

Green's function of the half-filled Landau level Chern-Simons theory in the temporal gauge

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We study the Green's function of the $\nu=1/2$ Chern-Simons system in the temporal (Weyl) gauge. We derive the Chern-Simons path integral in the temporal gauge. In order to do this, we gauge transform the path integral in the Coulomb gauge which represents the partition function of the correct normal ordered Chern-Simons Hamiltonian. We calculate the self-energy of this path integral in the random-phase approximation (RPA) for temperature $T=0$. This self-energy does not have the divergence with the logarithm of the area, which is known to imply the vanishing of the exact Green's function in the Coulomb gauge for an infinite area. By Chern-Simons retransforming the path integral representing the Green's function in the temporal gauge we calculate explicitly the exact Green's function by neglecting the interaction between the electrons, getting a finite value. Furthermore, we give arguments that the Green's function of the interacting system is also finite. The nonvanishing of the Green's function for infinite area is due to a dynamical creation of the phase factors linking the created and annihilated particles with the particles in the ground state. The absence of these phase factors is the reason for the vanishing of the Green's function in the Coulomb gauge.

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I. INTRODUCTION

The combination of an electronic interaction and a strong magnetic field in a two-dimensional electron system yields a rich variety of phases. These are best classified by the filling factor ν , which is the electron density divided by the density of a completely filled Landau level. In the case of $\nu \cong 1/2$, the behavior of the system resembles that of a Fermi liquid in the absence of a magnetic field, or at small magnetic fields. This effect can be explained with a new sort of quasiparticles: at $\nu=1/2$, each electron combines with two flux quanta of the magnetic field to form a composite fermion; these composite fermions then move in an effective magnetic field that is zero on the average. The interpretation of many experiments supports this picture. We mention transport experiments with antidots, in which features of the resistivity are related to closed loops of the composite fermions around the dots,¹ and also focusing experiments.² An overview of further experiments can be found in Ref. 3. A field theoretical formulation of this composite fermion picture was first established by Halperin, Lee, and Read⁴ (HLR) as well as Kalmeyer and Zhang.⁵

HLR studied many physical quantities within the random-phase approximation (RPA). Most prominent among these is the effective mass of the composite fermions, which is found to diverge at the Fermi surface.^{4,6} This is caused by the interaction of the composite fermions via transversal gauge interactions. Later on, Shankar and Murthy⁷ proposed a new theory of the $\nu=1/2$ system. Based upon a transformation of the Chern-Simons (CS) Hamiltonian one achieves a separation of the magnetoplasmon oscillators from the total interaction of the system. After restricting the number of the magnetoplasmon oscillators to the number of electrons Shankar and Murthy obtain a finite quasiparticle mass which scales with the inverse of the strength of the Coulomb repulsion. In their derivation they calculated a smaller number of self energy Feynman diagrams than in the RPA is calculated. Recently, Stern *et al.*⁸ calculated the self-energy of the theory of

Shankar and Murthy in RPA finding the same divergence of the effective mass as HLR. Furthermore, they examined the Lagrangian formalism in the temporal (Weyl) gauge of the CS theory. This appears to be very similar to the Hamilton theory of Murthy and Shankar⁹ which works with a Hilbert space consisting of the electron plus the magnetoplasmon degrees of freedom and an additional constraint on the Hilbert space. In their paper, Stern *et al.* show that the quasiparticles of both theories contain the dipole nature of the $\nu=1/2$ Rezayi-Read wave function,¹⁰ which has a good overlap with the $\nu=1/2$ exact ground state for small systems. In this paper we consider the Lagrangian formulation of the CS theory in the temporal gauge. Up to now it is not clear whether this theory and the Hamilton theory of Shankar and Murthy are in any relation. This is due to the difficulty in formulating a coherent state path integral for a Hamilton theory that contains a constraint including fermionic fields. We should mention that besides the theories of HLR and of Shankar and Murthy there are other alternative formulations of the CS theory that appear to be similar to the CS theory of Shankar and Murthy.¹¹

As mentioned first in Ref. 4 by semiclassical arguments and showed further by us non-perturbatively (calculated explicitly for the non-interacting system¹²) the Green's function of the $\nu=1/2$ CS system in the version of HLR (Coulomb gauge of the Lagrangian formulation of the CS theory) vanishes exponentially with an exponent proportional to $\ln(A)$, where A is the area of the system. This is caused by the CS transformation, which effectively gives a velocity boost to every electron. This velocity boost results in a one-particle energy that diverges proportional to $\ln(A)$. We further show in Ref. 12 that the A asymptotics of the Hartree-Fock approximation of the Green's function is in accordance with the exact Green's function (in contrast to the Green's function in RPA), which is the theoretical justification for formulating a perturbation theory around the Hartree-Fock mean field. This theory was discussed by us in Ref. 13. Due to this divergence it is difficult to formulate a quasiparticle language for such a

theory. When neglecting this $\ln(A)$ divergence in the self energy one gets the diverging effective mass on the fermi-surface discussed above. On the way to formulate a CS theory of the half-filled Landau level with meaningful quasiparticles we consider in this paper the Green's function of the CS theory in the temporal gauge. This is the Green's function for the new composite fermions. We will show that this Green's function does not vanish with an exponent proportional to $\ln(A)$ (at least for the Coulomb interacting system). We will calculate explicitly the Green's function in position-time representation by neglecting the Coulomb interaction. This yields a finite Green's function. Furthermore, we show that the Green's function should be also finite when taking into account the interaction between the electrons. These are the main results of this paper.

To this end we derive at first by a gauge transformation of the CS path integral in the Coulomb gauge¹⁴ the CS path integral in the temporal gauge. We have shown in Refs. 14 and 15 that one is not allowed to carry out the time slice continuum limit in CS path integrals of quantum Hall systems due to an additional term in the path integral that supplies the correct operator order in the correspondent CS Hamiltonian. When neglecting this additional term we get the familiar CS path integral, representing a gauge theory (e.g., Ref. 16). We will show that we get the same result either by writing down this gauge path integral in the temporal gauge or by gauge transforming the correct normal order CS path integral in the Coulomb gauge. By calculating the self energy of this path integral in RPA we get a singularity proportional to $1/T$ where T is the temperature. This was calculated earlier by Stern *et al.* in Ref. 6 for the Hamiltonian theory of Shankar and Murthy.

By retransforming the Green's function path integral to the electrons we get an effective path integral action of a time-dependent Hamiltonian. This time-dependent Hamiltonian describes electrons in a homogeneous magnetic field with two separated magnetic strings of opposite strength at the positions of the creation and the annihilation operator of the Green's function, which are adiabatically switched on until they get two flux quanta (for the $\nu=1/2$ system). By calculating the ground state energy and the ground state wave function of this Hamiltonian we derive explicitly for the noninteracting system a nonvanishing zero temperature Green's function. We will show further that the reason for the $T \rightarrow 0$ vanishing of the Green's function in RPA is caused by getting a difference in the ground state energy for the system taking into account the two strings in comparison to the system without the strings. The RPA corresponds to the energy correction in second-order perturbation theory in the string strength. Therefore, we calculate in this paper the energy difference for the interacting electron system, getting a zero energy difference. Thus, we see that the exact Green's function in the temporal gauge should be also finite when taking into account the Coulomb interaction between the electrons. Furthermore, we will see that by switching on the magnetic strings the creation and the annihilation operator in the Green's function gets additional CS phases linking them with all other electrons in the ground state. These phase factors do not exist in the comparable expression of the Green's func-

tion in the Coulomb gauge. This is the reason for the vanishing of the Coulomb gauged Green's function with an exponent proportional to $\log(A)$ not existent in the temporal gauge.¹²

The paper is organized as follows: In Sec. II we derive the CS path integral in the temporal gauge from the path integral in the Coulomb gauge. We compare the Green's functions in RPA of both gauges in Sec. III. In Sec. IV we consider the Green's function in the temporal gauge nonperturbatively.

II. THE CS PATH INTEGRAL IN THE COULOMB AS WELL AS THE TEMPORAL GAUGE

In this section we consider interacting spin-polarized electrons moving in two dimensions in a strong magnetic field B directed in the negative z direction. The electronic density is chosen such that the lowest Landau level of the noninteracting system is filled to a fraction $\nu=1/\tilde{\phi}$ where $\tilde{\phi}$ is an even number. We are mainly interested in $\tilde{\phi}=2$. The CS transformation is defined by¹⁷

$$\Psi^\dagger(\vec{r}) = \Psi_e^\dagger(\vec{r}) \exp \left[i \tilde{\phi} \int d^2 r' \arg(\vec{r} - \vec{r}') \rho(\vec{r}') \right], \quad (1)$$

where $\Psi_e^\dagger(\vec{r})$ is the electron creation operator, $\Psi^\dagger(\vec{r})$ is the creation operator of the transformed fermions (composite fermions), $\hat{\rho}(\vec{r})$ is the density operator of the fermion operators, and $\arg(\vec{r})$ is the angle that \vec{r} forms with the x axis. In this paper we use the convention that \arg has its cut on the negative real axis. The Hamiltonian is given after the transformation as

$$H_{CS}(\vec{a}_{CS}) = \int d^2 r \left\{ \frac{1}{2m} \Psi^\dagger(\vec{r}) (-i \vec{\nabla} + \vec{A} + \vec{a}_{CS})^2 \Psi(\vec{r}) + \frac{1}{2} \int d^2 r' [|\Psi(\vec{r})|^2 - \rho_B] V_{r,r'}^{ee} [|\Psi(\vec{r}')|^2 - \rho_B] \right\}. \quad (2)$$

The CS vector potential \vec{a}_{CS} is defined by $\vec{a}_{CS}(\vec{r}) = \tilde{\phi} \int d^2 r' \vec{f}(\vec{r} - \vec{r}') \Psi^\dagger(\vec{r}') \Psi(\vec{r}')$. Here $\Psi^\dagger(\vec{r})$ creates [and $\Psi(\vec{r})$ annihilates] a composite fermion with coordinate \vec{r} . $V_{r,r'}^{ee} = e^2/|\vec{r} - \vec{r}'|$ is the Coulomb interaction where $e^2 = q_e^2/\epsilon$. q_e is the charge of the electrons and ϵ is the dielectric constant of the background field ρ_B . $\vec{A}(\vec{r})$ is the vector potential $\vec{A} = \vec{B} \times \vec{r}/2$ and \vec{B} is a homogeneous magnetic field in the negative z direction $\vec{B} = -B \vec{e}_z$ where \vec{e}_z is the unit vector in the z direction. We suppose throughout this paper that B is a positive number. The function $\vec{f}(\vec{r})$ is given by $\vec{f}(\vec{r}) = \vec{e}_z \times \vec{r}/r^2$. We used the convention $\hbar=1$ and $c=1$ in the above formula (2). Furthermore, we set $q_e=1$ for the coupling of the magnetic potential to the electrons. We obtained in Ref. 14 the partition function of the Hamiltonian (2) in the path integral formalism. With the help of the real bosonic CS fields $[a^0(r,t), \vec{a}(r,t)]$ we get

$$Z_{\text{Coul}} = \lim_{\epsilon \rightarrow 0} \frac{1}{\bar{N}} \prod_{l=1}^{N_l} \int_{\text{BC}} \mathcal{D}[a_l^0, \vec{a}_l] \mathcal{D}[\Psi_l^*, \Psi_l] \times \exp[-\epsilon(L_l^{\text{Coul}} + L_{\text{CS},l} + L_{ee,l})]. \quad (3)$$

The various functions in Eq. (3) are given by

$$L_l^{\text{Coul}} = \int d^2r \Psi_l^*(\vec{r}) \frac{1}{\epsilon} [\Psi_l(\vec{r}) - \Psi_{l-1}(\vec{r})] - \Psi_l^*(\vec{r}) \left[\mu + \left(1 + i \frac{\epsilon}{2} a_l^0(\vec{r}) \right) i a_l^0(\vec{r}) \right] \Psi_{l-1}(\vec{r}) + \frac{1}{2m} \Psi_l^*(\vec{r}) [-i \vec{\nabla} + \vec{A}(\vec{r}) + \vec{a}_l(\vec{r})]^2 \Psi_{l-1}(\vec{r}), \quad (4)$$

$$L_{\text{CS},l} = \frac{1}{4\pi\tilde{\phi}} \int d^2r i a_l^0(\vec{r}) \vec{\nabla} \times \vec{a}_l(\vec{r}), \quad (5)$$

$$L_{ee,l} = \frac{1}{2} \int d^2r d^2r' [\Psi_l^*(\vec{r}) \Psi_{l-1}(\vec{r}') - \rho_B] \times V_{r,r'}^{ee}, [\Psi_l^*(\vec{r}') \Psi_{l-1}(\vec{r}') - \rho_B], \quad (6)$$

and

$$\bar{N} = \prod_{l=1}^{N_l} \int_{\text{BC}} \mathcal{D}[a_l^0, \vec{a}_l] \exp[-\epsilon(L_{\text{CS},l})]. \quad (7)$$

