaction is

$$\mathcal{H}_{\mathrm{SO}}=\alpha[\boldsymbol{\sigma},\nabla V]\mathbf{p}. \tag{3}$$

Here $\mathbf{p}$ is the operator of electron momentum, α is the effective SO coupling constant of bulk crystal. In A_3B_5 semiconductors $\alpha=(2E_g m)^{-1}[\Delta(2E_g+\Delta)/(E_g+\Delta)(3E_g+2\Delta)]$ (see, e.g., Ref. 16), E_g is the width of the forbidden band and Δ is the SO splitting of the valence band. Here and below we set $\hbar=1$.

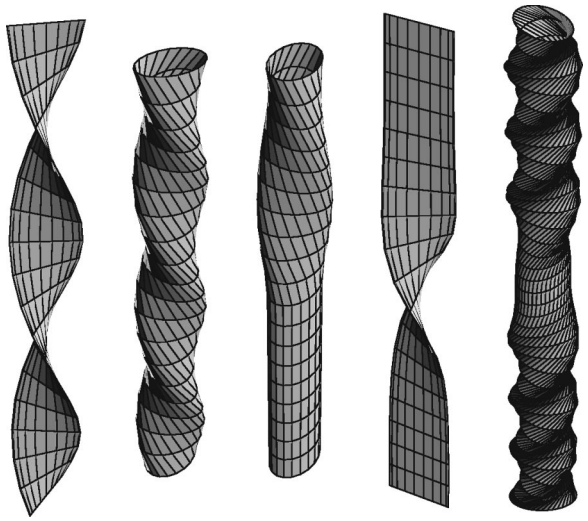

FIG. 1. Examples of twisted wires, from left to right: twisted 2D ribbon and twisted elliptic cylinder with constant torsion, conjuncture of twisted and nontwisted elliptic cylinders (step barrier for electrons), singly twisted band (smooth barrier), transition between uniformly twisted wires with the opposite signs of torsion through the domain of zero torsion (potential well).

To consider the problem we will first transform the Hamiltonian Eq. (2) to the new natural curvilinear coordinates $\overline{r_i}=U_{ij}r_j$, where $U(\phi)$ is the rotation operator about the axis z on the angle $\phi(z)$:

$$U=\begin{pmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}.$$

In this coordinate system (curvilinear and not orthogonal) the potential V does not depend on $\overline{z}$. The metric tensor is determined by

$$G_{ij}=\begin{pmatrix}1 & 0 & \overline{y}\overline{\tau}\\ 0 & 1 & -\overline{x}\overline{\tau}\\ \overline{y}\overline{\tau} & -\overline{x}\overline{\tau} & 1+\overline{\rho}^2\overline{\tau}^2\end{pmatrix}.$$

Here $\tau(z)\equiv d\phi(z)/dz$ is the internal torsion of the wire. The determinant of the metric tensor G_{ij} equals 1.

After transformation the Hamiltonian takes the form

$$\overline{\mathcal{H}}=\frac{1}{2m}\{\overline{\mathbf{p}}_\perp^2+[\overline{p_z}-\tau(\overline{z})L_z]^2\}+V(\overline{\boldsymbol{\rho}})+\overline{\mathcal{H}}_{\mathrm{SO}}. \tag{4}$$

Here $L_z=\overline{xp_y}-\overline{yp_x}$ is the z component of the angular momentum $\overline{p_i}=-i\partial/\partial\overline{r_i}$, $\overline{\mathbf{p}}_\perp=(\overline{p_x},\overline{p_y})$. The transformed $\overline{\mathcal{H}}_{\mathrm{SO}}$ can be expressed via the covariant components of vectors. The covariant components of any vector a_i transform as $\overline{a_i}=K_{ik}a_k$. The matrix of transformation $K_{ik}=\partial r_k/\partial\overline{r_i}$. For our transform,

$$K_{ij}=U_{ij}+\delta_{i,z}\overline{r_k}\frac{\partial U_{kj}}{\partial z}. \tag{5}$$

