

Fine structure of chiral symmetry breaking in the QED_3 theory of underdoped high- T_c superconductors

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We study the effects of the leading irrelevant perturbations on chiral symmetry breaking in the effective QED_3 theory of d -wave superconductor-insulator transition in underdoped cuprates. For weak symmetry breaking, the effect of a perturbation on energies of various insulating states can be classified according to its engineering dimension in the maximally symmetric theory. Considering the velocity anisotropy, repulsive interactions, and higher-order derivatives we show that the insulating state with the lowest energy is the spin density wave.

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I. INTRODUCTION

The d -wave superconducting state, besides the familiar rotational invariance, at low energies possesses an additional “chiral” symmetry for its quasiparticle excitations.¹ It was recently shown¹ that the quantum phase transition from the antiferromagnetic insulator to the d -wave superconductor may be understood in terms of this hidden symmetry: while present in the superconducting state, chiral symmetry is manifestly broken when the spin-density wave (SDW) order develops. The dynamical agent that brings about this change of symmetry was argued to be provided by the vortex fluctuations of the condensate, that can be represented by an effective gauge field minimally coupled to gapless quasiparticles of the d -wave state.^{2,1} That way the theory of the phase fluctuating d -wave superconductor can be mapped onto the three-dimensional quantum electrodynamics (QED_3), which was proposed as an effective low-energy description of the underdoped high-temperature superconductors.¹

Chiral symmetry, however, is not an exact symmetry of the d -wave superconducting state, but arises only in the low-energy limit of the standard quasiparticle action. At low energies one is allowed to linearize the spectrum near the nodes of the d -wave order parameter and neglect the higher-order derivatives and the short-range interactions between quasiparticles, which are both linearly irrelevant by power counting. In this approximation the action becomes symmetric under a global $U(2) \times U(2)$ transformation, where each $U(2)$ factor acts onto a four-component Dirac field that describes the gapless excitations near the pair of the diagonally opposed nodes¹ (see Fig. 1). If one would also neglect the intrinsic velocity anisotropy of the d -wave superconductor $v_F \gg v_\Delta$, with v_F being the Fermi velocity and v_Δ the velocity related to the amplitude of the superconducting order parameter, the chiral symmetry group would be enlarged into the 16 dimensional $U(4)$. The SDW insulator corresponds to breaking of this symmetry along one particular “direction,” while the broken generators of $U(4)$ rotate it towards one of the other possible broken symmetry states. Among these, one can discern three additional types of insulators, related to the SDW (and to each other) by chiral symmetry: the “ $d+ip$ ” and “ $d+is$ ” insulating states, and the stripelike one-

dimensional charge-density waves (CDW’s), in the classification of Refs. 1 and 3.

In the case of the maximal $U(4)$ chiral symmetry, all the above insulating states, and their various combinations,¹ are equally likely outcomes of the spontaneous symmetry breaking. This degeneracy is in reality removed by the symmetry-breaking perturbations to the QED_3 , the most prominent of which have already been mentioned above. For example, it was shown in Ref. 1 that the short-range repulsion between quasiparticles favors the SDW over the $d+ip$ insulator, and, moreover, enhances the SDW order deeper in the insulating state. In this paper we study the effects of weak velocity anisotropy, higher-order derivatives, and short-range repulsive interaction on the pattern of chiral symmetry breaking in the QED_3 theory of the phase fluctuating d -wave superconductors. All of these perturbations are irrelevant at low energies, and we show that their effects on the energies of various states with weakly broken chiral symmetry can be ordered according to their engineering dimensionality: velocity anisotropy, being only marginally irrelevant,^{4,5} provides then the dominant perturbation. Weak repulsion and the second-order derivatives are both equally (linearly) irrelevant, but to the first order it is only the repulsion that affects the energies of the insulating states. Fine structure of the chiral manifold of insulators is schematically depicted in Fig. 2.

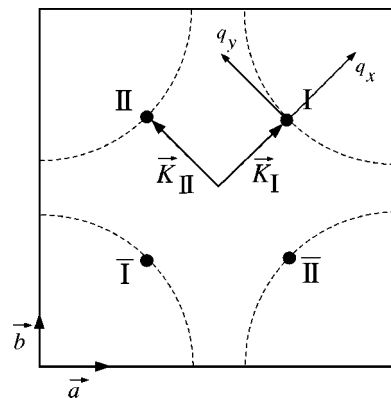


FIG. 1. Four nodes of the d -wave order parameter. Dashed line is the putative Fermi surface.