The path integral (3) is correct under the gauge condition $\vec{\nabla} \cdot \vec{a}_l = 0$ (Coulomb gauge). The time slice width ϵ is defined by $\epsilon = \beta/N_l$ where $\beta = 1/T$. The index l counts the discrete time slices. Furthermore, we have antiperiodic boundary conditions $\Psi_{N_l} = -\Psi_0$ (denoted by BC) for the Grassmann fields. The action of the path integral (3) is given by a fermionic term L_l^{Coul} , a bosonic term $L_{\text{CS},l}$ of the CS form, and a Coulomb interaction term $L_{ee,l}$. In comparison to the CS path integral of HLR⁴ we get an additional term proportional to $\epsilon(a_l^0)^2 \Psi_l^* \Psi_{l-1}/2$ in L_l^{Coul} , Eq. (4). This term is due to the non-normal-order of the Ψ^6 term in the CS Hamiltonian H_{CS} (2). This is best seen by integrating the path integral (3) over the CS fields. Due to the additive term one cannot perform the formal limit $\epsilon \rightarrow 0$ in Eq. (3). Now suppose that one may neglect the $\epsilon(a_l^0)^2 \Psi_l^* \Psi_{l-1}/2$ term in L_l^{Coul} , Eq. (4). Then one can take the formal limit $\epsilon \rightarrow 0$ in Eq. (3) getting the well known path integral describing a CS gauge theory in the Coulomb gauge. Without the gauge fixing condition $\vec{\nabla} \cdot \vec{a}_l(\vec{r}) = 0$ the path integral consists of the three independent CS fields a^0 and \vec{a} .¹⁶ The CS theory in the temporal gauge^{7,8} is then given by $a^0 = 0$. The neglect of the term $\epsilon(a_l^0)^2 \Psi_l^* \Psi_{l-1}/2$ in L_l^{Coul} is to our opinion not satisfactory because we showed in Refs. 14 and 15 that this results in the wrong RPA energy. Thus, it is important to determine whether the CS path integral in the temporal gauge used by Shankar and Murthy⁷ and Stern *et al.*⁸ is correct by considering the full Lagrangian (4) in the derivation. This will be done in the following:

We start from the path integral (3) by the gauge transformation of the fermionic fields:

$$\Psi_l \rightarrow \exp[i(g_l + F(a^0, \vec{a}))] \Psi_l,$$

$$\Psi_0 \rightarrow \exp[iF(a^0, \vec{a})] \Psi_0, \quad (8)$$

with

$$g_l = \sum_{k=1}^l \epsilon \left(a_k^0 - \frac{1}{\beta} \sum_{k=1}^{N_l} \epsilon a_k^0 \right). \quad (9)$$

The definition of g_l is chosen such that the Fourier transformation of g_l is $1/(i\omega)$ times the Fourier transformation of a^0 (for $\epsilon \rightarrow 0$). $F(a^0, \vec{a})$ is a function of the fields a^0 and \vec{a} , which does not depend explicitly on the time index l . From this transformation one sees that the new Grassmann fields keep the antiperiodic boundary condition $\Psi_{N_l} = -\Psi_0$. In the following, we define $F(a^0, \vec{a})$ such that the $\omega = 0$ term of the Fourier transformation of the function $g_l + F(a^0, \vec{a})$ is zero. This results in

$$F(a^0, \vec{a}) = - \left(\frac{1}{\beta} \sum_{k=1}^{N_l} \epsilon \sum_{k'=1}^{k'} \epsilon a_k^0 - \frac{1}{2} \sum_{k=1}^{N_l} \epsilon a_k^0 \right). \quad (10)$$

After inserting the transformation (8) of the Grassmann fields in (3) we expand the exponential function in relation (8). We do not have to consider all of the expansion terms for $\epsilon \rightarrow 0$. In order to determine which expansion terms have to be considered we further expand the exponential function in Eq. (3) of the exponent L_l^{Coul} containing at least one CS field a_l^0 or \vec{a}_l . Now one may assume that it is enough to consider only linear terms in the expansion of the exponential function in relation (8). This is not correct because one gets also terms of the order $O(1)$ after integrating out the Chern-Simons fields [e.g., a sum of Grassmann fields over the time slices times ϵ is of order $O(1)$]. By analyzing the terms carefully we see that one has to take into account up to the quadratic expansion terms in the exponential function in relation (8) to get all $O(1)$ terms in the path integral. Doing so, one can observe the interesting effect that the $\epsilon(a_l^0)^2 \Psi_l^* \Psi_{l-1}/2$ term in Eq. (4) is canceled by some of the expansion terms in relation (8). After an additional gauge transformation

$$\Psi_l(\vec{r}) \rightarrow \exp[i\tilde{\phi}\vec{f}(0) \cdot \vec{r}] \Psi_l(\vec{r}), \quad (11)$$

we obtain a path integral in which it is allowed to take the limit $\epsilon \rightarrow 0$. This path integral is given by

$$Z_{\text{Weyl}} = \frac{1}{\bar{N}} \int_{\text{BC}} \mathcal{D}[a^0, \vec{a}] \mathcal{D}[\Psi^*, \Psi] \times \exp \left[- \int_0^\beta dt (L^{\text{Weyl}} + L_{\text{CS}} + L_{ee}) \right], \quad (12)$$

with

$$L^{\text{Weyl}} = \int d^2r \Psi^*(\vec{r}, t) \left(\partial_t - \mu - \frac{i}{\beta} \int_0^\beta dt' a^0(\vec{r}, t') \right) \Psi(\vec{r}, t) \\ + \frac{1}{2m} \Psi^*(\vec{r}, t) (-i\vec{\nabla} + \vec{A}(\vec{r})) \\ + \vec{a}(\vec{r}, t) + \vec{\nabla} [g(\vec{r}, t) + F(a^0, \vec{a})]^2 \Psi(\vec{r}, t), \quad (13)$$

$$L_{CS} = \frac{1}{4\pi\tilde{\phi}} \int d^2r i a^0(\vec{r}, t) \vec{\nabla} \times \vec{a}(\vec{r}, t), \quad (14)$$

$$L_{ee} = \frac{1}{2} \int d^2r d^2r' [\Psi_l^*(\vec{r}) \Psi_{l-1}(\vec{r}') - \rho_B] \\ \times V_{r,r'}^{ee} [\Psi_l^*(\vec{r}') \Psi_{l-1}(\vec{r}) - \rho_B]. \quad (15)$$

By neglecting the third term in the first large parentheses in Eq. (13) for $T=0$ and the definition of the longitudinal CS gauge potential

$$\vec{a}_L(\vec{r}) = \vec{\nabla} [g(\vec{r}) + F(a^0, \vec{a})] \quad (16)$$

we get the well known CS path integral in the temporal gauge.^{7,8} This path integral was used by Stern *et al.* in Ref. 8 to show that the quasiparticles in the temporal gauge behave like dipoles with a dipole momentum perpendicular to their canonical momentum (for small momentum and frequency). This can be seen by calculating the response of the electrons in the RPA due to some external potential. This picture of the CS quasiparticles is very attractive¹⁸ due to a similar dipole interpretation of the Rezayi-Read wave function.¹⁰

It has been shown that this wave function has a very good overlap with the exact ground state for small systems.¹⁰

III. THE RPA GREEN'S FUNCTIONS

In this section we determine the RPA Green's functions in the Coulomb as well as the temporal gauge for temperature $T \rightarrow 0$. This was done earlier for the Coulomb gauge.¹² Since the Coulomb interaction has no influence on the singularity of the Green's function in the Coulomb as well as in the temporal gauge we will simplify the notation by considering explicitly only the interaction-free case of the Green's function. The Coulomb interaction can easily be taken into account by carrying out a Hubbard-Stratonovich decoupling¹⁹ of the Coulomb interaction term (15). In the following, we will mention explicitly where the results for the noninteracting system differ from those of the interacting system.

In Ref. 14, we calculated the grand canonical potential Ω_{Coul} from the CS path integral in the Coulomb gauge (3) in RPA. This was done by carrying out the integration of Eq. (3) over the fermionic fields and further by expanding the logarithms of the result in the CS fields. The restriction to quadratic order in the RPA for the CS fields results in

$$\Omega^{\text{Coul}} = \frac{1}{2\beta} \sum_{q, \Omega} \ln \left[1 - \Pi_{00} \left(\Pi_{TT} + \frac{\rho}{m} \right) \frac{(2\pi\tilde{\phi})^2}{q^2} \right]. \quad (17)$$

In this equation Π_{00} is the ideal gas density-density response and Π_{TT} is the transversal momentum-momentum response. These response functions can be calculated exactly.¹⁴ The grand canonical potential is then a functional of the interaction-free Green's function $G = -1/[i\omega - q^2/(2m) + \mu]$. The RPA self energy can be calculated by $\Sigma^{\text{Coul}} = \delta\Omega^{\text{Coul}}/\delta G$. By carrying out the calculation for the path integral (3), one gets for Σ^{Coul} one term that is divergent for $A \rightarrow \infty$.¹² This term corresponds to a self-energy Feynman-diagram with one density-density (a^0, a^0) RPA vertex. The other RPA self-energy diagram containing one transversal momentum-momentum (\vec{a}_T, \vec{a}_T) vertex is finite (here \vec{a}_T is the transversal component of \vec{a}). One obtains for the divergent self-energy term

$$\Sigma_{00}^{\text{Coul}}(k, \omega) = \Sigma_F(k, \omega) + \frac{1}{\beta} \sum_{q, \Omega} G(\vec{k} + \vec{q}, \omega + \Omega) \\ \times \left(D_{00}(q, \Omega) - \frac{\rho}{m} \right) \frac{(2\pi\tilde{\phi})^2}{q^2} \quad (18)$$

with the Fock self-energy

$$\Sigma_F(k, \omega) = - \sum_q n_F(|\vec{k} + \vec{q}|) \frac{\rho}{m} \frac{(2\pi\tilde{\phi})^2}{q^2} \quad (19)$$

and the (a^0, a^0) RPA vertex

$$D_{00}(q, \Omega) = \frac{(2\pi\tilde{\phi})^2}{q^2} \frac{\Pi_{TT} + \rho/m}{1 - \Pi_{00}(\Pi_{TT} + \rho/m)(2\pi\tilde{\phi})^2/q^2}. \quad (20)$$

ρ is the density of the system. The singular part of the (a^0, a^0) RPA vertex D_{00} in Eq. (18) has its parameter range in $\Omega \gg q\sqrt{\mu/m}$ and $q^2/(m\mu) \ll 1$.⁴ In this range one gets for the vertex

$$D_{00}(q, \Omega) \approx \frac{(2\pi\tilde{\phi})^2}{q^2} \frac{\rho}{m} \frac{\Omega^2}{\Omega^2 + \omega_c^2}. \quad (21)$$

Here ω_c is given by $B/m = (2\pi\tilde{\phi})\rho/m$. With the help of this expression we obtain for the $A \rightarrow \infty$ singular part of the Green's function

$$\Sigma_{00}^{\text{Coul}}(k, \omega) = \tilde{\phi}^2 \frac{\mu}{2} \ln \left(\frac{1}{A} \right) \\ \times \frac{i\omega - k^2/2m + \mu}{\omega_c + \text{sgn}(k^2/2m - \mu)(k^2/2m - i\omega - \mu)}. \quad (22)$$

$\text{sgn}[\cdot]$ is the sign of the argument.