The covariant transform $\overline{\sigma}$ of spin matrix is $\overline{\sigma_i}=K_{ij}\sigma_j=\tilde{\sigma}_i+\delta_{i,z}\overline{r_k}\partial\tilde{\sigma}_k/\partial z$. Here $\tilde{\boldsymbol{\sigma}}=U\boldsymbol{\sigma}$ are the rotated σ matrices

$$\tilde{\sigma}_x=\begin{pmatrix}0 & e^{i\phi}\\ e^{-i\phi} & 0\end{pmatrix},\quad \tilde{\sigma}_y=i\begin{pmatrix}0 & -e^{i\phi}\\ e^{-i\phi} & 0\end{pmatrix},$$

$$\tilde{\sigma}_z=\sigma_z. \tag{6}$$

The SO Hamiltonian can be rewritten as

$$\overline{\mathcal{H}}_{\mathrm{SO}}=\alpha\boldsymbol{\epsilon}^{ijk}\overline{\sigma_j}\frac{\partial\overline{V}}{\partial\overline{r_k}}\overline{p_i}, \tag{7}$$

where $\boldsymbol{\epsilon}^{ijk}$ is the antisymmetric tensor of Levi-Civitta.

Expanding the wave functions

$$\Psi(\overline{\mathbf{r}})=\sum_n\psi_n(\overline{\boldsymbol{\rho}})\chi_n(z)$$

in the eigenfunctions of transversal motion $\psi_n(\overline{\boldsymbol{\rho}})$, which satisfy the equations $[\overline{\mathbf{p}}_\perp^2/2m+V(\overline{\boldsymbol{\rho}})-E_n]\psi_n(\overline{\boldsymbol{\rho}})=0$, we get

$$(E_n+\overline{p_z^2}/2m-E)\chi_n+\sum_{n'}\mathcal{V}_{nn'}\chi_{n'}=0. \tag{8}$$

The Hamiltonian of perturbation $\mathcal{V}_{nn'}=\mathcal{V}^{\mathrm{tor}}_{nn'}+\mathcal{V}^{\mathrm{SO}}_{nn'}$ consists of the torsion-induced $\mathcal{V}^{\mathrm{tor}}_{nn'}$ and SO-induced $\mathcal{V}^{\mathrm{SO}}_{nn'}$ terms

$$\mathcal{V}^{\mathrm{tor}}_{nn'}=\frac{1}{2m}[2(L_z)_{nn'}\{\tau(z),\overline{p_z}\}+\tau^2(z)(L_z^2)_{nn'}], \tag{9}$$

$$\mathcal{V}^{\mathrm{SO}}_{nn'}=\alpha[\tilde{\sigma}_i(A_i)_{nn'}+(\tilde{\sigma}_x(F_y)_{nn'}-\tilde{\sigma}_y(F_x)_{nn'})\overline{p_z}], \tag{10}$$

where the figure brackets stand for the operation of symmetrization, $\{C,D\}=(CD+DC)/2$, $F_i=\partial V/\partial\overline{r_i}$,

$$A_z=F_x\overline{p_y}-F_y\overline{p_x},\quad A_x=\tau(z)\overline{y}A_z,\quad A_y=-\tau(z)\overline{x}A_z. \tag{11}$$

Below we explore the smallness of the torsion, $\tau d\ll 1$, as an adiabatic parameter. In this section we shall study the case of nondegenerate states n; the degenerate case will be considered later. If the torsion is small, an electron mainly conserves the definite transversal subband n, so that $\chi_n\gg\chi_{n'}$ if $n'\neq n$. The main contribution to the adiabatic Hamiltonian arises from the diagonal element $\mathcal{V}_{nn}$, providing $\mathcal{V}_{nn}\neq 0$, otherwise we need the second order corrections.