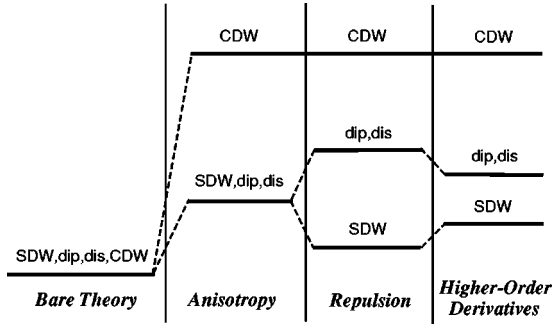


FIG. 2. Energies of the four types of insulators in the unperturbed (bare) action, with the velocity anisotropy, repulsive interaction, and the nonlinear (higher-order derivative) terms.

Our main conclusion is that, to the leading order, it is the SDW state that is selected by the weak perturbations to the QED₃. Although in real systems none of the above perturbations is truly weak, we believe our result provides a useful qualitative guide. In particular, it agrees with the standard picture of underdoped cuprates, upon identification of the SDW insulator as being continuously connected with the Mott insulating antiferromagnet near half filling.

In what follows we first briefly review the salient points of the QED₃ theory of phase fluctuating d -wave superconductor, and set up the convenient notation in terms of the eight component Dirac fields. We then proceed to calculate the lowest-order splittings of the energies due to velocity anisotropy, short-range repulsion, and higher-order derivative terms. We end with the summary and some concluding remarks on our results.

II. QED₃

We start with the finite temperature ($T \neq 0$) quantum mechanical action for the interacting d -wave quasiparticles,

$$S = S_{\text{BCS}} + S_U, \quad (1)$$

where

$$S_{\text{BCS}} = T \sum_{\omega_n} \int \frac{d^2 \vec{k}}{(2\pi)^2} \left(\sum_{\sigma} [i\omega_n + \xi(\vec{k})] c_{\sigma}^{\dagger}(\vec{k}, \omega_n) c_{\sigma}(\vec{k}, \omega_n) + \Delta(\vec{k}) c_{\uparrow}^{\dagger}(\vec{k}, \omega_n) c_{\downarrow}^{\dagger}(-\vec{k}, -\omega_n) + \text{H.c.} \right), \quad (2)$$

with ω_n 's denoting fermionic Matsubara frequencies, and with $\Delta(\vec{k})$ as the standard d -wave gap. S_U represents the short-range repulsion between electrons,

$$S_U = U \int_0^{\beta} d\tau \int d^2 \vec{r} \left(\sum_{\sigma} c_{\sigma}^{\dagger}(\vec{r}, \tau) c_{\sigma}(\vec{r}, \tau) \right)^2, \quad (3)$$

with $U > 0$. Shifting the momenta as $\vec{k} = \vec{K}_i + \vec{q}$, $i = I, II$, and rotating the coordinate frame as in Fig. 1, we define the eight-component Dirac field $\Psi^{\dagger} = (\Psi_I^{\dagger}, \Psi_{II}^{\dagger})$ where for each node $i = I, II$,

$$\Psi_i^{\dagger}(\vec{q}, \omega_n) \equiv [c_{\uparrow}^{\dagger}(\vec{K}_i + \vec{q}, \omega_n), c_{\downarrow}^{\dagger}(-\vec{K}_i - \vec{q}, -\omega_n), c_{\uparrow}^{\dagger}(-\vec{K}_i + \vec{q}, \omega_n), c_{\downarrow}^{\dagger}(\vec{K}_i - \vec{q}, -\omega_n)]. \quad (4)$$

At $T = 0$, with $q_0 \equiv \omega$, the action may be then written as

$$S = \int \frac{d^3 \vec{q}}{(2\pi)^3} \bar{\Psi}(q) \Gamma_0 \{i q_0 + i M(\vec{q})\} \Psi(q) + U \int d\tau d^2 \vec{r} [\bar{\Psi}(\vec{r}, \tau) B_U \Psi(\vec{r}, \tau)]^2, \quad (5)$$

with $M(\vec{q}) = \text{diag}[M_I(q_x, q_y), M_{II}(q_y, q_x)]$, where $iM_i = \text{diag}(\mathcal{H}_i, \mathcal{H}_i)$ is a 4×4 matrix defined as

$$\mathcal{H}_i = \begin{pmatrix} \xi(\vec{K}_i + \vec{q}) & \Delta(\vec{K}_i + \vec{q}) \\ \Delta^*(\vec{K}_i + \vec{q}) & -\xi(-\vec{K}_i - \vec{q}) \end{pmatrix}. \quad (6)$$