We may now carry out a similar calculation for the CS path integral in the temporal gauge (12) yielding for the grand canonical potential Ω^{Weyl} in RPA

$$\begin{aligned} \Omega^{\text{Weyl}} = & \frac{1}{2\beta} \sum_{\vec{q}, \Omega \neq 0} \ln \left[1 - \left(\Pi_{LL} + \frac{\rho}{m} \right) \left(\Pi_{TT} + \frac{\rho}{m} \right) \frac{(2\pi\tilde{\phi})^2}{\Omega^2} \right] \\ & + \frac{1}{2\beta} \sum_{\vec{q}} \ln \left[1 - \Pi_{00}(\vec{q}, 0) \left(\Pi_{TT}(\vec{q}, 0) + \frac{\rho}{m} \right) \right. \\ & \left. \times \frac{(2\pi\tilde{\phi})^2}{q^2} \right]. \end{aligned} \quad (23)$$

Here Π_{LL} is the ideal gas longitudinal momentum-momentum response function. With the help of the ideal gas continuity equation $(\Pi_{LL} + \rho/m)/\Omega^2 = \Pi_{00}/q^2$ we get $\Omega^{\text{Coul}} = \Omega^{\text{Weyl}}$. Nevertheless, Ω^{Coul} and Ω^{Weyl} are not identical as a function of G . As in the Coulomb gauge we calculate the RPA self-energy by $\Sigma^{\text{Weyl}} = \delta\Omega^{\text{Weyl}}/\delta G$. Then we get one term that corresponds to the RPA self-energy diagram containing one transversal momentum-momentum vertex (\vec{a}_T, \vec{a}_T) . This finite term is the same as that in the Coulomb

gauge. The other term corresponds to the self-energy diagrams containing one longitudinal momentum-momentum vertex (\vec{a}_L, \vec{a}_L) . It is given by

$$\begin{aligned} \Sigma_{LL}^{\text{Weyl}}(k, \omega) = & \frac{1}{\beta} \sum_{\vec{q}} G(\vec{k} + \vec{q}, \omega) \mathcal{D}_{00}(q, 0) \\ & + \frac{1}{\beta} \sum_{\vec{q}, \Omega \neq 0} G(\vec{k} + \vec{q}, \omega + \Omega) \mathcal{D}_{00}(q, \Omega) \frac{q^2}{\Omega^2} \\ & \times \left(\frac{(2\vec{k} + \vec{q}) \cdot \vec{q}}{2mq} \right)^2. \end{aligned} \quad (24)$$

Here the first term originates from the third term in the first large parentheses in Eq. (13). Contrary to the Coulomb gauge, one finds an infinite self-energy for $T=0$ in the parameter range $\Omega \ll q\sqrt{\mu/m}$ and $q^2/(m\mu) \ll 1$ because of the additional $1/\Omega^2$ factor in the second term in Eq. (24). In this parameter range the vertex \mathcal{D}_{00} is given by

$$\mathcal{D}_{00}(q, \Omega) \approx \frac{q^2}{24\pi m} \frac{1}{\sqrt{2m\mu m} |\Omega| / (2\pi)^2 q + [e^2 m / (2\pi\tilde{\phi})^2] q + q^2 (1/48\pi^2 + 1/(2\pi\tilde{\phi})^2)}. \quad (25)$$

This expression contains a correction due to the Coulomb interaction. Inserting relation (25) in Eq. (24) results in a self-energy term proportional to β , which is given by

$$\Sigma_{LL}^{\text{Weyl}}(k, \omega) \approx \frac{\beta}{12} \sum_{|\vec{q}| < \kappa} G(\vec{k} + \vec{q}, \omega) \mathcal{D}_{00}(q, 0) \left(\frac{(2\vec{k} + \vec{q}) \cdot \vec{q}}{2m} \right)^2. \quad (26)$$

Here $\kappa \ll \sqrt{m\mu}$ is a momentum cutoff. Thus, one finds that $\Sigma_{LL}^{\text{Weyl}}$ is proportional to β and diverges as $T \rightarrow 0$. This results in a divergent self-energy for temperature $T=0$. By comparing relation (26) with Eq. (22) one sees that the self-energy in the temporal gauge is not divergent for $A \rightarrow \infty$. In Ref. 12 we showed that the Green's function in the Coulomb gauge vanishes with an exponent proportional to $\ln(A)$ in the position-time representation. This is caused by an effective velocity boost obtained for every CS quasiparticle by the CS transformation.⁴ This results in a self-energy proportional to $\ln(A)$. Due to the missing $\ln(A)$ term in the RPA self energy in the temporal gauge the first question we want to answer by nonperturbative methods is the following.

A. Is it true that the exact CS Green's function in the temporal gauge does not show a similar $A \rightarrow \infty$ vanishing asymptotics as the Green's function in the Coulomb gauge?

The β divergence of the self-energy in relation (26) has its origin in the form of the \vec{a}_L coupling to the fermionic fields in L^{Weyl} , Eq. (13), which results in an additional q^2/ω^2 factor for every (\vec{a}_L, \vec{a}_L) vertex in comparison to the (a_0, a_0) vertices in the Coulomb gauge. From this it is clear that a

similar kind of divergence should also be given in the self-energy diagrams beyond RPA.

Stern *et al.* mentioned first in Ref. 8 (for the case of the Hamiltonian formulation of the CS theory in the temporal gauge) that this divergence in the self-energy is caused by the additional gauge freedom of the CS path integral (12) with respect to time-independent gauge transformations. This causes the partition function to be independent of the zero frequency \vec{a}_L variable by additionally carrying out the integration over the fermionic fields. Nevertheless, one cannot deduce from this the behavior of the partition function for \vec{a}_L with frequencies approximately zero by additionally carrying out the integration over the fast \vec{a}_L modes and \vec{a}_T . This would be more relevant for the behavior of the Green's function than the additional gauge freedom. Thus, we are led to the second question we would like to answer by nonperturbative methods.

B. Is it true that the exact CS Green's function in the temporal gauge is zero for temperature $T=0$?

A first approach to a solution of this question is given by the following observation: By carrying out a gauge retransformation (8) of the path integral representing the Green's function $G(\vec{r}, t)$ in the temporal gauge the Grassmann fields representing the created and the annihilated particle in the Green's function obtain exponential prefactors of the form (8). By an expansion of the exponents up to quadratic order in the a^0 fields we get for the $T \rightarrow 0$ diverging part of the Green's function a term proportional to $\beta [D_{00}^{\text{ex}}(\vec{r}, \omega=0)$

$-\mathcal{D}_{00}^{\text{ex}}(0, \omega=0)$]. $\mathcal{D}_{00}^{\text{ex}}(\vec{r}, \omega=0)$ is the exact (a^0, a^0) vertex corresponding to Eq. (20) in RPA. $\mathcal{D}_{00}^{\text{ex}}(\vec{r}, \omega=0)$ is given by

$$\begin{aligned} \mathcal{D}_{00}^{\text{ex}}(\vec{r}, \omega=0) = & -\tilde{\phi}^2 \int d^2r' \vec{f}(\vec{r}-\vec{r}') \cdot \vec{f}(\vec{r}') \frac{\langle \hat{\rho}(\vec{r}') \rangle}{m} \\ & + \tilde{\phi}^2 \int dt \int d^2r' d^2r'' \vec{f}(\vec{r}-\vec{r}') \cdot \langle T \hat{J}(\vec{r}', t) \\ & \times \hat{J}(\vec{r}'', 0) \rangle_c \cdot \vec{f}(\vec{r}''). \end{aligned} \quad (27)$$

T is the time ordering operator. The right-hand side of Eq. (27) can be calculated in the electronic system. Thus, $\hat{\rho}(\vec{r})$ is the density operator and $\hat{J}(\vec{r})$ is the current operator of the electrons. $\langle \cdot \rangle_c$ is the connected average with respect to the electronic ground state (not the CS ground state). It is clear from the derivation above that the Feynman diagrams of the Green's function in first order in $\Sigma_{LL}^{\text{Weyl}}$ are contained in this Green's function. We now restrict our considerations to the noninteracting electron system. In Appendix A we calculate $\mathcal{D}_{00}^{\text{ex}}(\vec{r}, \omega=0)$ without any approximation for this noninteracting system. We obtain

$$\mathcal{D}_{00}^{\text{ex}}(\vec{r}, 0) = \frac{\tilde{\phi}}{l_0^2 m} \left\{ \left[\ln \left(\frac{r^2}{2l_0^2} \right) + \gamma \right] e^{-r^2/2l_0^2} + \frac{3}{2} E_1 \left(\frac{r^2}{2l_0^2} \right) \right\}. \quad (28)$$

Here $l_0 = 1/\sqrt{B}$ is the magnetic length. γ is Euler's constant and E_1 is the exponential integral function. We obtain that $\mathcal{D}_{00}^{\text{ex}}(\vec{r}, \omega=0) - \mathcal{D}_{00}^{\text{ex}}(0, \omega=0)$ diverges for the noninteracting system. This divergence was regularized in the RPA self-energy formula (24) with the help of a momentum cutoff in the uv region. It is easy to see that the expression $\mathcal{D}_{00}^{\text{ex}}(\vec{r}, \omega=0) - \mathcal{D}_{00}^{\text{ex}}(0, \omega=0)$ agrees with the energy formula of the second-order perturbation theory of electrons in a homogeneous magnetic field B under the perturbation of two magnetic strings of flux $\tilde{\phi}$ and $-\tilde{\phi}$ at the origin and at the position \vec{r} , respectively (see, e.g., Appendix A). It is well known that the energy of electrons in a homogeneous magnetic field B with two magnetic strings is finite. Therefore, this energy cannot be calculated perturbatively. We will show in the following subsections that this is in fact the reason for the temporal gauged Green's function in RPA to be zero for $T=0$. Furthermore, we show that the exact Green's function is finite for $T=0$ because the ground state energy corrections due to the two magnetic strings is zero.

IV. THE EXACT CS GREEN'S FUNCTION IN THE TEMPORAL GAUGE

In this section, we calculate the CS Green's function in the temporal gauge nonperturbatively. This was done by us in Ref. 20 for the CS Green's function in the Coulomb gauge. There we determined the Green's function in the position-time representation by CS retransforming (1) the Green's function to the electronic Hilbert space. A similar procedure will be done in this section for the Green's function in the temporal gauge. It is defined by

$$G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau') = \langle \Psi(\vec{x}, \tau) \Psi^*(\vec{x}', \tau') \rangle_{\text{Weyl}}. \quad (29)$$

Here $\langle \cdot \rangle_{\text{Weyl}}$ is the average with respect to the CS path integral in the temporal gauge (12). We now carry out the inverse of the gauge transformation (8) and (11) on the fermionic fields in this expression. After integrating out the CS fields one gets the following expression

$$\begin{aligned} G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau') = & \frac{1}{N_G} \int_{\text{BC}} \mathcal{D}[\Psi^*, \Psi] \Psi(\vec{x}, \tau) \Psi^*(\vec{x}', \tau') \\ & \times \exp \left[- \int_0^\beta dt L_G(\vec{A}_{ss}^t) \right]. \end{aligned} \quad (30)$$

with the norm N_G

$$N_G = \int_{\text{BC}} \mathcal{D}[\Psi^*, \Psi] \exp \left[- \int_0^\beta dt L_G(0) \right] \quad (31)$$

and the Lagrangian

$$\begin{aligned} L_G(\vec{A}_{ss}^t) = & \int d^2r \left\{ \Psi^*(\vec{r}, t) (\partial_t - \mu) \Psi(\vec{r}, t) \right. \\ & + \frac{1}{2m} \tilde{\phi}^2 \int d^2r' \vec{f}^2(\vec{r}-\vec{r}') \Psi^*(\vec{r}, t) \Psi(\vec{r}, t) \\ & \times \Psi^*(\vec{r}', t) \Psi(\vec{r}', t) + \frac{1}{2m} \Psi^*(\vec{r}, t) \\ & \times [-i\vec{\nabla} + \vec{A}(\vec{r}) + \vec{a}_{\text{CS}}(\vec{r}, t) + \vec{A}_{ss}^t(\vec{r})]^2 \Psi(\vec{r}, t) \\ & + \frac{1}{2} \int d^2r' [|\Psi(\vec{r}, t)|^2 - \rho_B] \\ & \left. \times V_{r, r'}^{ee} [|\Psi(\vec{r}', t)|^2 - \rho_B] \right\}. \end{aligned} \quad (32)$$

The string configuration \vec{A}_{ss}^t in $L_G(\vec{A}_{ss}^t)$ is given by

$$\begin{aligned} \vec{A}_{ss}^t(\vec{r}) = & -\tilde{\phi} \vec{f}(\vec{r}-\vec{x}) \left(\Theta(\tau-t) - \frac{\tau-t}{\beta} - \frac{1}{2} \right) \\ & + \tilde{\phi} \vec{f}(\vec{r}-\vec{x}') \left(\Theta(\tau'-t) - \frac{\tau'-t}{\beta} - \frac{1}{2} \right). \end{aligned} \quad (33)$$

Here $\Theta(x)$ is the Heaviside function. We see from this formula that the effective Lagrangian of the Green's function is given by the well-known CS Lagrangian and additionally two time-dependent strings with opposite fluxes centered at the coordinates \vec{x} and \vec{x}' . Furthermore, we see from Eq. (33) that the Green's function depends on $\tau - \tau'$ (time translational invariance). We see from Eqs. (30)–(33) that one has to solve a complicated time-dependent Schrödinger equation to get the Green's function in the temporal gauge. Nevertheless, we will derive a solution of the problem for temperature $T=0$ below. Thus, in the following we restrict the calculation of the Green's function to temperature $T=0$. We treat at first the Green's function for time ordering $\tau - \tau' > 0$. Then the path integral (30) can be interpreted as follows.