Another small parameter is the relativistic SO parameter α. The effective Hamiltonian will be found in the first order of α. Within this approximation SO interaction vanishes in nontwisted wires. The diagonal element of this perturbation also vanishes in the twisted wires, therefore the adiabatic SO interaction arises in the second perturbation order from the interference of $\mathcal{V}^{\mathrm{tor}}_{nn'}$ and $\mathcal{V}^{\mathrm{SO}}_{nn'}$.

To obtain the effective Hamiltonian we shall proceed similarly to the k-p perturbation theory. Let an electron state be formed mainly from the subband n. The nondiagonal elements of $\mathcal{V}_{nn'}$ result in admixing other subbands. As in, Ref. 17 we express the ''small'' components $\chi_{n'}$ via the ''large'' components χ_n and substitute them into the equation for χ_n. Terminating the iteration we find

$$\left(E_n+\frac{1}{2m}\overline{p_z^2}-E+\mathcal{V}_{nn}\right)\chi_n - \sum_{n'\neq n}\mathcal{V}_{nn'}\left(E_{n'}+\frac{1}{2m}\overline{p_z^2}-E\right)^{-1}\mathcal{V}_{n'n}\chi_n=0. \tag{12}$$

The smoothness of the function $\phi(z)$ yields a smallness of momentum transfer in Fourier transform of $\mathcal{V}_{nn'}$. Hence typical differences in longitudinal energies in Eq. (12) are less than intersubband distance and the denominators in the resolvent $(E_{n'}+\overline{p_z^2}/2m-E)^{-1}$ can be replaced by $(E_n-E_{n'})^{-1}$. The final perturbational formula yields

$$\left(E_n+\frac{1}{2m}\overline{p_z^2}-E+\mathcal{V}_{nn}\right)\chi_n-\sum_{n'\neq n}\mathcal{V}_{nn'}\frac{1}{E_{n'}-E_n}\mathcal{V}_{n'n}\chi_n=0. \tag{13}$$

After substitution of $\mathcal{V}_{nn'}$ we find the effective 1D Schrödinger equation for electrons in the nth subband of the transversal quantization

$$\left(E_n+\frac{1}{2m}\overline{p_z^2}+\mathcal{U}_n(z)-\gamma_n\{\tau(z),\overline{p_z}\}^2+H_{\mathrm{SO}}^{(n)}\right)\chi_n=0. \tag{14}$$

Here

$$\mathcal{U}_n(z)=\beta_n\tau^2(z),$$

$$\beta_n=\frac{1}{2m}(L_z^2)_{nn},\quad \gamma_n=\frac{1}{m^2}\sum_{n'\neq n}\frac{|(L_z)_{nn'}|^2}{E_{n'}-E_n}. \tag{15}$$

Note that the contribution to the diagonal element $\mathcal{V}_{nn}^{\mathrm{tor}}$ from the first term in Eq. (9) vanishes for nondegenerate states. The quantity $\mathcal{U}_n(z)$ is the torsion-induced effective geometric potential in the nth subband. The parameter γ_n determines the corrections to the longitudinal kinetic energy. The geometric potential is positive and grows with the torsion.

In the general case, when $E_{n'}-E_n$ and E_n have same orders, the contribution proportional to γ_n has the order of magnitude of kinetic energy along the wire, $p^2/2m$, multiplied by the small parameter $(\tau d)^2$. Hence in this case the correction to the kinetic energy is negligible. The correction becomes essential in the near-degenerate spectrum due to the small denominator in Eq. (13). In this case (and for the degenerate case also) we should explore the near-degenerate theory of perturbations (see below).

The torsion can be considered as the way to provide the potential for longitudinal control on electrons by producing the longitudinal confinement, barriers, etc. The solvable one-dimensional problems from the quantum mechanics textbooks are simply transcribed to the systems with nonuniform torsion $\tau(z)$. Hence the problems with the function $[\tau(z)]^2$ of the form $a\,\theta(z)$, $a\,\theta(z)\,\theta(b-z)$, $a\tanh(z/b)$, $a/\cosh^2(z/b)$ are solvable.