Here $\bar{\Psi} = \Psi^{\dagger} \Gamma_0$, and $\Gamma_0 = \text{diag}(\gamma_0, \gamma_0)$, with $\gamma_0 = \sigma_1 \otimes \mathbf{1}$, $\gamma_1 = -\sigma_2 \otimes \sigma_3$, $\gamma_2 = -\sigma_2 \otimes \sigma_1$, $\gamma_3 = -\sigma_2 \otimes \sigma_2$, satisfying the Clifford algebra $\{\gamma_{\mu}, \gamma_{\nu}\} = 2\delta_{\mu\nu}$. Finally, $B_U = \Gamma_0 A_U$, with $A_U = \text{diag}(\sigma_3, \sigma_3, \sigma_3, \sigma_3)$.

One may next expand $iM(\vec{q})$ around the nodes of the superconducting order parameter in powers of \vec{q} . It is convenient to define the following matrices: $M_1 = -i\sigma_3 \otimes \sigma_3$, $M_2 = -i\sigma_3 \otimes \sigma_1$. Using the symmetry property of the d -wave gap and the quasiparticle dispersion $\Delta(-\vec{k}) = \Delta(\vec{k})$ and $\xi(-\vec{k}) = \xi(\vec{k})$, one finds

$$S_{\text{BCS}} = S_0 + S_A + S_{NL}, \quad (7)$$

$$S_0 = \int d\tau d^2 \vec{r} \bar{\Psi}(\vec{r}, \tau) \{ \Gamma_0 \partial_{\tau} + \Gamma_1 \partial_x + \Gamma_2 \partial_y \} \Psi(\vec{r}, \tau), \quad (8)$$

$$S_A = \int d\tau d^2 \vec{r} \bar{\Psi}(\vec{r}, \tau) \{ B_1 \partial_x + B_2 \partial_y \} \Psi(\vec{r}, \tau), \quad (9)$$

where $\Gamma_i = \Gamma_0 A_i$, $B_i = \Gamma_0 \delta A_i$; $A_1 = \text{diag}(M_1, M_2)$, $A_2 = \text{diag}(M_2, M_1)$, $\delta A_1 = \text{diag}(\lambda_F M_1, \lambda_{\Delta} M_2)$, $\delta A_2 = \text{diag}(\lambda_{\Delta} M_2, \lambda_F M_1)$ and $\lambda_F = v_F - 1$, $\lambda_{\Delta} = v_{\Delta} - 1$ with $v_F = \partial_{q_x} \xi(\vec{K}_I + \vec{q})|_{q=0}$, $v_{\Delta} = \partial_{q_y} \Delta(\vec{K}_I + \vec{q})|_{q=0}$. v_F and v_{Δ} are the two characteristic velocities of the linearized spectrum, in units of some fixed velocity c , which we set to $c = 1$. The isotropic theory is recovered in the limit $v_F = v_{\Delta}$. We will discuss the form of the S_{NL} that represents the higher-order derivative terms shortly. Note that the matrices A_i 's and δA_i 's are independent of Γ_0 .

In the rest of the paper we will consider the linear, isotropic, noninteracting S_0 to be the maximally symmetric action. It remains invariant under a global chiral rotation,

$$\Psi \rightarrow e^{i \sum_{i=1}^{16} \theta_i J_i} \Psi, \quad (10)$$

with the generators J_i , $i = 1 \dots 16$, forming the U(4) algebra. The explicit form of the generators is given in Ref. 1. The connection to the QED₃ is finalized by coupling the gapless quasiparticles to the fluctuating vortex excitations, which

upon a singular gauge transformation may be represented by an effective gauge field in S_0 , by $\partial_\mu \rightarrow \partial_\mu + ia_\mu$. The charge of the gauge field is proportional to the vortex condensate.¹ Such a gauge transformation also turns the d -wave quasiparticles into neutral spin-1/2 excitations. In the phase coherent, i.e., superconducting state, charge vanishes and the gauge field is effectively decoupled, leaving the sharp spinon (or quasiparticle) excitations behind. When the vortices condense, on the other hand, superconductivity is lost, and simultaneously the spinon “mass” term $\sim m\bar{\Psi}\Psi$ becomes dynamically generated.⁶ Here, $m = \Sigma(q \rightarrow 0) \neq 0$, with $\Sigma(q)$ representing the spinon self-energy due to the interaction with the gauge field. Rewriting the mass term in terms of the electron operators one recognizes that it simply represents a two-dimensional SDW with the periodicity determined by the wave vectors $2\vec{K}_i$.¹