With the help of

$$H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^t) = \int d^2r \left\{ \frac{1}{2m} \Psi^\dagger(\vec{r}) (-i\vec{\nabla} + \vec{A} + \vec{a}_{CS} + \vec{A}_{ss,2}^t)^2 \Psi(\vec{r}) + \frac{1}{2} \int d^2r' [|\Psi(\vec{r}')|^2 - \rho_B] V_{r,r'}^{ee} [|\Psi(\vec{r}')|^2 - \rho_B] \right\} \quad (34)$$

and

$$\vec{A}_{ss,2}^t = \left(\frac{1}{2} - \frac{t}{\beta} \right) \vec{\phi} f(\vec{r} - \vec{x}) - \left(\frac{3}{2} - \frac{t}{\beta} \right) \vec{\phi} f(\vec{r} - \vec{x}'), \quad (35)$$

we define the time evolution operator $U_t(\vec{a}_{CS}, \vec{A}_{ss,2}^t)$ by

$$- \frac{\partial}{\partial t} U_t(\vec{a}_{CS}, \vec{A}_{ss,2}^t) = H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^t) U_t(\vec{a}_{CS}, \vec{A}_{ss,2}^t) \quad (36)$$

with the boundary condition $U_0 = 1$. Because the flux quantum numbers of the strings of the vector potentials $\vec{A}_{ss,2}^\beta$ and $\vec{A}_{ss,2}^0$ differ only by an integer value we know that the eigenspaces of $H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^0)$ and $H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^\beta)$ are linked by the unitary phase transformation

$$\text{Ph}_{\vec{\phi}}(\vec{x}, \vec{x}') = \exp \left(-i \vec{\phi} \int d^2r \arg(\vec{x} - \vec{r}) \hat{\rho}(\vec{r}) \right) \times \exp \left(i \vec{\phi} \int d^2r \arg(\vec{x}' - \vec{r}) \hat{\rho}(\vec{r}) \right) \quad (37)$$

acting on the states of $H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^\beta)$. The eigenvalues are invariant under this transformation. With the help of the phase factors (37) we get for the Green's function for $\tau - \tau' > 0$

$$G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau') = e^{-i \vec{\phi} f(0) \cdot (\vec{x} - \vec{x}')} \times \lim_{\beta \rightarrow \infty} \frac{1}{\text{Tr} \{ \exp [-(\beta + \tau - \tau') H_{ss}(\vec{a}_{CS}, 0)] \}} \times \text{Tr} \{ U_\beta(\vec{a}_{CS}, \vec{A}_{ss,2}^t) \text{Ph}_{\vec{\phi}}(\vec{x}, \vec{x}') \Psi_e(\vec{x}) \exp [-(\tau - \tau') H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^\beta)] \Psi_e^\dagger(\vec{x}') \}, \quad (38)$$

where Ψ_e^\dagger is the electron creation operator (1). Now assume that $H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^t)$ has the same ground state degeneracy and ground state energy as the Hamiltonian $H_{ss}(\vec{a}_{CS}, 0)$. This will be shown for the noninteracting as well as for the interacting electron system in the following subsections. Then it is intuitively clear that the operator $U_\beta(\vec{a}_{CS}, \vec{A}_{ss,2}^t) / \text{Tr} \{ \exp [-(\beta + \tau - \tau') H_{ss}(\vec{a}_{CS}, 0)] \}$ has its support on the lowest energy eigenspace for $\beta \rightarrow \infty$. We will also show this explicitly in the following subsection. On the other hand it is clear that we get an infinite or zero Green's function, respectively, if

the integral of the ground state energy of $H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^t) - H_{ss}(\vec{a}_{CS}, 0)$ over the time t is nonzero. As mentioned above the incorrect reproduction of the ground state energy within second-order perturbation theory is the reason that the RPA self-energy is infinite for $T=0$.

Now we compare the expression (38) of the Green's function with the corresponding expression in the Coulomb gauge for $\tau - \tau' > 0$. This function is given by the expression (38) with the substitutions $U_\beta(\vec{a}_{CS}, \vec{A}_{ss,2}^t) \text{Ph}(\vec{x}, \vec{x}') \rightarrow U_\beta(\vec{a}_{CS}, 0)$, $\Psi_e(\vec{x}) \rightarrow \Psi(\vec{x})$, $\Psi_e^\dagger(\vec{x}') \rightarrow \Psi^\dagger(\vec{x}')$, $H_{ss}(\vec{a}_{CS}, \vec{A}_{ss,2}^\beta) \rightarrow H_{ss}(\vec{a}_{CS}, 0)$ and $\exp[-i \vec{\phi} f(0) \cdot (\vec{x} - \vec{x}')] \rightarrow 1$. As mentioned above we calculated in Ref. 20 a CS Green's function in the Coulomb gauge that vanishes exponentially with an exponent proportional to $\ln(A)$. The reason for this is that a quasiparticle $\Psi^\dagger(\vec{x}')$ created at time τ' gets a velocity boost from all other particles in the ground state. This results in an infinite quasiparticle energy. This is not the case for the quasiparticle $\Psi_e^\dagger(\vec{x}')$ created at time τ' in the Green's function of the temporal gauge. Thus, we see that the CS phases between the created particle and all other electrons of the ground state that led to the velocity boost in the Coulomb gauge are automatically annihilated through the dynamical creation of an opposite phase by turning on the magnetic strings in H_{ss} in the temporal gauge. In this sense one can understand the missing $\ln(A)$ terms in the RPA self-energy of the temporal gauge. Now we carry out a CS retransformation (1) of the expression (38). This results in

$$G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau') = e^{-i \vec{\phi} f(0) \cdot (\vec{x} - \vec{x}')} \times \lim_{\beta \rightarrow \infty} \frac{1}{\text{Tr} \{ \exp [-(\beta + \tau - \tau') H_{ss}(0, 0)] \}} \times \text{Tr} \{ U_\beta(0, \vec{A}_{ss,2}^t) \text{Ph}_{\vec{\phi}}(\vec{x}, \vec{x}') \Psi(\vec{x}) \exp [-(\tau - \tau') H_{ss}(0, \vec{A}_{ss,2}^\beta)] \Psi^\dagger(\vec{x}') \}. \quad (39)$$

In the following section we will calculate this expression for the case of a noninteracting electron system. Then we see from Eq. (39) that by the CS retransformation one loses all many-particle interaction terms. Thus, we can determine the Green's function by calculating the one-particle ground state wave function and the ground state energy of the Hamiltonian $H_{ss}(0, \vec{A}_{ss,2}^t)$. From this we get $U_\beta(0, \vec{A}_{ss,2}^t)$. In Sec. IV A, we calculate the ground state wave function and the ground state energy of an electron in a homogenous magnetic field in the background of two opposite magnetic strings. By using this result, we calculate in Sec. IV B the Green's function in the temporal gauge without Coulomb interaction. Section IV C is devoted to show that the Green's function of the interacting electron system should be also finite.

A. The quantum mechanics of an electron in a homogeneous magnetic field and a background of one or two separated magnetic strings

In this section we discuss the eigenfunctions and eigenvalues of an electron in a homogenous magnetic field with a background of one or two separated strings. The eigenvalues and eigenfunctions of these systems have to be calculated to get an expression for the one-particle propagator $U_\beta(0, \vec{A}_{ss,2})$, Eq. (36), as well as for the exact density-density propagator $\mathcal{D}_{00}^{\text{ex}}$, Eq. (28).

We will solve at first the simpler problem of an electron in a homogeneous magnetic field and a background of *one* magnetic string at the origin with flux quantum ϕ . The one-particle Hamiltonian is given by

$$H_s^1(\phi) = \frac{1}{2m} [-i\vec{\nabla} + \vec{A}(\vec{r}) - \phi\vec{f}(\vec{r})]^2. \quad (40)$$

We now seek a solution of the the form $\Psi(r, \varphi) = (1/\sqrt{2\pi})f(r)e^{i p \varphi}$ (we used cylindrical polar coordinates). Then one gets for the eigenvalue equation in polar coordinates with $\vec{A} = B(y, -x)/2$

$$\frac{1}{2m} \left(f'' + \frac{1}{r} f' - \frac{(p-\phi)^2}{r^2} f \right) + \left[E - \frac{1}{8} m \omega_c^2 r^2 + \frac{1}{2} \omega_c (p-\phi) \right] f = 0. \quad (41)$$

This differential equation is similar to the differential equation of electrons in a homogeneous magnetic field B without a magnetic string which can be recovered by replacing $p - \phi \rightarrow p$. The eigenfunctions and eigenvalues of this system are well known (e.g., Ref. 21). By using an analogous method to solve the differential equation (41) we get for the regular eigenfunctions and eigenvalues, which are finite at $r=0$,

$$\Psi_{n,p}^\phi(\vec{r}) = \left[\frac{n!}{2\pi l_0^2 |p-\phi| \Gamma(n+1+|p-\phi|)} \right]^{1/2} e^{i p \varphi} \times \left(\frac{r}{l_0} \right)^{|p-\phi|} L_n^{|p-\phi|} \left(\frac{r^2}{2l_0^2} \right) e^{-r^2/4l_0^2}, \quad (42)$$

$$E_{n,p}^\phi = \omega_c \left[n + \frac{1}{2} |p-\phi| - \frac{1}{2} (p-\phi) + \frac{1}{2} \right] \quad (43)$$

with $n \in \mathbb{N}_0$ and $p \in \mathbb{Z}$. Beside these eigenfunctions we also have eigenfunctions that are not finite at $r=0$ (but nevertheless square integrable). These irregular eigenfunctions and eigenvalues are given by (for $0 \leq \phi \leq 1$) $\Psi_{n,0}^{\phi,\text{sing}} \propto (r/l_0)^{-\phi} L_n^{-\phi} (r^2/(2l_0^2)) e^{-r^2/(4l_0^2)}$ with eigenvalues $E_{n,0}^{\phi,\text{sing}} = \omega_c (n+1/2)$ and $\Psi_{n,1}^{\phi,\text{sing}} \propto e^{i\varphi} (r/l_0)^{-(1-\phi)} L_n^{-(1-\phi)} (r^2/(2l_0^2)) e^{-r^2/(4l_0^2)}$ with eigenvalues $E_{n,1}^{\phi,\text{sing}} = \omega_c (n+\phi-1/2)$. It is well known for an electron in the background of a magnetic string^{22,23} that there is no domain of the Hamiltonian H_s^1 that contains the eigenfunctions $\Psi_{n,p}^\phi(\vec{r})$ as well as the

singular eigenfunctions $\Psi_{n,0}^{\phi,\text{sing}}$ and $\Psi_{n,1}^{\phi,\text{sing}}$ in a way that the operator H_s^1 is self-adjoint. This is the reason for the non-orthogonality of the regular and the irregular eigenfunctions.²³ It is now possible to restrict the domain of the Hamiltonian to get a self-adjoint extension of H_s^1 . This restriction is not unique. The concrete extension has to be determined by physical arguments. For example, in the Aharonov-Bohm case²⁴ (i.e., electrons in the background of a magnetic string) one can show rather generally that the correct self-adjoint extension of the Hamiltonian consists of the domain of wave functions which are zero at the origin²⁵ (for $0 < \phi < 1$). This is done by a regularization of the magnetic string field at the origin (the string width R_0 being finite). After a calculation of the inner and outer solutions of this spread out string and a calculation of the matching conditions one gets for $R_0 \rightarrow 0$ only a square integrable nonzero eigenfunction in the case where the function is zero at the origin (for $0 < \phi < 1$). We have done a similar calculation for the case of an electron in the background of a finite number of homogeneously spread out magnetic strings in a homogeneous magnetic field. It is then easily seen that the asymptotics of the wave function at the border R_0 of a string does not depend on the existence of the homogeneous magnetic field and the other strings for $R_0 \rightarrow 0$ [this could be also seen from Eq. (41)]. By an examination of the matching conditions we get a square integrable nonzero eigenfunction only in the case where the function is zero at the origin of the strings. Thus, we have to use the regular eigenfunctions (42) as solutions. One can see from these eigenvalues that in contrast to the case of an electron in a string background without a homogeneous magnetic field we have an energy splitting of the Landau levels due to the string background.