The effective SO interaction is given by the expression

$$\begin{aligned}H_{\mathrm{SO}}^{(n)}&=-\sum_{n'\neq n}\frac{\mathcal{V}_{nn'}^{\mathrm{tor}}\mathcal{V}_{n'n}^{\mathrm{SO}}+\mathcal{V}_{nn'}^{\mathrm{SO}}\mathcal{V}_{n'n}^{\mathrm{tor}}}{E_{n'}-E_n}\\&=-\frac{\alpha}{m}\sum_{n'\neq n}\frac{(L_z)_{n'n}}{E_{n'}-E_n}((\mathbf{A}_{nn'}-\mathbf{A}_{n'n})\cdot\{\tilde{\boldsymbol{\sigma}},\{\tau(z),\overline{p_z}\}\}\\&\quad+[\{(\tilde{\boldsymbol{\sigma}}\times\mathbf{F}_{nn'})_z,\overline{p_z}\},\{\tau(z),\overline{p_z}\}]).\end{aligned} \tag{16}$$

Here the square brackets stand for the commutator. Near the subband bottoms the contribution $\propto\sigma_z$ alone survives, and we get

$$H_{\mathrm{SO}}^{(n)}=\alpha_n\sigma_z\{\tau(z),\overline{p_z}\}, \tag{17}$$

where

$$\alpha_n=\frac{2\alpha}{m}\sum_{n'\neq n}\frac{(F_y\overline{p_x}-F_x\overline{p_y})_{nn'}(L_z)_{n'n}}{E_{n'}-E_n}. \tag{18}$$

Equation (18) shows that the spectrum decays onto two independent spin states with the definite spin projections $\pm 1/2$ on z axis.

The simplest case occurs for the uniform torsion $\tau(z)=\mathrm{const}$. In that event the longitudinal momentum $\overline{p_z}$ is the conserving quantity, and the energy spectrum becomes

$$E_n(\overline{p})=E_n+\overline{p^2}/2m\pm\alpha_n\tau\overline{p}+\tau^2\beta_n. \tag{19}$$

It should be emphasized that the momentum $\overline{p_z}$ in the coordinates $\overline{\mathbf{r}}$ is not the same as the momentum p_z in the laboratory coordinate system $\mathbf{r}$. The operator p_z generates infinitesimal translations along z axis while $\overline{p_z}=p_z+\tau L_z$ determines in the laboratory system the superposition of infinitesimal translation and rotation (spiral translation) that corresponds to the spiral symmetry of an uniformly twisted wire.

Equation (19) is valid if the spin splitting $2\alpha_n\tau\overline{p}$ is less than the intersubband distance. If the wire form approaches to a cylindrical one, the quantities α_n do not vanish, and the linear in $\overline{p}$ term in spectrum Eq. (19) remains. This looks strange, if not to consider that the spectrum of transversal states tends to degenerate with the resulting inapplicability of Eq. (19). In fact, the case of the circular symmetric wire should be studied using the near-degenerate perturbation theory.

Let us discuss the physical reason of the spin-orbit interaction in the twisted wire. Let an electron be described by a wave packet propagating along the wire. The transversal shape of the packet repeats the wire cross section. During the propagation the packet rotates with the angular velocity $\tau(z)\overline{p}/m$. The rotating packet produces the magnetic field $B_z\sim\tau(z)\overline{p}/m$ in the frame translationally accompanying the electron. Interaction of this field with the electron spin results in the SO Hamiltonian $H_{\mathrm{SO}}\sim\tau(z)\sigma_z\overline{p_z}/m$.

The twisted wire can provide a way of spin control. Consider the wire with constant torsion surrounded by nontwisted domains with reflectionless smooth transitions between different parts. This system can serve as a "spin rotator." If electron states with spin projections ± 1 on the x axes denote the information states 1 and 0, correspondingly, π-angle spin rotator carries out the function NOT.

