Phase transition and critical behavior of the $d=3$ Gross-Neveu model

F. Höfling, C. Nowak, and C. Wetterich
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany
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A second-order phase transition for the three-dimensional Gross-Neveu model is established for one fermion species, $N=1$. This transition breaks a paritylike discrete symmetry. It constitutes its peculiar universality class with critical exponent $\nu=0.63$ and scalar and fermionic anomalous dimensions $\eta_\sigma=0.31$ and $\eta_\psi=0.11$, respectively. We also compute critical exponents for other N. Our results are based on exact renormalization-group equations.

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An understanding of systems with many fermionic degrees of freedom is one of the big challenges in statistical physics. Due to the anticommuting nature of the variables, numerical simulations are not straightforward—analytical methods are crucially needed. One, typically, has to solve a functional integral for a d-dimensional system with Grassmann variables. Approximate solutions for "test models" would be of great value. The Gross-Neveu (GN) model[1] is one of the simplest fermionic models. In three dimensions, a discrete symmetry forbids a mass term unless it is spontaneously broken. In the symmetric phase, the GN model is, therefore, a realization of a statistical system of gapless fermions. For a large number N of fermion species, it is known[2–5] that a second-order phase transition separates the symmetric phase from an ordered phase, where the symmetry is spontaneously broken and the fermions become massive. Using methods based on an exact renormalization-group equation,[6] a second-order transition for $N\geqslant 2$ was confirmed. To the best of our knowledge, we know, however, of no previous work that clarifies the existence and nature of the phase transition in the simplest model with only one fermion species. The model with one fermion species is inaccessible to lattice simulations due to the fermion doubling problem and the $1/N$ expansion is not expected to give reasonable results for $N=1$. The case $N=1$ is also of special interest since an order parameter $\langle \bar{\psi}_j \ \psi^j\rangle \neq 0$ leads to a ground state, which does not admit any discrete symmetry involving the reflection of all coordinates, in contrast to the models with $N\geqslant 2$.

In this paper, we improve the exact renormalization-group approach and establish a second-order phase transition for $N=1$. We also compute the critical exponents. This is important beyond a possible relevance for real physical systems: the GN model constitutes a peculiar universality class due to the presence of massless fermions at the critical point. Just as the O(N)-Heisenberg models for bosons, the GN model could in the future become a benchmark for our understanding of critical systems in the presence of fermions.

The GN model describes N-fermionic fields with local quartic interaction. Here ψ_j, $j=1\ldots N$, are irreducible representations of the group O(d) including parity reflections, i.e., $2^{d/2}$ component Dirac spinors for d even and $2^{(d-1)/2}$ for d odd. The classical Euclidean action

$$S=\int \mathrm{d}^d x \ \left\{ \bar{\psi}_j(x) i \partial\!\!\!/ \psi^j(x) + \frac{\bar{G}}{2}[\bar{\psi}_j(x) \ \ \psi^j(x)]^2 \right\} \qquad (1)$$

is symmetric under a coordinate reflection

$$\psi(x)\mapsto -\psi(-x), \quad \bar{\psi}(x)\mapsto \bar{\psi}(-x).$$

(We note that ψ and $\bar{\psi}$ are independent variables in an Euclidean formulation.) A nonvanishing expectation value of $\bar{\psi}_j\psi^j$ spontaneously breaks this symmetry. If the spinors ψ_j contain more than one irreducible representation of the rotation (or Lorentz) group SO(d), we can find alternative definitions of the coordinate reflection, where $\bar{\psi}_j\psi^j$ remains invariant. In particular, this is realized for Dirac spinors in even dimensions, where a mass term couples two irreducible representations ψ_L and ψ_R. One may then define a standard parity transformation, under which $\bar{\psi}\psi$ is invariant. In this case, however, $\bar{\psi}_R\psi_L\neq 0$ breaks a discrete chiral symmetry $\psi_L\mapsto -\psi_L$, $\bar{\psi}_L\mapsto -\bar{\psi}_L$. We will concentrate here on one two-component spinor in three dimensions. For $N=1$, the above "parity transformation" is the only possible choice of coordinate reflections and $\bar{\psi}_j\psi^j\neq 0$ spontaneously breaks this symmetry.