Next, we will calculate the eigenfunctions and eigenvalues of an electron in a homogeneous magnetic field B in the background of *two* magnetic strings of opposite strength separated by a distance d . In contrast to the one-string system above we do not have a rotational symmetry. This makes it much more complicated to get the eigenfunctions and eigenvalues of the system. Therefore, we will restrict us in the following to the ground state. This is enough because one can calculate the Green's function for $T=0$ from the knowledge of the ground state eigenfunctions and eigenvalues due to the denominator in Eq. (39). The Hamiltonian of an electron in a homogeneous magnetic field in the background of two magnetic strings with flux quanta $-\phi$ and ϕ located at the origin and $d\vec{e}_x$ is given by

$$H_{ss}^1(\phi) = \frac{1}{2m} [-i\vec{\nabla} + \vec{A}(\vec{r}) - \phi\vec{f}(\vec{r}) + \phi\vec{f}(\vec{r}-d\vec{e}_x)]^2. \quad (44)$$

We assume that $\phi \geq 0$. We now carry out a phase transformation on the eigenfunctions Ψ of H_{ss}^1 . It is given by

$$\Psi_p(\vec{r}) = e^{-i\phi \text{arg}[r]} e^{+i\phi \text{arg}[\vec{r}-d\vec{e}_x]} \Psi(\vec{r}). \quad (45)$$

In the following we denote the one-dimensional subspace $y=0$, $0 \leq x \leq d\vec{e}_x$ of the plane by C . With the help of the transformation (45) one easily can show that Ψ_p are eigen-

functions of the Hamiltonian without the strings $H^1 = [-i\vec{\nabla} + \vec{A}(\vec{r})]^2/2m$ for $r = \mathbb{R}^2 \setminus C$ with the matching conditions

$$\begin{aligned}\Psi_p(\vec{r} + \epsilon\vec{e}_y) &= e^{i\phi 2\pi} \Psi_p(\vec{r} - \epsilon\vec{e}_y), \\ \vec{\nabla} \Psi_p(\vec{r} + \epsilon\vec{e}_y) &= e^{i\phi 2\pi} \vec{\nabla} \Psi_p(\vec{r} - \epsilon\vec{e}_y)\end{aligned}\quad (46)$$

for $\vec{r} \in C$ and $\epsilon \rightarrow 0^+$. By using the complex variables $z = x + iy$ and $\bar{z} = x - iy$ we get for the Hamiltonian H^1

$$H^1 = -\frac{2}{m} \partial_z \partial_{\bar{z}} - \frac{B}{2m} (z \partial_z - \bar{z} \partial_{\bar{z}}) + \frac{B^2}{8m} \bar{z} z. \quad (47)$$

With the help of the ansatz $\Psi_p(z, \bar{z}) = u(z, \bar{z}) e^{-|z|^2 B/4}$ we have to solve on $\mathbb{R}^2 \setminus C$ the eigenvalue equation $H_z^1 u(z, \bar{z}) = E u(z, \bar{z})$ with

$$H_z^1 = -\frac{2}{m} \partial_z \partial_{\bar{z}} + \frac{B}{m} (\bar{z} \partial_{\bar{z}}) + \frac{B}{2m} \quad (48)$$

and the scalar product

$$\langle u, v \rangle_1 = \int d^2 r e^{-|z|^2 B/2} \bar{u} v \quad (49)$$

for two wave functions u, v . The energy of a normalized wave function u on $\mathbb{R}^2 \setminus C$ is given by

$$\begin{aligned}E &= \langle u(z, \bar{z}), H_z^1(z, \bar{z}) u(z, \bar{z}) \rangle_1 \\ &= \frac{2}{m} \langle \partial_{\bar{z}} u(z, \bar{z}), \partial_z u(z, \bar{z}) \rangle_1 + \frac{B}{2m}.\end{aligned}\quad (50)$$

From this equation we find that the ground state wave functions are the normalized holomorphic wave functions on $\mathbb{C} \setminus C$ that fulfill the transformed matching conditions corresponding to Eq. (46) (here we identified \mathbb{R}^2 with the complex plane \mathbb{C}).

We now determine a linearly independent basis of the ground state wave functions. To this end we carry out the following transformations on the ground state wave functions

$$u_t(z) = z^\phi (z - z_0)^{-\phi} u(z). \quad (51)$$

Here $z_0 = d$ and $z \in \mathbb{C}$. The scalar product of the transformed wave functions is given by

$$\langle u_t, v_t \rangle_2 = \int d^2 r e^{-|z|^2 B/2} |z|^{-2\phi} |z - z_0|^{2\phi} \bar{u}_t v_t. \quad (52)$$

Using the matching conditions (46) we find that the ground state wave functions are the holomorphic functions on the whole complex plane \mathbb{C} with a finite norm corresponding to the scalar product (52). It is shown in Ref. 26 that this space as well as the space of the holomorphic functions on \mathbb{C} with a finite norm (49) known as the Segal-Bargmann space²⁶ are Hilbert spaces. We will denote the first by HL_2^2 and the Segal-Bargmann space by HL_1^2 . One can see easily that both spaces consists of the same holomorphic functions. It is well

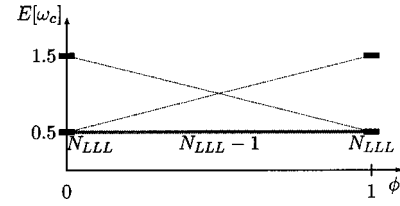


FIG. 1. Ground state energy E of an electron in a magnetic field B in the background of two magnetic strings with flux ϕ and $-\phi$. In the figure we show E as a function of the flux ϕ . N_{LLL} is the number of lowest Landau level eigenstates for $\phi = 0$. This number is proportional to the area of the system.

known that the functions $\{z^p\}$ ($p \in \mathbb{N}_0$) are a basis²⁷ of the Segal-Bargmann space HL_1^2 . We obtain from the definition of the scalar product (49) that the basis functions are orthogonal. This is no longer the case for $\{z^p\}$ ($p \in \mathbb{N}_0$) in the Hilbert space with the scalar product (52). Nevertheless, we will show in Appendix C that these functions are indeed a basis of this Hilbert space.

Then, by carrying out the retransformations (51) and (45) we get a basis for the ground state eigenfunctions of H_{ss}^1 , Eq. (44), for $0 \leq \phi \leq 1$. Due to the considerations below (43) concerning the domain of the Hamiltonian we find for this basis

$$\Psi_p^\phi = e^{ip\varphi} r^{p-\phi} (\sqrt{r^2 - 2dr\cos\varphi + d^2})^\phi e^{-r^2 B/4} \text{ for } p \geq 1, \quad (53)$$

where $0 \leq \phi \leq 1$. We find one additional basis state for the limiting cases $\phi = 0, 1$

$$\Psi_0^\phi = e^{-r^2 B/4} \text{ for } \phi = 0, 1 \quad (54)$$

We see from Eq. (50) that the energy eigenvalues E_p of the wave functions Ψ_p^ϕ (the ground state energy for string strength ϕ) are given by

$$E_p = \frac{\omega_c}{2}. \quad (55)$$

By comparing the ground state degeneracy as a function of ϕ and the energy of an electron in a homogeneous magnetic field in the background of one string [Eqs. (42) and (43)] and in the background of two magnetic strings [Eqs. (53), (54) and (55)] we get agreement of these two systems for string distance $d \rightarrow \infty$. In Fig. 1 we show the ground state energy as a function of the magnetic flux for the Hamiltonian of an electron in the background of two magnetic strings. We see from this figure that only one wave function ($p=0$) of the degenerate ground state for $\phi=0$ increases in energy. The rest of the ground states ($p \neq 0$) keep their lowest Landau level energy.

B. The calculation of the Green's function

In this section, we will calculate the Green's function $G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau')$, Eq. (38), for $\tau - \tau' > 0$ and $\beta \rightarrow \infty$. Afterwards we will generalize the results to times $\tau - \tau' < 0$. For doing this we have to calculate the thermodynamic time evo-

lution U_β of the time dependent Hamiltonian $H_{ss}(0, \vec{A}_{ss,2}^t)$, Eq. (34). We first will calculate the one-particle transition matrix of the adiabatic time-dependent Hamiltonian $H_{ss}^1((t/\beta)\vec{\phi})$ (t is the time parameter) divided by the one-particle partition function $\text{Tr}\{\exp[-\beta(H_{ss}^1(0) - \mu)]\}$ for $\beta \rightarrow \infty$. We will denote this quantity by $\text{UP}^1(\phi)$ where $\phi = (t/\beta)\vec{\phi}$. This can be calculated from the finite wave functions for $\beta \rightarrow \infty$ of the transformed Schrödinger wave equation

$$\partial_\phi \varphi(\phi) = -\frac{\beta}{\vec{\phi}} \left(H_{ss}^1(\phi) - \frac{\omega_c}{2} \right) \varphi(\phi) \quad (56)$$

for $0 \leq \phi \leq \vec{\phi}$. Then the solutions of the Schrödinger equation of motion for imaginary times of the Hamiltonian H_{ss}^1 are given by $\Psi(\phi) = \exp[-(\beta/\vec{\phi})(\omega_c/2)\phi] \varphi(\phi)$. We will solve Eq. (56) first for $0 \leq \phi \leq 1$ and $\beta \rightarrow \infty$. We now define $\varphi = \varphi_0 + \varphi_A$ where φ_0 is that part of the wave function φ which is in the eigenspace H_{ss}^1 of the lowest eigenvalue $\omega_c/2$ for $\phi \neq 0$ and $\phi \neq 1$ (i.e., $\text{span}\{\Psi_p^\phi\}$ for $p \geq 1$). For $\phi = 0$ or $\phi = 1$, respectively, we denote φ_0 by that part of the ground state wave function that is linked continuously as a function of ϕ to a ground state wave function for $0 < \phi$ or $\phi < 1$, respectively. Thus, φ_A is that part of the wave function φ that is in the subspace spanned by the higher energy eigenfunctions.

In Appendix C we show that $\langle \varphi_A | \varphi_A \rangle(\phi)$ vanishes as $O(1/\beta)$ for $0 < \phi < 1$ and as $O(1/\beta^{1/(n+1)})$ at $\phi = 1$ where n is defined by the order of the intersection of the eigenvalues at $\phi = 1$. In the case of a smooth intersection we find that $\langle \varphi_A | \varphi_A \rangle(\phi)$ vanishes as $o(1)$ at $\phi = 1$. This corresponds to the well known adiabatic theorem²⁸ in the case of the solutions of the Schrödinger equation (56) for real times. Furthermore, we obtain in Appendix C that the transition operator $\text{UP}^1(\phi)$ is nonzero only on the subspace spanned by Ψ_p^0 , Eq. (53), for $p \geq 1$. One can then get the finite part of $\text{UP}^1(\phi)$ by solving the projected Schrödinger equation

$$\langle \Psi_p^\phi | \partial_\phi \varphi_0(\phi) \rangle = 0 \quad (57)$$

for $p = 1, \dots, \infty$. The set of equations (57) shows that the time evolution of $\varphi_0(\phi)$ is given by a parallel transport in the submanifold of the ground states. This is a well-known transport in quantum mechanics that is responsible for the Berry phase²⁹ in the case of a nondegenerate ground state. Assuming that $\varphi_0(0) = \Psi_p^0$, we get $\text{UP}^1(1)\varphi_0(0) = \varphi_0(1)$ for the evolution where the background strings are switched on adiabatically up to one flux quantum. In the following, we will calculate this quantity for small string separation d . At first, we have to calculate the overlap matrix S_{nl}^d :

$$\begin{aligned} S_{nl}^d(\phi) := & \int d^2r \Psi_n^\phi(\vec{r}) \Psi_l^\phi(\vec{r}) = \frac{2\pi 2^n n!}{B^{n+1}} \left[\delta_{nl} \left(1 + \frac{\phi^2 B}{2n} d^2 \right) \right. \\ & - \delta_{n+1,l} \phi d - \delta_{n-1,l} \frac{\phi B}{2n} d + \delta_{n+2,l} O(1/n^0) O(d^2) \\ & \left. + \delta_{n-2,l} O(1/n^2) O(d^2) + O(d^3) \right]. \quad (58) \end{aligned}$$