The SDW order therefore corresponds to the particular form of the Γ_0 matrix we have chosen. By applying a unitary $U(4)$ transformation on our Γ_0 one recovers all other insulating states that are related to the SDW by chiral symmetry. In particular, transforming Γ_0 as

$$\Gamma_0 \rightarrow e^{i(\pi/4)J} \Gamma_0 e^{-i(\pi/4)J} \quad (11)$$

with $J = \mathbf{1} \otimes \gamma_3$ produces the $d+ip$ insulator: the phase incoherent, translationally symmetric state with an additional p -wave pairing between spinons. Similarly, $J = (1/\sqrt{2})\sigma_1 \otimes \{[\gamma_1 - \gamma_2 + i\gamma_0(\gamma_1 + \gamma_2)]\}$ yields the $d+is$ insulator, and $J = (1/\sqrt{2})\sigma_2 \otimes (\gamma_1 - \gamma_2)$ gives the CDW with the periodicity set by the wave vector $\vec{K}_I - \vec{K}_{II}$. Of course, there is a continuum of other chiral rotations that are possible, and which lead to various linear combinations of the four fundamental states defined above.

In the rest of the paper we study how the degeneracy between the SDW, CDW, $d+ip$, and $d+is$ states as defined above is removed by the leading perturbations to S_0 : anisotropy, repulsion, and higher-order derivatives.

III. ANISOTROPY

We set $U=0$ in S to study first the effect of weak anisotropy ($\lambda_F, \lambda_\Delta \ll 1$) on the energy of the degenerate ground states of the chiral symmetry broken, isotropic QED₃. Since the action S_{BCS} is symmetric under the exchange $v_F \leftrightarrow v_\Delta$, the energies of various states should be invariant under the same exchange. Moreover, if $v_F = v_\Delta$, both velocities can be rescaled out of the problem by an appropriate choice of c . The energy splittings between the states must therefore be proportional to $(\lambda_F - \lambda_\Delta)^2$, to the lowest order. Indeed, defining the three-dimensional volume as $V = (2\pi)^{-3} \int d\tau d^2\vec{r}$, the first-order correction of the energies per volume due to the velocity anisotropy is

$$\Delta E_A^{(1)} = \frac{1}{V} \langle S_A \rangle_0 = \sum_i \text{tr}(B_i \Gamma_i) I_0 \quad (12)$$

$$= 8m^3 (\lambda_F + \lambda_\Delta) I_0, \quad (13)$$

independently of the choice of Γ_0 . I_0 is a positive, dimensionless integral

$$I_0 = \frac{1}{3} \int^{\Lambda/m} \frac{d^3\vec{x}}{(2\pi)^3} \frac{x^2}{\sigma^2(x) + x^2}, \quad (14)$$

where $\vec{x} = \vec{q}/m$ and $\sigma(x) = \Sigma(x)/m$ is the rescaled self-energy. Λ denotes the upper cutoff. $\Delta E_A^{(1)}$ therefore provides only an overall energy shift, the same for all states.

To the second order, however, we find

$$\begin{aligned} \Delta E_A^{(2)} &= -\frac{1}{2V} (\langle S_A^2 \rangle_0 - \langle S_A \rangle_0^2) \\ &= -\frac{1}{2} m^3 \sum_{ij} [\text{tr}(B_i B_j) I_{ij} - \text{tr}(B_i \Gamma_\alpha B_j \Gamma_\beta) I_{ij, \alpha\beta}], \end{aligned} \quad (15)$$

where

$$I_{ij} = \int^{\Lambda/m} \frac{d^3\vec{x}}{(2\pi)^3} \frac{\sigma^2(x) x_i x_j}{(\sigma^2(x) + x^2)^2}, \quad (16)$$

$$I_{ij, \alpha\beta} = \int^{\Lambda/m} \frac{d^3\vec{x}}{(2\pi)^3} \frac{x_i x_j x_\alpha x_\beta}{(\sigma^2(x) + x^2)^2}. \quad (17)$$

Since the rescaled self-energy $\sigma(x)$ in the QED₃ falls off rapidly for $x \gg 1$,⁶ we will set $\Lambda = m$ in I_{ij} , and then approximate $\sigma(x) = 1$ under the integral. This “constant mass” approximation seems not to be appropriate for the integral $I_{ij, \alpha\beta}$, which is without $\sigma(x)$ in the numerator of the integrand. Luckily, however, $\text{tr}(B_i \Gamma_\alpha B_j \Gamma_\beta)$ is independent of the choice of Γ_0 and therefore the term with $I_{ij, \alpha\beta}$ does not contribute to the energy differences, but only to the overall shift of the energies. This will be the generic situation in all further calculations. We will then take $\Lambda = m$ and $\sigma(x) = 1$ in all the integrals hereafter.