Depending on the value of the coupling $\bar{G}$, the model will be in the symmetric phase or exhibit spontaneous symmetry breaking (SSB) with a nontrivial expectation value of the order parameter $\langle G\bar{\psi}_j \ \psi^j\rangle$. We will describe a space dependent fermion bilinear $-i \ \overline{G\psi_j}(x)\psi^j(x)$ by a real scalar field $\sigma(x)$ such that SSB is indicated by nonzero homogeneous $\langle\sigma\rangle\neq 0$. By a partial bosonization, we express the GN model (1) as an equivalent Yukawa model with

$$S_\sigma=\int \mathrm{d}^d x \left(\bar{\psi}_j i\partial\!\!\!/\psi^j + i\sigma\bar{\psi}_j\psi^j + \frac{1}{2\bar{G}}\sigma^2 \right). \qquad (2)$$

The equivalence of the partition function can be seen by performing the Gaussian σ integration [$\bar{\eta}\psi$ means $\int \mathrm{d}^d x \ \bar{\eta}_j(x) \ \psi^j(x)$],

$$Z[\bar{\eta},\eta]=\int \mathcal{D}\sigma\mathcal{D}\psi\mathcal{D}\bar{\psi}\exp(-S_\sigma[\sigma,\psi,\bar{\psi}]+\bar{\eta}\psi+\eta\bar{\psi})$$
$$=\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\exp\left[-\bar{\psi}i\partial\!\!\!/\psi+\bar{\eta}\psi+\eta\bar{\psi}-\frac{\bar{G}}{2}(\bar{\psi}\psi)^2\right]$$
$$\times\int \mathcal{D}\sigma\exp\left[-\frac{1}{2\bar{G}}(\sigma+i\bar{G}\bar{\psi}\psi)^2\right], \tag{3}$$

where the last factor yields an irrelevant constant.

A powerful tool for nonperturbative examinations are exact renormalization-group equations for the effective action Γ, i.e., the generating functional of the 1PI Green's functions.[7,8] Starting with the classical action S at the UV cutoff Λ, we obtain a type of coarse-grained free energy Γ_k by integrating out the quantum fluctuations with momenta larger than a given scale k. Eventually, we reach the macroscopic thermodynamic potential[8] Γ at $k\to 0$. The IR cutoff k is implemented in a smooth way by introducing a masslike term ΔS_k into the classical action, which gives extra masses to modes with momenta smaller than k. In the limit $k\to 0$, the IR cutoff is absent, and fluctuations at all scales have been taken into account. At $k\to\Lambda$, all fluctuations are suppressed and Γ_Λ approaches S.

This procedure should be explained more precisely. For notation purposes, we combine the fermionic and (real) bosonic fields to a column vector $\chi=(\sigma,\psi,\bar{\psi}^T)$, and the row $\mathcal{J}$ contains all the external sources $\mathcal{J}=(J,\bar{\eta},\eta^T)$. We start with the generating functional for the connected correlation functions in the presence of the IR cutoff,

$$W_k[\mathcal{J}]=\ln\int \mathcal{D}\chi\exp(-S_\sigma[\chi]-\Delta S_k[\chi]+\mathcal{J}\chi). \tag{4}$$

For vanishing ΔS_k (at $k\to 0$), this matches exactly with the free energy of the Gross-Neveu model (1). The effective action is then defined via a modified Legendre transformation by

$$\Gamma_k[\Phi]:=-W_k[\mathcal{J}[\Phi]]+\mathcal{J}[\Phi]\Phi-\Delta S_k[\Phi], \tag{5}$$

which depends on the expectation values of the fields $\Phi=\langle\chi\rangle$.

The infrared cutoff takes the form $\Delta S_k[\Phi]=\frac{1}{2}\Phi^T R_k\Phi$, and R_k is a matrix

$$R_k(p,q)=\begin{pmatrix} R_{kB}(q) & 0 & 0\\ 0 & 0 & -R_{kF}^T(-q)\\ 0 & R_{kF}(q) & 0\end{pmatrix}(2\pi)^d\delta^d(p-q),$$

with $R_{kB}(q)=Z_{\sigma,k}q^2 r_{kB}(q)$ and $R_{kF}(q)=-Z_{\psi,k}q\!\!\!/ r_{kF}(q)$. ($Z_{\sigma,k}$ and $Z_{\psi,k}$ are wave-function renormalizations.) With these definitions, an exact renormalization-group equation for the scale dependence of Γ_k can be found[7–9]:

$$\partial_t\Gamma_k=\frac{1}{2}\mathrm{STr}\{(\Gamma_k^{(2)}+R_k)^{-1}\ \partial_t R_k\}, \tag{6}$$

where $t=\ln(k/\Lambda)$. The supertrace runs over momenta and all internal indices, and provides appropriate minus signs for the fermionic sector. The heart of the flow equation is the fluctuation matrix,

$$[\Gamma_k^{(2)}(p,q)]_{ab}:=\frac{\vec{\delta}}{\delta\Phi_a^T(-p)}\ \Gamma_k\ \frac{\overleftarrow{\delta}}{\delta\Phi_b(q)}. \tag{7}$$

Together with R_k, it represents the exact inverse propagator at the scale k.

Equation (6) is an exact but complicated functional differential equation. There is no way around some approximation by truncating the most general form of Γ_k. We work here in the lowest order of a systematic derivative expansion, where Γ_k contains a scalar potential, kinetic terms, and a Yukawa coupling. In momentum space, it is given by [$\int dq=\int d^dq/(2\pi)^d$]:

$$\Gamma_k[\sigma,\psi,\bar{\psi}]=\int d^dx U_k(\sigma)+\int dq\left\{\frac{Z_{\sigma,k}}{2}\sigma(-q)q^2\sigma(q)\right.$$
$$-Z_{\psi,k}\bar{\psi}_j(q)\ q\!\!\!/\ \psi^j(q)$$
$$\left.+\int dp\, i\bar{h}_k\bar{\psi}_j(p)\ \sigma(p-q)\ \psi^j(q)\right\}. \tag{8}$$

The connection between $\bar{G}$ and $\bar{h}$ becomes clear if we rescale σ in Eq. (2) to $\bar{h}\sigma$ and set $\bar{G}=\bar{h}^2/m_\sigma^2$, m_σ denoting the boson mass.

Using a truncation of the effective action causes the limit $k\to 0$ to depend on the precise form of the cutoff functions. In order to take control of this, we have used two different choices, an exponential[8] and a linear[10] cutoff,

$$yr_B^{exp}(y)=\frac{y}{\exp(y)-1},\quad yr_B^{lin}(y)=(1-y)\Theta(1-y),$$

where $y=q^2/k^2$, and $r_F(y)$ is chosen in both cases such that $y(1+r_B)=y(1+r_F)^2$. We introduce renormalized, dimensionless quantities $h_k^2=Z_\sigma^{-1}Z_\psi^{-2}k^{d-4}\bar{h}_k^2$, $\tilde{\rho}=\frac{1}{2}Z_\sigma k^{2-d}\sigma^2$, $u_k=U_k k^{-d}$, and we use $u_k'=\partial u_k/\partial\tilde{\rho}$, etc.

We obtain a set of evolution equations for the effective parameters of the theory by inserting Eq. (8) in the exact flow Eq. (6). The evolution equation of the effective scalar potential u_k is found by evaluating $\Gamma_k^{(2)}$ for a constant scalar background field,

$$\partial_t u_k=-du_k+(d-2+\eta_\sigma)\tilde{\rho}u_k'+2v_d\{l_0^d(u_k'+2\tilde{\rho}u_k'';\eta_\sigma)$$
$$-N'l_0^{(F)d}(2h_k^2\tilde{\rho};\eta_\psi)\}, \tag{9}$$

where we use $N'=d_\gamma N$, with d_γ the dimension of the γ matrices, and $v_d^{-1}=2^{d+1}\pi^{d/2}\Gamma(d/2)$. The definition of the threshold functions l_n^d and $l_n^{(F)d}$ as well as $l_{n_1,n_2}^{(FB)d}$, m_{n_1,n_2}^d, $m_{2/4}^{(F)d}$, and $m_{n_1,n_2}^{(FB)d}$ used below [see Eqs. (11)–(13)] is given in the appendix of Ref. 8. These functions contain all the momentum integrations. For the linear cutoff, the integrations can be performed analytically—an enormous advantage for the subsequent numerics. We obtain