With the help of this overlap matrix it is easy to solve the parallel transport equation (57). Defining the matrix $[\text{UP}^1](\phi)$ by $\vec{c}(\phi) = [\text{UP}^1](\phi) \vec{c}(0)$ where $\varphi_0(\phi) = \sum_p c_p(\phi) \Psi_p^\phi$ we get as a solution of Eq. (57)

$$\begin{aligned} [\text{UP}^1](\phi) = & T \exp \left[-\frac{1}{2} \int_0^\phi d\phi' [S^d(\phi')]^{-1} [\partial_{\phi'} S^d(\phi')] \right] \\ = & T \exp \left[-\frac{1}{2} \int_0^\phi d\phi' \{ 1 + [S^0(\phi')]^{-1} \Delta S^d(\phi') \}^{-1} \right. \\ & \left. \times [S^0(\phi')]^{-1} \partial_{\phi'} \Delta S^d(\phi') \right]. \quad (59) \end{aligned}$$

Here, $S = S^0 + \Delta S^d$ where S^0 is the overlap matrix S^d for $d = 0$. Now we expand the exponential function in Eq. (59) and the term $[1 + (S^0)^{-1} \Delta S^d]^{-1}$ in the exponent. Then we get an expansion of $\text{UP}^1(1)$ in d . $\text{UP}^1(\vec{\phi})$ is then given by $\text{UP}^1(\vec{\phi}) = [\text{Ph}_{-\vec{\phi}}^1(0, d\vec{e}_x)] [\text{Ph}_1^1(0, d\vec{e}_x) \text{UP}^1(1)]^{\vec{\phi}}$. Ph^1 is the phase transformation operator, Eq. (37), calculated in the one-particle sector in the first quantized language. By using $\text{UP}^1(1)\Psi_0^0 = 0$ we get

$$\begin{aligned} & [\text{Ph}_{\vec{\phi}}^1(0, d\vec{e}_x) \text{UP}^1(\vec{\phi})]_{nl} \\ = & (1 - \delta_{n0})(1 - \delta_{l0}) \left\{ \delta_{nl} \left(1 - \frac{\vec{\phi}^2 B}{16} \frac{(2n+1)}{n(n+1)} d^2 \right) \right. \\ & + \delta_{n1} \delta_{l1} \frac{\vec{\phi}(\vec{\phi}-1)B}{16n} d^2 + \left(\delta_{n-1,l} \frac{\vec{\phi}B}{4n} \sqrt{2nB} \right. \\ & \left. - \delta_{n+1,l} \frac{\vec{\phi}}{2} \frac{1}{\sqrt{2(n+1)B}} \right) d \left. - \delta_{n,0} \delta_{l,1} \frac{1}{\sqrt{2B}} d \right. \\ & + \delta_{n+2,l} O(1/n^1) O(d^2) + \delta_{n-2,l} O(1/n^1) O(d^2) \\ & \left. + O(d^3) \right\}, \quad (60) \end{aligned}$$

where $[\text{Ph}_{\vec{\phi}}^1(0, d\vec{e}_x) \text{UP}^1(\vec{\phi})]_{nl}$ is the matrix operator with respect to the orthonormal basis $\{\Psi_n^0\}$, Eq. (53), for $n = 0, \dots, \infty$. With the help of this matrix we will calculate the Green's function G^{Weyl} , (39). For doing this we first calculate the overlap function $\text{Ov}(d)$ by the use of this matrix, yielding

$$\begin{aligned} \text{Ov}(d) := & \frac{\sum_{\Psi_p^- \in \nu_{1/\vec{\phi}}, p_i \neq 0} \text{Det}[\text{Ph}_{\vec{\phi}}^1(0, d\vec{e}_x) \text{UP}^1(\vec{\phi})]_p}{\sum_{\Psi_p^- \in \nu_{1/\vec{\phi}}} 1} \\ = & \frac{\sum_{\Psi_p^- \in \nu_{1/\vec{\phi}}, p_i \neq 0} \exp(\text{Tr}\{\ln[\text{Ph}_{\vec{\phi}}^1(0, d\vec{e}_x) \text{UP}^1(\vec{\phi})]_p\})}{\sum_{\Psi_p^- \in \nu_{1/\vec{\phi}}} 1} \\ = & \left(1 - \frac{1}{\vec{\phi}} \right) \exp \left[-(\vec{\phi}-1) \frac{B}{8} \ln(A) d^2 \right] \end{aligned}$$

$$+ O(1/A^0)O(d^2) \Big]. \quad (61)$$

Here $\nu_{1/\bar{\phi}}$ are the ground states of the $\nu=1/\bar{\phi}$ system. The ground state $\Psi_{\vec{p}}^-$ is then given by the Slater determinant $\Psi_{\vec{p}}^- = S[\Psi_{0,p_1}^0, \dots, \Psi_{0,p_N}^0]$. The wave functions Ψ_{0,p_N}^0 can be seen in Eq. (42). $[\cdot]_{\vec{p}}$ is the sub matrix of the argument with line and column indices p_i . For deriving this result we used techniques of averaging over the ground states developed in Ref. 12. In Eq. (61) we mean by $O(1/A^0)O(d^2)$ that these terms are finite for $A \rightarrow \infty$ and of order $O(d^2)$. The finiteness of the exponents of order $O(d^3)$ for $A \rightarrow \infty$ is obtained by extending the analysis of the overlap matrix S_{nl}^d , Eq. (58), as a function of n, l to all orders of d . We see from Eq. (61) that the overlap function $\text{Ov}(d)$ is zero for $d > 0$ and $A \rightarrow \infty$. This is in accordance with the general inequality $-1 \leq \text{Ov}(d) \leq 1$ that can be derived from the parallel transport equation (57) by showing that the norm of the vector φ_0 is invariant under the transport.

It is now easy to calculate the Green's function $G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau')$, Eq. (39), for $\tau - \tau' > 0$. This is done by writing down the expression (39) in the one-particle basis. With the help of Eqs. (60) and (61) and by using the above-mentioned averaging techniques as well as (gauge) transformation properties of the path integral Eqs. (12) to get the Green's function also for $\vec{x} \neq 0$, we obtain

$$\begin{aligned} G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau') &= (-1)^{\bar{\phi}/2} \text{Ov}(|\vec{x} - \vec{x}'|) e^{-i\bar{\phi}\vec{f}(0) \cdot (\vec{x} - \vec{x}')} \\ &\times \left[G^0(\vec{x}, \vec{x}'; \tau, \tau') + e^{-i\vec{A}(\vec{x}) \cdot (\vec{x} - \vec{x}')} e^{-|\vec{x} - \vec{x}'|^2 B/4} \right. \\ &\times e^{-(\tau - \tau')(\omega_c/2 - \mu)} \\ &\left. \times \left(\frac{1}{\bar{\phi}} \frac{B}{4\pi} |\vec{x} - \vec{x}'|^2 + O(B^2 |\vec{x} - \vec{x}'|^4) \right) \right]. \quad (62) \end{aligned}$$

G^0 is the Green's function of the Hamiltonian $H_{ss}(0,0)$, calculated with respect to the vacuum ground state (zero particle ground state), i.e., $G^0(\vec{x}, \vec{x}'; \tau, \tau') = \Theta(\tau - \tau') \int d^2r \delta(\vec{x} - \vec{r}) \exp[-(\tau - \tau')\{-i\vec{\nabla} + \vec{A}(\vec{r})\}^2/2m - \mu] \delta(\vec{x}' - \vec{r})$. One can deduce from Eq. (30) that the expression (62) is also valid for the Green's function $G^{\text{Weyl}}(\vec{x}, \vec{x}'; \tau, \tau')$ for $\tau - \tau' < 0$ (we have $G^0 = 0$ for this time order). The first term in the brackets in Eq. (62) is a typical term for noninteracting electrons representing that term in Eq. (39) in the first quantized language where the two δ functions corresponding to the created and annihilated particle in Eq. (39) carry the same particle index. Due to the enormous effort of calculation we determined the terms where the two δ functions carry different particle indices represented by the rest of the summands in Eq. (62) only for small distances $|\vec{x}' - \vec{x}|$.

Because G^0 or \vec{A} , respectively, is not translation invariant we obtain that the Fourier transform of the Green's function G^{Weyl} does not have the form $G^{\text{Weyl}}(\vec{k}, \vec{k}'; \omega, \omega')$

$= G^{\text{Weyl}}(\vec{k}, \omega) \delta_{\vec{k}, -\vec{k}'} \delta_{\omega, -\omega'}$. During the calculation of the Green's function of the $\nu=1/\bar{\phi}$ system within CS theories one usually uses the condition

$$(2\pi\bar{\phi})\rho = \vec{\nabla} \times \langle \vec{a} \rangle = \vec{\nabla} \times \vec{A} \quad (63)$$

and the fact that \vec{A} is transversal (Coulomb gauge). Moreover, in the Coulomb gauge we have the freedom to choose the origin (x_0, y_0) of the vector potential $\vec{A} = 1/2B(y - y_0, -(x - x_0))$. We see from Eq. (62) that the explicit result of the Green's function depends on the origin of the symmetric gauge potential. By using the condition (63) during perturbational calculations we lose the dependence of the Green's function³⁰ on the external magnetic field \vec{A} and thus also on the origin (x_0, y_0) . Nevertheless, this dependence must appear in through the limit $A \rightarrow \infty$ in integrals $\int_A d^2r \dots$ when calculating a Feynman diagram. In Fourier space the dependence of the result on the concrete limit $A \rightarrow \infty$ should also be seen in the transition going from momentum sums to momentum integrals. Carrying out this transition is not straightforward for integrals of functions which fall off very slowly such as the integral of f , Eq. (2). The result depends on the way of calculating the limit $A \rightarrow \infty$. Because the integrands of perturbational calculations within RPA fall off rapidly enough the explicit dependence on (x_0, y_0) has to be chosen when going beyond RPA. We see from Eq. (62) that there are several ways to choose (x_0, y_0) or the integral limit $\int_{A \rightarrow \infty} d^2r \dots$, respectively, to get a translationally invariant Green's function [e.g., $x_0 = (x_1 + x'_1)/2, y_0 = (x_2 + x'_2)/2$, or $x_0 = x'_1, y_0 = x'_2$, or . . .], which is necessary when defining the effective mass of quasi-particles. One can see from Eq. (62) that all these choices differ in their Green's function by a pure phase factor $\exp[i\vec{A}(\vec{r}_{0,2}) \cdot (\vec{x} - \vec{x}')] (r_{0,2}$ is a vector not depending on \vec{x}, \vec{x}'), which in momentum space results in different translations of the momentum variable of the Green's function. Thus, we see that all these choices result in the same effective mass. It is then clear that by an additional shift of the origin (x_0, y_0) of value $-\vec{f}(0)$ we can make the gauge factor $\exp[-i\bar{\phi}\vec{f}(0) \cdot (\vec{x} - \vec{x}')] in Eq. (62) vanish. This gauge transformation does not destroy the translational invariance of the Green's function (62) when choosing either of the origins (x_0, y_0) discussed above.$

Summarizing, in contrast to the RPA result of the Green's function in the temporal gauge calculated in Sec. III the exact Green's function in the temporal gauge is finite. Nevertheless, we see from Eqs. (61) and (62) that the Green's function has its support on the one-dimensional subspace $\vec{x} = \vec{x}'$ for system area $A \rightarrow \infty$. This is due to the infinite degeneracy of the ground state of noninteracting electrons in a homogeneous magnetic field. More precisely, at the end of the parallel transport calculated in Eq. (57) the ground state is orthogonal to its phase transformed starting value resulting in the vanishing of $\text{Ov}(|\vec{x} - \vec{x}'|)$ at values $|\vec{x} - \vec{x}'| \neq 0$. This degeneracy of the ground state usually is not present in the case of a Coulomb-interacting system. Now assume that the nondegeneracy of the ground state is present for all string values and that the ground state energy correction due to the

additional strings is zero. Then we can deduce that the Green's function in the temporal gauge has no degeneracy having the support in a one dimensional subspace of the \vec{x} , \vec{x}' plane. This is immediately clear because in this case the overlap function $\text{Ov}(|\vec{x}-\vec{x}'|)$ consists of a phase factor for all $|\vec{x}-\vec{x}'|$ corresponding to a Berry phase.²⁹ This will be shown in the following section.