The lowest (second)-order effect of the velocity anisotropy is to increase the energy of the CDW relative to the SDW, $d+ip$ and $d+is$, which remain degenerate:

$$\Delta E_{A, \text{CDW}} - \Delta E_{A, \text{other}} = 4m^3 (v_F - v_\Delta)^2 I > 0,$$

$$I = \frac{1}{3} \int^1 \frac{d^3\vec{x}}{(2\pi)^3} \frac{x^2}{(1+x^2)^2} = \frac{10-3\pi}{48\pi^2}. \quad (18)$$

The energy splitting vanishes when $v_F = v_\Delta$, as expected.

The fact that $d+ip$ and $d+is$ insulators have the same energy in presence of velocity anisotropy can be shown to be generally true to any order of the perturbation theory. To see this, note that $d+ip$ and $d+is$ states are represented by $\Gamma_{0, d+ip} = \text{diag}(\sigma_2, -\sigma_2, \sigma_2, -\sigma_2)$, and $\Gamma_{0, d+is} = \text{diag}(\sigma_2, \sigma_2, \sigma_2, \sigma_2)$, respectively. When written in the 2×2 from, the only difference between the two is in the signs of some terms, which always may be changed by a unitary transformation. Put differently, the choice of Γ_0 enters the energy calculation only through the combination $B_i = \Gamma_0 \delta A_i$. Matrices B_i , on the other hand, have to appear in

even numbers in our calculation, as in Eq. (15), otherwise the accompanying integrals will be odd in some momentum component and vanish. Since A_i 's are block diagonal, the sign of the block-diagonal elements in B_i 's, then, cannot matter: $d+ip$ and $d+is$ states remain degenerate to all orders in weak anisotropy.

IV. REPULSION

We now set $v_F = v_\Delta = 1$ to work out the first finite energy contribution of the short-range repulsion S_U to the degenerate ground states of the isotropic action. It is found that

$$\Delta E_U^{(1)} = \frac{1}{V} \langle S_U \rangle_0 = m^4 U J^2 \{ [\text{tr}(B_U)]^2 - \text{tr}(B_U^2) \},$$

$$J = \int_0^1 \frac{d^3 x}{(2\pi)^3} \frac{1}{1+x^2} = \frac{4-\pi}{8\pi^2}. \quad (19)$$

The first term in Eq. (19) vanishes identically for all states; the second term also vanishes for the CDW, but it increases the energy of the $d+ip$ and the $d+is$, while decreasing the energy of the SDW,

$$\Delta E_{U,d+ip}^{(1)} = \Delta E_{U,d+is}^{(1)} = +8m^4 U J^2, \quad (20)$$

$$\Delta E_{U,SDW}^{(1)} = -8m^4 U J^2, \quad (21)$$

$$\Delta E_{U,CDW}^{(1)} = 0. \quad (22)$$

Note that $d+ip$ and $d+is$ remain degenerate in presence of the repulsive interaction as well.

V. HIGHER-DERIVATIVE TERMS

We may expand $iM(\vec{q})$ beyond the linear terms that led to the anisotropic action, Eqs. (8) and (9). For instance, defining $M_\xi = \mathbf{1} \otimes \sigma_3, M_\Delta = \mathbf{1} \otimes \sigma_1$, the second-derivative term is given by

$$S_{NL} = - \int d\tau d^2 r \vec{\Psi}_I^\dagger \{ M_\xi \xi_0''(\partial^2) + M_\Delta \Delta_0''(\partial^2) \} \Psi_I$$

$$+ \{ I \rightarrow II, x \leftrightarrow y \} + \dots, \quad (23)$$

where $\xi_0''(x_1, x_2) = \sum_{ij} x_i x_j \partial_{x_i} \partial_{x_j} \xi(x_1, x_2)|_{\vec{x}=0}$ and similarly for Δ_0'' . Ellipsis indicates higher than second derivative terms. It seems, then, that we need to specify the dispersion relation and the gap function to determine the effect of the higher-order derivative terms on the energies of the degenerate ground states. Interestingly, it turns out that up to the first nonvanishing (second) order of perturbation, the qualitative effect of these terms is to raise the energy of the SDW, lower that of $d+ip$ and $d+is$, and keep the energy of the CDW unchanged, independently of the functional form of the dispersion and the gap function.