$$l_n^d(\omega;\eta_\sigma)=\frac{2(\delta_{n,0}+n)}{d}\left(1-\frac{\eta_\sigma}{d+2}\right)\frac{1}{(1+\omega)^{n+1}},$$

$$l_n^{(F)d}(\omega;\eta_\psi)=\frac{2(\delta_{n,0}+n)}{d}\left(1-\frac{\eta_\psi}{d+1}\right)\frac{1}{(1+\omega)^{n+1}},$$

$$l_{n_1,n_2}^{(FB)d}(\omega_1,\omega_2;\eta_\psi,\eta_\sigma)$$
$$=\frac{2}{d}\frac{1}{(1+\omega_1)^{n_1}(1+\omega_2)^{n_2}}\left[\frac{n_1}{1+\omega_1}\left(1-\frac{\eta_\psi}{d+1}\right)+\frac{n_2}{1+\omega_2}\left(1-\frac{\eta_\sigma}{d+2}\right)\right],$$

$$m_{n_1,n_2}^d(\omega_1,\omega_2;\eta_\sigma)=\frac{1}{(1+\omega)^{n_1}(1+\omega)^{n_2}},$$

$$m_2^{(F)d}(\omega;\eta_\psi)=\frac{1}{(1+\omega)^4}\ ,\quad m_4^{(F)d}(\omega;\eta_\psi)=\frac{1}{(1+\omega)^4}+\frac{1-\eta_\psi}{d-2}\frac{1}{(1+\omega)^3}-\left(\frac{1-\eta_\psi}{2d-4}+\frac{1}{4}\right)\frac{1}{(1+\omega)^2},$$

$$m_{n_1,n_2}^{(FB)d}(\omega_1,\omega_2;\eta_\psi,\eta_\sigma)=\left(1-\frac{\eta_\sigma}{d+1}\right)\frac{1}{(1+\omega_1)^{n_1}(1+\omega_2)^{n_2}}.$$

The anomalous dimensions η_σ and η_ψ are defined as

$$\eta_\sigma(k)=-\partial_t \ln Z_{\sigma,k},\qquad \eta_\psi(k)=-\partial_t \ln Z_{\psi,k}. \tag{10}$$

Taking second derivatives of Eq. (6) with respect to the fields by means of Eq. (7), we obtain evolution equations for the exact inverse boson and fermion propagators. Expanding the momentum dependence at $q^2=0$ for $\sigma=\sigma_{0k}$ at the potential minimum yields equations for η_σ and η_ψ:

$$\eta_\sigma(k)=8\frac{v_d}{d}\{\kappa_k(3u_k''+2\kappa_k u_k''')^2 m_{4,0}^d(u_k'+2\kappa_k u_k'',0;\eta_\sigma)+N' h_k^2[m_4^{(F)d}(2h_k^2\kappa_k;\eta_\psi)-2h_k^2\kappa_k m_2^{(F)d}(2h_k^2\kappa_k;\eta_\psi)]\}, \tag{11}$$

$$\eta_\psi(k)=8\frac{v_d}{d}h_k^2 m_{1,2}^{(FB)d}(2h_k^2\kappa_k,u_k'+2\kappa_k u_k'';\eta_\psi,\eta_\sigma). \tag{12}$$

Here κ_k denotes the position of the minimum of $u_k(\tilde\rho)$ and all derivatives of the potential are evaluated at $\tilde\rho=\kappa_k$. Dividing the evolution equation of the fermion propagator by σ and evaluating it at zero momentum, and $\sigma=\sigma_{0k}$ yields the evolution equation of h_k^2,

$$\partial_t h_k^2=(2\eta_\psi+\eta_\sigma+d-4)h_k^2+8v_d h_k^4 l_{1,1}^{(FB)d}(2h_k^2\kappa_k,u_k'+2\kappa_k u_k'';\eta_\psi,\eta_\sigma). \tag{13}$$

We study the flow of the effective potential in two different ways that correspond to two different truncations of Γ_k. First, we expand the evolution equation of the potential (9) in a Taylor series up to third order in $\tilde\rho$ around its minimum and evolve the resulting coupled ordinary differential equations.[6] In a second, more involved approach, the full equation for u_k' is discretized on a grid, yielding coupled ordinary differential equations (one for each grid point), which are solved simultaneously. We use the equation for u_k' instead of the one for u_k for numerical reasons.