C. The exact Green's function with Coulomb interaction

It is clear that in contrast to the noninteracting case above one cannot solve exactly the (ground state) eigenvalue problem of N electrons in a homogeneous magnetic field in the background of two separated magnetic strings in the presence of Coulomb interaction. Nevertheless, we will show that we get no energy correction due to the two additive opposite magnetic strings and that when having a nondegenerate ground state for $\phi=0$ the system has a nondegenerate ground state for all ϕ . In this subsection, we assume the commonly believed nondegeneracy of the ground state of the interacting half-filled Landau system. We denote by $\Psi_{\phi,d}^n$ a normalized eigenstate with energy $E^n(\phi,d)$, where n labels the states. Furthermore, we denote the Hamiltonian of the system by $H_{ss}^{e,N}$, i.e.,

$$H_{ss}^{e,N}(\phi,d) = \sum_i^N \frac{1}{2m} [-i\vec{\nabla}_i + \vec{A}(\vec{r}_i) + \phi\vec{f}(\vec{r}_i + d\vec{e}_x/2) - \phi\vec{f}(\vec{r}_i - d\vec{e}_x/2)]^2 + \frac{1}{2} \sum_{i \neq j} V_{r_i, r_j}^{ee}. \quad (64)$$

Here N is the number of electrons in the $\nu=1/\bar{\phi}$ system. For simplicity we used a version of the many particle Hamiltonian in Eq. (64) where the magnetic field is symmetric around the origin and the strings are positioned at $d\vec{e}_x/2$ and $-d\vec{e}_x/2$. With the help of the current operator $\hat{J}_{\phi,d}(\vec{r}) = (1/m) \sum_i \delta(\vec{r}-\vec{r}_i) [-i\vec{\nabla}_i + \vec{A}(\vec{r}_i) + \phi\vec{f}(\vec{r}_i + d\vec{e}_x/2) - \phi\vec{f}(\vec{r}_i - d\vec{e}_x/2)]$ (the physical current being the real part of the expectation value of this current operator) we get

$$\partial_{\xi} E^n = \int d^2r [\partial_{\xi} \vec{F}(\vec{r})] \cdot \langle \hat{J}(\vec{r}) \rangle_{\phi,d}^n \quad (65)$$

with

$$\vec{F}_{\phi,d}(\vec{r}) = \frac{\phi}{m} [\vec{f}(\vec{r} + d\vec{e}_x/2) - \vec{f}(\vec{r} - d\vec{e}_x/2)], \quad (66)$$

where ξ equals d or ϕ , respectively. Here we denoted $\langle \Psi_{\phi,d}^n | \hat{J}_{\phi,d}(\vec{r}) | \Psi_{\phi,d}^n \rangle$ by $\langle \hat{J}(\vec{r}) \rangle_{\phi,d}^n$. One can interpret Eq. (65) as the energy variation that is due to the induced electrical field resulting from a variation in the string magnetic field. We have the following relation for the eigenfunctions:

$$\begin{aligned} \Psi_{1-\phi,d}^n(\vec{r}_1, \dots, \vec{r}_N) \\ = \Psi_{\phi,d}^n(-\vec{r}_1, \dots, -\vec{r}_N) \exp\left(-i \sum_i \arg[\vec{r}_i + d\vec{e}_x/2]\right) \\ \times \exp\left(+i \sum_i \arg[\vec{r}_i - d\vec{e}_x/2]\right) \end{aligned} \quad (67)$$

with $E^n(\phi,d) = E^n(1-\phi,d)$ (we denote these related states by the same label). Thus, we obtain

$$\langle \hat{J}(\vec{r}) \rangle_{1-\phi,d}^n = -\langle \hat{J}(-\vec{r}) \rangle_{\phi,d}^n. \quad (68)$$

Summarizing, due to the different signs of the strengths of the two strings we obtain an inversion symmetry of the energy spectrum with respect to the strength $\phi=0.5$. This inversion symmetry does not depend on the string distance d . In the following, we calculate the derivative of the energy $E_{\phi,d}^n$ with respect to d . By using Eqs. (65), (66), and (68) we get

$$\begin{aligned} \partial_d E^n(\phi,d) &= \int d^2r [\partial_d \vec{F}_{\phi,d}(\vec{r})] \cdot \langle \hat{J}(\vec{r}) \rangle_{\phi,d}^n \\ &= -\frac{\phi}{1-\phi} \int d^2r [\partial_d \vec{F}_{1-\phi,d}(\vec{r})] \cdot \langle \hat{J}(\vec{r}) \rangle_{1-\phi,d}^n \\ &= -\frac{\phi}{1-\phi} \partial_d E^n(1-\phi,d). \end{aligned} \quad (69)$$

Here we used $\vec{F}_{\phi,d}(\vec{r}) = \vec{F}_{\phi,d}(-\vec{r})$. Since $E^n(\phi,d) = E^n(1-\phi,d)$ we get

$$\partial_d E^n(\phi,d) = 0 \quad (70)$$

for $\phi \neq 0,1$. From Eq. (70) we observe that one should not have any energy correction due to the two separated magnetic strings. This is in contrast to the exact derivation of the ground state energy in Sec. IV A where we showed that one of the ground state energy levels rises in its energy when switching on the two strings (see Fig. 1). To find the reason for this discrepancy we have to examine the above derivation a little more carefully. First, as mentioned in Sec. IV A [below Eq. (43)] the domain of the Hamiltonian in Eq. (64) is different for different d s. Thus, we have to spread out the magnetic strings (at string width R_0) before carrying out the analysis above. It is then clear that one can deduce Eq. (70) only when the energy levels and wave functions are smooth for string width $R_0 \rightarrow 0$. This should be the case for $d > 0$. It is then clear from Sec. IV A that some eigenvalues are discontinuous at $d=0$ (for $R_0 \rightarrow 0$), i.e., $\partial_d E^n(\phi,0)$ is infinite. This follows from the fact that $\partial_d \vec{F}_{\phi,d}(\vec{r})|_{d=0} = (\phi/m) \times [\vec{e}_y/(x^2+y^2) - \vec{f}(\vec{r})2x/(x^2+y^2)]$ scales like $O(1/r^2)$. Thus, we obtain from Eq. (65) that $\partial_d E^n(\phi,0)$ can be infinite for states having a nonzero current expectation value. Since we assumed that the ground state of the Coulomb interacting system without strings is nondegenerate and thus $\langle \hat{J}(\vec{r}) \rangle_{\phi,0} = 0$, we have one energy level that stays invariant when the magnetic strings are switched on. This level has the same energy as the ground state of the system without the strings.

Nevertheless, a level crossing with this state could occur for $\phi \neq 0, 1$. This should not happen because we see from Eq. (65) that if we have an energy discontinuity for some ϕ then we have a discontinuity for all $\phi > 0$ and the energy discontinuity is an increasing function of ϕ (see also Fig. 1). Then we obtain from the considerations above and the knowledge that the energy spectrum for $\phi = 0$ is in accordance with $\phi = 1$ that we have a nondegenerate ground state for all $0 \leq \phi \leq 1$ and $d \geq 0$. This ground state should have the same energy as the ground state of the system without strings.

To get a better physical insight into the fact that the energy of the ground state remains invariant for $\phi \neq 0$ in the following we give a second derivation. In this derivation we use the Wigner–von Neumann theorem.³¹ This theorem states that for a Hamiltonian that depends on one parameter one has a level crossing only in the case where the two states that take part in this level crossing belong to different representations of the symmetry group of the Hamiltonian. Because of the nonzero overlap of the ground state for $\phi = 0$ and $\phi = 1$ for all $d \geq 0$ and large system area (by using that the density of the ground state wave function is homogenous for $\phi = 0$) we find that these two states belong to the same representation of the symmetry group of the Hamiltonian. Thus, we see that these two states are connected when switching on the flux ϕ [furthermore, during the parallel transport (57) the system remains in this state]. Now we calculate the second derivative of the energy of this state with respect to ϕ for some $d > 0$. This term is proportional to the magnetic field correction at the positions of the two strings that is created by the electrons from the first-order current correction which is itself caused by an infinitesimal variation $\delta\phi$ of ϕ [we have a term similar to expression (27)]. This current correction is due to the acceleration of the electrons by the induced electric field resulting from the variation $\delta\phi$. Because we have no spin degree of freedom we know from Lenz's rule that the magnetic field correction tends to reduce the flux variation. This results in a nonnegative second-order derivative of the energy with respect to ϕ for all ϕ and d . Because we have $\partial_\phi E(0, d) = \partial_\phi E(1, d) = 0$ for the nondegenerate ground state, we find that the energy of the state connecting the ground states for $\phi = 0$ and $\phi = 1$ remains invariant when switching on the two magnetic strings.

It is clear that the two derivations above that make predictions on the spectrum of the interacting system are not exact proofs in the sense that they use only the Hamiltonian (64) as an input. The exact proof is an outstanding problem. Nevertheless, both derivations presented above lead to according predictions based on meaningful physical input.

Now we can use the results at the beginning of Secs. IV and Sec. IV B to obtain the Green's function for the Coulomb interacting system. We obtain for the Green's function of the CS system in the temporal gauge

$$G^{\text{Weyl}, e}(\vec{x}, \vec{x}'; \tau, \tau') = (-1)^{\bar{\phi}/2} \text{Ov}^e(|\vec{x} - \vec{x}'|) G^e(\vec{x}, \vec{x}'; \tau, \tau'). \quad (71)$$

Here we neglected a factor $e^{-i\bar{\phi}f(0) \cdot (\vec{x} - \vec{x}')}$ [see the discussion below Eq. (62)]. G^e is the many-particle Green's func-

tion for Coulomb interacting electrons in a Coulomb gauged vector potential A [the many-particle Green's function of the Hamiltonian $H_{ss}(0, 0)$]. We should mention that the expression (71) is valid for all τ, τ' . $\text{Ov}^e(|\vec{x} - \vec{x}'|)$ is the overlap function corresponding to Ov , Eq. (61), in the noninteracting case. As mentioned at the end of Sec. IV C, Ov^e is a phase factor which is calculated by the parallel transport equation (57) (Ψ_p^ϕ in this equation has to be substituted by the nondegenerate ground state of string strength ϕ). Thus, we see from this equation that the Green's function does not have the same degeneracy as in the case of the noninteracting system.

V. SUMMARY AND OUTLOOK

The CS theory of the $\nu = 1/2$ system in the Coulomb gauge established by HLR does not allow the formulation of a quasiparticle picture of the CS fermions without a physical motivated cancellation of diverging terms in Feynman diagrams. This is due to a vanishing Green's function for infinite area. Motivated by this, we consider in this paper the CS theory in the temporal gauge. Our intention is to formulate a CS theory of the half-filled Landau level which has meaningful quasiparticles. At first, we derive the CS path integral in the temporal gauge by a gauge transformation from the correct normal ordered CS path integral in the Coulomb gauge. We show that one has to be careful with the time slices of the path integrals to get the correct result. From this we calculate the self-energy in RPA for both gauges. For the self-energy in the temporal gauge, we obtain a scaling with $1/T$. With the help of a CS retransformation of the path integral representing the Green's function we calculate the Green's function nonperturbatively for the noninteracting electron system. We get a finite result for this Green's function. The reason for this shortcoming in RPA is a wrong result when calculating the ground state energy up to second-order perturbation theory in the string strength for a system of electrons in a homogeneous magnetic field and two separated magnetic strings of opposite strength. We obtain exactly a zero energy correction of the ground state. Furthermore, we calculate explicitly the ground state wave functions of this system. By considering methods also used in deriving the Berry phase we obtain the exact Green's function in the temporal gauge. Apart from this we get the reason for the missing of the $\ln(A)$ divergence in the self-energy in the temporal gauge in RPA. This is due to a dynamical creation of phase factors linking the created and annihilated electrons in the Green's function to all other electrons when switching on adiabatically the two magnetic strings. These phase factors do not exist in the Coulomb gauge, thereby leading to the $\ln(A)$ behavior of the exact Green's function. As a generalization of these results, we find that the ground state energy of an interacting electron system (in the homogeneous magnetic background field) should be the same with and without the two strings. From this we deduce that also the Green's function of the interacting electron system is finite.