Let the electron spin at the entrance of the system be oriented along the x axis. While propagating along the wire the spin will rotate in the x-y plane. Using the SO Hamiltonian (17) we find that the angle of spin rotation $\theta(z)$ is $\theta(z)=2m\alpha_n\phi(z)$, where, in accordance with the assumptions, $m\alpha_n\ll 1$.

II. EFFECTIVE HAMILTONIAN FOR A PARABOLIC QUANTUM WELL

Here we study a specific quantum well with confining potential $V(\boldsymbol{\rho})=m(\omega_x^2x^2+\omega_y^2y^2)/2$. The coefficients β_n, γ_n, and α_n are determined solely by the transversal wave function of the wire. The simple calculation gives

$$\alpha_n=\frac{\alpha}{2}\left[(\omega_x-\omega_y)^2\frac{n_x+n_y+1}{\omega_x+\omega_y}+(\omega_x+\omega_y)^2\frac{n_x-n_y}{\omega_x-\omega_y}\right], \tag{20}$$

$$\beta_n=\frac{1}{8m\omega_x\omega_y}[(2n_x+1)(2n_y+1)(\omega_x^2+\omega_y^2)-2\omega_x\omega_y],$$

$$[n=(n_x,n_y),n_{x,y}=0,1,2,\ldots]. \tag{21}$$

The applicability of Eq. (20) is limited by the proximity of the denominators to zero. If α_n becomes too large, the perturbative approach fails and the degenerate perturbation theory should be used.

We shall estimate the value of $\alpha_{0,0}$ for the lowest subband of a InAs helix scrolled from a strip, similar to the one obtained in Ref. 1. InAs has strong enough basic SO interaction with $\alpha=0.145/mE_g$. In fact, the helixes obtained in Ref. 1 contain both monolayers of InAs and bilayers of GaAs, but apparently it is easy to produce similar structures with many InAs layers and only few GaAs layers. These wires have curvature, but nevertheless can be described by the potential of the form Eq. (1) with the shift of origin in the $(\bar{x},\bar{y})$ plane. Providing that the confining potential is harmonic it can be written as $m[\omega_x^2(x-x_0)^2+\omega_y^2(y-y_0)^2]/2$. For this potential the Eqs. (20),(21) prove also correct.

For $\omega_x\gg\omega_y$, that corresponds to a strip helix, $m\alpha_{00}=0.145E_{00}/E_g$. If we set $E_{00}=0.1$ eV, the parameter $m\alpha_{00}=0.0354$. The spiral wires obtained in Ref. 1, have $\tau\sim10^6$ cm^{-1}. For such value of τ, the above-mentioned π-angle spin rotator should be 4×10^{-5} cm long.

III. DEGENERATE STATES

The degenerate spectrum appears, e.g., for a hard-wall wire with square cross-sections or in a parabolic potential well with multiple frequencies. In this case the resonant denominators vanish and the theory of perturbation fails. The same is true in the near-degenerate spectrum. In these cases one should use the initial expression (8), where the sum includes the states from the degenerate (or near-degenerate) group only, numerated by a number ν:

$$\left(E_\nu+\frac{\bar{p}_z^2}{2m}-E\right)\chi_\nu+\sum_{\nu'\neq\nu}\mathcal{V}_{\nu\nu'}\chi_{\nu'}=0. \tag{22}$$