The only critical parameter in the theory is the four-fermion coupling $\bar G_{\Lambda cr}$. This parameter has to be tuned in order to be near to the phase transition. The flow Eqs. (9)–(13) are evolved from an UV cutoff scale Λ to $k\to 0$. We concentrate on the three-dimensional case with one fermion species ($d=3$ and $N=1$). The initial values of the parameters are chosen such that $\Gamma_\Lambda=S_\sigma$: $Z_{\sigma\Lambda}=10^{-10}\simeq 0$, $Z_{\psi\Lambda}=1$, $\bar h_\Lambda^2=c\ \Lambda$, and $u_\Lambda'(\tilde\rho)=e:=(Z_{\sigma\Lambda}\bar G_\Lambda\Lambda)^{-1}$ for all $\tilde\rho$. The GN model corresponds to $c=1$, whereas for $c\neq 1$, we investigate Yukawa-type theories in the same universality class.

Very close to the phase transition, we find scaling solutions for all evolving parameters corresponding to vanishing β functions. This behavior indicates a second-order phase transition. The symmetric regime is characterized by $u_k'(0)>0$ as well as $\kappa_k=0$, while after spontaneous symmetry breaking $u_k'(0)$ becomes negative and $\kappa_k>0$. Starting in the symmetric regime, the system evolves into the SSB regime and reaches the scaling solution. Further down the flow, it evolves back into the symmetric regime for $\bar G_\Lambda<\bar G_{\Lambda_{cr}}$ or it remains in the SSB regime for $\bar G_\Lambda>\bar G_{\Lambda_{cr}}$. This behavior contrasts the one for $N\geqslant 2$, where the scaling solution is located in the symmetric regime.

Figure 1 shows the critical flow of κ_k and $u_k'(0)$ for two different values of c, namely $c=1$ (full and dashed lines, respectively) and $c=10^{-10}$ (thin dashed lines). It nicely demonstrates that only the beginning of the flow is affected by the choice of the noncritical parameters. All universal quantities such as critical exponents and mass gaps are independent of c. The polynomial expansion of the potential works well only for small enough c. The value of the critical coupling $\bar G_{\Lambda cr}$ depends, however, on the choice of c. In the limit $c\to 1$, it converges towards a constant value, which is reported in Table I.

We calculate the critical exponents characterizing a second-order phase transition. In order to test the reliability of the numerical algorithms, we determine the exponents in the symmetric as well as in the ordered phase. The scale dependent renormalized boson mass $m_{\sigma R}^2(k)$ is defined as the curvature of the potential at its minimum. In the SSB regime, it is given by $m_{\sigma R}^2(k)=2k^2\kappa_k u_k''(\kappa_k)$, in the symmetric re-

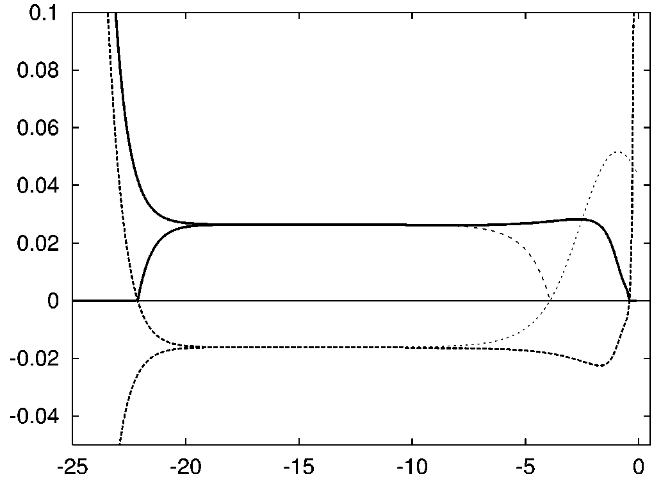

FIG. 1. Critical flow of $10\kappa_k$ (full line) and $u'_k(0)$ (dashed line) plotted as functions of $t=\ln(k/\Lambda)$. The thin dashed lines show the flow for $c=10^{-10}$. We have chosen two initial values of e very near to the phase transition. The flow separates only for very small k, according to the respective phase.