Summarizing, we showed in this paper that the CS theory in the temporal gauge contains a meaningful Green's function apart from the physical meaningful dipole feature of the

quasiparticles. To our understanding, this is the premise to examine the widely discussed effective mass of the CS fermions. The next stage would be the formulation of a perturbation theory of the temporal gauge which shows the finiteness of the Green's function. This should be a conserved approximation.³² The knowledge of the exact Green's function of the non-interacting system derived in Sec. IV should be useful in finding such a formulation. Such a procedure should yield meaningful results not only for the one particle sector given by the Green's function but also new results for the higher particle sectors representing most of the physics of the composite fermions. This is work in progress.

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APPENDIX A: THE CALCULATION OF THE EXACT (a^0, a^0) VERTEX

In this section we calculate $\mathcal{D}_{00}^{\text{ex},2}(\vec{r}, \omega=0)$ [Eq. (27)] for a noninteracting electron system without any approximation. With the help of the eigenfunctions $\Psi_{n,p}^0$ [Eq. (42)] and $E_{n,p}^0$ [Eq. (43)] we get for the second summand in Eq. (27) by the insertion of a complete set of eigenfunctions

$$\begin{aligned} \mathcal{D}_{00}^{\text{ex},2}(\vec{r}, \omega=0) &= \tilde{\phi}^2 \frac{1}{\sum_{\Psi_{p'} \in \nu_{1/\tilde{\phi}}} 1} \sum_{\Psi_{p'} \in \nu_{1/\tilde{\phi}}} \sum_{i=1}^N \sum_{n=1}^{\infty} \frac{1}{n\omega_c m^2} \\ &\times 2 \operatorname{Re} \left[\int d^2 r'' \Psi_{0,p_i}^{0*}(\vec{r}'') \left(\frac{\vec{\nabla}''}{i} + \vec{A}(\vec{r}'') \right) \right. \\ &\times \Psi_{n,p_i}^0(\vec{r}'') \vec{f}(\vec{r} - \vec{r}'') \int d^2 r' \Psi_{n,p_i}^{0*}(\vec{r}') \\ &\left. \times \left(\frac{\vec{\nabla}'}{i} + \vec{A}(\vec{r}') \right) \Psi_{0,p_i}^0(\vec{r}') \vec{f}(\vec{r}') \right]. \end{aligned} \quad (\text{A1})$$

Here N is the number of electrons in the $\nu=1/\tilde{\phi}$ system. Re is the real part of its argument. The subexpression $-\sum_{n=1}^{\infty} [1/(n\omega_c m)] \Psi_{n,p_i}^0(\vec{r}'') \int d^2 r' \Psi_{n,p_i}^0(\vec{r}') [\vec{\nabla}'/i + \vec{A}(\vec{r}')] \Psi_{0,p_i}^0(\vec{r}') \vec{f}(\vec{r}')$ in Eq. (A1) is the first-order correction to the ground state wave function Ψ_{0,p_i}^0 and is due to a magnetic string at the origin. Thus, we see that $\mathcal{D}_{00}^{\text{ex},2}(\vec{r}, \omega=0)$ corresponds to the first-order correction (in $\tilde{\phi}'$) of the operator $\tilde{\phi} \int d^2 r' \vec{f}(\vec{r} - \vec{r}') \cdot \vec{J}(\vec{r}', t)$ to the perturbation $\tilde{\phi}' \int d^2 r' \vec{J}(\vec{r}', 0) \cdot \vec{f}(\vec{r}')$ (setting at last $\tilde{\phi}' = \tilde{\phi}$). From Eq. (42) we obtain that the eigenfunctions $\Psi_{n,p}^{\phi}$ of the one-string system are analytic in ϕ at $\phi \approx 0$ for $p \neq 0$. Therefore, we can split the n sum in $\mathcal{D}_{00}^{\text{ex},2}(\vec{r}, \omega=0)$ [Eq. (A1)] in terms

with $p_i=0$ and terms with $p_i \neq 0$. The term in the square brackets in Eq. (A1) for $p_i=0$ can be calculated easily giving zero. Thus, we get for $\mathcal{D}_{00}^{\text{ex},2}(\vec{r}, \omega=0)$

$$\begin{aligned} \mathcal{D}_{00}^{\text{ex},2}(\vec{r}, \omega=0) &= -\frac{\tilde{\phi}^2}{m} \frac{1}{\sum_{\Psi_{0,p'} \in \nu_{1/\tilde{\phi}}} 1} \sum_{\Psi_{0,p'} \in \nu_{1/\tilde{\phi}}} \sum_{i=1}^N (1 - \delta_{p_i,0}) \\ &\times \partial_{\tilde{\phi}'} \left[\int d^2 r'' \Psi_{0,p_i}^{\tilde{\phi}'*}(\vec{r}'') \left(\frac{\vec{\nabla}''}{i} + \vec{A}(\vec{r}'') \right) \right. \\ &\left. \times \Psi_{0,p_i}^{\tilde{\phi}'}(\vec{r}'') \vec{f}(\vec{r} - \vec{r}'') \right]_{\tilde{\phi}'=0}. \end{aligned} \quad (\text{A2})$$

This expression as well as the first term in Eq. (27) can be calculated analytically. After some straight forward calculations we get Eq. (28).

APPENDIX B: A MONOMIAL BASIS FOR HL_2^2

In this appendix we prove that $\{z^p\}$ ($p \in \mathbb{N}_0$) is a basis of HL_2^2 . For simplicity we use $B=1$ in the following considerations. Then we have

$$HL_i^2(\mathbb{C}, \alpha_i) := \left\{ F \in H(\mathbb{C}) \left| \int dz |F(z)|^2 \alpha_i(z) < \infty \right. \right\}, \quad (\text{B1})$$

where $H(\mathbb{C})$ are the holomorphic functions of the complex plane \mathbb{C} and

$$\alpha_1(z) = e^{-|z|^{2/2}}, \quad (\text{B2})$$

$$\alpha_2(z) = e^{-|z|^{2/2} r^{-2} \phi (r^2 - 2rd \cos \varphi + d^2)^\phi}. \quad (\text{B3})$$

It is shown in Ref. 28 that HL_1^2 , HL_2^2 are Hilbert spaces [with the scalar products (49) and (52)] and that the functions $\{z^p\}$ ($p \in \mathbb{N}_0$) form an orthogonal basis of HL_1^2 . We will show in the following that $\{z^p\}$ is also basis of HL_2^2 .

To this end we define

$$s(R) := \sup_{r \geq R} \{ |r^{-2} \phi (r^2 - 2rd \cos \varphi + d^2)^\phi| \},$$

$$\|F\|_{R,i} := \int_{|z| \geq R} dz |F(z)|^2 \alpha_i(z). \quad (\text{B4})$$

Here sup is the supremum of the argument. It holds that $\|g\|_{R,2} \leq s(R) \|g\|_{R,1}$ for $R \geq 0$. Due to the holomorphy of the functions in HL_1^2 , HL_2^2 we know that for $u(z) \in HL_1^2$ there exist a series $\sum_{n \geq 0} a_n z^n = u(z)$ of uniform convergence for all finite R with $|z| \leq R$ and point wise convergence in the whole complex plane. Thus, we have $\lim_{N \rightarrow \infty} (\|\sum_{n \geq 0}^N a_n z^n - g\|_2 - \|\sum_{n \geq 0}^N a_n z^n - g\|_{R,2}) = 0$. Now we have to show that $\lim_{N \rightarrow \infty} \|\sum_{n \geq 0}^N a_n z^n - g\|_{R,2} = 0$. Then we have the following inequality

$$\left\| \sum_{n \geq 0}^N a_n z^n - g \right\|_{R,2} \leq s(R) \left\| \sum_{n \geq 0}^N a_n z^n - g \right\|_{R,1} \leq s(R) \left\| \sum_{n \geq 0}^N a_n z^n - g \right\|_1. \quad (\text{B5})$$

We see from Eqs. (B1), (B2), and (B3) that HL_1^2 and HL_2^2 contain the same holomorphic functions. Because $\{z^p\}$ is a basis of the Segal-Bargmann space HL_1^2 we have from relation (B5) that $\lim_{N \rightarrow \infty} \left\| \sum_{n \geq 0}^N a_n z^n - g \right\|_{R,2} = 0$ and thus the proof that $\{z^p\}$ is a basis of HL_2^2 .

APPENDIX C: THE ASYMPTOTIC SOLUTIONS OF THE TIME DEPENDENT SCHRÖDINGER EQUATION (56)

In this appendix we discuss the solutions of the time-dependent Schrödinger equation (56) for $\beta \rightarrow \infty$. We will show that the solutions of this equation have a vanishing part in the eigensubspace corresponding to higher eigenvalues, i.e., $\varphi_A(\phi)$ is zero for $0 < \phi \leq 1$. First, we get from Eq. (56)

$$\partial_\phi \langle \varphi_A | \varphi_A \rangle = -\partial_\phi \langle \varphi_0 | \varphi_0 \rangle - 2 \frac{\beta}{\phi} \langle \varphi_A | \left(H_{ss}^1(\phi) - \frac{\omega_c}{2} \right) | \varphi_A \rangle. \quad (\text{C1})$$

Denoting by $\Delta E(\phi)$ the difference between the energy of the first excited state and the ground state which is zero for $\phi = 0, 1$ we define $\langle \varphi'_A | \varphi'_A \rangle$ by the following differential equation

$$\partial_\phi \langle \varphi'_A | \varphi'_A \rangle = -\partial_\phi \langle \varphi_0 | \varphi_0 \rangle - 2 \frac{\beta}{\phi} \Delta E(\phi) \langle \varphi'_A | \varphi'_A \rangle. \quad (\text{C2})$$

By comparing Eqs. (C1) and (C2) we find that $\langle \varphi'_A | \varphi'_A \rangle \geq \langle \varphi_A | \varphi_A \rangle$ for $0 \leq \phi \leq 1$. By a straight forward calculation

we obtain the following solution of the differential equation (C2):

$$\begin{aligned} \langle \varphi'_A | \varphi'_A \rangle(\phi) &= \langle \varphi'_A | \varphi'_A \rangle(0) \exp \left[-2 \frac{\beta}{\phi} \int_0^\phi d\phi' \Delta E(\phi') \right] \\ &\quad - \int_0^\phi d\phi' (\partial_{\phi'} \langle \varphi_0 | \varphi_0 \rangle(\phi')) \\ &\quad \times \exp \left[-2 \frac{\beta}{\phi} \int_{\phi'}^\phi d\phi'' \Delta E(\phi'') \right]. \quad (\text{C3}) \end{aligned}$$

We see from Eq. (C3) that $\partial_\phi \langle \varphi_0 | \varphi_0 \rangle(\phi)$ has to be negative and infinite for some value $\phi > 0$ in order to get a finite value $\langle \varphi'_A | \varphi'_A \rangle(\phi) > 0$ for $\beta \rightarrow \infty$ and $\phi > 0$. In the following we will show that this can not be the case. From Eq. (56) we get

$$\begin{aligned} \partial_\phi \langle \varphi_0 | \varphi_0 \rangle(\phi) &= 2 \operatorname{Re}[\langle \partial_\phi \varphi_0(\phi) | \varphi_A(\phi) \rangle] = 2 \operatorname{Re}[\langle \varphi_0(\phi) | \\ &\quad \ln(\sqrt{r^2 - 2dr \cos \varphi + d^2}/r) | \varphi_A(\phi) \rangle]. \quad (\text{C4}) \end{aligned}$$

Here we used the basis functions (53). We see from Eq. (C4) that $\partial_\phi \langle \varphi_0 | \varphi_0 \rangle(\phi)$ is finite for $0 \leq \phi \leq 1$. From this, (C3) and the spectrum in Fig. 1 we get that $\langle \varphi_A | \varphi_A \rangle(\phi)$ vanishes as $O(1/\beta)$ for $0 < \phi < 1$ and as $O(1/\beta^{1/(n+1)})$ at $\phi = 1$ where n is defined by the order of the intersection of the eigenvalues at $\phi = 1$, i.e., $\Delta E(1 - \phi) = O((1 - \phi)^n)$. In the case that the intersection is smooth we get that $\langle \varphi_A | \varphi_A \rangle(\phi)$ vanishes as $o(1)$ at $\phi = 1$. Furthermore, we see from Eqs. (C3) and (C4) that the solutions of the Schrödinger equation (56) are only nonzero for starting values $\varphi_0(0) \neq 0$ in the case $\beta \rightarrow \infty$.

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