For definiteness, we present here the results for the tight-binding model on a square lattice, for which the dispersion and the d -wave gap are given by

$$\xi(k_a, k_b) = -2t(\cos k_a a + \cos k_b a) - \mu,$$

$$\Delta(k_a, k_b) = \Delta_0(\cos k_a a - \cos k_b a), \quad (24)$$

where t is the nearest-neighbor hopping matrix element, a is the lattice spacing, μ is the chemical potential, and Δ_0 is the amplitude of the d -wave superconducting order parameter. The result is then

$$\Delta E_{NL,d+ip}^{(2)} = \Delta E_{NL,d+is}^{(2)} = -8m^3(ma)^2 L, \quad (25)$$

$$\Delta E_{NL,SDW}^{(2)} = +8m^3(ma)^2 L, \quad (26)$$

$$\Delta E_{NL,CDW}^{(2)} = 0. \quad (27)$$

Again L is a positive, dimensionless integral, given by

$$L = \frac{(\cot \tilde{k}_F)^2}{4(ma)^4} \int_0^1 \frac{d^3 x}{(2\pi)^3} \frac{1}{(1+x^2)^2}$$

$$\times \{ \{ v_F [\cos(max^+) + \cos(max^-) - 2] \}^2$$

$$+ \{ v_\Delta [\cos(max^+) - \cos(max^-)] \}^2 \}, \quad (28)$$

where $\tilde{k}_F = (1/\sqrt{2})k_F a$ and $x^\pm = (1/\sqrt{2})(q_x \pm q_y)/m$. Assuming that near the transition, $ma \ll 1$, to the zeroth order in ma yields,

$$L = \frac{1}{16} (\cot \tilde{k}_F)^2 \int_0^1 \frac{d^3 x}{(2\pi)^3} \frac{1}{(1+x^2)^2}$$

$$\times [v_F^2(x_1^2 + x_2^2)^2 + v_\Delta^2(2x_1 x_2)^2] + O[(ma)^2]$$

$$= \frac{\tilde{L}}{16} (\cot \tilde{k}_F)^2 (2v_F^2 + v_\Delta^2) + O[(ma)^2], \quad (29)$$

where $\tilde{L} = (15\pi - 46)/180\pi^2$.

VI. SUMMARY

Collecting our results together, the shift in energy of a given state may be written as

$$\Delta E = 8m^3 \left\{ \frac{1}{2} \theta_A (v_F - v_\Delta)^2 I + \theta_U (mU) J^2 + \theta_{NL} (ma)^2 L \right\}, \quad (30)$$

where each $\theta_{A,U,NL} = 0$ or ± 1 , depending on the change in energy of the state in consideration. In our constant mass approximation all three integrals I, J , and L are mass independent positive constants, so that Eq. (30) represents an expansion of the energies of the insulating states in powers of the dynamically generated mass m .

For weak chiral symmetry breaking, there is a hierarchy of the perturbation terms, in which each symmetry-breaking perturbation assumes a place according to the degree of its irrelevancy. Velocity anisotropy then, being the marginal perturbation at the bare level, provides the dominant contribution, while the repulsive interaction, is the subdominant one. Second-order derivatives, although equally irrelevant as the repulsion, contribute to the energies of the insulating states

only to the second order, and therefore are the least important perturbation. Breaking of the degeneracy between the insulators due to each perturbation is schematically given in Fig. 2.

Our conclusion is that the degeneracy of the chiral manifold is broken in favor of the SDW, which is the lowest energy state when the weak perturbations to the maximally chirally symmetric action are taken into account. The translationally symmetric $d+ip$ and $d+is$ insulators remain degenerate, as one would expect, since the translational symmetry remains intact even in presence of all three

perturbations. Assuming that d -wave superconductor-insulator transition is continuous, or possibly weakly first order, implies then that the insulating state is the SDW. If the transition is strongly first order,⁷ chiral mass m increases and it is conceivable that there could be some level crossings in our fine structure of the chiral manifold.

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