gime by $m^2_{\sigma R}(k)=k^2u'_k(0)$. In the ordered phase, the running of $m_{\sigma R}(k)$ essentially stops once k becomes much smaller than $m_{\sigma R}(k)$. The situation is slightly more difficult in the symmetric phase since the fermions are massless. Their fluctuations induce a scale dependence of $Z_{\sigma,k}$ even for very small momenta k: $\eta_\sigma\to 1$ for $k\to 0$ in contrast to $\eta_\sigma\to 0$ for $k\to 0$ in the ordered phase. To get rid of this problem, we define the renormalized boson mass at some fixed small ratio $r_c=k/\overline{m}_{\sigma R}$ (Ref. 6) as

$$\overline{m}^2_{\sigma R}=k_c^2[u'_{k_c}(0)-u'^{cr}_{k_c}(0)],\quad k_c=r_c\overline{m}_{\sigma R}, \tag{14}$$

where $u'^{cr}_k(0)$ denotes $u'_k(0)$ on the critical trajectory. In the numerical calculations we have used a ratio $r_c=0.01$, but our results do not depend on r_c for $r_c\lesssim 0.1$. Thus, we define the critical exponent ν,

$$\nu=\frac{1}{2}\lim_{\delta e\to 0}\frac{\partial\ln\overline{m}^2_{\sigma R}(\delta e)}{\partial\ln\delta e}, \tag{15}$$

where $\delta e=|e-e_{cr}|$. The exponent γ is defined as usual, since the unrenormalized boson mass $m^2_\sigma=Z_\sigma m^2_{\sigma R}$ is not affected by the fluctuations of the massless fermions,

$$\gamma=\lim_{\delta e\to 0}\frac{\partial\ln m^2_\sigma(\delta e)}{\partial\ln\delta e}. \tag{16}$$

In the ordered phase, both exponents are defined as usual, using $m^2_{\sigma R}$ instead of $\overline{m}^2_{\sigma R}$ for the definition of ν. The critical exponent β is defined as

$$\beta=\frac{1}{2}\lim_{\delta e\to 0}\frac{\partial\ln\sigma_0^2}{\partial\ln\delta e}, \tag{17}$$

with $\sigma_0=\lim_{k\to 0}\sigma_{0k}$. The exponents η_σ and η_ψ for the critical correlation functions are computed by taking the values of the scale dependent anomalous dimensions $\eta_\sigma(k)$ and $\eta_\psi(k)$ at the scaling solution (Sec. 4.2 of Ref. 8). Table I lists our results for the critical exponents. We find a good match of the values in the two different phases. Besides, we have checked the index relations $\gamma=\nu(2-\eta_\sigma)$ and $\beta=\frac{1}{2}\nu(d-2+\eta_\sigma)$ which are well fulfilled. The dependence of the exponents on the cutoff functions r_B and r_F as well as on the truncation can also be seen from the table. The latter one seems to be weaker when using the linear cutoff. The error in the exponent could be larger than the difference between the results for different truncations of the potential and

TABLE I. Critical exponents and mass gaps for $N=1$ in three dimensions.

Truncation	Full equation for u'_k		Taylor expansion	
Cutoff	lin	exp	lin	exp
ν_{symm}	0.621	0.640	0.623	0.633
γ_{symm}	1.051	1.077	1.053	1.062
$\nu(2-\eta_\sigma)$	1.051	1.076	1.054	1.064
ν_{ssb}	0.620	0.637	0.622	0.632
γ_{ssb}	1.050	1.071	1.053	1.062
β_{ssb}	0.406	0.420	0.407	0.417
$(\nu/2)(1+\eta_\sigma)$	0.405	0.420	0.407	0.417
η_σ	0.308	0.319	0.308	0.319
η_ψ	0.112	0.114	0.112	0.113
Δ_σ	16.0	17.6	16.8	18.1
Δ_ψ	14.2	14.9	14.4	15.2
$\overline{G}_{\Lambda cr}\Lambda$	43.13	26.68		

TABLE II. Critical exponents for various N in three dimensions.