As an example, we shall consider the near-degenerate parabolic confining potential with $\omega_x-\omega_y\ll\omega_x$. For simplicity, we neglect the SO interaction. The lowest level $(\omega_x+\omega_y)/2$ is not degenerate, while the second $(\omega_x+3\omega_y)/2$ and the third $(3\omega_x+\omega_y)/2$ compose the group of near-degenerate levels. (For definiteness, we set $\omega_x>\omega_y$.) It is convenient to define $\nu=n_x-n_y$, $\nu=\pm1$. From Eq. (22) it follows that

$$\bar{p}_z^2\chi_{-1}+2i\{\tau,\bar{p}_z\}\chi_1+\tau^2\chi_{-1}-k_{-1}^2\chi_{-1}=0,$$

$$\bar{p}_z^2\chi_1-2i\{\tau,\bar{p}_z\}\chi_{-1}+\tau^2\chi_1-k_1^2\chi_1=0,$$

$$k_{\pm1}^2=2m\left(E-(\omega_x+\omega_y)\mp\frac{1}{2}(\omega_x-\omega_y)\right). \tag{23}$$

It can be shown that for strict degeneracy ($\omega_x=\omega_y$, the wire with circular symmetry) the torsion can be excluded from the effective Schrödinger equation. To accomplish this one should carry out the transformation $\chi_\pm=\exp(\mp i\phi)(\chi_1\pm i\chi_{-1})$. Equation (23) takes the form

$$(\bar{p}_z^2/m-2E+\omega_x+\omega_y)\chi_\pm=(\omega_y-\omega_x)e^{\mp2i\phi(z)}\chi_\mp. \tag{24}$$

If $\omega_x=\omega_y$, Equation (24) becomes independent of τ in accordance with the physical meaning. Equation (23) immediately yields the spectrum for the case of uniform torsion

$$E=\omega_x+\omega_y+\frac{\bar{p}^2+\tau^2}{2m}\pm\sqrt{\frac{(\omega_x-\omega_y)^2}{4}+\frac{\tau^2\bar{p}^2}{m^2}}. \tag{25}$$

IV. TWISTED WIRE IN MAGNETIC FIELD

In this section we generalize the problem without the SO interaction to include the magnetic field along the wire axis. Choosing the vector potential of the magnetic field **B** as **A** $=(-By,Bx,0)/2$, we have instead of Eq. (2)

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}_\perp^2+\frac{1}{2m}p_z^2+\frac{\omega_c}{2}L_z+\frac{m\omega_c^2\rho^2}{2}+V(\boldsymbol{\rho},z), \tag{26}$$

where $\omega_c=eB/mc$ is the cyclotron frequency. Transformation to the curvilinear coordinates leads to

$$\bar{\mathcal{H}}=\frac{1}{2m}[\bar{\mathbf{p}}_\perp^2+(\bar{p}_z-\tau(z)L_z)^2]+\frac{\omega_c}{2}L_z+\frac{m\omega_c^2\bar{\rho}^2}{2}+V(\bar{\boldsymbol{\rho}}). \tag{27}$$

Further we can proceed in the same manner we used to obtain Eq. (14). The resulting 1D Schrodinger equation in the presence of longitudinal magnetic field has the form

$$\left(E_n-E+\frac{1}{2m}\bar{p}_z^2+\mathcal{U}_n(z)-\gamma_n\{\tau(z),\bar{p}_z\}^2-\gamma_nm\omega_c\{\tau(z),\bar{p}_z\}+\frac{m\omega_c^2(\bar{\rho}^2)_{nn}}{2}\right)\chi_n=0. \tag{28}$$

Equation (28) is valid for the bottom states only. Note that the interaction of longitudinal motion with the magnetic field is characterized by the same constant γ_n as the torsion-induced additions to the kinetic energy.

V. CONCLUSIONS

In conclusion, we have solved the problem about the effective single-subband Hamiltonian of electrons in the adiabatically twisted straight wire. Both nonrelativistic and relativistic cases were studied. The main contribution to the nonrelativistic Hamiltonian is the effective potential caused by the wire torsion. The potential of nonuniform torsion can reflect or localize electrons propagating along the wire. The

spin-orbit interaction between the translational momentum of electron along the wire and the same component of electron spin has been found. This interaction can be used for electron spin control. We have also constructed the effective Hamiltonian in the presence of longitudinal magnetic field.

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