N	2	4	12
ν	0.927[a] 0.962[b] 0.738[c]	1.018[a] 1.016[b] 0.903[c] 0.948[d] 1.00(4)[e]	1.018[a] 1.011[b] 1.007[c]
η_σ	0.525[a] 0.554[b] 0.635[c]	0.756[a] 0.786[b] 0.776[c] 0.763[d] 0.754(8)[e]	0.927[a] 0.935[b] 0.914[c] 0.913[f]
η_ψ	0.071[a] 0.067[b] 0.105[f]	0.032[a] 0.028[b] 0.044[f]	0.0087[a] 0.0057[b] 0.0124[f]
Δ_σ	15.1[a] 17.3[b]	12.0[a] 14.0[b]	5.5[a] 6.4[b]
Δ_ψ	10.8[a] 11.4[b]	7.5[a] 7.9[b]	3.3[a] 3.4[b]

[a]Linear cutoff, full equation for u'_k.
[b]Exponential cutoff, Taylor expansion of u'_k.
[c]$1/N$ expansion (Padé-Borel resummed).
[d]ϵ expansion (Padé-Borel resummed).
[e]Monte Carlo simulations.
[f]Conformal techniques.

different cutoffs. This issue could be investigated by extending the truncation (8), e.g., by including a quartic fermion interaction $(\bar{\psi}\psi)^2$ along the lines discussed in Ref. 11.

Since the fluctuations generate bosonic and fermionic masses, one might be interested in the resulting gaps. They are proportional to the order parameter $\rho_0=\frac{1}{2}Z_\sigma\sigma_0^2$:

$$m_{\sigma R}=\Delta_\sigma\rho_0, \quad m_{\psi R}=\Delta_\psi\rho_0 \tag{18}$$

The gaps Δ_σ and Δ_ψ are shown in Table I.

We have also calculated the critical exponents in the three-dimensional GN model for $N>1$. They are compared with results from other methods in Table II. For $N=2$,4, and 12, critical exponents have been calculated in the $1/N$ expansion to $O(1/N^2)$[12,13] and anomalous dimensions to $O(1/N^3)$ using conformal techniques.[14] Monte Carlo methods have been used to calculate critical exponents for $N=4$, which are compared with the results from the $\epsilon=4-d$ expansion to $O(\epsilon^2)$.[13] Our values for the Taylor expansion with exponential cutoff (footnote b) agree well with Ref. 6, where the precise numerical implementation was different.

The largest discrepancy between different approaches concerns the values of the anomalous dimensions. This is similar as for the bosonic O(N) models and is expected to improve substantially in the next order in the derivative expansion.[15] The overall picture is, however, quite satisfactory and lends support to the validity of our method based on the exact renormalization-group equation. We believe that our finding of a second-order phase transition for $N=1$ is quite robust.

[1] D.J. Gross and A. Neveu, Phys. Rev. D **10**, 3235 (1974).
[2] G. Gat, A. Kovner, and B. Rosenstein, Nucl. Phys. B **385**, 76 (1992).
[3] S. Hands, A. Kocić, and J.B. Kogut, Ann. Phys. **224**, 29 (1993).
[4] T. Reisz, hep-lat/9712017 (unpublished).
[5] E. Babaev, Phys. Lett. B **497**, 323 (2001).
[6] L. Rosa, P. Vitale, and C. Wetterich, Phys. Rev. Lett. **86**, 958 (2001).
[7] C. Wetterich, Phys. Lett. B **301**, 90 (1993).
[8] J. Berges, N. Tetradis, and C. Wetterich, Phys. Rep. **363**, 223 (2002).
[9] K.I. Aoki, K.I. Morikawa, J.I. Sumi, H. Terao, and M. Tomoyose, Prog. Theor. Phys. **97**, 479 (1997).
[10] D.F. Litim, Phys. Rev. D **64**, 105007 (2001).
[11] H. Gies and C. Wetterich, Phys. Rev. D **65**, 065001 (2002).
[12] J.A. Gracey, Int. J. Mod. Phys. A **9**, 567 (1994).
[13] L. Kärkkäinen, R. Lacaze, P. Lacock, and B. Petersson, Nucl. Phys. B **415**, 781 (1994); **438**, 650(E) (1994).
[14] J.A. Gracey, Int. J. Mod. Phys. A **9**, 727 (1994); A.N. Vasilev, S.E. Derkachev, N.A. Kivel, and A.S. Stepanenko, Theor. Math. Phys. **94**, 127 (1993).
[15] S. Seide and C. Wetterich, Nucl. Phys. B **562**, 524 (1999).